EWGFM 50 MEETING

EWGFM Paris Award

The Board of the Euro Working Group for Financial Modeling created an award in the memory of Francesco Paris.

The Second annual award of €500 will be made at the 50th meeting.

Scholars are urged to submit their recent research papers.to *daniela.pastres@uniroma1.it* referencing "EWGFM Paris Award"

DEADLINE

April 1st 2012

The 50th Meeting of the Euro Working Group for Financial Modelling is organized in Rome by Sapienza University and sponsored by Sapienza Research Funds of 2011.

Contributions were also made by the Faculty of Economics, University of Rome *Roma Tre.*

SAPIENZA UNIVERSITÀ DI ROMA

ROMA TRE UNIVERSITÀ DEGLI STUDI

Centro Congressi - Sapienza University
Via Salaria, 113 - Rome

EWGFM 50 MEETING

50th

EWGFM Meeting

May 3-5 2012

Rome, Italy

Scientific Committee
R.L. D'Ecclesia, Sapienza University of Rome, Italy
R. Castellano, University of Macerata., Italy
L. Mastroeni, University Rome Tre. Italy

Organizing Committee
F. Barcellona, Sapienza University of Rome, Italy
R. Cerqueti, University of Macerata, Italy
M. Ligios, Sapienza University of Rome, Italy
R. Monaco, Sapienza University of Rome, Italy
C. Perruolo, Sapienza University of Rome, Italy

A selection of the presented papers will be published in the European Journal of Finance (IF 0,486), edited by R.L. D'Ecclesia and Jaap Spronk

May 3rd

08:30
Registration

09:00-09:30
R.L. D'Ecclesia, Sapienza University of Rome,
Welcome and Opening Remarks

Opening Session

09:30-10:15
A. Malliaris, Loyola University, Chicago
"Asset Bubbles Monetary Policy and Risks to the Real Economy"

10:15-11:00
J. V. Andersen, INLN, Nice
"Inter-disciplinarity: finance applied to physics- physics applied to finance"

11:00-11:30
Coffee Break

11:30-12:30
W. De Bondt, De Paul University of Chicago
"Financial expertise: fact or fiction"
13:00-14:30
Lunch at Piccolo Abruzzo on Via Sicilia 237

Session 1

14:30-18:30
Contributed papers

20:00 Complimentary Dinner
Sala degli Angeli, Palazzo Barberini
Via XX Settembre

May 4th

Session 2

09:00-11:00
Contributed papers

11:00-11:30 Coffee Break

11:30-13:00 Contributed papers

13:00-14:30 Lunch at Piccolo Abruzzo

Special Session
How quantitative models adjusted to changing risk management issues

14:30-16:00 M. Minenna, G. Boi, P. Verzella
Probabilistic scenarios and risk measurement: "*A quantitative framework to assess the risk-reward profile of non equity products*".

16:00-16:30 Tea Break

16:30-18:30 Round Table

S. Galluccio, BNP Paribas

A. Guglielmi, Mediobanca Securities London

F. Fiordelisi, Roma Tre, University of Rome

G. Fusai, Cass Business School of London and University Piemonte Orientale

R. Stein, Moody's Credit Agency.

M. Minenna, Risk Analyst, Rome

20:00 Conference Dinner

May 5th

Session 4

09:00-11:00 Contributed papers

11:00-11:30 Coffee Break

11:30-13:00 Contributed papers

13:00 R.L. D'Ecclesia
Closing Remarks and Future EWGFM programs

13:30-14:30
Lunch at Piccolo Abruzzo on Via Sicilia 237

Book Presentation at:

EWGFM 50 MEETING

A quantitative methodology for risk assessment in financial products

Marcello Minenna

Rome, 4th April 2012

Opinions expressed in this work are exclusively of the author

Syllabus

- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon

Unbundling and Probabilistic performance scenarios

The returns evaluation requires the estimate of all the relevant risk factors connected with the financial structure of each product

DEFAULTABLE BOND	LOW-RISK BOND	VPPI PRODUCT	INDEX-LINKED CERTIFICATE
Interest Rate Volatility Significant exposure to credit risk	Interest Rate Volatility Limited exposure to credit risk	Interest Rate Volatility Limited exposure to Market risk	Interest Rate Volatility Significant exposure to Market risk

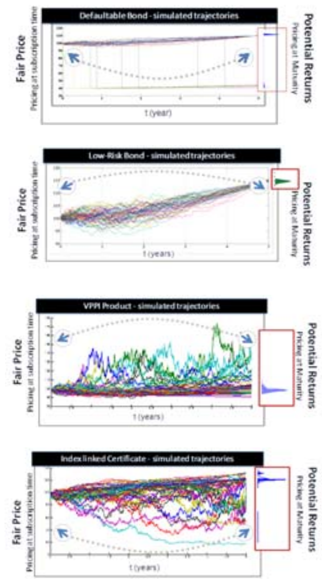
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

LOW-RISK BOND

VPPI PRODUCT

INDEX LINKED CERTIFICATE



Fair Price at time zero is a **Weighted average**

Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND | LOW-RISK BOND | VPPI PRODUCT | INDEX LINKED CERTIFICATE

Fair Price at time zero is a **weighted average**

first moment of the probability distribution at expiry date is also a **weighted average**

Probability distribution of the Defaultable Bond | Probability distribution of the Low Risk Bond | Probability distribution of the VPPI Product | Probability distribution of the Index Linked Certificate

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond | Probability distribution of the Low Risk Bond | Probability distribution of the VPPI Product | Probability distribution of the Index Linked Certificate

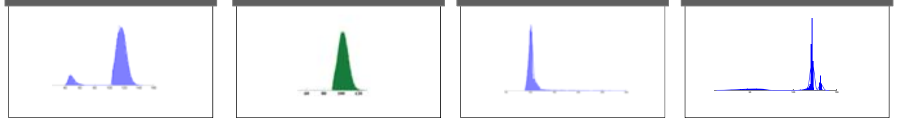
discounted + expected | discounted + expected | discounted + expected | discounted + expected

Defaultable Bond **Fair Price** at time zero = Low Risk Bond **Fair Price** at time zero = VPPI Product **Fair Price** at time zero = Index Linked Certificate **Fair Price** at time zero

Working Hypothesis: The calculated fair price is the same for completely different financial structures

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond | Probability distribution of the Low Risk Bond | Probability distribution of the VPPI Product | Probability distribution of the Index Linked Certificate



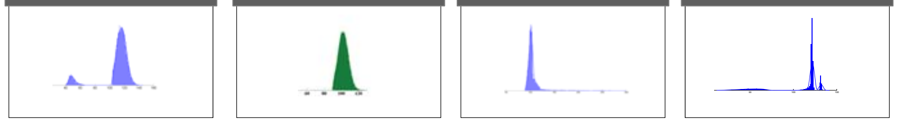
discounted + expected | discounted + expected | discounted + expected | discounted + expected

Defaultable Bond **Fair Price** at time zero = Low Risk Bond **Fair Price** at time zero = VPPI Product **Fair Price** at time zero = Index Linked Certificate **Fair Price** at time zero

Question: How much information about the original probability distribution the price will convey in each case analyzed?

