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CONSOB

Three Pillars for Risk-Transparency in Structured Finance

Syllabus

Preliminaries
- regulatory framework
- products' risk-return profile VS investors' risk-return profile

Three-pillars approach
- financial structures
- 1st Pillar: unbundling and performance scenarios
  - return target products
    - unbundling
    - probabilistic performance scenarios
  - risk target and benchmark products
  - model risk assessment
- 2nd Pillar: the degree of risk
  - risk target and benchmark products
    - mapping
    - migration
  - return target products
- 3rd Pillar: recommended investment time horizon
  - risk target and benchmark products
    - first passage time
    - connection between probability, volatility and costs
    - characterization of the necessary condition in the space of returns
    - how to determine a consistent series of Time Horizons
  - return target products

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Preliminaries

The transparency on the risk profile of non-equity investment products is based on three synthetic indicators (three pillars) – defined through the development of specific quantitative methods – in order to allow investors to take informed investment decisions.

Traditional narrative description of all possible risks associated with a financial product	Synthetic indicators robust, objective and backward verifiable

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Traditional narrative description of all possible risks associated with a financial product	Synthetic indicators robust, objective and backward verifiable

Consob Annual Report 2008
Speech by the Chairman to the Financial Market

"The inclusion of indicators on performance scenarios, the degree of risk, costs and recommended investment time horizons in information documents will allow investors to assess and compare investments based on standard criteria.

This is a new approach on the international scene that meets the needs of a market, such as in Italy, where a high capacity for investment tends to privilege direct forms of investment".

European Commission
The EU Single Market
Communication from the Commission on Packaged Retail Investment Products

The level of protection afforded to the retail investor should not vary according to the legal form of these products […]

This work:

- will provide a market (for packaged retail investment products) in which regulatory arbitrage does not drive savings towards particular products;
- has the objective to introduce a horizontal approach that will provide a coherent basis for the regulation of mandatory disclosures and selling practices at European level, irrespective of how the product is packaged or sold.

QdF Consob n. 63: A Quantitative Risk-Based Approach to the Transparency on Non-Equity Investment Products

Transparency regulation on the risk profile of non-equity investment products should be standard and translate into suitable regulatory provisions a coherent approach to risk measurement and to its correct representation to the potential investors.

This will create a context compatible with the concrete realization of a levelled playing field and with the prevention of any regulatory arbitrage which could arise due to the fragmentation of the current regulation.

[…] the only solution is represented by a thorough revision of both the European and the Italian regulatory framework in the direction of a single directive on the transparency for non-equity investment products.

European Commission
The EU Single Market
Communication from the Commission on Packaged Retail Investment Products

Update on Commission work on Packaged Retail Investment Products
16 december 2009

Pre-contractual disclosures

Common elements to allow for comparisons to include the structure of documents, order of sections, use of plain language, and focus on key information about nature of product, its risks, potential performance and costs.

QdF Consob n. 63: A Quantitative Risk-Based Approach to the Transparency on Non-Equity Investment Products

The regulatory choices Consob has made over time reflect its view of the prospectus as the privileged channels to realize an effective transparency both in the offering and in the distribution of non-equity investment products.

Such approach, developed and progressively implemented by Consob, is based on three pillars, corresponding to three synthetic indicators defined through the application of specific quantitative methods.

The three pillars fully define the contents of a product information sheet which should become the core of the prospectus and of the other transparency documentation intended to effectively.

Proposal of the European Commission for a DIRECTIVE OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL amending Directive 2003/71/EC on the PROSPECTUS (September 2009)

Whereas (10):

"The summary of the prospectus is a key source of information for retail investors. It should be short, simple and easy for targeted investors to understand. It should focus on the key information that investors need in order to be able to make informed investment decisions. Its content should not be restricted to any predetermined number of words. The format and content of the summary should be determined in a way that ensures comparability with other investment products that are similar to the investment proposal described in the prospectus.".

FINANCIAL REGULATORY REFORM: A NEW FOUNDATION

Protect consumers and investors from financial abuse.

To rebuild trust in our markets, we need strong and consistent regulation and supervision of consumer financial services and investment markets. …

We must promote transparency, simplicity, fairness, accountability, and access. We propose:

…

- Stronger regulations to improve the transparency, fairness, and appropriateness of consumer and investor products and services
- A level playing field and higher standards for providers of consumer financial products and services, whether or not they are part of a bank.

FINANCIAL REGULATORY REFORM: A NEW FOUNDATION

Transparency.

We propose a new proactive approach to disclosure.

[…] all disclosures and other communications with consumers be reasonable: balanced in their presentation of benefits, and clear and conspicuous in their identification of costs, penalties, and risks.

Mandatory disclosure forms should be clear, simple, and concise.

Moreover, reasonableness does not mean a litany of every conceivable risk, which effectively obscures significant risks. It means identifying conspicuously the more significant risks. It means providing consumers with disclosures that help them to understand the consequences of their financial decisions.

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    - o how to determine a consistent series of Time Horizons
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The implementation of the disclosure regulation on the risk-profile of non-equity investment products should allow the investor, even assisted by a financial advisor, to choose the financial product more suitable to his investment objectives.

Equity Mutual Funds
Liquidity Mutual Funds
Certificates
Bond Unit Linked
Covered Warrants
Structured Bond
Equity Unit Linked
BOND ETF
LIQUIDITY ETF
Bond
Bond Mutual Funds
EQUITY ETF
Index Linked
Structured ETF

Three different directives for the same financial engineering

UCITS Directive	Prospectus Directive	Life Assurance Directive
Equity Mutual Funds BOND ETF Liquidity Mutual Funds EQUITY ETF Structured ETF Bond Mutual Funds LIQUIDITY ETF	Certificates Bond Structured Bond Covered Warrants	Equity Unit Linked Bond Unit Linked Index Linked

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The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

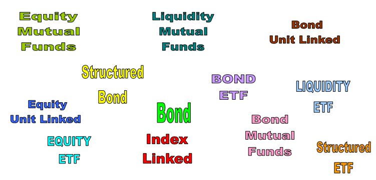
Preliminaries: products' risk-return profile VS investors' risk-return profile

The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

Time goal:
liquidity/investment horizon

INVESTMENT HORIZON

(less than 3 years)



Preliminaries: products' risk-return profile VS investors' risk-return profile

The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

Time goal:
liquidity/investment horizon

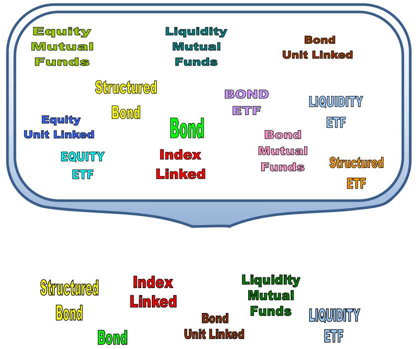
INVESTMENT HORIZON

(less than 3 years)

Risk profile:
risk limit in terms of downside

RISKS

(medium-low)



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The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

Time goal:
liquidity/investment horizon

INVESTMENT HORIZON

(less than 3 years)

Risk profile:
risk limit in terms of downside

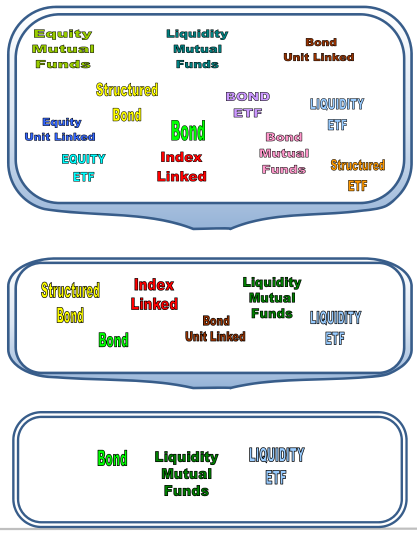
RISKS

(medium-low)

Return goal:
target returns

RETURNS

(maximum return)



Preliminaries: products' risk-return profile VS investors' risk-return profile

RETURNS | RISKS | INVESTMENT HORIZON

… allow the investor to match his needs with the features of the financial products and to make an informed investment decision

Preliminaries: products’ risk-return profile VS investors’ risk-return profile

RETURNS | RISKS | INVESTMENT HORIZON

… allow the investor to match his needs with the features of the financial products and to make an informed investment decision

PREVENT MISBUYING

Preliminaries: products’ risk-return profile VS investors’ risk-return profile

INVESTOR NEEDS

INVESTMENT OBJECTIVES
- INVESTMENT HORIZON
- RISKS
- RETURNS

FINANCIAL SITUATION

EXPERIENCE AND KNOWLEDGE

MATCHING

PRODUCT FEATURES

New Prospectus Directive

The format and content of the summary should be determined in a way that ensures comparability with other investment products that are similar to the investment proposal described in the prospectus.

KID
Key Investor Document
- INVESTMENT HORIZON
- RISKS
- RETURNS

Preliminaries: products’ risk-return profile VS investors’ risk-return profile

INVESTOR NEEDS

INVESTMENT OBJECTIVES
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FINANCIAL SITUATION

EXPERIENCE AND KNOWLEDGE

MATCHING

INVESTMENT FIRMS

(MiFID suitability test)
Client
Profiling

Investment firms interpret the needs of client according to their internal procedures that may differ from company to company

Potential Mismatch?

PRODUCT FEATURES

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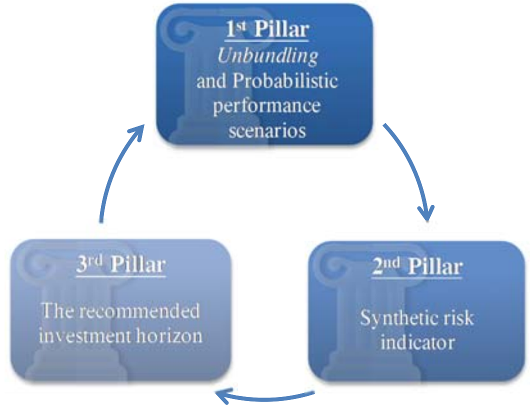
Three-pillars approach

RETURNS | RISKS | INVESTMENT HORIZON

The key qualitative information is made objective by using a three-pillars approach based on quantitative measures.

Three-pillars approach

The three pillars are closely linked together and offer to investors an organic and internally consistent representation of the risks, costs and potential performances of the product over the recommended investment horizon.



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Three-pillars approach: financial structures

The three-pillars approach is based on the preliminary classification of the products into three types of financial structures:

Three-pillars approach: financial structures

“Risk target” products

“Risk target” products invest in any market and any financial instrument in order to optimize over time a given target in terms of risk exposure.

Three-pillars approach: financial structures

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“Benchmark” products

“Benchmark” products have an investment policy which is anchored to a benchmark, and in relation to this benchmark the asset management style may be either passive or active.

Three-pillars approach: financial structures

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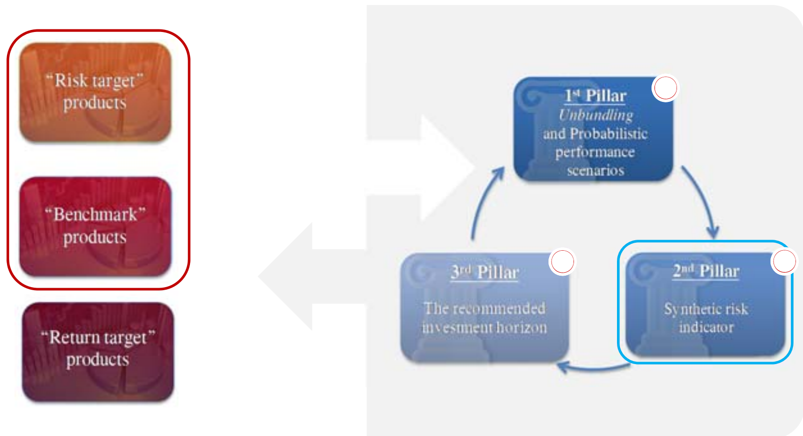
“Return target” products

“Return target” products feature a financial engineering (and, in some cases, a consequent investment policy) aimed at pursuing a minimum target return on the financial investment.

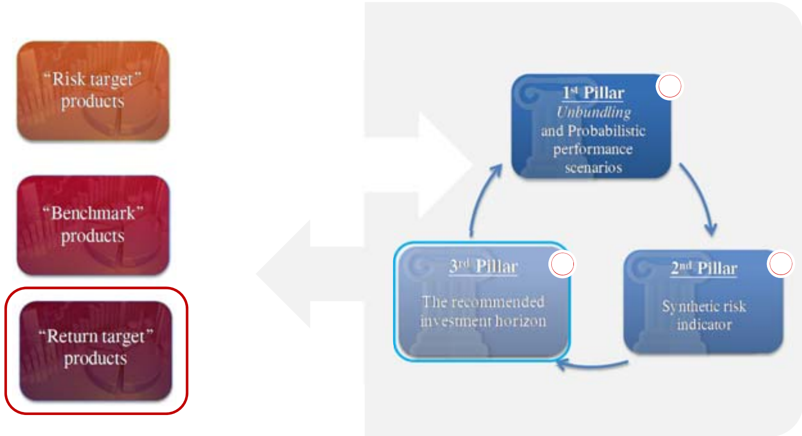
Three-pillars approach: financial structures

In “risk target” or “benchmark” products the degree of risk, together with the costs applied, allows to determine the recommended minimum investment time horizon. This horizon is used as the reference period to calculate the probability scenarios.



Three-pillars approach: financial structures

In "return target" products the target return at a given maturity clearly identifies the investment time horizon (a shorter holding period would compromise the liquidability of the product) w.r.t. which the probability scenarios and the degree of risk are determined.



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1st Pillar: *unbundling* and performance scenarios

Unbundling and Probabilistic Performance Scenario

Performance risk
w.r.t. the risk-free asset
under the risk-neutral probability measure

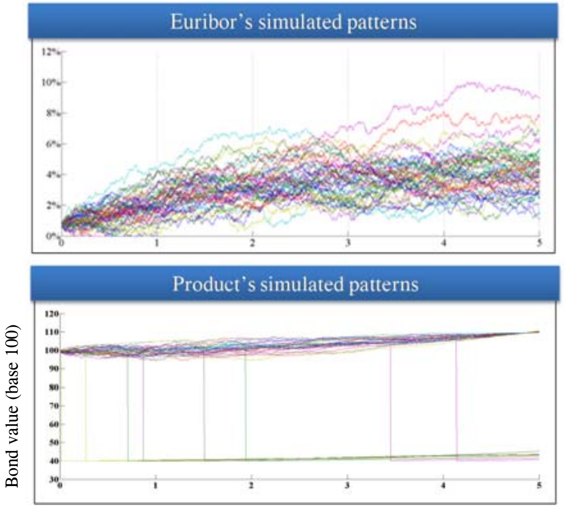
… illustrates the unbundling of the price of the non-equity investment product at the time of subscription and provides a clear and concise information about its possible outcomes and costs.

1st Pillar: return target products

In "return target" products (e.g. corporate bonds) the connection between the pricing at time zero and the pricing at maturity is evident, as the probability table is a necessary step to obtain the unbundling of the price of the product at time 0.

