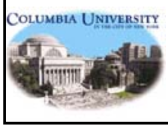
Book Presentation at:

MATHEMATICS OF FINANCE MA PROGRAM
DEPARTMENT OF MATHEMATICS, COLUMBIA UNIVERSITY

A quantitative methodology for risk assessment in financial products

Marcello Minenna



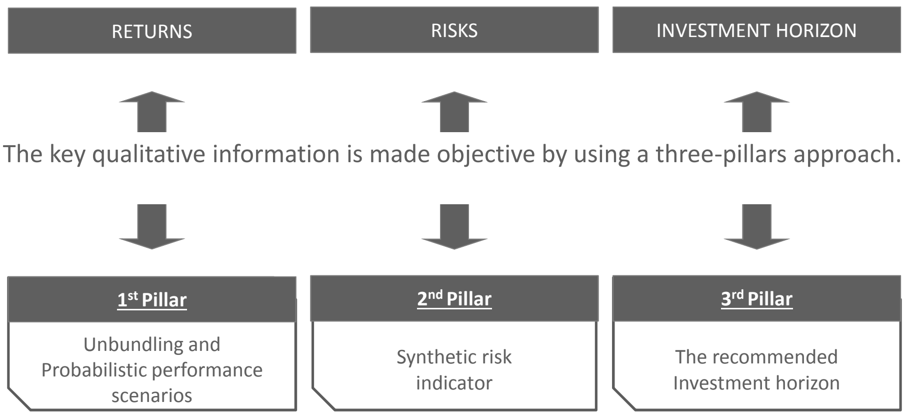
New York, 14th November 2012

Opinions expressed in this work are exclusively of the author

Syllabus

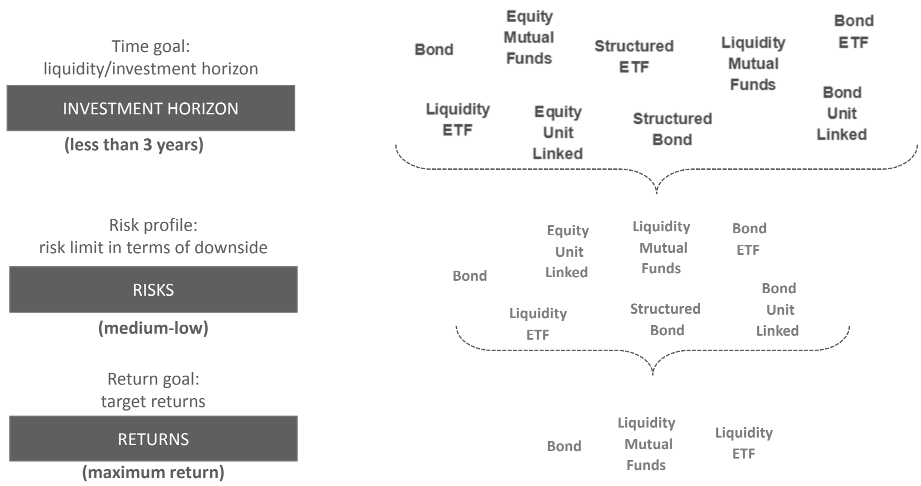
- Preliminaries: the three pillars
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon
- An Application of the methodology

Preliminaries



Preliminaries

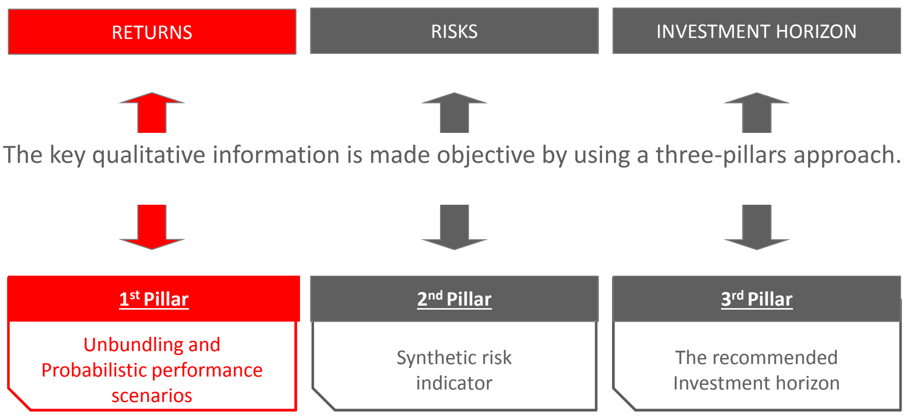
These metrics provide a guide in the interpretation of complex information relating to financial products, by supporting the process of selection by means of a sequential filtering procedure:



Syllabus

- Preliminaries: the three pillars
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon
- An Application of the methodology

Unbundling and Probabilistic performance scenarios



Unbundling and Probabilistic performance scenarios

The returns evaluation requires the estimate of all the relevant risk factors connected with the financial structure of each product