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond | Probability distribution of the Low Risk Bond | Probability distribution of the VPPI Product | Probability distribution of the Index Linked Certificate

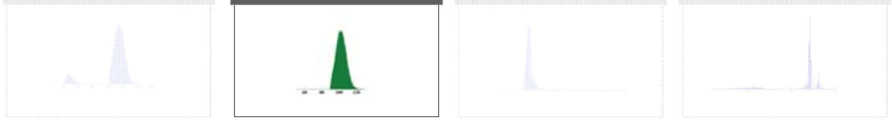


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	Low Risk Bond	VPPI Product	Index Linked Certificate
Bimodality	Regular	Asymmetry	Multimodality
High dispersion	Symmetry	Kurtosis	Asymmetry
	Low dispersion		Kurtosis
			High dispersion

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate

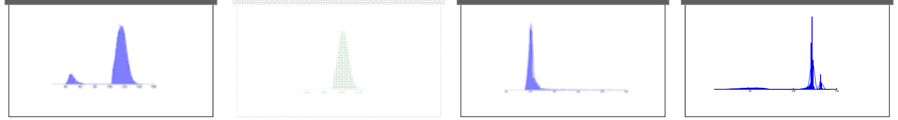


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
	High significance of the price information		

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate

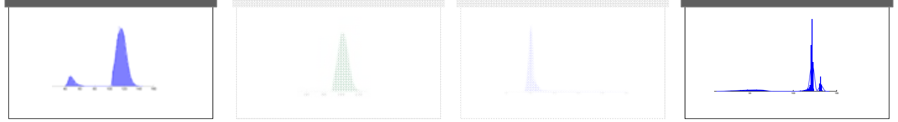


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
	High significance of the price information	Limited significance of the price information	

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate

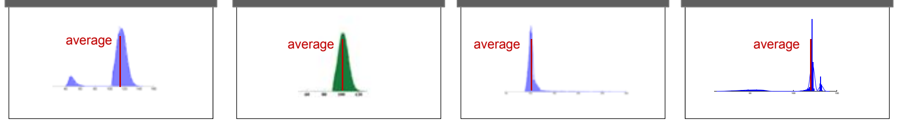


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
Poor significance of the price information	High significance of the price information	Limited significance of the price information	Poor significance of the price information

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate



STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion

The price and the corresponding average at expiry date – in presence of IRREGULAR distributions – qualify a partial and misleading information

Unbundling and Probabilistic performance scenarios

Significance of the price information → **As a weighted average, the price is strictly connected with the first moment of the probability distribution**

As the literature suggests, in presence of multimodality and irregular shapes for the probability distributions, the number of moments necessary to properly describe the probability distribution increases drammatically.

See:

(1) Shohat, Tamarkin, 1943 - American Mathematical Survey
(2) Szego, 1959 - American Mathematical Society
(3) Totik, 2000 – Journal of Analytical Mathematics
(4) Gavriliadis, Athanassoulis, 2009 – Journal of Computational and Applied Mathematics

Unbundling and Probabilistic performance scenarios

Significance of the price information → **Mathematical Basis to test the significance of the price information**

Given a finite number of moments 2k, it's possible to derive the following approximate relationship between the probability function f (x) and its Christoffel function of degree k:

$$f(x) \approx f_{AP,k}(x) = \frac{k}{c_0 \pi \sqrt{(x-a)(b-x)}} \lambda_k(x)$$

con $x \in [a,b]$. c_0 è un fattore di normalizzazione.

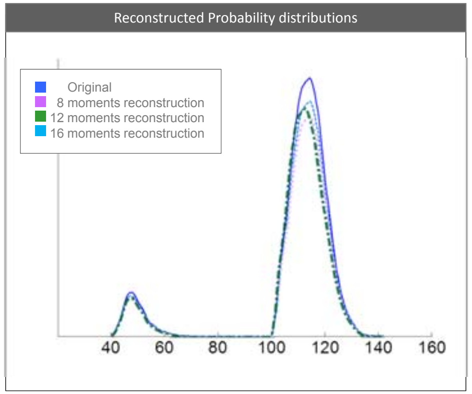
→ It' s then immediate to apply the approximating formula for different values of k in order to test the accuracy of the approximation for the probability distributions corresponding to our different financial products

Unbundling and Probabilistic performance scenarios

Bimodality
High dispersion

Significance test of the price information

DEFAULTABLE BOND



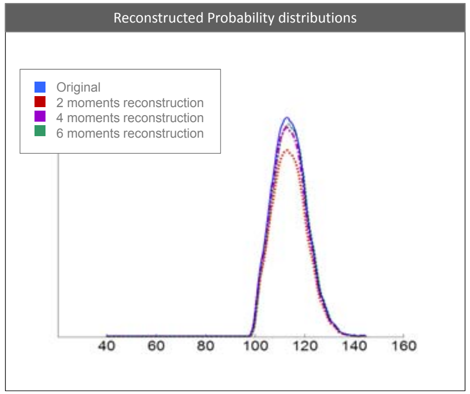
At least 16 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

Unbundling and Probabilistic performance scenarios

Regular symmetry
Low dispersion

Significance test of the price information

LOW-RISK BOND



Only 4 moments are sufficient in order to describe properly the original distribution. The information content of the first moment can be considered adequate.

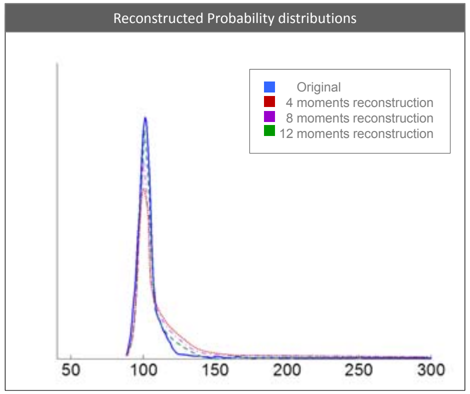
Unbundling and Probabilistic performance scenarios

Asymmetry

kurtosis

Significance test of the price information

VPPI PRODUCT



12 moments describe correctly the pattern of the original distribution. The information content of the first moment needs to be integrated.

Unbundling and Probabilistic performance scenarios

Multimodality

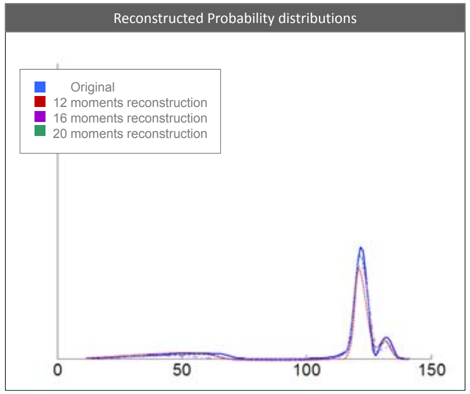
Asymmetry

kurtosis

High dispersion

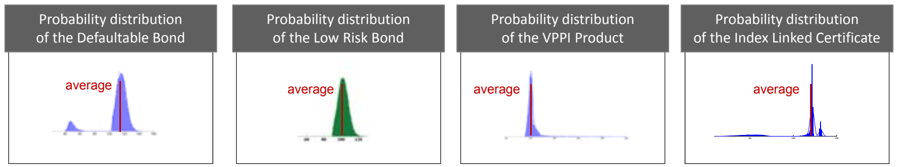
Significance test of the price information

INDEX LINKED CERTIFICATE



At least 20 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

Unbundling and Probabilistic performance scenarios

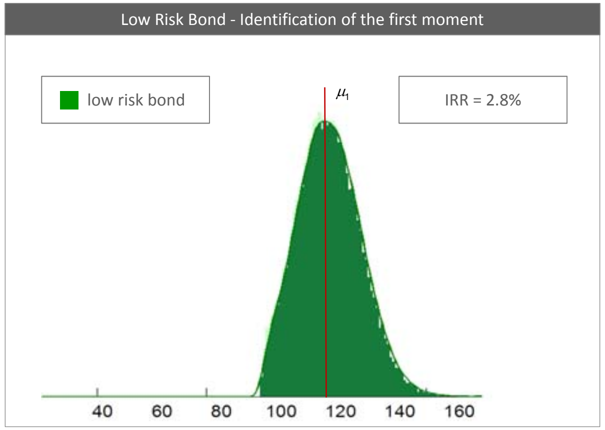


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	Low Risk Bond	VPPI Product	Index Linked Certificate
Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
16 moments needed	4 moments needed	12 moments needed	20 moments needed

From a pure statistical point of view, a proper reconstruction of the original distribution needs at least 4 moments even for the most regular one