1st Pillar
Unbundling
and Probabilistic performance scenarios

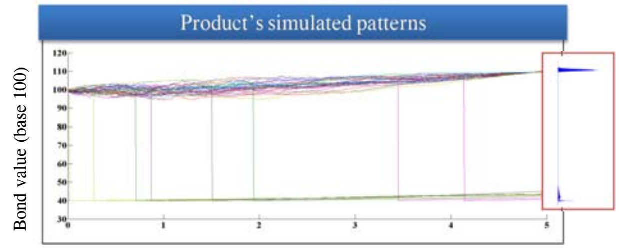
5 year fixed-rate bond



t (year)

1st Pillar
Unbundling
and Probabilistic performance scenarios

The final values of the bond at the end of the 5th year provide the probability distribution of potential returns (so-called *pricing* at maturity).



t (year)

Possible outcomes
Pricing at maturity

1st Pillar
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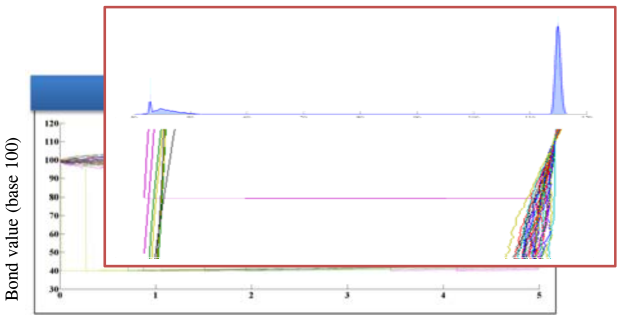

t (year)

Possible outcomes
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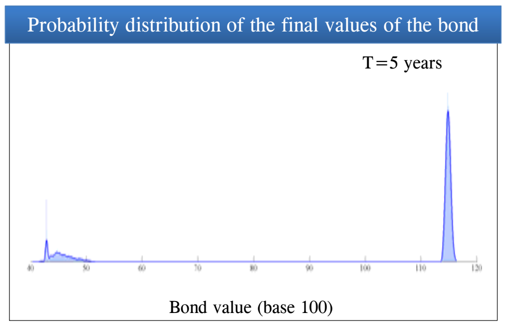

Possible outcomes
Pricing at maturity

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1st Pillar: return target products (*unbundling*)

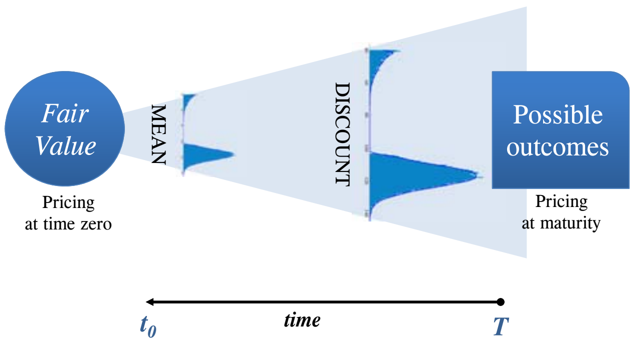
1st Pillar
Unbundling
and Probabilistic performance scenarios

The *unbundling* table shows the fair value of the product at time zero …

1st Pillar: return target products (*unbundling*)

1st Pillar
Unbundling
and Probabilistic performance scenarios

The *unbundling* table shows the fair value of the product at time zero … which is equal to the expected value, under the risk-neutral probability measure, of the possible outcomes discounted at the risk-free rate.



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1st Pillar
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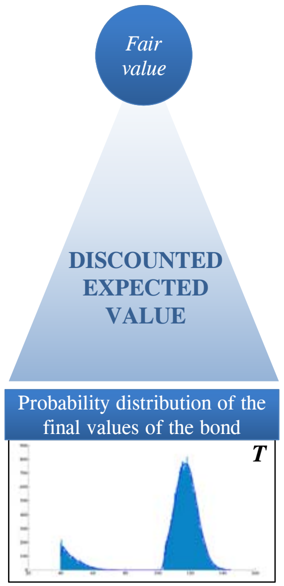

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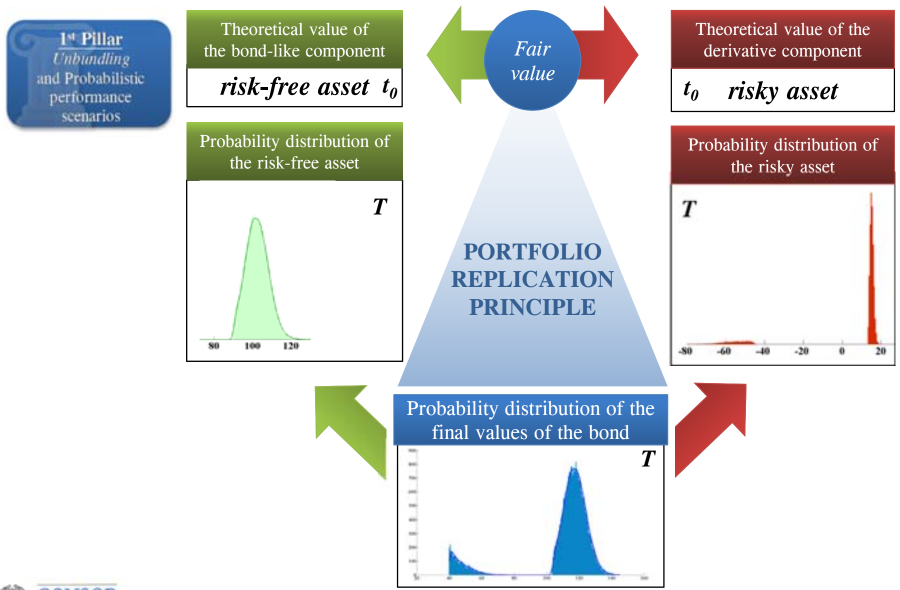

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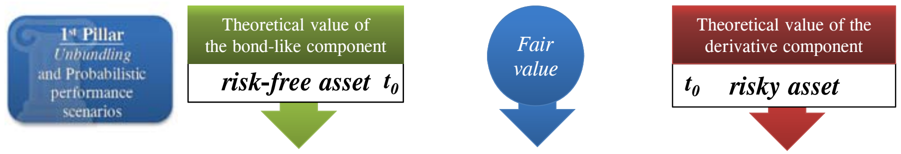

A	Theoretical value of the bond-like component	…
B	Theoretical value of the derivative component	…
C = A + B	*Fair value*	…
D	Explicit costs	…
E	Implicit costs	…
F = C + D + E	Issue price	100

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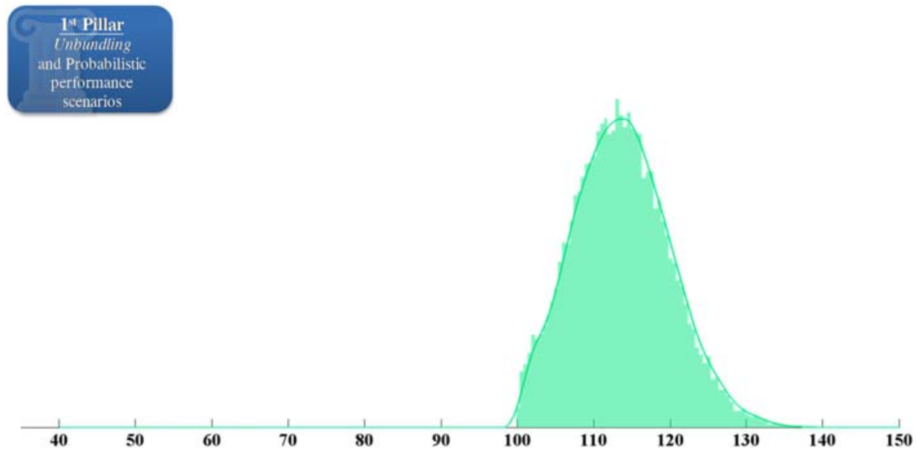
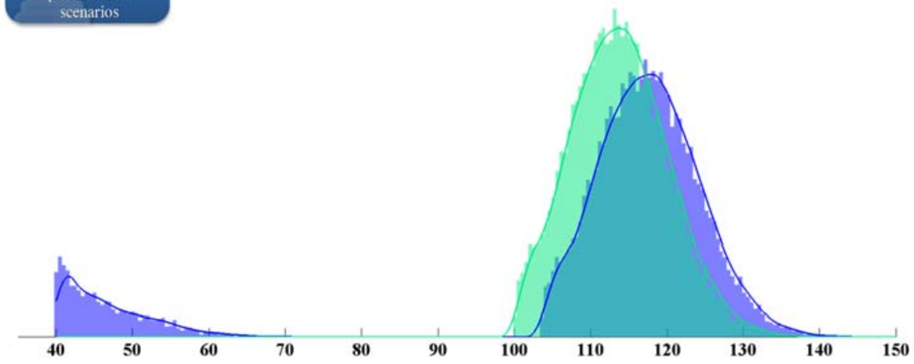
1st Pillar: return target products (probabilistic performance scenarios)



Probability Distribution of the final value of the Notional Capital invested in the risk-free asset

1st Pillar: return target products (probabilistic performance scenarios)

1st Pillar
Unbundling
and Probabilistic performance scenarios



Probability Distribution of the final value of the Invested Capital

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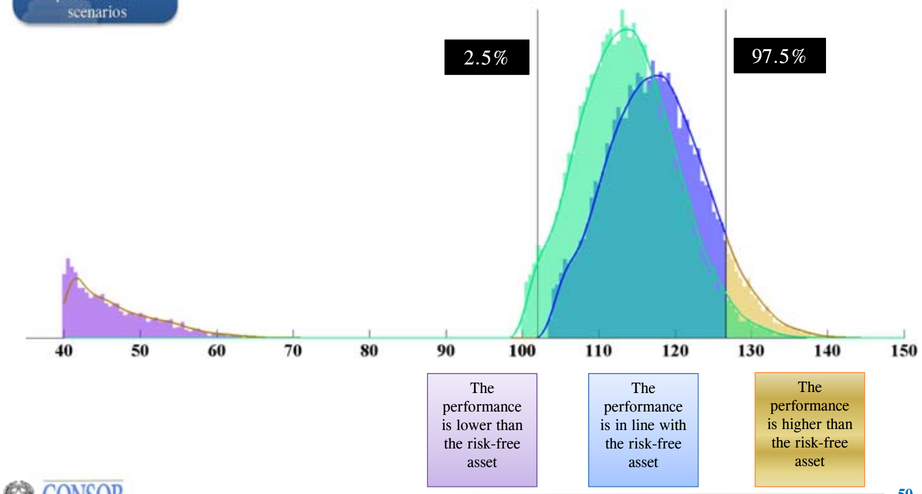

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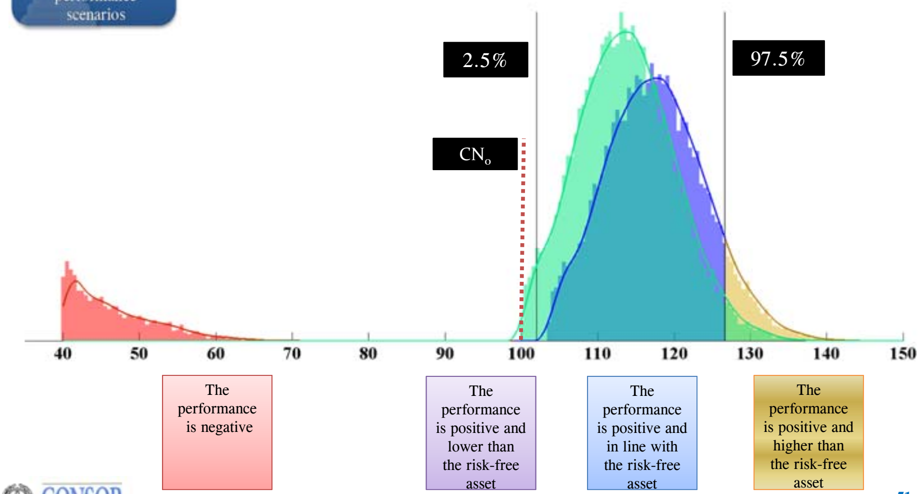

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Unbundling
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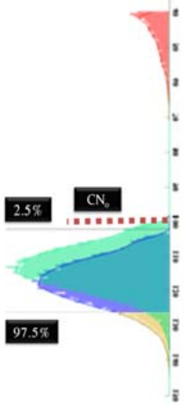

SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
The performance is positive and in line with the risk-free asset	%	€
The performance is positive and higher than the risk-free asset	%	€

1st Pillar: return target products (*unbundling* and performance scenarios)

1st Pillar
Unbundling
and Probabilistic performance scenarios

Connection between the pricing at time zero and the pricing at the end of recommended investment horizon

Time Zero — Financial investment table

A	Theoretical value of the bond-like component	…
B	Theoretical value of the derivative component	…
C = A + B	*Fair value*	…
D	Explicit costs	…
E	Implicit costs	…
F = C + D + E	Issue price	100

End of the recommended investment horizon — Table of probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
The performance is positive and in line with the risk-free asset	%	€
The performance is positive and higher than the risk-free asset	%	€

1:1 Relationship

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    - first passage time
    - connection between probability, volatility and costs
    - characterization of the necessary condition in the space of returns
    - how to determine a consistent series of Time Horizons
  - return target products

1st Pillar: risk target and benchmark products

1st Pillar
Unbundling
and Probabilistic performance scenarios

"Risk target" products | "Benchmark" products | "Return target" products

In "risk target" and "benchmark" products, the above described connection between fair value and possible outcomes is satisfied at any time. In these products, the calculation of the returns' probability distribution is an intermediate step of the process carried out to determine the recommended minimum investment time horizon.

1st Pillar: risk target and benchmark products

1st Pillar
Unbundling
and Probabilistic performance scenarios

Connection between the pricing at time zero and the pricing at the end of recommended minimum investment horizon

Time Zero — Financial investment table

A	*Fair value*	…
B	Explicit costs	…
C	Implicit costs	…
D = A + B + E	Issue price	100

End of the recommended investment horizon — Table of probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
The performance is positive and in line with the risk-free asset	%	€
The performance is positive and higher than the risk-free asset	%	€

1:1 Relationship

Syllabus

Preliminaries
- regulatory framework
- products' risk-return profile VS investors' risk-return profile

Three-pillars approach
- financial structures
- **1st Pillar: unbundling and performance scenarios**
  - return target products
    - unbundling
    - probabilistic performance scenarios
  - risk target and benchmark products
  - **model risk assessment**
- 2nd Pillar: the degree of risk
  - risk target and benchmark products
    - mapping
    - migration
  - return target products
- 3rd Pillar: recommended investment time horizon
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1st Pillar: model risk assessment

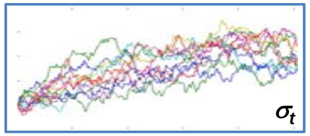
1st Pillar
Unbundling
and Probabilistic performance scenarios

Model Risk Assessment

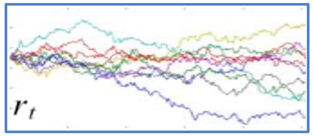
The recommended time horizon has a significant influence on the choice of the model

For Time Horizons greater than 1 year.....