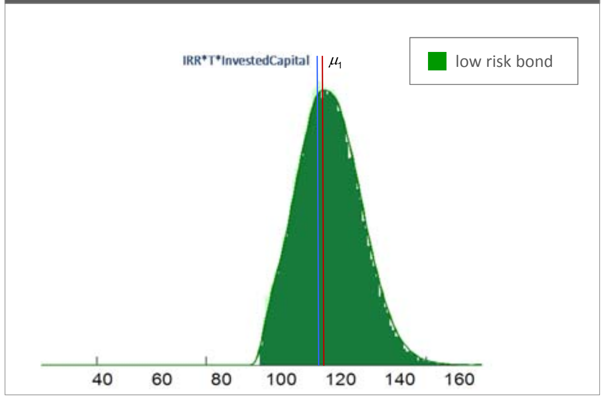
Unbundling and Probabilistic performance scenarios



IRR ⟷ First moment of the probability distribution

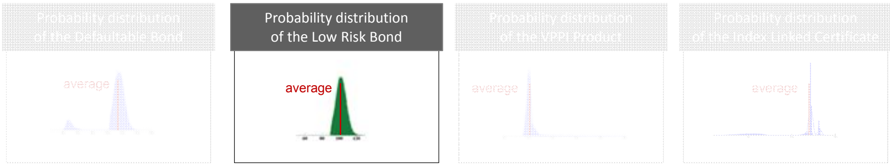
Unbundling and Probabilistic performance scenarios

Low Risk Bond - First moment and Internal Rate of Return



IRR = 2.8% $\mu_1 \approx$ IRR*T*InvestedCapital = 114

Unbundling and Probabilistic performance scenarios



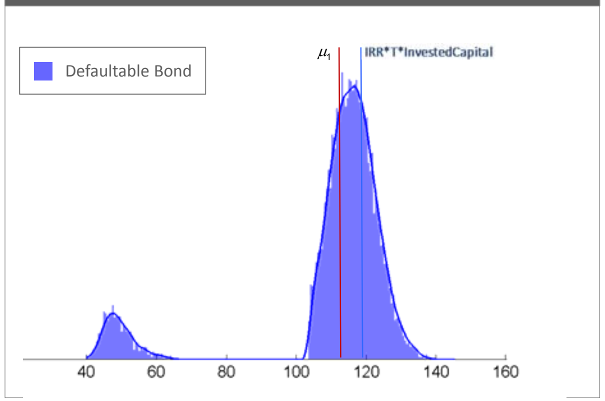
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	Low Risk Bond	VPPI Product	Index Linked Certificate
Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
16 moments needed	Price ≈ Average ≈ IRR	12 moments needed	16 moments needed

Even if 4 moments are needed for a proper reconstruction of the probability distribution, the average and its related measures (IRR and price), convey sufficient information for the investor decision process

Unbundling and Probabilistic performance scenarios

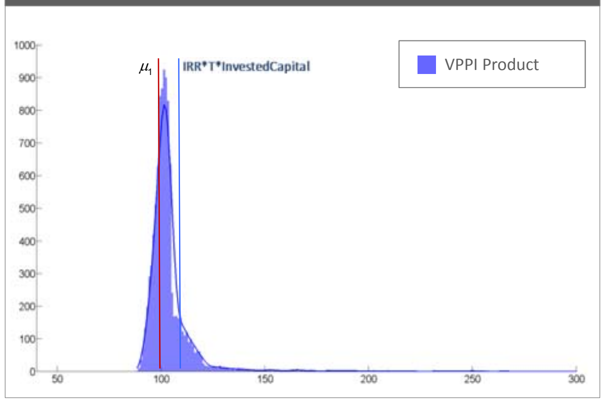
Defaultable Bond - First moment and Internal Rate of Return



IRR = 3.85% $\mu_1 \neq$ IRR*T*InvestedCapital = 119.25

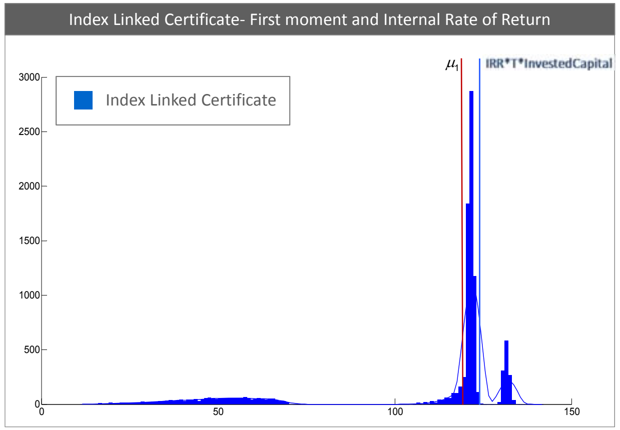
Unbundling and Probabilistic performance scenarios

VPPI Product- First moment and Internal Rate of Return



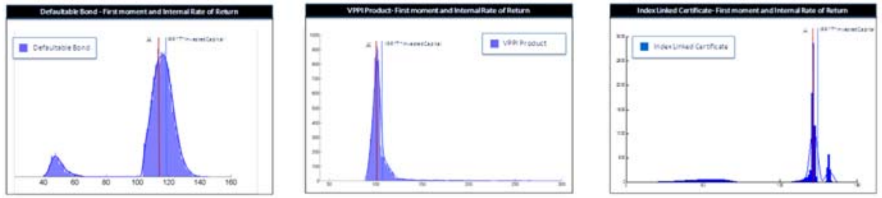
IRR = 2.53% $\mu_1 \neq$ IRR*T*InvestedCapital = 112.65

Unbundling and Probabilistic performance scenarios



IRR = 5.91% $\mu_1 \neq$ IRR*T*InvestedCapital = 129.55

Unbundling and Probabilistic performance scenarios



For more complex financial structures, the average progressively looses its connection with the internal rate of return of the investment, so reducing its usefulness as an effective tool for the decision process

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate

STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	Low Risk Bond	VPPI Product	Index Linked Certificate
Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
Price Average IRR	Price = Average = IRR	Price Average IRR	Price Average IRR

The price and the corresponding average and IRR at expiry date – in presence of IRREGULAR distributions – need to be complemented with additional information related to the shape of the probability distribution

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

- be easy to understand
- capture efficiently all the main statistical characteristics of the probability distribution of the product

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

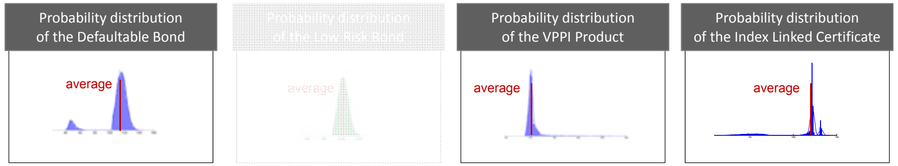
The additional information to be supplemented must

be easy to understand

capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 1: **Convey the entire probability distribution**

Unbundling and Probabilistic performance scenarios



MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

2 Factor Short Interest Rate Hull-White Model

Short Interest Rate Cox Ingersoll Ross Model

Heston Stochastic Volatility Model for the Equity component

Barndorff Nielsen Normal Inverse Gaussian Model for the Equity component

Merton Jump Diffusion Model for the Equity component

Variance Gamma Model for the Equity component

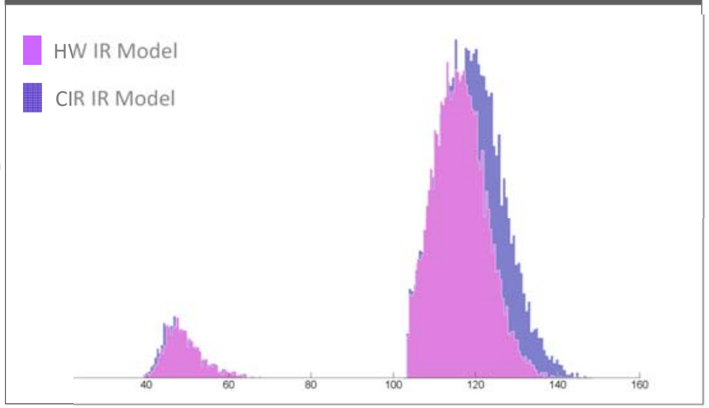
The shape of the probability distribution of the potential returns is obviously dependent on the modelling assumptions.

Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distributions of the final values of the defaultable bond

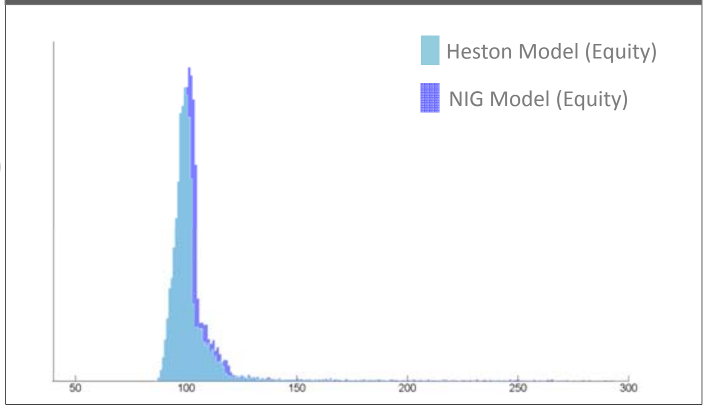


Unbundling and Probabilistic performance scenarios

VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

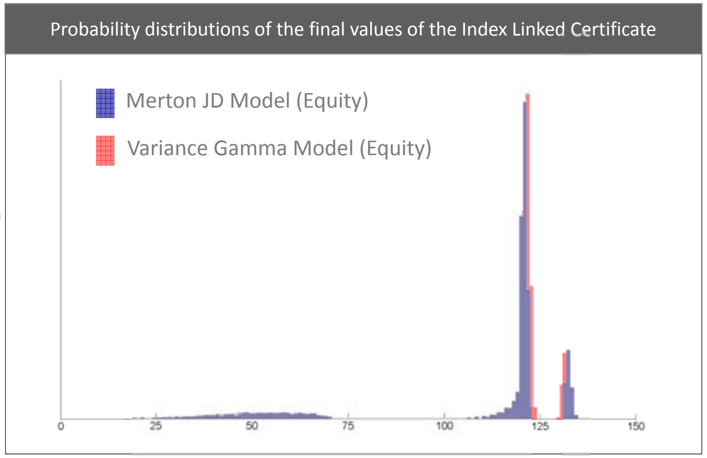
Probability distributions of the final values of the VPPI product



Unbundling and Probabilistic performance scenarios

INDEX LINKED CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

- be easy to understand ✗
  - the probability distribution is an abstract object not easy to handle
- capture efficiently all the main statistical characteristics of the probability distribution of the product ✗
  - the shape of the probability distribution is dependent on the modelling assumptions

Proposal 1: Convey the entire probability distribution

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

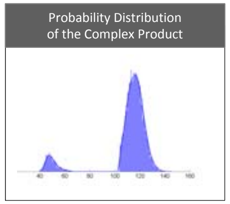
- be easy to understand
- capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 2: Unbundling the information content of the price

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Unbundling the information content of the price

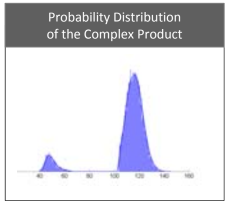


DISCOUNTED EXPECTED VALUE → **Fair Value** (Complex Product)

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Unbundling the information content of the price

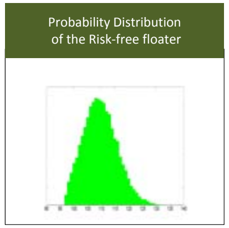


DISCOUNTED EXPECTED VALUE

Fair Value
(Complex Product)

A risk-free floater with same fair value and coupon payment dates of the complex product is defined

=



DISCOUNTED EXPECTED VALUE

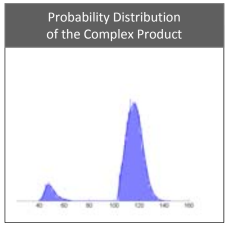
Fair Value
(Risk-free floater)

Unbundling and Probabilistic performance scenarios

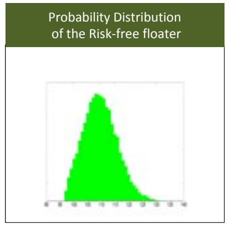
COMPLEX PRODUCT

Unbundling the information content of the price

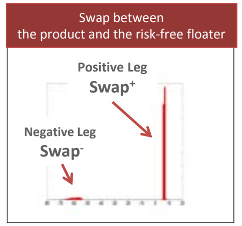
Any non-elementary return-target product can be replicated by a portfolio composed of the associated risk-free floater and of a zero-value swap which transforms the cash flow structure of the risk-free security into the cash flow structure of the product itself, ie, denoting by $\{swap_t\}_{t\in[0,T]}$ the value process of the swap



=



+



Fair Value
(Complex Product)

Fair Value
(Risk-free floater)

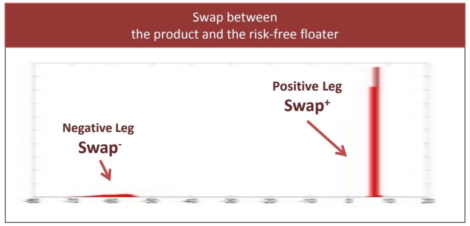
Fair Value
(Swap = 0)

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Unbundling the information content of the price

Fair Value
(Swap = 0)



$|FV(Swap^-)| = |FV(Swap^+)|$

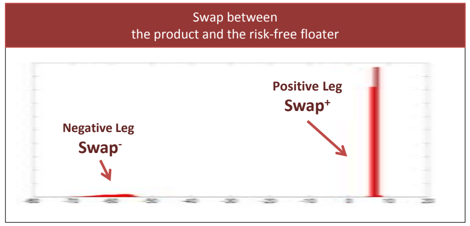
Theoretical value of the Risky component

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Unbundling the information content of the price

Fair Value
(Swap = 0)



$|FV(Swap^-)| = |FV(Swap^+)|$

Theoretical value of the Risky component

C	*Fair value*
B	Theoretical value of the Risky component
A=C-B	Theoretical value of the Risk-Free component

Financial investment table
(*Price Unbundling*)

DEFAULTABLE BOND

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	*Fair value*	96.3
D	Costs	3.7
E = C + D	Issue price	100

VPPI PRODUCT

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	*Fair value*	96.5
D	Costs	3.5
E = C + D	Issue price	100

INDEX LINKED CERTIFICATE

A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	*Fair value*	96.1
D	Costs	3.9
E = C + D	Issue price	100

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand

capture efficiently all the main statistical characteristics of the probability distribution of the product

the unbundling represented by using a table is first level tool useful to appreciate the impact of the costs and the riskiness of the product

The unbundling exploits only the information contained in the first order moment of the probability distribution

Proposal 2: Unbundling the information content of the price

COMPLEX PRODUCT

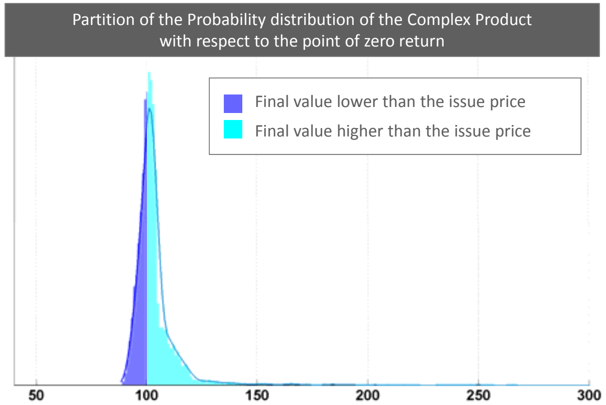
The additional information to be supplemented must

be easy to understand

capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 3: **Perform a reduction in granularity by implementing a partition of the probability distribution**

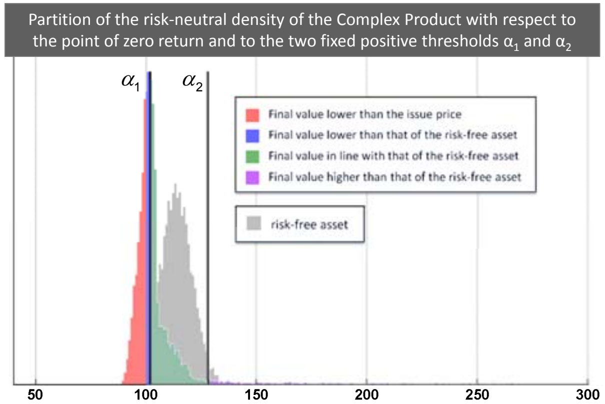
COMPLEX PRODUCT



The assessment of the probability of recovering at least the amount paid for the product is of great significance for the investor.