I



II



1st Pillar: model risk assessment

1st Pillar
Unbundling
and Probabilistic performance scenarios

Model Risk Assessment

The recommended time horizon has a significant influence on the choice of the model

Two possible choices...

I

HESTON

Stochastic Volatility Model

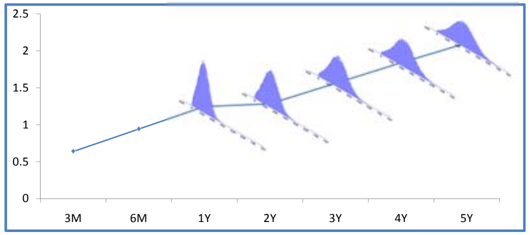
II

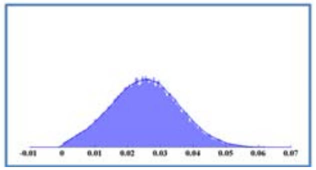
BAKSHI CAO CHEN
Stochastic Volatility - Stochastic Interest Rate - Jump Model

1st Pillar: model risk assessment

1st Pillar
Unbundling
and Probabilistic performance scenarios

Step 1: Calculation of the Probability Distribution of the Notional Capital at the end of recommended time horizon

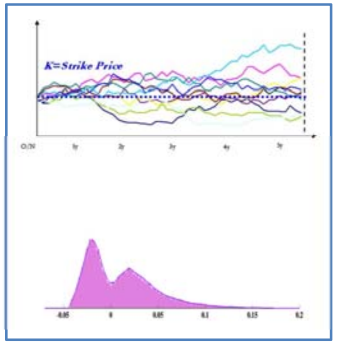




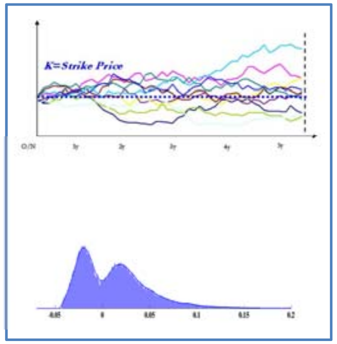
1st Pillar
Unbundling
and Probabilistic performance scenarios

Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

Heston

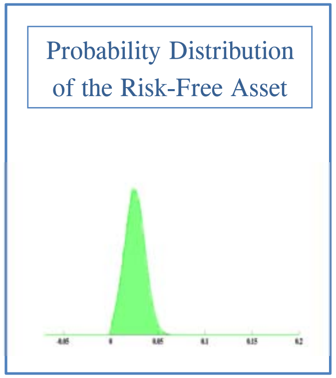


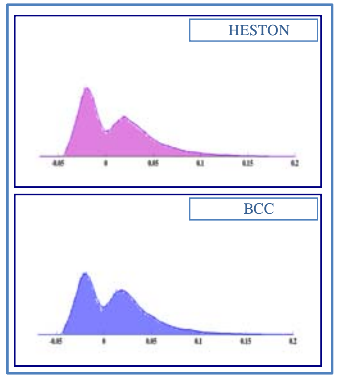
BCC



1st Pillar
Unbundling
and Probabilistic performance scenarios

Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon



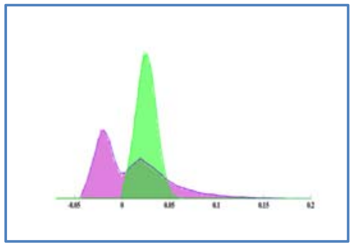


1st Pillar
Unbundling
and Probabilistic performance scenarios

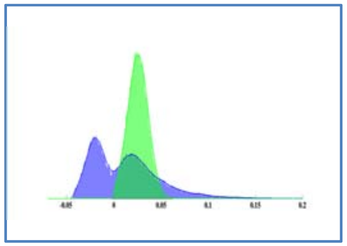
Step 3: Probabilistic comparison with the Risk-Free Asset

Analysing the two probability distributions…

Heston



BCC



1st Pillar
Unbundling
and Probabilistic performance scenarios

Step 3: Probabilistic comparison with the Risk-Free Asset

… the following output is obtained:

Heston

Scenarios	Probability	Median Values
The performance is negative	46.61%	€ 90.50
The performance is positive but lower than the risk-free asset	3.39%	€ 101.27
The performance is positive and in line with the risk-free asset	33.28%	€ 112.19
The performance is positive and higher than the risk-free asset	16.71%	€ 139.93

BCC

Scenarios	Probability	Median Values
The performance is negative	43.47%	€ 90.54
The performance is positive but lower than the risk-free asset	4.50%	€ 101.35
The performance is positive and in line with the risk-free asset	34.92%	€ 111.95
The performance is positive and higher than the risk-free asset	17.11%	€ 138.79

1st Pillar: model risk assessment

1st Pillar
Unbundling
and Probabilistic performance scenarios

Assessing the model risk

Heston

Scenarios	Probability	Median Values
The performance is negative	46.61%	€ 90.50
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|Δ| = 3.14

1st Pillar: model risk assessment

1st Pillar
Unbundling
and Probabilistic performance scenarios

Assessing the model risk

Heston

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|Δ| = 1.11

1st Pillar: model risk assessment

1st Pillar
Unbundling
and Probabilistic performance scenarios

Assessing the model risk

Heston

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|Δ| = 1.64

1st Pillar: model risk assessment

1st Pillar
Unbundling
and Probabilistic performance scenarios

Assessing the model risk

Heston

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|Δ| = 0.40

Syllabus

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Synthetic Risk Indicator

… provides a description, on a qualitative scale, of the risk level of the financial products based on volatility measures.

… represents in an explicit way the riskiness of the product embedded in the probabilistic performance scenarios of the first pillar.

The degree of risk of "risk target" and "benchmark" products is initially identified by the intermediary choosing the risk class which he deems to better match the specific features of the product's financial engineering over the recommended investment time horizon.

During this horizon, the intermediary monitor any possible migration of the degree of risk to a different risk class or, for "benchmark" products, to a different management class (i.e. the intensity of the asset management activity in terms of deviation from the chosen benchmark).

Synthetic Risk Indicator
(The degree of risk)

Six qualitative risk classes

Low
Medium-Low
Medium
Medium-High
High
Very High

2nd Pillar

Synthetic risk indicator

Synthetic Risk Indicator
(The degree of risk)

Time evolution of the volatility

↓

Mapping of the qualitative risk classes into corresponding volatility intervals

Syllabus

Preliminaries
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2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

The mapping is performed according to the following steps:

↓

Step 1: Definition of Loss Intervals

Step 2: Mapping of Loss Intervals to the corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals

2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 1: Definition of Loss Intervals

What is a loss in a financial investment?

RISK NEUTRALITY PRINCIPLE

$$\text{LOSS} \in (-100\%, \overline{r^{rf}}\,]$$

$\overline{r^{rf}}$ = average of the probability distribution of the risk-free rate

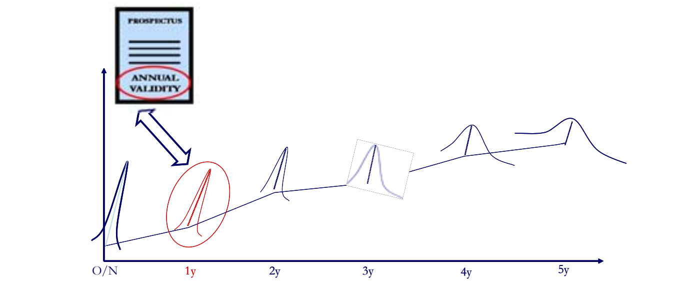
2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 1: Definition of Loss Intervals

given the risk-free yield curve and the associated volatility surface…

Risk-free yield curve and volatility surface



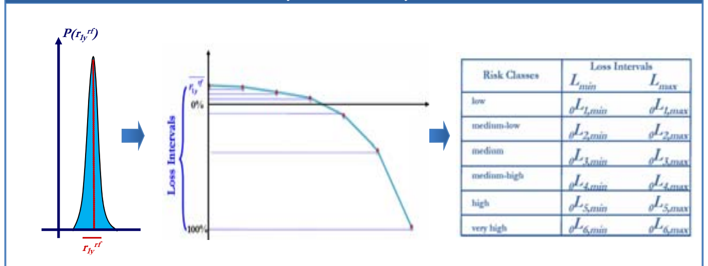
2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 1: Definition of Loss Intervals

the corresponding annual loss interval (multiple of r_{1y}^{rf} according to an exponential function) is associated to each risk class

From risk-free yield curve to one year loss intervals



2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 2: Mapping into Initial Volatility Intervals

Risk Classes	Loss Intervals L_{min}	L_{max}
low	$_0L_{1,min}$	$_0L_{1,max}$
medium-low	$_0L_{2,min}$	$_0L_{2,max}$
medium	$_0L_{3,min}$	$_0L_{3,max}$
medium-high	$_0L_{4,min}$	$_0L_{4,max}$
high	$_0L_{5,min}$	$_0L_{5,max}$
very high	$_0L_{6,min}$	$_0L_{6,max}$

Risk Classes	Volatility Intervals σ_{min}	σ_{max}
low	$_0\sigma_{1,min}$	$_0\sigma_{1,max}$
medium-low	$_0\sigma_{2,min}$	$_0\sigma_{2,max}$
medium	$_0\sigma_{3,min}$	$_0\sigma_{3,max}$
medium -high	$_0\sigma_{4,min}$	$_0\sigma_{4,max}$
high	$_0\sigma_{5,min}$	$_0\sigma_{5,max}$
very high	$_0\sigma_{6,min}$	$_0\sigma_{6,max}$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals

TOOLS

√ GARCH Diffusive Models

√ Non linear Stochastic Programming

2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

The Weak Convergence Theorem on R^2

The jump-continuous process $\{X_t^h\}$, whose measurable space is $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$, converges weakly for h↓ 0 to the continuous process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x,t)dt + \sigma(x,t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4 hereafter are satisfied.

2nd Pillar: risk target and benchmark products

2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

Condition

n. 1

If ∃ a δ>0 s.t.: $\lim_{h\downarrow 0}\begin{pmatrix} c_{h,\delta}(x_1,t) \\ c_{h,\delta}(x_2,t) \end{pmatrix} = 0$

then

$\exists\, a(x,t)$ and $b(x,t)$ s.t.:

$$\lim_{h\downarrow 0}\begin{pmatrix} b_h(x_1,t) \\ b_h(x_2,t) \end{pmatrix} = \begin{pmatrix} b(x_1,t) \\ b(x_2,t) \end{pmatrix}$$

$$\lim_{h\downarrow 0}\begin{pmatrix} a_h(x_1,t) & a_h((x_1,x_2),t) \\ a_h((x_2,x_1),t) & a_h(x_2,t) \end{pmatrix} = \begin{pmatrix} a(x_1,t) & 0 \\ 0 & a(x_2,t) \end{pmatrix}$$

2nd Pillar: risk target and benchmark products

2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

Condition

n. 2

$\exists\ \sigma(x,t)$ s.t.: $\forall x_1 \in \mathbb{R}^1, \forall x_2 \in \mathbb{R}^1,$

it holds

$$\begin{pmatrix} \sigma(x_1,t) & 0 \\ 0 & \sigma(x_2,t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1,t)} & 0 \\ 0 & \sqrt{a(x_2,t)} \end{pmatrix}$$

2nd Pillar: risk target and benchmark products

2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

Conditions

n. 3

For h↓ 0, X_0^h converges in distribution to a random variable X_0 with probability measure v_0 on $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$

n. 4

v_0, $a(x,t)$ and $b(x,t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution v_0, a conditional second moment $a(x,t)$ and a conditional first moment $b(x,t)$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

The Continuous Limit of the M-GARCH(1,1)

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \widetilde{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

$\widetilde{Z}_k$ and Z_k are i.i.d. $N(0,1)$

Weak Convergence theorem

$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = \left(\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right) dt + 2|\beta_1| \sqrt{Var(\ln |Z_t|)} dW_t^*$$

Z_t is $N(0,1)$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

The Prediction Interval for the Volatility

key point

then

From the Diffusion Limit of the M-GARCH(1,1) Process it is possible to establish a *Predictive Interval for* σ_t

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

The Prediction Interval for the Volatility

distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d \ln \sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right] dt + 2|\beta_1| \sqrt{Var(\ln |Z_t|)} dW_t^*$$

O.U. Process

$$\ln \sigma_t^2 \sim N\left(\left(\ln \sigma_s^2 + \frac{\beta_0 + 2\beta_1 E(\ln|Z_t|)}{(\beta_1 - 1)}\right) e^{(\beta_1 - 1)(t-s)} - \frac{\beta_0 + 2\beta_1 E(\ln|Z_t|)}{(\beta_1 - 1)}; \sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2 \left(e^{2(\beta_1 - 1)(t-s)} - 1\right)}{2(\beta_1 - 1)}} \right)$$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

matching of the first two conditional moments

discrete process

$$E\left(\ln \sigma_k^2\right) = \beta_0^{(k)} + \beta_1^{(k)} \ln \sigma_{k-1}^2 + 2\beta_1^{(k)} E\left(\ln |Z_{k-1}|\right)$$

$$Var\left(\ln \sigma_k^2\right) = 4\left(\beta_1^{(k)}\right)^2 Var\left(\ln |Z_{k-1}|\right)$$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

matching of the first two conditional moments

continuous process

$$E\left(\ln \sigma_t^2\right) = \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E\left(\ln |Z_t|\right)}{(\beta_1 - 1)}\right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E\left(\ln |Z_t|\right)}{(\beta_1 - 1)}$$

$$Var\left(\ln \sigma_t^2\right) = \frac{4\beta_1^2 Var\left(\ln |Z_t|\right)}{2\left(\beta_1 - 1\right)} \left(e^{2(\beta_1 - 1)} - 1\right)$$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

matching of the first two conditional moments

matching of the parameters

$$\left|\beta_1^{(k)}\right| = |\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)} - 1}{2\left(\beta_1 - 1\right)}}$$

$$\beta_0^{(k)} = -2|\beta_1| \sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} E\left(\ln |Z_{k-1}|\right) - |\beta_1| \sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} \ln \sigma_{k-1}^2 + e^{(\beta_1-1)} \ln \sigma_{k-1}^2 + \frac{[\beta_0 + 2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1}$$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

matching of the first two conditional moments

the discrete process can be written as:

$$\begin{aligned}\ln \sigma_k^2 - \ln \sigma_{k-1}^2 = \; & \frac{[\beta_0 + 2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1} - \\ & -2|\beta_1| \sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} E\left(\ln |Z_{k-1}|\right) + \\ & + \left(e^{(\beta_1-1)} - 1\right) \ln \sigma_{k-1}^2 + \\ & + 2|\beta_1| \sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} \ln |Z_{k-1}|\end{aligned}$$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

maximum likelihood estimation

setting:

$$y_k := \ln \sigma_k^2 - \ln \sigma_{k-1}^2$$

$$\varepsilon := \ln |Z_{k-1}|$$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

maximum likelihood estimation

then:

$$y_k = \frac{(\beta_0 - 1.27\beta_1)(e^{(\beta_1-1)}-1)}{\beta_1-1} + 1.27\,|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} + \left(e^{(\beta_1-1)}-1\right)\ln\sigma^2_{k-1} + 2\,|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}}\varepsilon$$

where we used: $E\left(\ln|Z_{k-1}|\right) = -0.6351$

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

maximum likelihood estimation

the likelihood function

$$L\left(w;\underline{\beta}\right) = \prod_{k=2}^{K}\left[\frac{1}{|\beta_1|\sqrt{2\pi}}\sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}}\cdot e^{\left(\frac{1}{2|\beta_1|}\sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}}\cdot w_k\right)} \cdot e^{\left(-\frac{1}{2}\exp\left(\frac{1}{|\beta_1|}\sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}}\cdot w_k\right)\right)}\right]$$

where: $\underline{\beta} := (\beta_0, \beta_1)$

$$w_k := y_k - \frac{(\beta_0-1.27\beta_1)(e^{(\beta_1-1)}-1)}{\beta_1-1} - 1.27\,|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} - \left(e^{(\beta_1-1)}-1\right)\ln\sigma^2_{k-1}$$

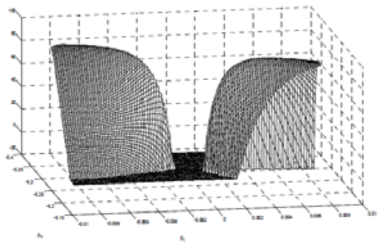
2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

maximum likelihood estimation

shape of the associated log-likelihood



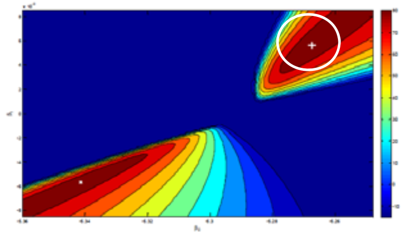
2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

maximum likelihood estimation

β_0 and β_1 estimates



2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models

the estimated parameters enter in the bounds of the volatility prediction interval

$$\sigma^G_{t,\min} = e^{\frac{-z_{\frac{1+\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}}+\left(\ln\sigma^2_{t-1}+\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}{2}}$$

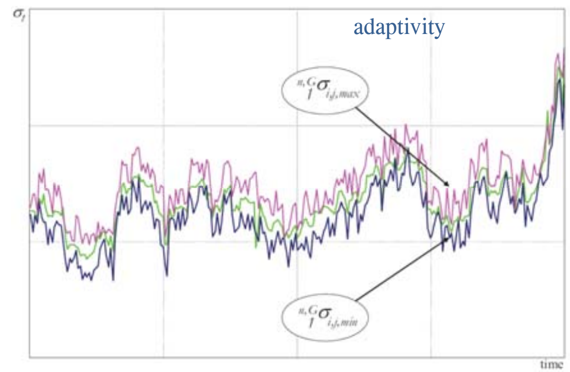
$$\sigma^G_{t,\max} = e^{\frac{z_{\frac{1+\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}}+\left(\ln\sigma^2_{t-1}+\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}{2}}$$

2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: GARCH Diffusive Models



2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

BEGIN PROCEDURE

2nd Pillar

Synthetic risk indicator

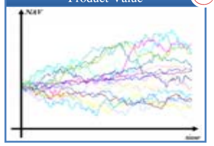
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

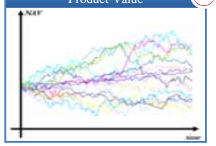
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

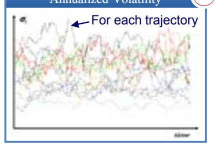
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Annualized Volatility



2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

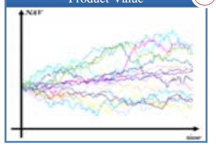
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

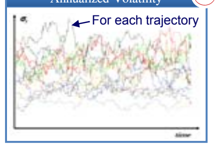
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

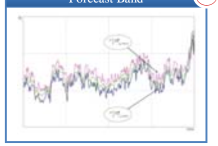
Product Value



Annualized Volatility



Forecast Band



2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

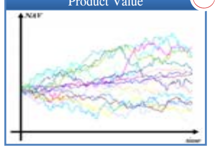
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

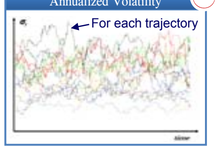
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Annualized Volatility



Initial Interval

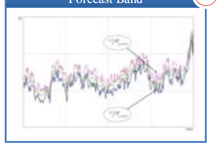
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval

$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band



2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

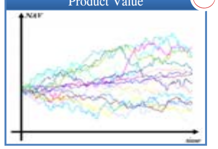
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

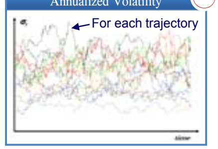
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Annualized Volatility



$\Delta > 5\%$ and $\Delta_{up} > \Delta_{down}$

Initial Interval

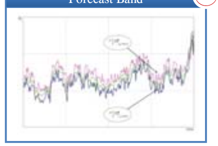
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval

$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band



2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

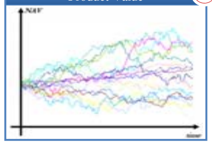
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

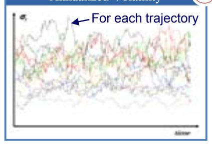
Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Update
$[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$

$\Delta > 5\%$ and $\Delta_{up} > \Delta_{down}$

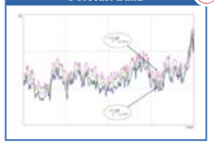
Product Value


Annualized Volatility


Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band


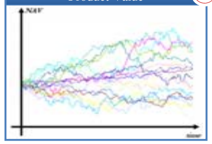
2nd Pillar: risk target and benchmark products

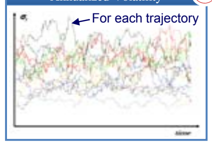
2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$

Product Value


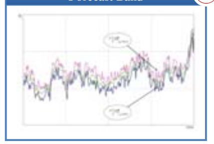
Annualized Volatility


$\Delta > 5\%$ and $\Delta_{up} < \Delta_{down}$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band


2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

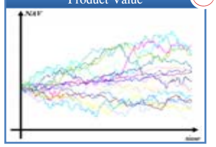
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

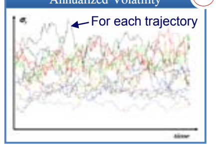
Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$

Update
$[{}_1\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

$\Delta > 5\%$ and $\Delta_{up} < \Delta_{down}$

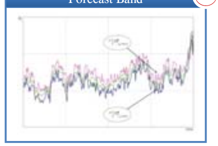
Product Value


Annualized Volatility


Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

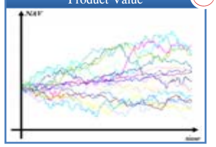
Forecast Band


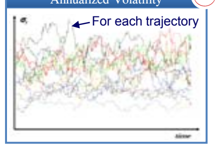
2nd Pillar: risk target and benchmark products

2nd Pillar
Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

Product Value


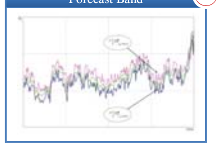
Annualized Volatility


$\Delta \leq 5\%$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band


2nd Pillar

Synthetic risk indicator

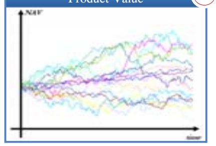
Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals: Non Linear Stochastic Programming

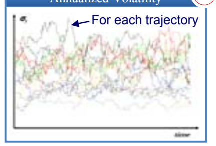
END PROCEDURE

$[{}_k\sigma_{4,min} \quad {}_k\sigma_{4,max}] = [\sigma_{4,min} \quad \sigma_{4,max}]$

Product Value



Annualized Volatility



$\Delta \leq 5\%$

Initial Interval

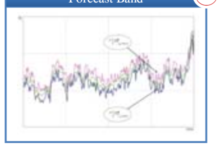
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval

$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

Forecast Band



2nd Pillar

Synthetic risk indicator

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 3: Fine-tuning of Volatility Intervals

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Low	0.01%	0.49%
Medium-Low	0.50%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

2nd Pillar

Synthetic risk indicator

Synthetic Risk Indicator
(The degree of risk)

Time evolution of the volatility

Mapping of the qualitative risk classes into corresponding volatility intervals

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Low	0.01%	0.49%
Medium-Low	0.50%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

2nd Pillar

Synthetic risk indicator

"Risk target" products | "Benchmark" products | "Return target" products

For benchmark products the degree of risk is supplemented by a synthetic indicator of the asset management style:

passive or active

In this second case, the intensity of the active management style depends on the extent of the deviation from the benchmark and on its direction

2nd Pillar
Synthetic risk indicator

Synthetic Risk Indicator
(The degree of deviation from the benchmark)

Three qualitative management classes

Limited
Intermediate
Considerable

2nd Pillar
Synthetic risk indicator

Synthetic Risk Indicator
(The degree of deviation from the benchmark)

Time evolution of the volatility

Mapping of the qualitative risk classes into corresponding volatility intervals

Time evolution of the "intensity" of the management style

Mapping of each management class into corresponding intervals of a suitable measure

2nd Pillar
Synthetic risk indicator

Mapping of each management class into corresponding intervals of a suitable measure

Choice of a proper Volatility Measure:
the *delta-vol*
$\Delta\sigma = \sigma_F - \sigma_B$

Risk Classes	Delta-Vol Intervals					
	Limited		Intermediate		Considerable	
	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$
Low	-0.118%	0.118%	-0.176%	0.176%	-0.235%	0.235%
Medium-Low	-0.239%	0.239%	-0.358%	0.358%	-0.477%	0.477%
Medium	-0.600%	0.600%	-0.900%	0.900%	-1.200%	1.200%
Medium-High	-1.250%	1.250%	-1.875%	1.875%	-2.500%	2.500%
High	-3.125%	3.125%	-4.668%	4.668%	-6.249%	6.249%
Very High	-6.250%	6.250%	-9.375%	9.375%	-12.500%	12.500%

Syllabus

Preliminaries
- ❑ regulatory framework
- ❑ products' risk-return profile VS investors' risk-return profile

Three-pillars approach
- ❑ financial structures
- ❑ 1st Pillar: unbundling and performance scenarios
  - ➢ return target products
    - o unbundling
    - o probabilistic performance scenarios
  - ➢ risk target and benchmark products
  - ➢ model risk assessment
- ❑ 2nd Pillar: the degree of risk
  - ➢ risk target and benchmark products
    - o mapping
    - • **migration**
  - ➢ return target products
- ❑ 3rd Pillar: recommended investment time horizon
  - ➢ risk target and benchmark products
    - o first passage time
    - o connection between probability, volatility and costs
    - o characterization of the necessary condition in the space of returns
    - o how to determine a consistent series of Time Horizons
  - ➢ return target products

2nd Pillar
Synthetic risk indicator

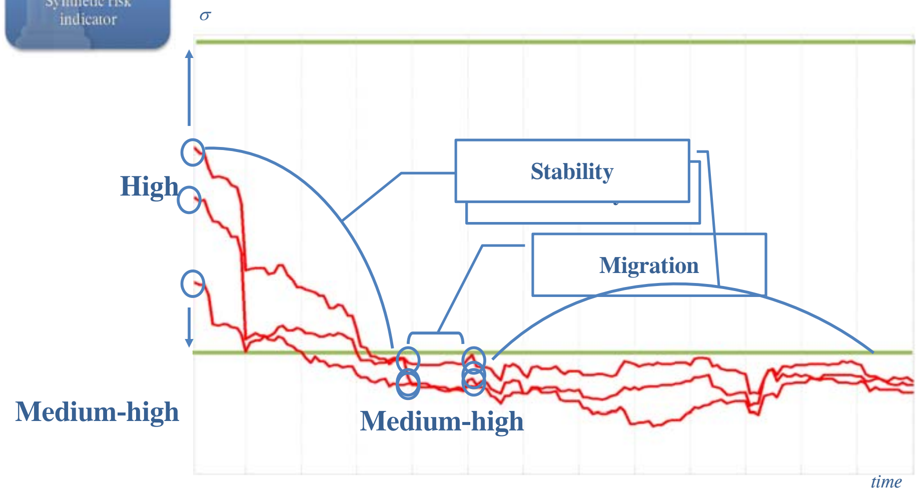
Migration of the Synthetic Risk Indicator

Migrations of the risk profile are persistent changes either of the degree of risk or of the degree of deviation from the benchmark which can significantly affect investors assessment of the non-equity product.

2nd Pillar
Synthetic risk indicator

Migration of the Synthetic Risk Indicator

(degree of risk)



2nd Pillar
Synthetic risk indicator

Migration of the Synthetic Risk Indicator

In order to <u>correctly detect migrations</u>, the width of both volatility and *delta-vol* intervals must be adequately set with respect to the period taken as a reference to assess the occurrence of these phenomena.

<u>Too wide intervals</u> could result in an <u>artificial reduction</u> in the number of migrations detected.

<u>Too narrow intervals</u> could result in an <u>excessive</u> number of <u>migrations</u>, many of them being spurious.