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



It is appropriate to explore further partitions of the macro-event “the final value of the investment is higher than the issue price” by performing a direct comparison with the final values of the risk-free asset.

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

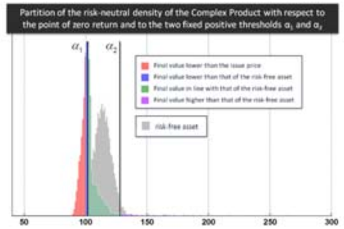


Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	…	…
The performance is positive but lower than the risk-free asset	…	…
The performance is positive and in line with the risk-free asset	…	…
The performance is positive and higher than the risk-free asset	…	…

MEAN VALUES

$$E^{P}(S_T \mid S_T < 100) = \frac{1}{P(S_T < 100)} \int_{-\infty}^{100} x f_{S_T}(x)\,dx$$

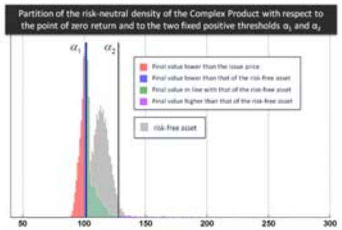
$$E^{P}(S_T \mid 100 \leqslant S_T < \alpha_1) = \frac{1}{P(100 \leqslant S_T < \alpha_1)} \int_{100}^{\alpha_1} x f_{S_T}(x)\,dx$$

$$E^{P}(S_T \mid \alpha_1 \leqslant S_T < \alpha_2) = \frac{1}{P(\alpha_1 \leqslant S_T < \alpha_2)} \int_{\alpha_1}^{\alpha_2} x f_{S_T}(x)\,dx$$

$$E^{P}(S_T \mid S_T \geqslant \alpha_2) = \frac{1}{P(S_T \geqslant \alpha_2)} \int_{\alpha_2}^{+\infty} x f_{S_T}(x)\,dx$$

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

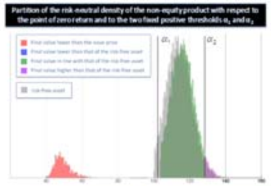


Benefits of this solution:

1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information and the table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;

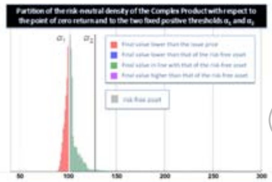
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND



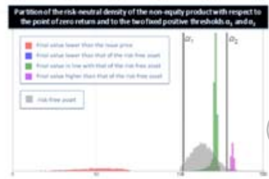
SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

VPPI PRODUCT



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

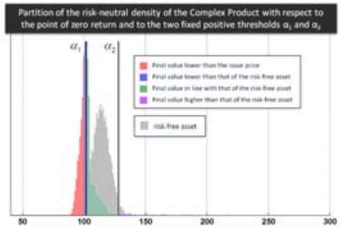
INDEX LINKED CERTIFICATE



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Benefits of this solution:

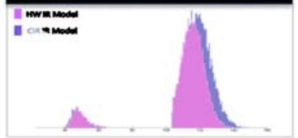
1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information; The table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;
2. The model risk arising from the different proprietary models of the issuers has a limited impact.

Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the Defaultable Bond



Difference less than 2%

HW IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

CIR IR MODEL

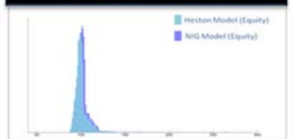
SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	8.3%	49.9
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	86.8%	117.9
The performance is positive and higher than the risk-free asset	4.9%	135.4

Unbundling and Probabilistic performance scenarios

VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the VPPI Product



Difference less than 2%

HESTON MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	38.9%	95.5
The performance is positive but lower than the risk-free asset	18.9%	100.2
The performance is positive and in line with the risk-free asset	38.4%	106.3
The performance is positive and higher than the risk-free asset	3.8%	182.5

NIG MODEL

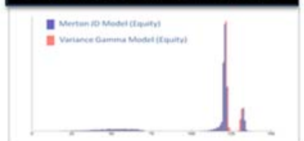
SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

Unbundling and Probabilistic performance scenarios

INDEX LINKED CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the Index Linked Certificate



Difference less than 4%

MERTON JD MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	48.2
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	65.8%	117.6
The performance is positive and higher than the risk-free asset	15.3%	132.7

VARIANCE GAMMA MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

✓ be easy to understand

✓ capture efficiently all the main statistical characteristics of the probability distribution of the product

the partition should be done by choosing events that have a strong financial meaning

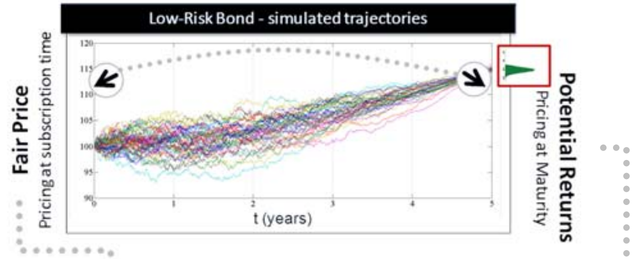
the reduction in granularity mitigates in a significant way the model risk

Proposal 3: **Perform a reduction in granularity by implementing a partition of the probability distribution**

Unbundling and Probabilistic performance scenarios

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

COMPLEX PRODUCT



Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	
B	Theoretical value of the Risky component	
C = A + B	*Fair value*	
D	Costs	
E = C + D	Issue price	

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	...	...
The performance is positive but lower than the risk-free asset	...	...
The performance is positive and in line with the risk-free asset	...	...
The performance is positive and higher than the risk-free asset	...	...

Unbundling and Probabilistic performance scenarios

This approach allows to explain the "CONVERTIBILITY RISK" that actually affects the pricing of European sovereign bond.

Market quotes the event of the breaking of the Eurozone.

PIIGS countries suffer
DEVALUATION RISK

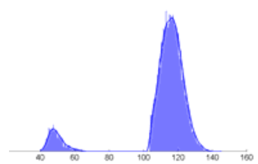
CORE countries take advantage of
APPRECIATION RISK

Unbundling and Probabilistic performance scenarios

This approach allows to explain the "CONVERTIBILITY RISK" that actually affects the pricing of European sovereign bond.

Market quotes the event of the breaking of the Eurozone.