2nd Pillar
Synthetic risk indicator

Migration Rule

(degree of risk)

the iterative procedure guarantees that a product belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 year

↕

no more than 16 days over 250

2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of risk)

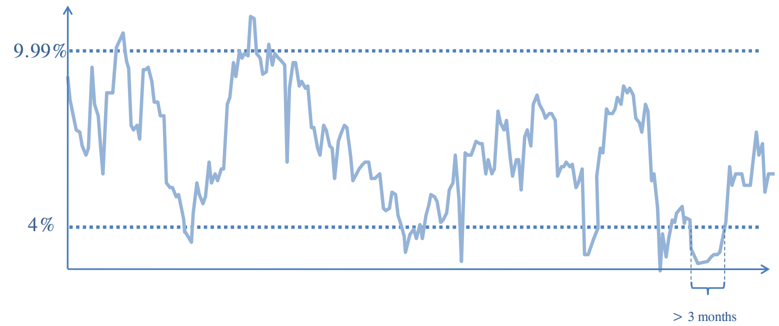
BUT

migration risk is measured against fixed volatility intervals

2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of risk)

as confirmed by back-testing simulations:

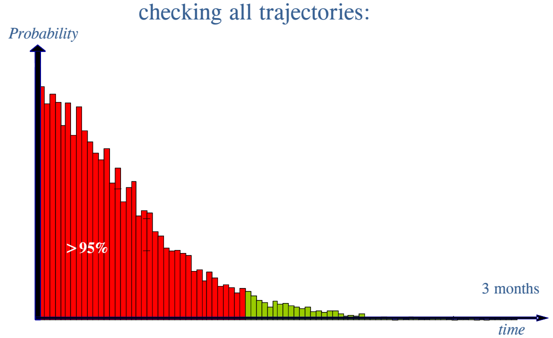


only 1 outlier lastis more than 3 months

2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of risk)

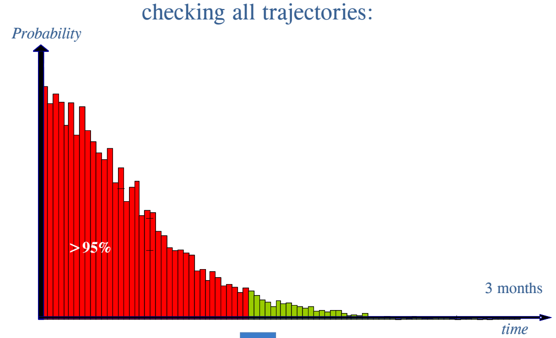
checking all trajectories:



2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of risk)

checking all trajectories:



rolling daily check over the last 3 months

2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of risk)

hence

output intervals are inherently prudential w.r.t. the 3 months migration rule

hence

output intervals are wide enough to avoid spurious migrations…

2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of deviation from the benchmark)

In “benchmark” products, for a given qualitative risk class, the migration to a new management class occurs when for three consecutive months the *delta-vol* lays:

A. on values falling within the intersection of the original class and one or more other classes with a lower intensity of the management activity, in the case the original class is either intermediate or considerable;

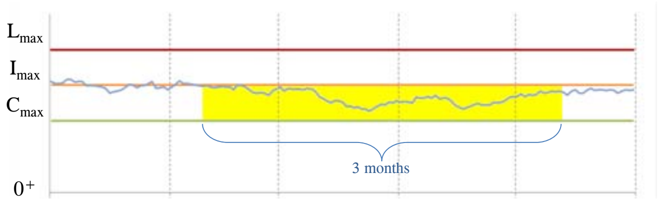
B. outside the bounds of the interval associated with the original class, if this is either the limited or the intermediate class.

2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of deviation from the benchmark)

Case A/B: no migration

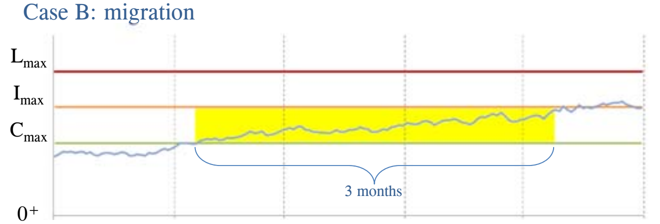
Case A: migration



2nd Pillar
Synthetic risk indicator

Migration Rule
(degree of deviation from the benchmark)

Case B: migration



Syllabus

Preliminaries
- ❑ regulatory framework
- ❑ products' risk-return profile VS investors' risk-return profile

Three-pillars approach
- ❑ financial structures
- ❑ 1st Pillar: unbundling and performance scenarios
  - ➢ return target products
    - o unbundling
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  - ➢ risk target and benchmark products
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- ❑ 2nd Pillar: the degree of risk
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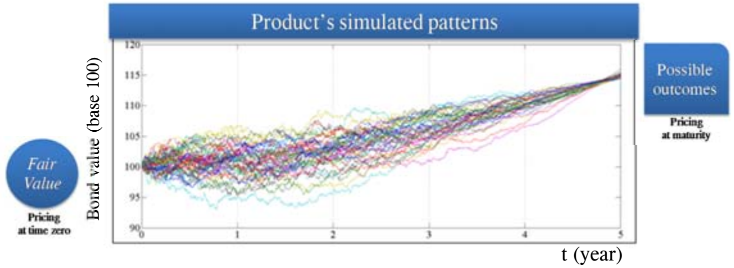
    - o first passage time
    - o connection between probability, volatility and costs
    - o characterization of the necessary condition in the space of returns
    - o how to determine a consistent series of Time Horizons
  - ➢ return target products

2nd Pillar: return target products

In "return target" products the analysis of the volatility measures implicit in the probability distribution of the potential returns makes it possible to determine the risk class



2nd Pillar: return target products

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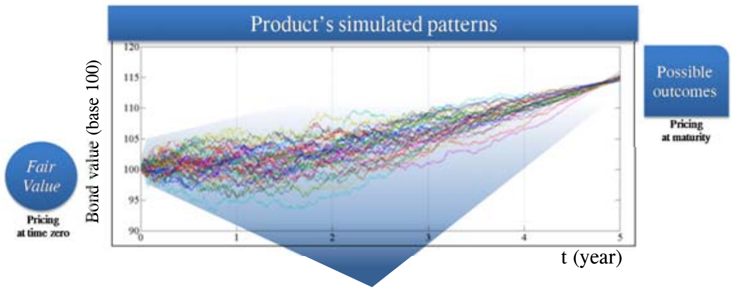

DEGREE OF RISK

Syllabus

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    - o characterization of the necessary condition in the space of returns
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  - ➢ return target products

3rd Pillar
The recommended investment horizon

The Recommended Investment Time Horizon

Investment time horizon consistent with the risk-return profile and the costs associated with the product.

3rd Pillar
The recommended investment horizon

"Risk target" products | "Benchmark" products | "Return target" products

The recommended investment time horizon

…for "risk-target" and benchmark products, the recommended investment time horizon is calculated as the *break-even* time, i.e. the minimum time required to recover initial costs and to off-set running costs, *at least once*, from a probabilistic point of view.

Syllabus

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    - characterization of the necessary condition in the space of returns
    - how to determine a consistent series of Time Horizons
  - return target products

3rd Pillar
The recommended investment horizon

The recommended investment time horizon

In analytical terms, the probability of the event:

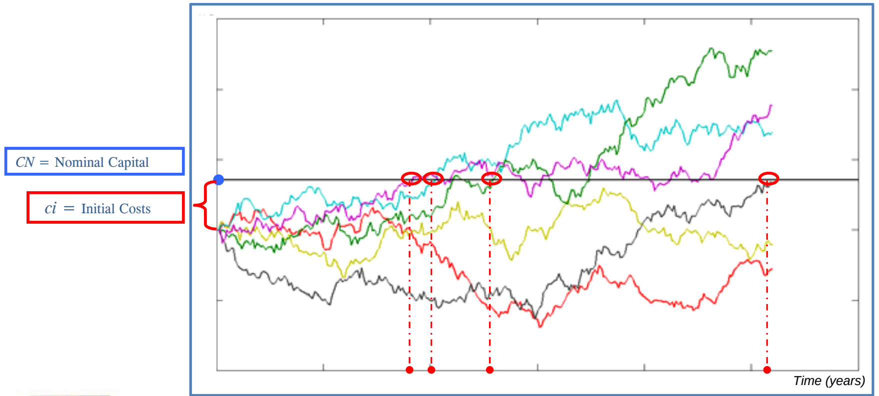
The investment recovers the initial costs and to off-sets the running costs at least once

can be calculated through the concept of

First Passage Time

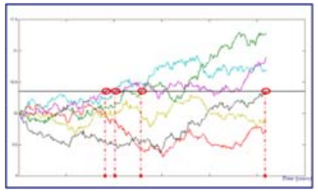
3rd Pillar
The recommended investment horizon

First Passage Time:
First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



3rd Pillar
The recommended investment horizon

First Passage Time:



The Invested Capital is modelled through a generalized Itô process including all the running costs, under the risk-neutral measure:

$$dCI_t = (r_t - c_t)CI_t dt + \sigma_t CI_t dW_t^{(P)}$$

3rd Pillar
The recommended investment horizon

The probability of the event:

The investment recovers the initial costs and off-sets the running costs at least once

is perfectly represented by the cumulative distribution function of the first passage times, i.e.:

$$P[t^* \le T] = \alpha$$

where

$$t^* = \inf[t \in \Re^+ : CI_t > CN]$$

is the first passage time

3rd Pillar
The recommended investment horizon

The probability of the event:

The investment recovers the initial costs and off-sets the running costs at least once

given a confidence level α, uniquely identifies a time T^* on the cumulative distribution function of the first passage times, i.e.:

$$T^* = \{T \in \Re^+ : P[t^* \le T] = \alpha\}$$

where

$$t^* = \inf[t \in \Re^+ : CI_t > CN]$$

is the first passage time

3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \leq T\right] = \alpha \right\}$$

is defined as the recommended minimum investment time horizon

3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

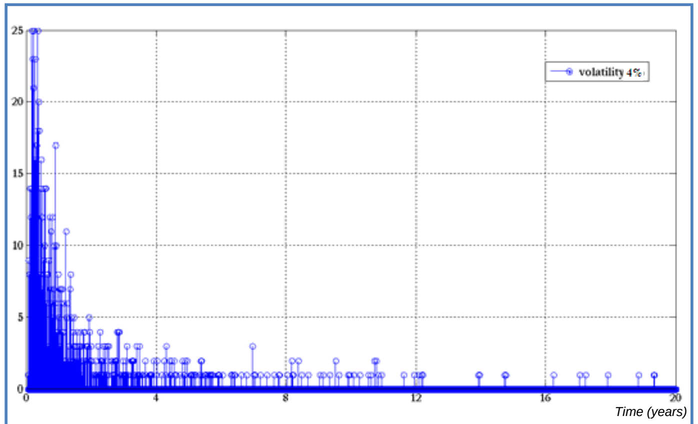
$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \leq T\right] = \alpha \right\}$$

Operative Calculation procedure:

3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

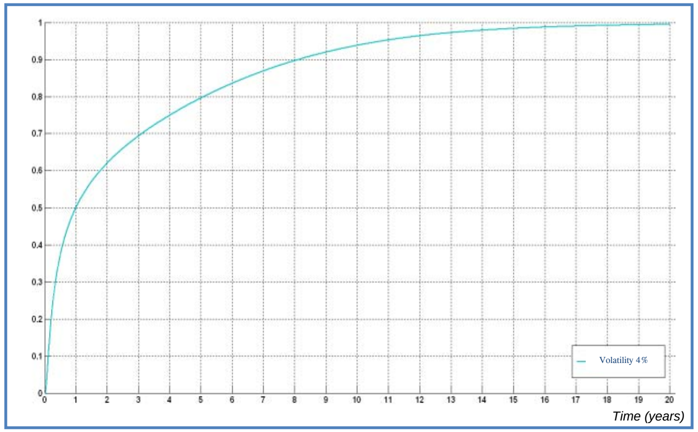
1. Calculation of the probability distribution of the first passage times:



3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

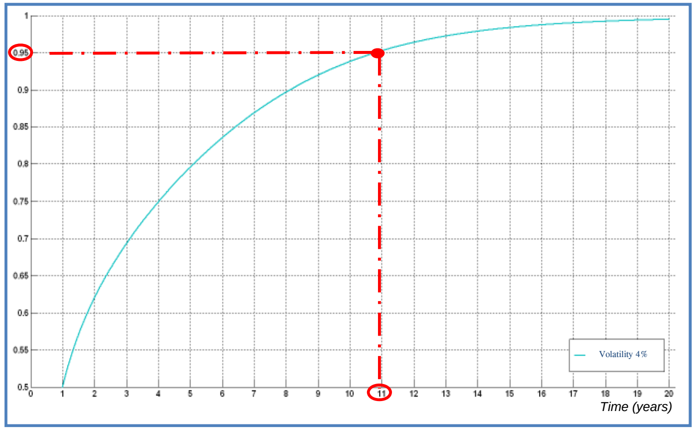
2. Derivation of the cumulative distribution function of the first passage times:



3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

3. The confidence level α uniquely identifies T^* on the cumulative distribution function of the first passage times:



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3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

First passage times for the break-even barrier are monitored at infinitesimal time intervals:

$dt \to 0$

Constant interest rate
$r_t = \bar{r}$

$$T^* = \left\{ T \in \Re^+ : P\left[t^* \le T\right] = \alpha \right\}$$

$$P\left[t^* \le T\right] = N\left(d_2\left(\frac{CI_0}{CN}\right)\right) + \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot N\left(-d_2\left(\frac{CN}{CI_0}\right)\right)$$

$$d_2(x) = \frac{\log x + \left(\bar{r} - cr - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

cr : recurrent costs as a fixed %

Constant interest rate
$r_t = \bar{r}$

$dt \to 0$

$$\lim_{T\to\infty} P\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

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Constant interest rate

$r_t = \bar{r}$

For high volatility products, given the current level of the yield curve and the average level of running costs, this is the most relevant hypothesis.

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

cr : recurrent costs as a fixed %

$dt \to 0$

$$\lim_{T\to\infty} \mathrm{P}\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

Constant interest rate

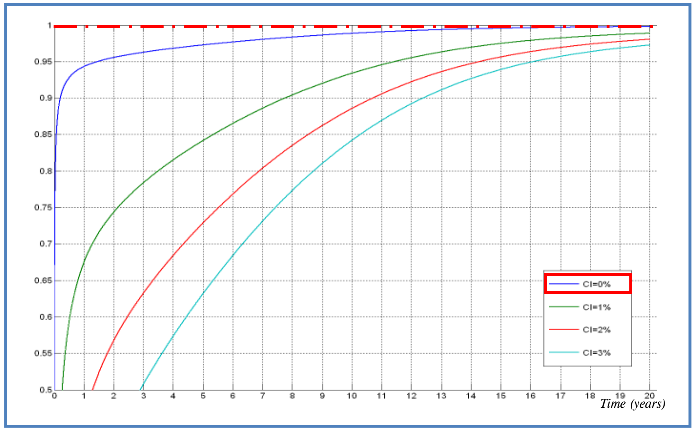
$r_t = \bar{r}$

The higher are *up-front* costs applied to the Invested Capital, the lower is the asymptotic limit of the cumulated probability

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$



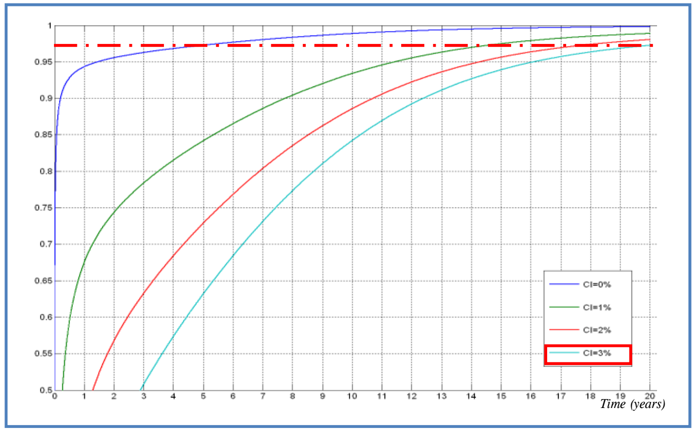
Constant interest rate

$r_t = \bar{r}$

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$



Constant interest rate

$r_t = \bar{r}$

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

cr : recurrent costs as a fixed %

Constant interest rate

$r_t = \bar{r}$

$dt \to 0$

$$\lim_{T\to\infty} P\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

The higher is the volatility increases, the lower is the asymptotic limit of the cumulated probability

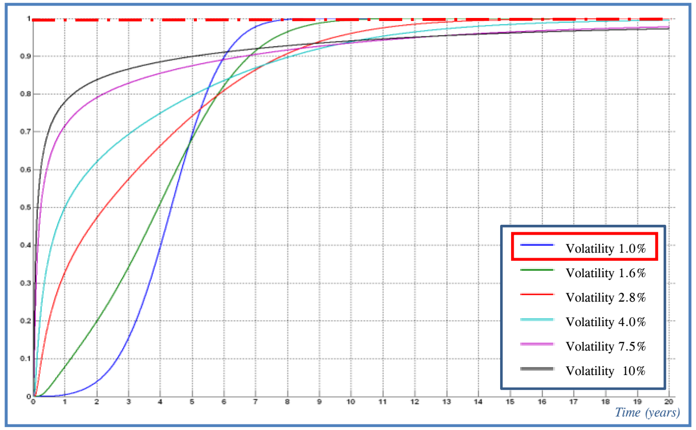
3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

Constant interest rate

$r_t = \bar{r}$



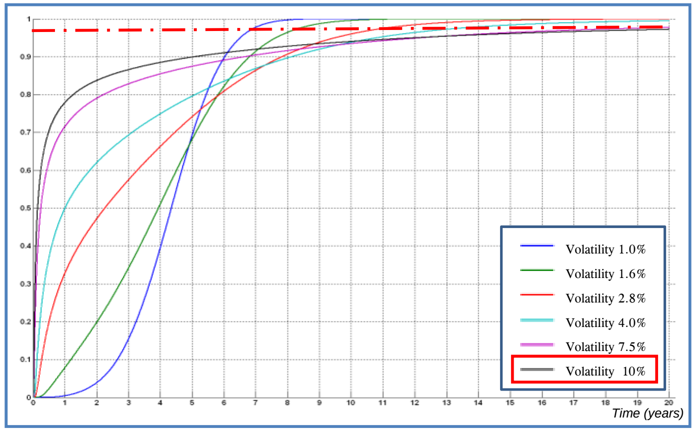
3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

Constant interest rate

$r_t = \bar{r}$



3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Under our assumptions:

Constant interest rate

$r_t = \bar{r}$

$dt \to 0$

$$\lim_{T\to\infty} P\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

For a given level of costs, it is possible to analytically derive the connection between volatility and time horizon

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

First Order Sensitivity Analysis

$$\frac{dP}{d\sigma} = \left(-4 \frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right) \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

FIRST ORDER ASYMPTOTIC CONDITION

Constant interest rate

$r_t = \bar{r}$

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4 \frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right) \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

1. $(\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $(\bar{r} - cr) \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$

Constant interest rate

$r_t = \bar{r}$

The existence of two alternative states of nature requires to verify whether both of them make sense in financial terms under the risk-neutral measure.

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4 \frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right) \left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$

Constant interest rate

$r_t = \bar{r}$

Being running costs a specific feature of any financial product they would interfere with the task of idenfying which of the two conditions has a sound financial meaning. Therefore, they will be temporarily neglected.

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4 \frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right) \left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

Constant interest rate

$r_t = \bar{r}$

Since it is safe to assume a positive interest rate *r* in financial markets, only condition 1. correctly captures the connection between volatility and time horizon.

3rd Pillar – The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

Constant interest rate

$r_t = \bar{r}$

As $T \to \infty$ condition 1. implies that the cumulative distribution function P is a strictly decreasing of the volatility, i.e.:

$$\forall \sigma_i, \sigma_j \in \Re^+, \sigma_j > \sigma_i \Rightarrow P(\sigma_j) < P(\sigma_i)$$

3rd Pillar: recommended investment time horizon

3rd Pillar – The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

Constant interest rate

$r_t = \bar{r}$

In other words, for a given a confidence level, as the volatility grows, the recommended investement time horizon increases as well:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

3rd Pillar: recommended investment time horizon

3rd Pillar – The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

Constant interest rate

$r_t = \bar{r}$

Furthermore, condition 1. alone is sufficient to guarantee a minimum time T^* beyond which the following strong condition holds:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

3rd Pillar: recommended investment time horizon

3rd Pillar – The recommended investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{(\bar{r}-cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

$cr = 0$

1. $(\bar{r}-cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$(\bar{r}-cr) \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

Constant interest rate

$r_t = \bar{r}$

Generalizing…

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

The analysis can be extended to second order conditions:

$$\frac{d^2 P}{d\sigma^2}$$

Constant interest rate

$r_t = \bar{r}$

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

$$\frac{d^2 P}{d\sigma^2} = \frac{4}{\sigma^4}(\bar{r} - cr)\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot \left[1 + \frac{4(\bar{r} - cr)}{\sigma^2}\ln\left(\frac{CN}{CI_0}\right)\right]$$

Constant interest rate

$r_t = \bar{r}$

Second Order Sensitivity Analysis

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

$$\frac{d^2 P}{d\sigma^2} = \frac{4}{\sigma^4}(\bar{r} - cr)\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot \left[1 + \frac{4(\bar{r} - cr)}{\sigma^2}\ln\left(\frac{CN}{CI_0}\right)\right]$$

$$(\bar{r} - cr) > 0 \Rightarrow \frac{d^2 P}{d\sigma^2} > 0$$

SECOND ORDER ASYMPTOTIC CONDITION

Constant interest rate

$r_t = \bar{r}$

Second Order Sensitivity Analysis

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

1.
$$\begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \frac{d^2 P}{d\sigma^2} > 0 \end{cases}$$

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

~~2. $(\bar{r} - cr) \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$~~

Constant interest rate

$r_t = \bar{r}$

Summarizing the results of the asymtptic analisys in continuous time:

- As T $\to \infty$, for given a confidence level, more volatility implies a larger recommended investment time horizon
- It is always possible to find a minimum and finite time T^*, beyond which the strong condition

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

holds

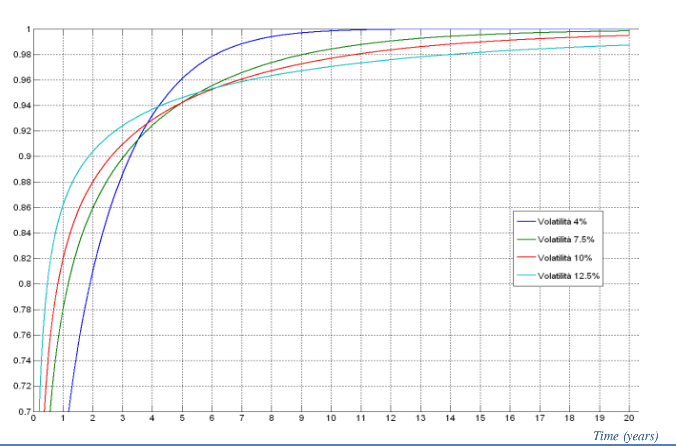
3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON $(\bar{r}-cr)>0$

Constant interest rate

$r_t=\bar{r}$



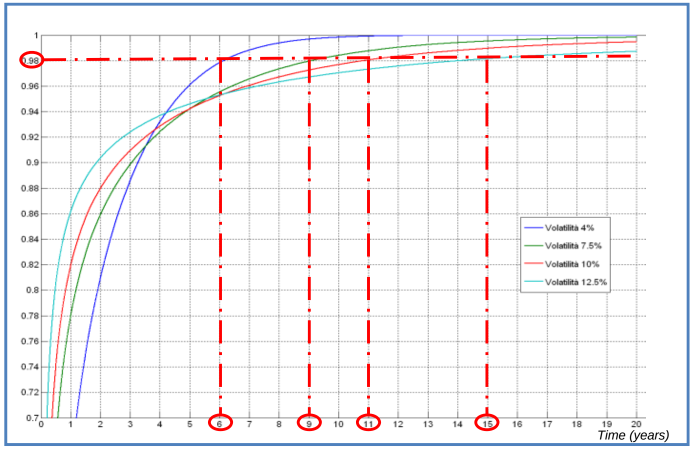
3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON $(\bar{r}-cr)>0$

Constant interest rate

$r_t=\bar{r}$



3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$$T\rightarrow\infty, dt\rightarrow 0$$

1.
$$\begin{cases}(\bar{r}-cr)>0 \Leftrightarrow \dfrac{dP}{d\sigma}<0 \\ (\bar{r}-cr)>0 \Rightarrow \dfrac{d^2P}{d\sigma^2}>0\end{cases}$$

2. $(\bar{r}-cr)\leq 0 \Leftrightarrow \dfrac{dP}{d\sigma}\geq 0$

Constant interest rate

$r_t=\bar{r}$

If the structural condition 1. is not satisfied, the outcomes of the analysis must be disregarded and the analytical tool available in continuous time yields meaningless results.

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

$$T\rightarrow\infty, dt\rightarrow 0$$

1.
$$\begin{cases}(\bar{r}-cr)>0 \Leftrightarrow \dfrac{dP}{d\sigma}<0 \\ (\bar{r}-cr)>0 \Rightarrow \dfrac{d^2P}{d\sigma^2}>0\end{cases}$$

2. $(\bar{r}-cr)\leq 0 \Leftrightarrow \dfrac{dP}{d\sigma}\geq 0$

$\bar{T}=x\ years$

Constant interest rate

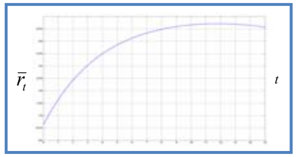
$r_t=\bar{r}$

It is necessary to drop from the analysis those cases which yield condition 2 (i.e. whenever the drift positiveness is not satisfied). Under such a condition, the recommended time horizon is set *by default* equal to a pre-defined limit *x*.

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

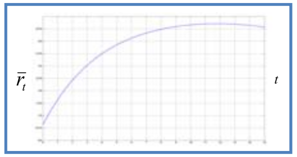
Time-varying, deterministic interest rate
$r_t = \bar{r}_t$



3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Time-varying, deterministic interest rate
$r_t = \bar{r}_t$



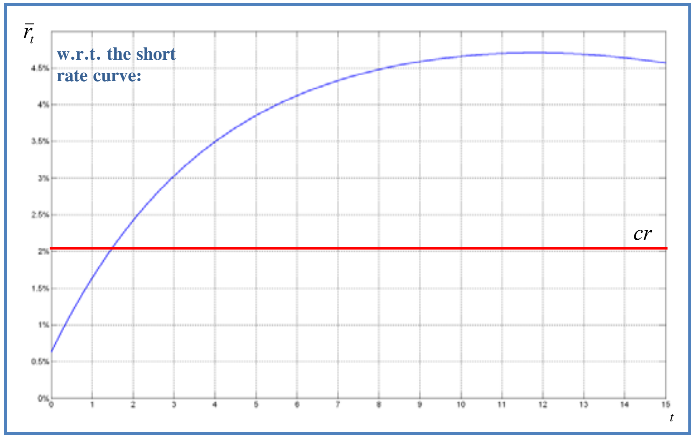
In presence of time-varying, deterministic interest rates, the condition given by $\bar{r}_t - cr > 0$ is NOT ALWAYS satisfied, due to the time dependence of the *short rate* curve.

It is necessary to identify a criterion which allows to discard all times such that the condition $\bar{r}_t - cr < 0$ is satisfied.

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 0% $\forall \sigma \geq 0$

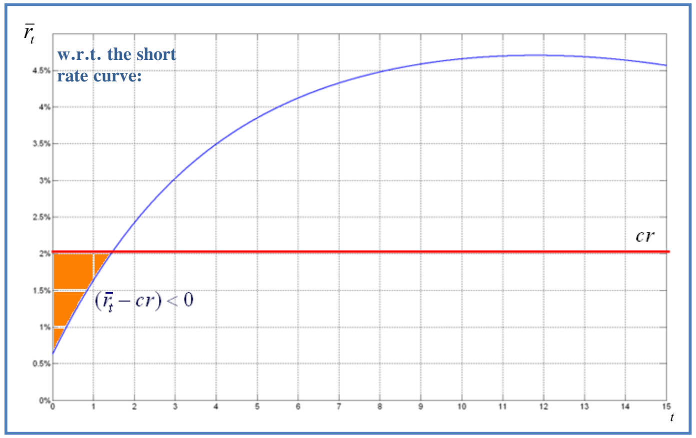


Deterministic rate
ci=0%, cr=2%

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 0% $\forall \sigma \geq 0$



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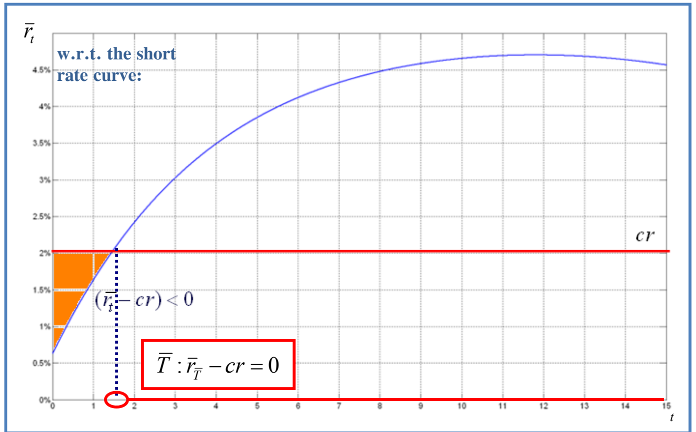


Deterministic rate
ci=0%, cr=2%

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 0% $\forall \sigma \geq 0$

Starting from $\bar{T} : r_{\bar{T}} - cr = 0$ the recommended minimum time horizon must be calculated as the break-even time of the costs of the financial investment. Hence running costs must be off-set.

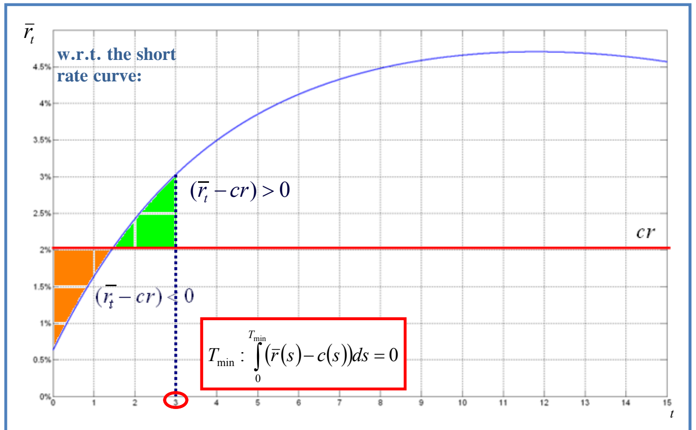
It is possible to define the NECESSARY CONDITION valid for any level of volatility as follows:

Deterministic rate
ci=0%, cr=2%

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 0% $\forall \sigma \geq 0$



Deterministic rate
ci=0%, cr=2%

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 2% $\forall \sigma \geq 0$

In presence of initial costs, the recommended minimum time horizon must be calculated as the break-even time of the costs of the financial investment. Hence, initial cost must be recovered and running costs must be off-set.