PIIGS countries suffer
DEVALUATION RISK



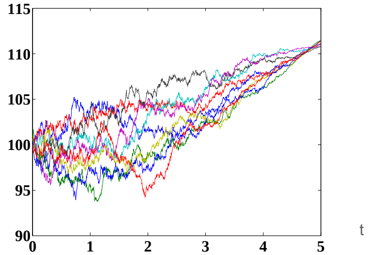
CORE countries take advantage of
APPRECIATION RISK

Syllabus

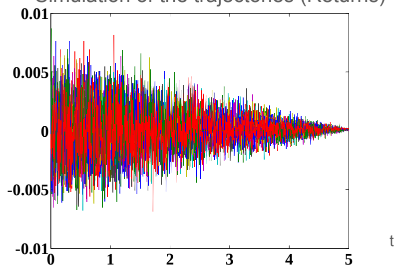
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon

Synthetic risk indicator

Simulation of the trajectories (Price)

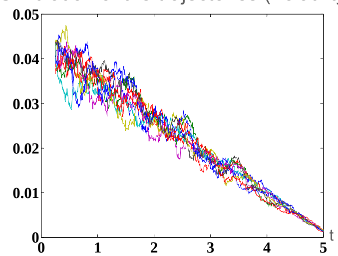


Simulation of the trajectories (Returns)



Non-*equity product*: Fixed bond like

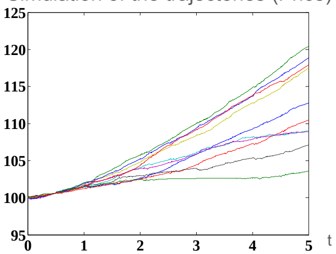
Simulation of the trajectories (Volatility)



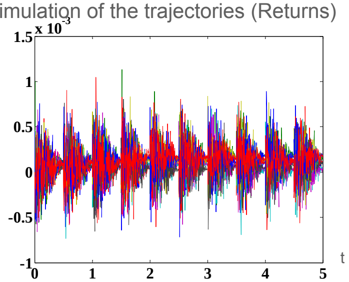
Degree of risk

Synthetic risk indicator

Simulation of the trajectories (Price)

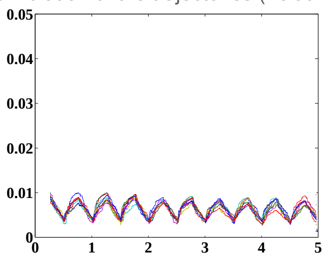


Simulation of the trajectories (Returns)



Non-*equity product*: Floater bond like

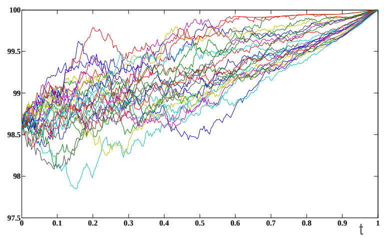
Simulation of the trajectories (Volatility)



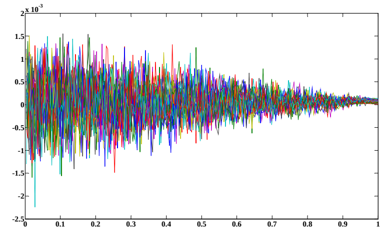
Degree of risk

Synthetic risk indicator

Simulation of the trajectories (Price)

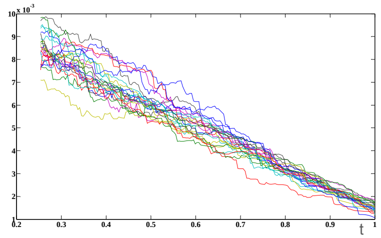


Simulation of the trajectories (Returns)



Non-*equity product*: Zero Coupon Bond

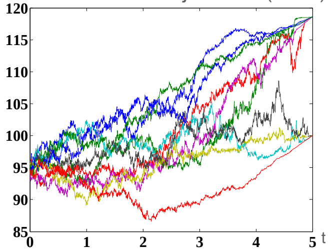
Simulation of the trajectories (Volatility)



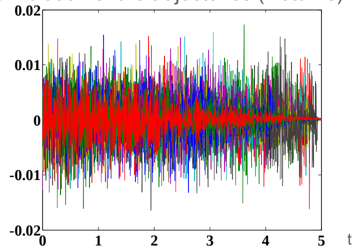
Degree of risk

Synthetic risk indicator

Simulation of the trajectories (Price)

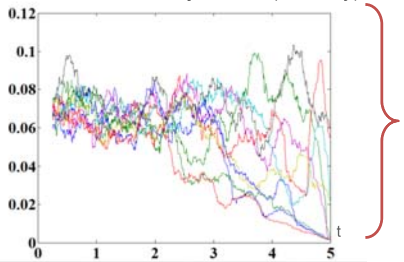


Simulation of the trajectories (Returns)



Simulation of the trajectories (Volatility)

Non-*equity product*: Structured Bond

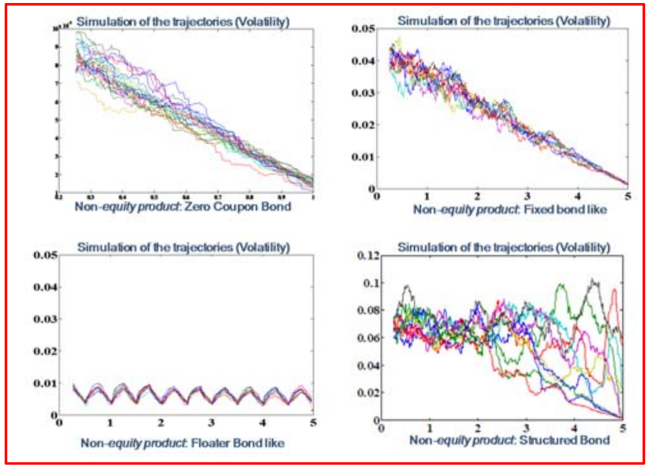


Degree of risk

Synthetic risk indicator

COMPLEXITY FOR RETAIL INVESTORS

The volatility patterns are abstract objects that an average investor cannot handle.



Synthetic risk indicator

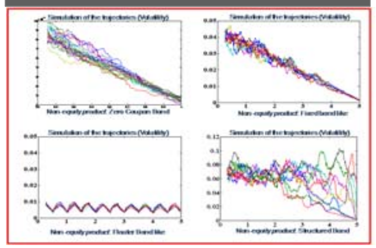
Conversely, a table with qualitative labels that characterizes the risk classes is very easy to understand

Risk Classes
Very Low
Low
Medium-Low
Medium
Medium-High
High
Very High

The assignment of the degree of risk is made according to a quantitative criterion that maps coherently any volatility interval into a corresponding qualitative risk class

Synthetic risk indicator

Volatility of the product's potential returns



DEGREE OF RISK

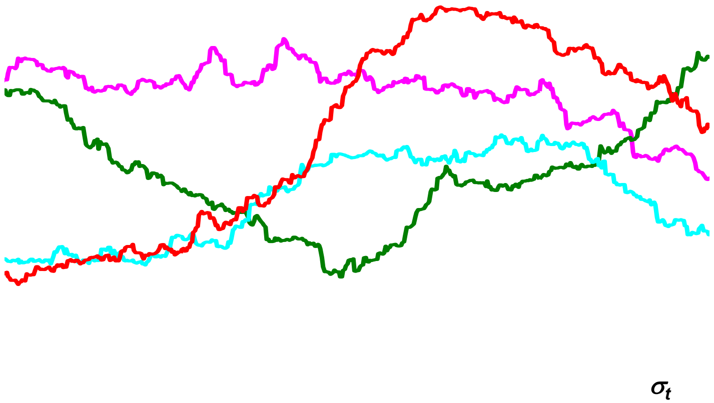
MEASUREMENT: product's positioning inside a grid of *n* volatility intervals

REPRESENTATION: mapping of any volatility interval into a corresponding qualitative risk class

Risk Classes	Volatility Intervals	
Very Low	$\sigma_{1,min}$	$\sigma_{1,max}$
Low	$\sigma_{2,min}$	$\sigma_{2,max}$
Medium-Low	$\sigma_{3,min}$	$\sigma_{3,max}$
Medium	$\sigma_{4,min}$	$\sigma_{4,max}$
Medium-High	$\sigma_{5,min}$	$\sigma_{5,max}$
High	$\sigma_{6,min}$	$\sigma_{6,max}$
Very High	$\sigma_{7,min}$	$\sigma_{7,max}$