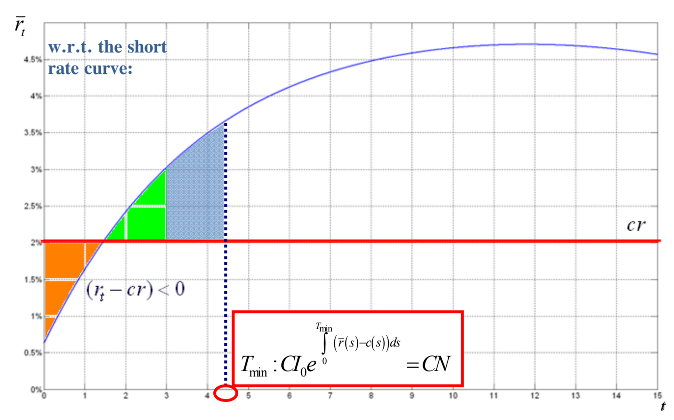
In this case the NECESSARY CONDITION valid for any level of volatility becomes:

Deterministic rate
ci=2%, cr=2%

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 2% $\forall \sigma \geq 0$



Deterministic rate
ci=2%, cr=2%

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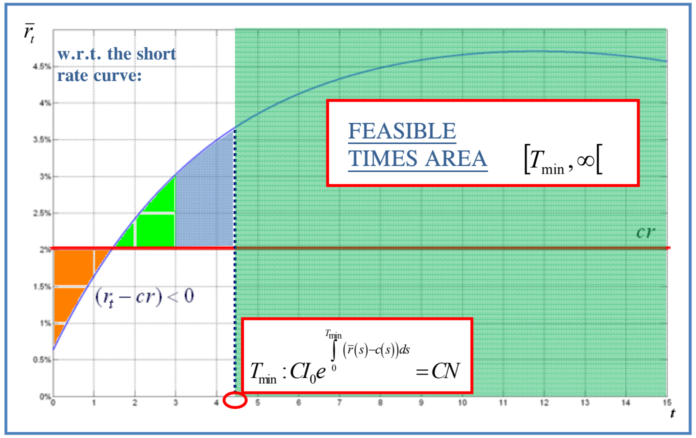

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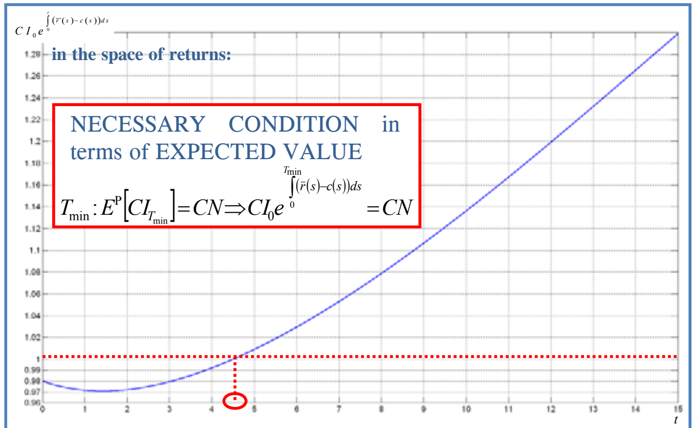

Deterministic rate
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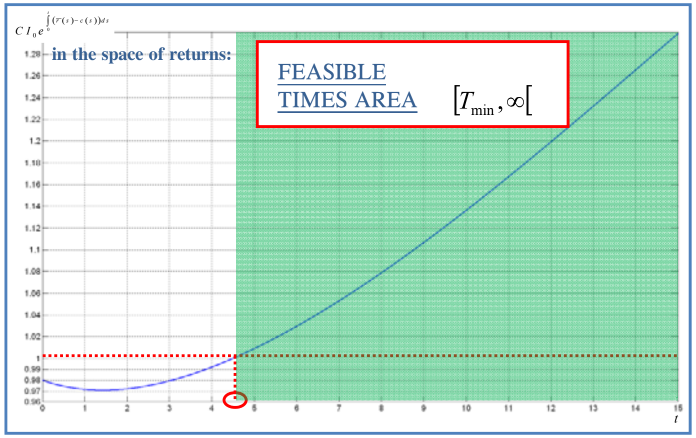
Hp: Initial Costs = 2% $\forall \sigma \geq 0$



Deterministic rate
ci=2%, cr=2%

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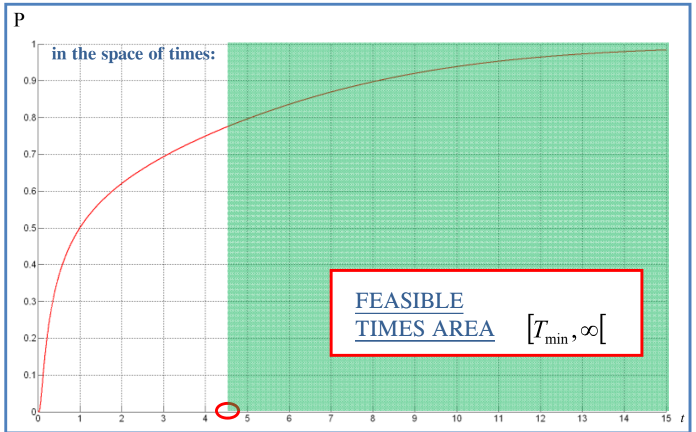
Connection between probability, volatility and costs

Hp: Initial Costs = 2% $\forall \sigma \geq 0$

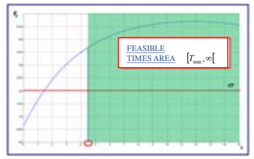


Deterministic rate

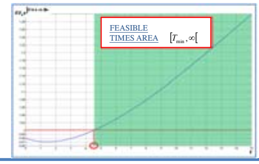
ci=2%, cr=2%

3rd Pillar
The recommended investment horizon

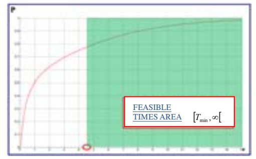
Connection between probability, volatility and costs



NECESSARY CONDITION CHARACTERIZATION w.r.t. THE SHORT RATE CURVE



NECESSARY CONDITION CHARACTERIZATION IN THE SPACE OF RETURNS



NECESSARY CONDITION CHARACTERIZATION IN THE SPACE OF TIMES

Deterministic rate

ci=2%, cr=2%

3rd Pillar
The recommended investment horizon

Connection between probability, volatility and costs

Hp: Initial Costs = 2% $\forall \sigma \geq 0$

Starting from $T_{\min}$ the recommended investment time horizon is calculated as the *break-even* time of the costs the financial investment, i.e. the minimum time required to recover initial costs and to off-set running costs, at least once, from a probabilistic point of view.

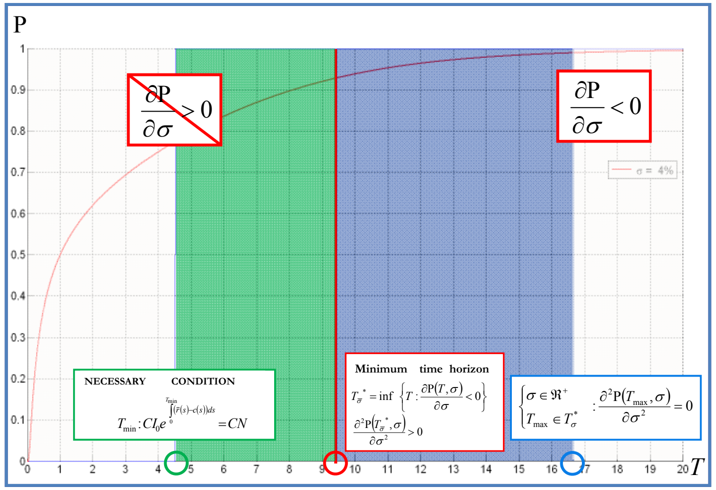
It is possible to define the SUFFICIENT CONDITION as follows:

Deterministic rate

ci=2%, cr=2%

3rd Pillar
The recommended investment horizon

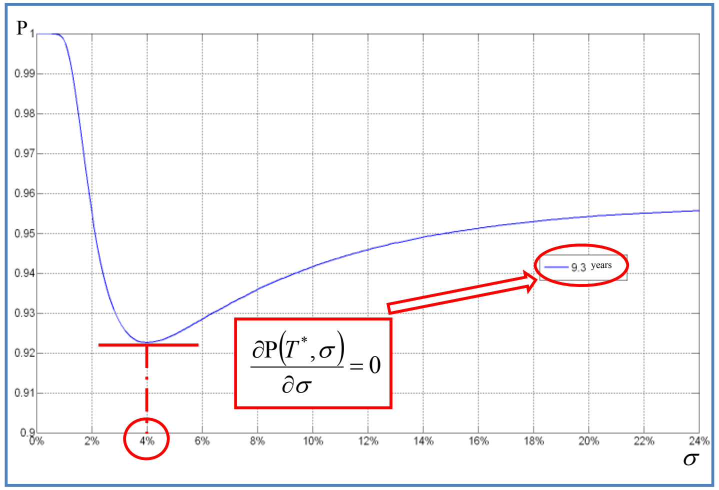
Connection between probability, volatility and costs



Deterministic rate

ci=2%, cr=2%

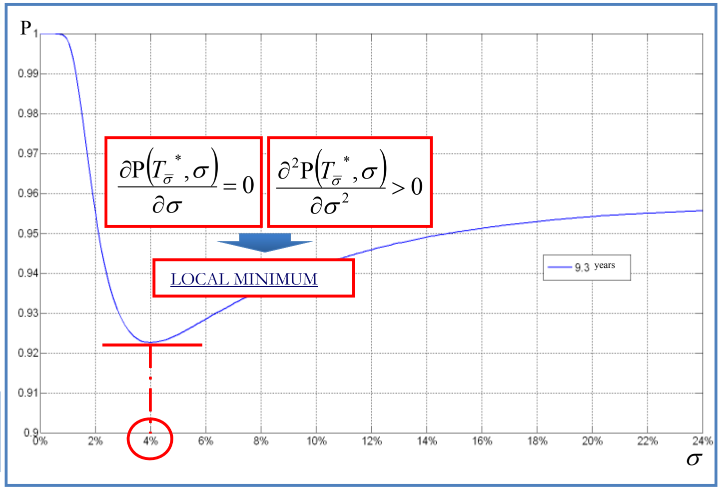
Connection between probability, volatility and costs



Deterministic rate

ci=2%, cr=2%

Connection between probability, volatility and costs

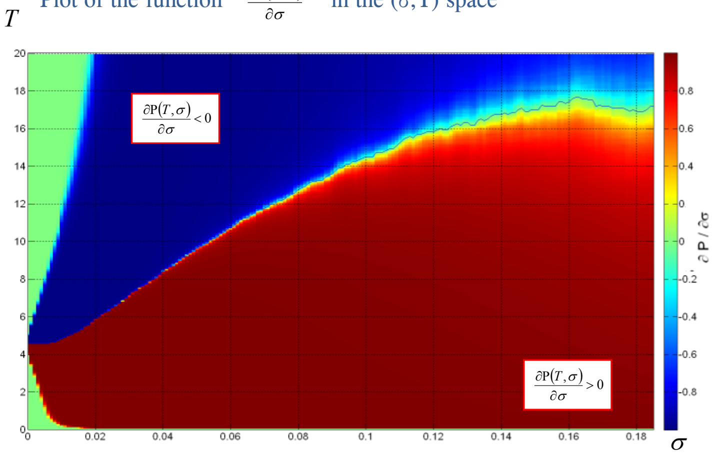


Deterministic rate

ci=2%, cr=2%

Connection between probability, volatility and costs

Plot of the function $\frac{\partial P(T,\sigma)}{\partial \sigma}$ in the (σ,T) space

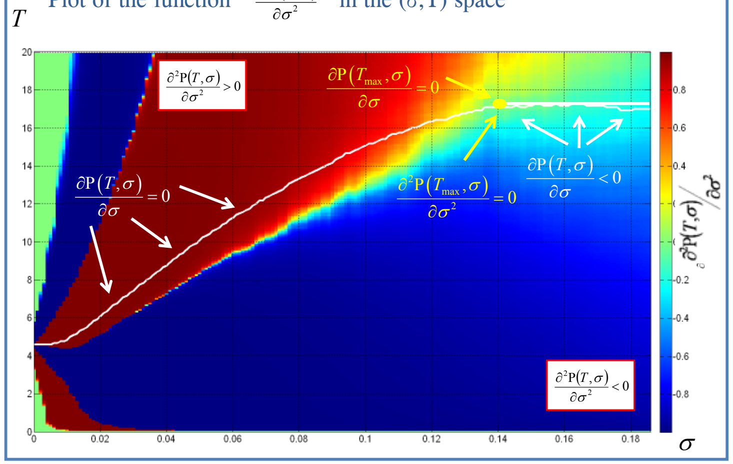


Deterministic rate

ci=2%, cr=2%

Connection between probability, volatility and costs

Plot of the function $\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$ in the (σ,T) space



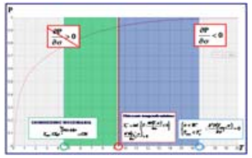
Deterministic rate

ci=2%, cr=2%

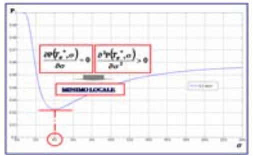
3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

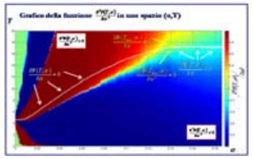
Connection between probability, volatility and costs



SUFFICIENT CONDITION CHARACTERIZATION IN THE SPACE OF TIMES



SUFFICIENT CONDITION CHARACTERIZATION IN THE (σ,P) SPACE



SUFFICIENT CONDITION CHARACTERIZATION IN THE (σ,T) SPACE

Deterministic rate

ci=2%, cr=2%

Syllabus

Preliminaries
- ❑ regulatory framework
- ❑ products' risk-return profile VS investors' risk-return profile

Three-pillars approach
- ❑ financial structures
- ❑ 1st Pillar: unbundling and performance scenarios
  - ➢ return target products
    - o unbundling
    - o probabilistic performance scenarios
  - ➢ risk target and benchmark products
  - ➢ model risk assessment
- ❑ 2nd Pillar: the degree of risk
  - ➢ risk target and benchmark products
    - o mapping
    - o migration
  - ➢ return target products
- ❑ 3rd Pillar: recommended investment time horizon
  - ➢ risk target and benchmark products
    - o first passage time
    - o connection between probability, volatility and costs
    - • **characterization of the necessary condition in the space of returns**
    - o how to determine a consistent series of Time Horizons
  - ➢ return target products

3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

Characterization of the necessary condition in the space of returns

The NECESSARY CONDITION:

$$\forall \sigma \geq 0,\ E^P\left[CI_{T_{\min}}\right] = CN$$

defined on the weak event:

The investment recovers the initial costs and off-sets the running costs *at least* once by the time T

has a significative characterization with respect to the strong event:

The investment *at least* recovers the initial costs and off-sets the running costs by the time T

3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

Characterization of the necessary condition in the space of returns

The investment recovers the initial costs and off-sets the running costs *at least* once by the time T

↓

Stochastic Time Distribution

The investment *at least* recovers the initial costs and off-sets the following ones by the time T

↓

Stochastic Returns Distribution

3rd Pillar – The recommended investment horizon

Characterization of the necessary condition in the space of returns

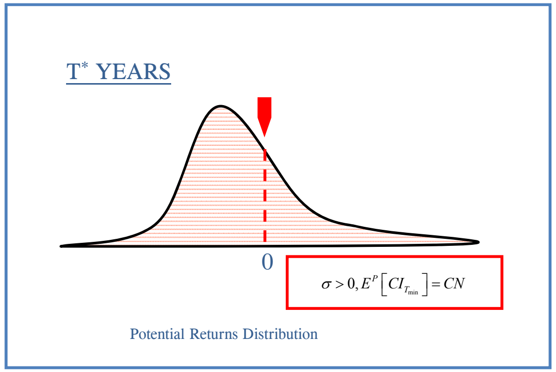
The NECESSARY CONDITION:

$$\forall \sigma \geq 0,\ E^{P}\left[CI_{T_{\min}}\right] = CN$$

which identifies a minimum time in the space of stochastic times, in the space of stochastic returns corresponds to the strong event of costs' break-even on average for the investment in the financial product.