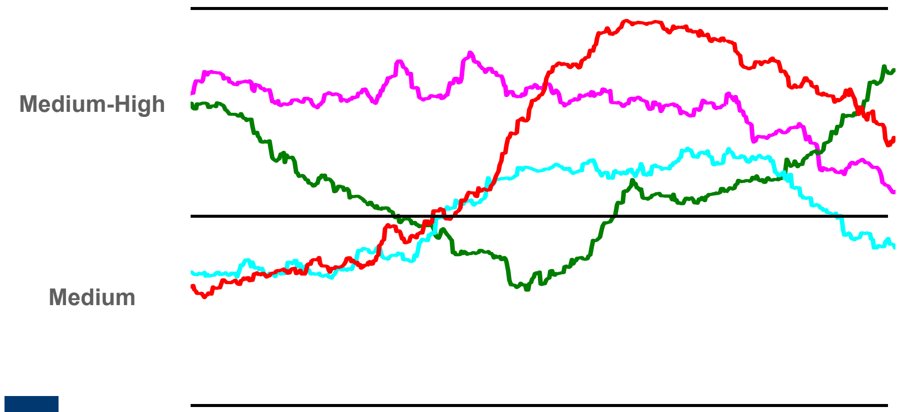
Products with the same risk budget

must have the same degree of risk



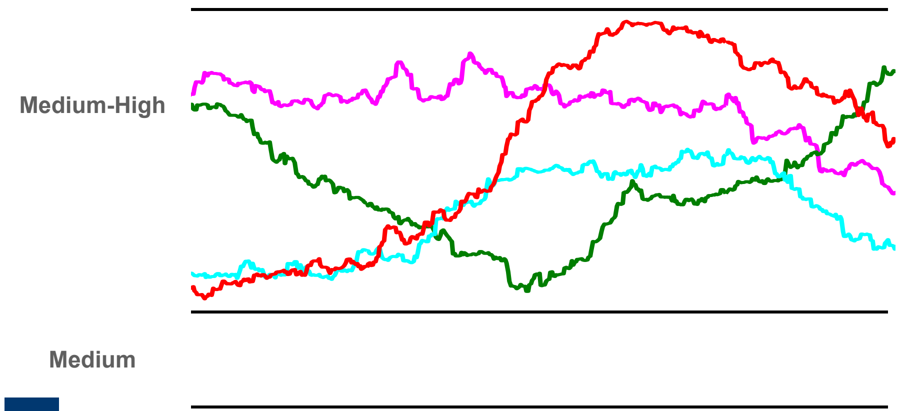
Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations



Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations



Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations

THE ISSUE

Defining suitable requirements to partition the volatility space $[0,+\infty)$ into an optimal number n^* of subsequent intervals with optima extrema

Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations

Requirement n.1

the **optimal grid** of volatility intervals
has to be **consistent** with the **principle**:

+ RISK + LOSSES

VOLATILITY INTERVALS MUST HAVE AN INCREASING WIDTH IN ABSOLUTE TERMS

Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations

Requirement n.2

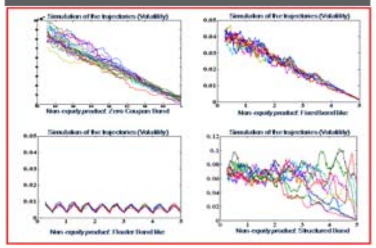
the optimal grid of volatility intervals must be

market feasible

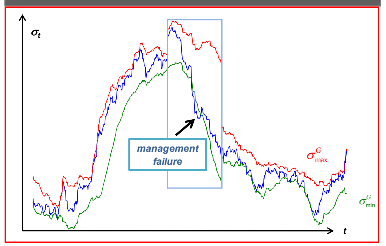
REALIZED VOLATILITY CONSISTENT WITH MARKET EXPECTATIONS OF FUTURE VOLATILITY

(UNLESS FOR SIGNIFICANT SUDDEN SHOCKS)

Volatility of the product's potential returns



Diffusive GARCH



DEGREE OF RISK

Risk Classes	Volatility Intervals	
Very Low	$\sigma_{1,min}$	$\sigma_{1,max}$
Low	$\sigma_{2,min}$	$\sigma_{2,max}$
Medium-Low	$\sigma_{3,min}$	$\sigma_{3,max}$
Medium	$\sigma_{4,min}$	$\sigma_{4,max}$
Medium-High	$\sigma_{5,min}$	$\sigma_{5,max}$
High	$\sigma_{6,min}$	$\sigma_{6,max}$
Very High	$\sigma_{7,min}$	$\sigma_{7,max}$

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

The **optimal grid** of volatility intervals is **consistent** with the **1st requirement**:

+ RISK + LOSSES

Syllabus

- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon

The recommended investment time horizon

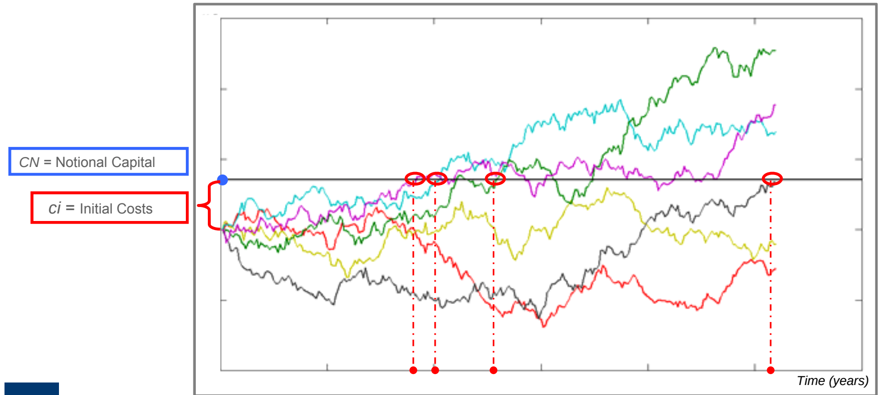
The event to study from a probabilistic point of view is related to possible exit strategies after having recovered all the costs of the product :

The investment recovers the initial costs and off-sets the running costs at least once

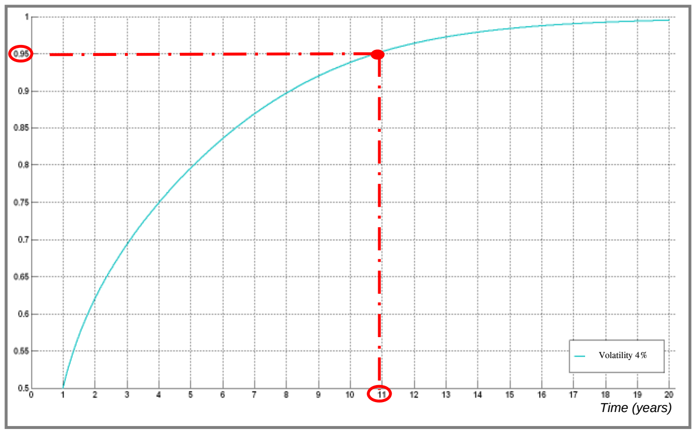
that can be calculated through the concept of

First Passage Time for the cost recovery barrier

First Passage Time:
First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.