3rd Pillar – The recommended investment horizon

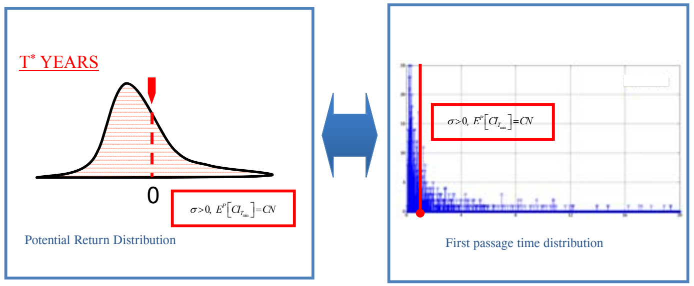
Characterization of the necessary condition in the space of returns



Potential Returns Distribution

3rd Pillar – The recommended investment horizon

Characterization of the necessary condition in the space of returns



Potential Return Distribution

First passage time distribution

Syllabus

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  - ➢ return target products

3rd Pillar

The recommended investment horizon

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

STRONG CONVERGENCE LEMMA for times

Given a sequence of financial products F_j with volatility σ_j and recalling the first order sufficient condition:

$$T^*_\sigma = \max\left\{T_{\min}, T : \frac{\partial P(T,\sigma)}{\partial \sigma} = 0\right\}, \quad \forall \sigma \in \Re^+$$

the first order sufficient condition can be specified for the class of products F_j in the following form:

$$T^{\varepsilon_j}_{\sigma_j} : P\left(T^{\varepsilon_j}_{\sigma_j}, \sigma_{j+1}\right) = P\left(T^{\varepsilon_j}_{\sigma_j}, \sigma_j\right)$$

It therefore holds the following strong convergence relation with respect to times:

$$\lim_{\sigma_{j+1} \to \sigma_j} T^{\varepsilon_j}_{\sigma_j} = T^*_{\sigma_j}$$

where $\varepsilon_j = (\sigma_{j+1} - \sigma_j) > 0.$

3rd Pillar

The recommended investment horizon

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

Formally, for any sequence of products with volatility σ_j, defined in a given class of costs (ci,cr):

Strong convergence lemma for times
First order sufficient condition

Strong monotony condition of times w.r.t. volatility
Second order sufficient condition

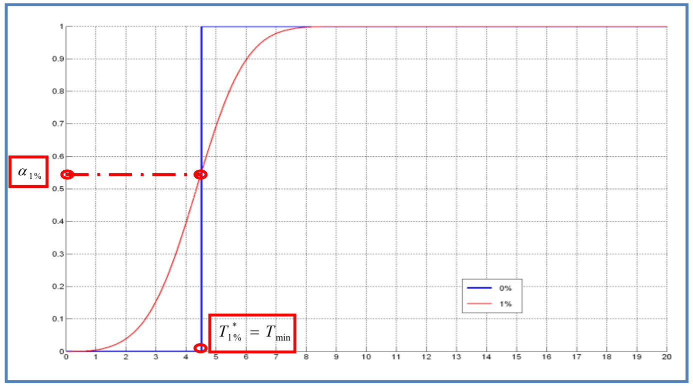
$$\forall j = 1,\ldots N, \sigma_{j+1} > \sigma_j,$$
$$T^*_{j+1} = \max\left\{ T^*_j,\ T \in [T_{\min}, T_{\max}] : P\left[t^*_{\sigma_{j+1}} \le T\right] = P\left[t^*_{\sigma_j} \le T\right] = \alpha^*_{j+1} \right\}$$

3rd Pillar

The recommended investment horizon

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

Practical Method to derive a sequence of time horizons

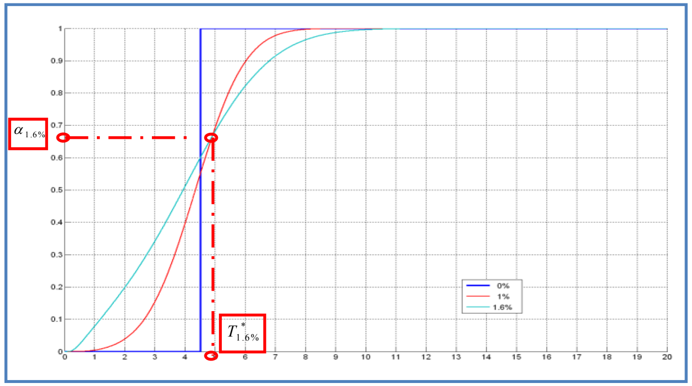


3rd Pillar

The recommended investment horizon

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

Practical Method to derive a sequence of time horizons

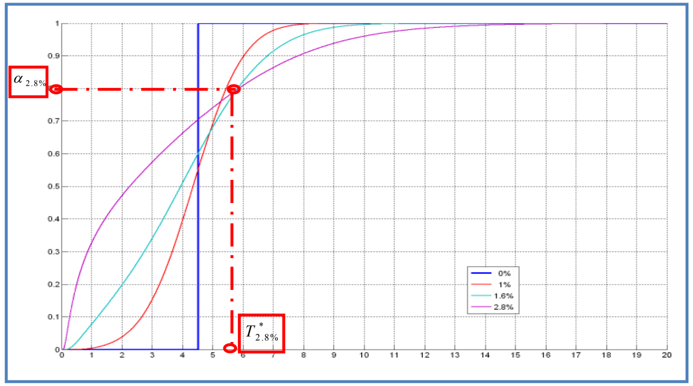


3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

Practical Method to derive a sequence of time horizons



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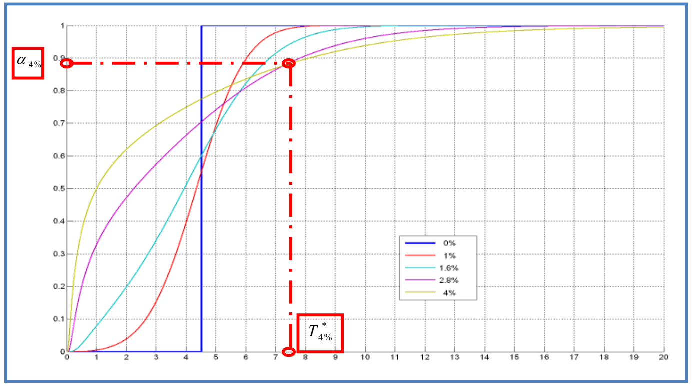

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The recommended investment horizon

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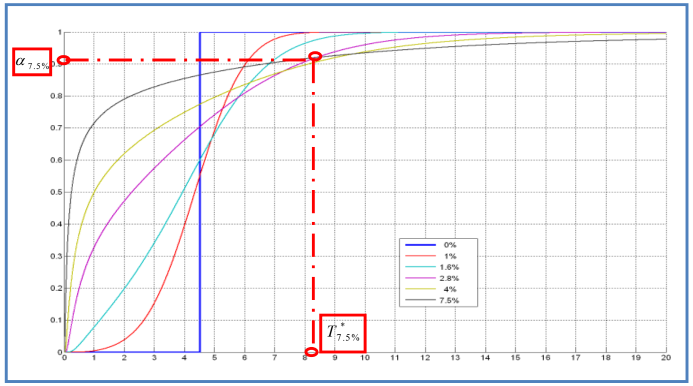

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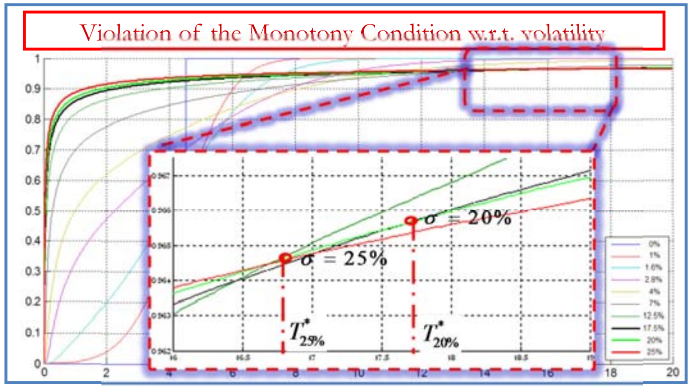

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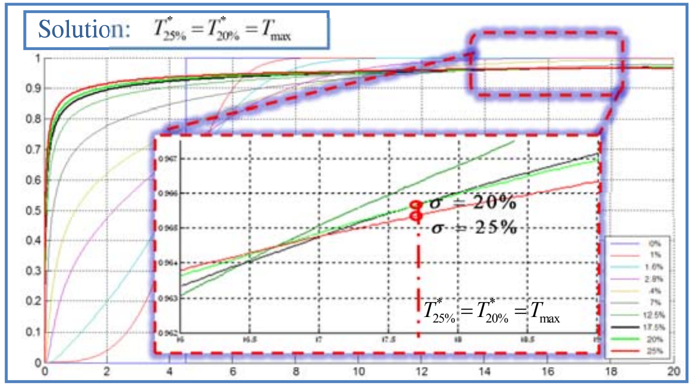

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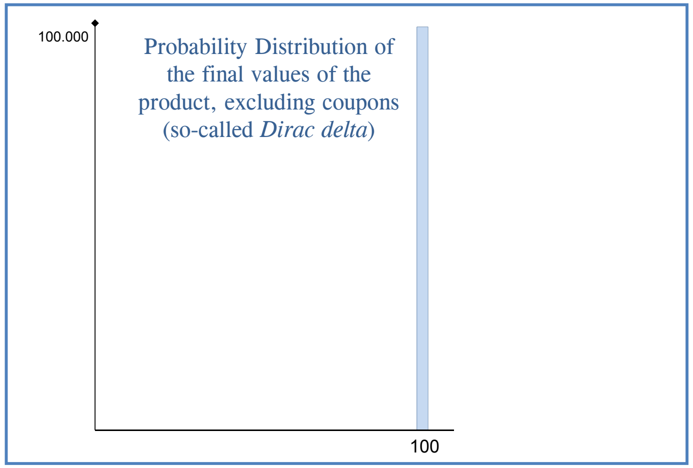
3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

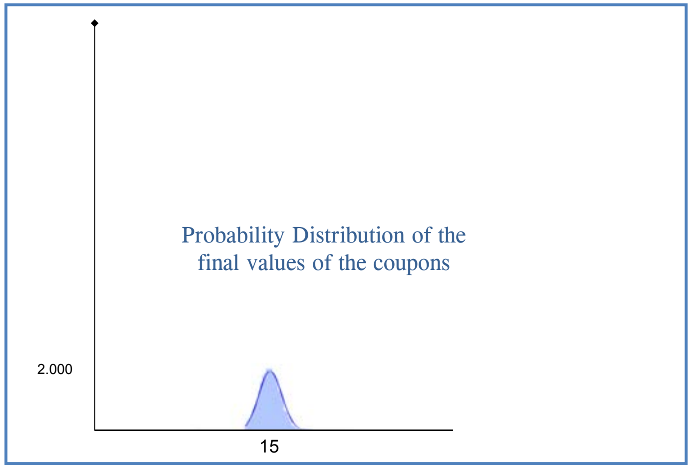
In return target products (such as corporate bonds) the redemption at maturity generates a peak in the probability distribution of the final value of the Invested Capital



3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon

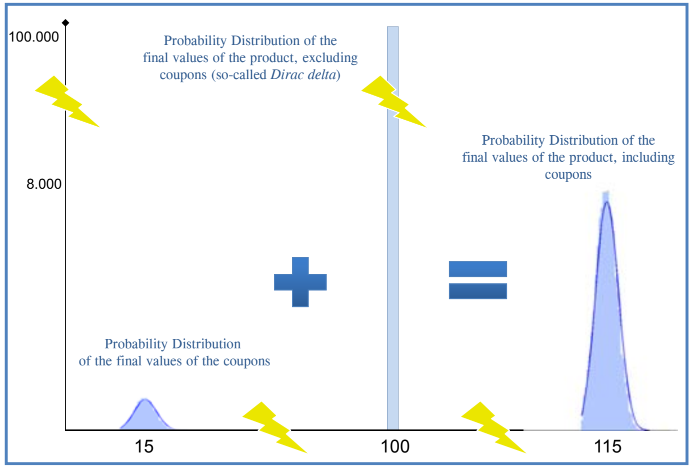
If the product pays coupons this increases the variability of the final distribution of the Invested Capital due to the capitalization of the coupons at the risk-free rate.



3rd Pillar: recommended investment time horizon

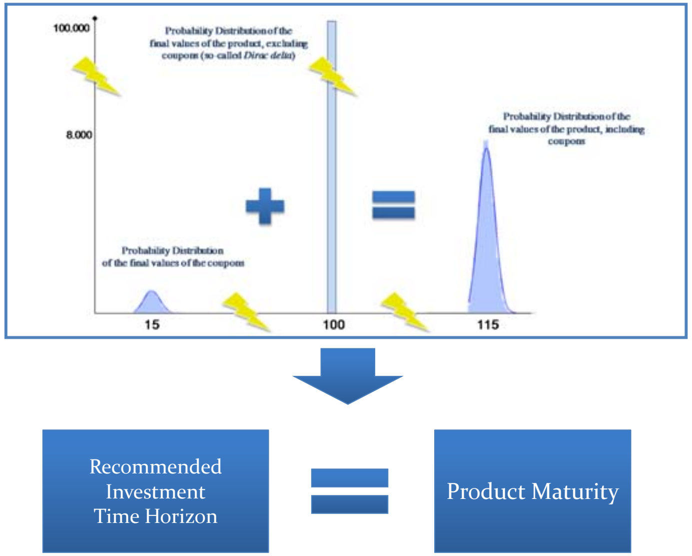
3rd Pillar
The recommended investment horizon

The sum of two distributions provides the final probability distribution of the Invested Capital



3rd Pillar: recommended investment time horizon

3rd Pillar
The recommended investment horizon



Marcello Minenna

Head of Quantitative Analysis Unit
CONSOB

Three Pillars for Risk-Transparency in Structured Finance