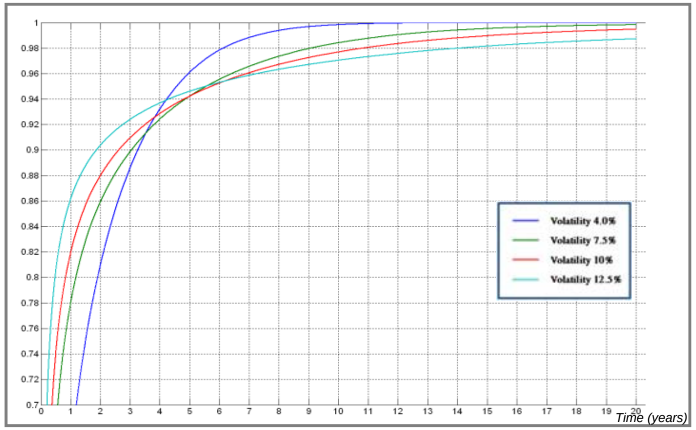


The confidence level α uniquely identifies T^* on the cumulative distribution function of the first passage times:



Recommended Investment Time Horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



Recommended Investment Time Horizon

minimum investment time horizon …

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \leq T\right] = \alpha \right\}$$

…. must be coherent with the principle

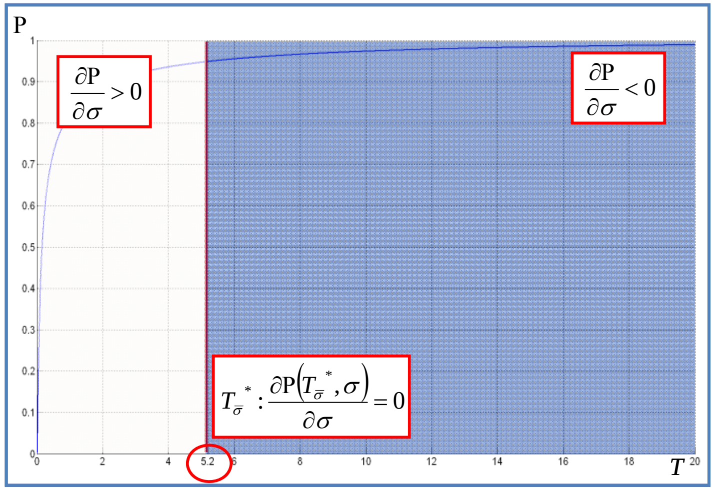
+ VOLATILITY + TIME HORIZON

$$\exists T^* \in [0, \infty[: \frac{d\mathrm{P}}{d\sigma} = 0$$

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

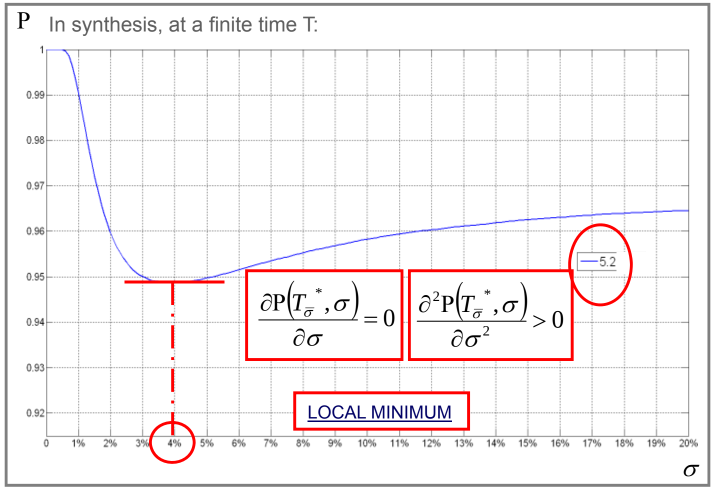
Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



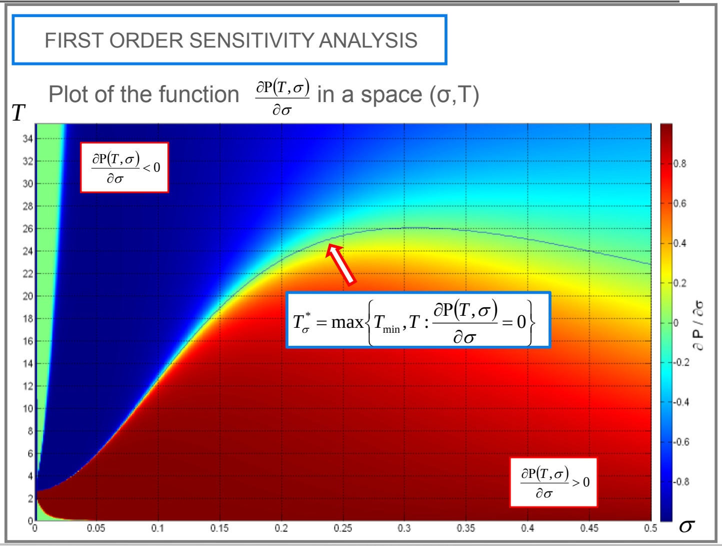
Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



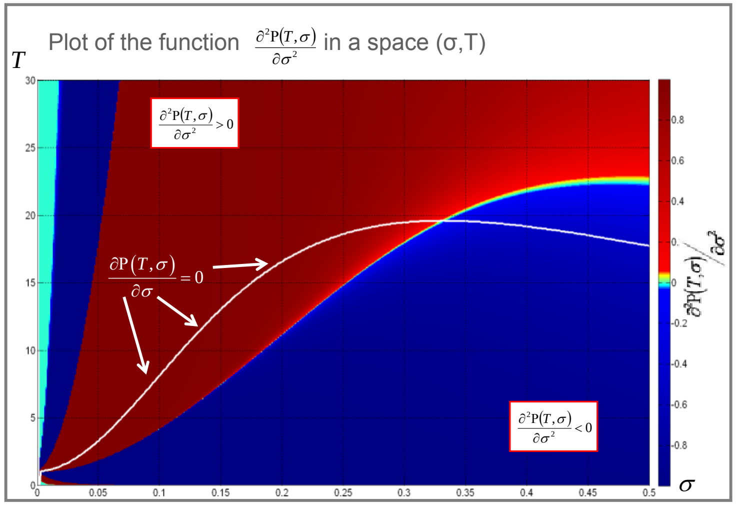
Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



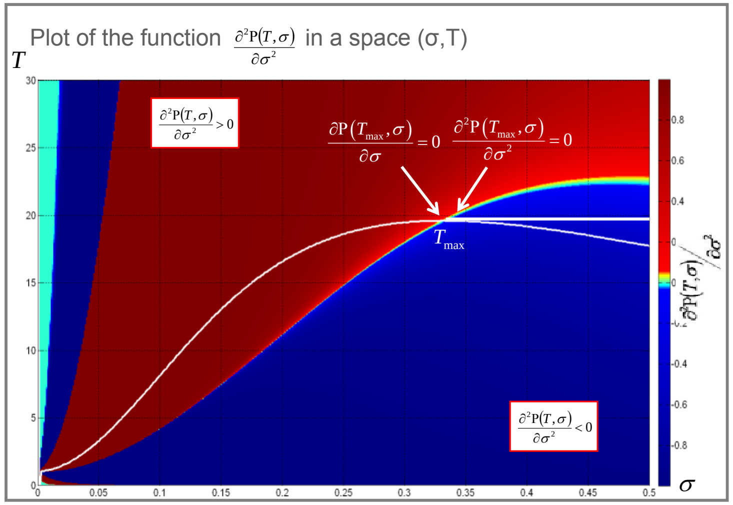
Recommended Investment Time Horizon

Searching the minimum: **the 2nd order condition**



Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



Examples

DEFAULTABLE BOND

DESCRIPTION: Senior bond with a 5 year maturity, paying bi-annual step-up coupons ranging from 4.7% to 5.30%.

1st PILLAR:

Financial investment table
(Price Unbundling)

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	*Fair value*	96.3
D	Costs	3.7
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

2nd PILLAR: Degree of Risk: Medium-High

3rd PILLAR: Recommended investment time horizon: 5 years

Examples

VPPI PRODUCT

DESCRIPTION: VPPI technique is aimed at protecting the initial value of the financial investment over a specified time horizon and obtaining possible gains by limited exposure to the equity markets.

1st PILLAR

Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	*Fair value*	96.5
D	Costs	3.5
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

2nd PILLAR: Degree of Risk: Medium

3rd PILLAR: Recommended investment time horizon: 5 years

Examples

INDEX LINKED CERTIFICATE

DESCRIPTION: The index-linked certificate is characterised by a complex financial engineering that makes intensive use of diverse derivatives components. These derivatives link the performances of the product to the variability of an equity index.

1st PILLAR

Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	*Fair value*	96.1
D	Costs	3.9
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

2nd PILLAR: Degree of Risk: High

3rd PILLAR: Recommended investment time horizon: 5 years