DEFAULTABLE BOND	LOW-RISK BOND	VPPI PRODUCT	INDEX-LINKED CERTIFICATE
Interest Rate Volatility	Interest Rate Volatility	Interest Rate Volatility	Interest Rate Volatility
Significant exposure to credit risk	Limited exposure to credit risk	Limited exposure to Market risk	Significant exposure to Market risk

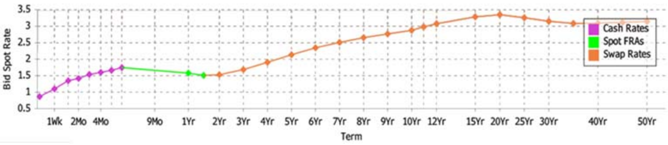
Unbundling and Probabilistic performance scenarios

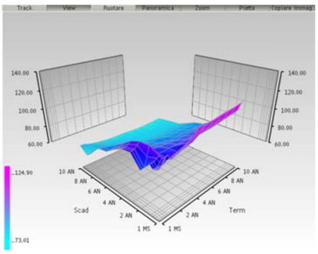
DEFAULTABLE BOND

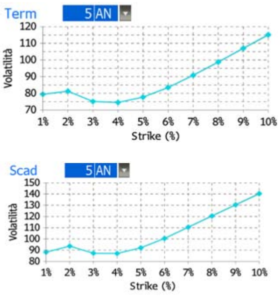
Interest Rate Volatility

Significant exposure to credit risk

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product



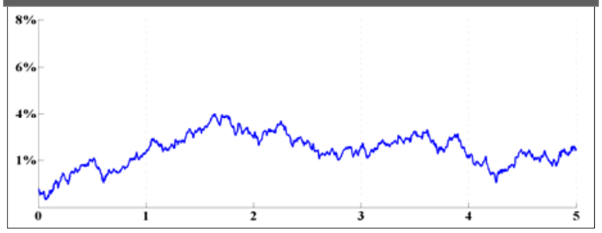




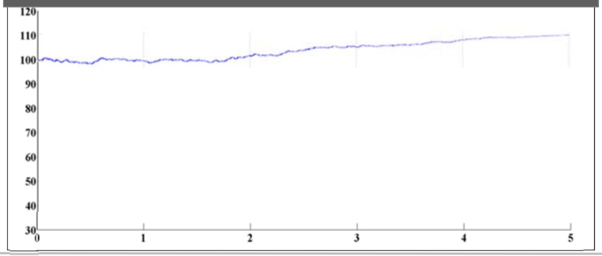
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

3 Months Euribor - simulated trajectories



Defaultable Bond - simulated trajectories



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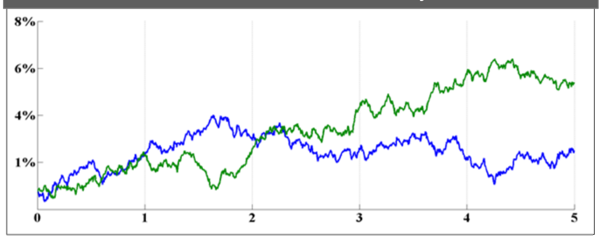
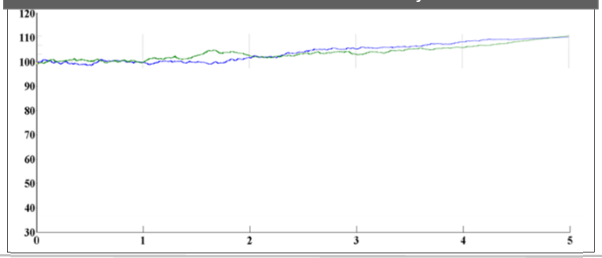
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

3 Months Euribor - simulated trajectories



Defaultable Bond - simulated trajectories



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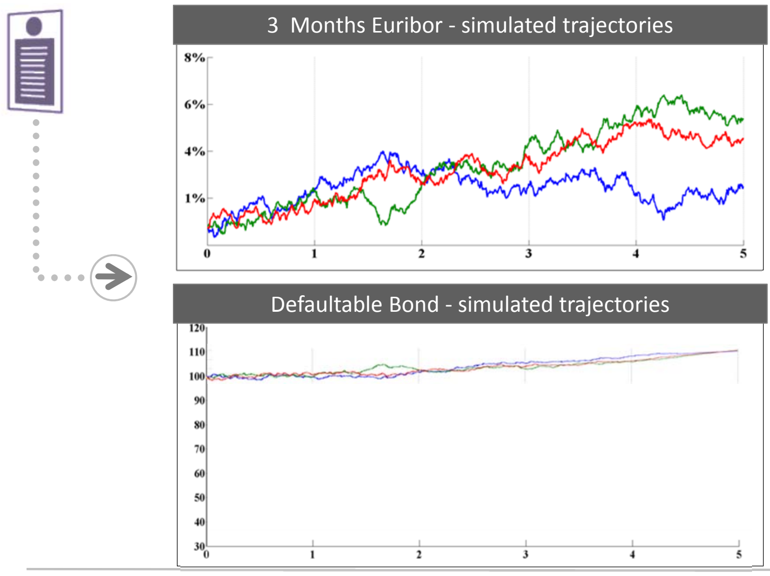
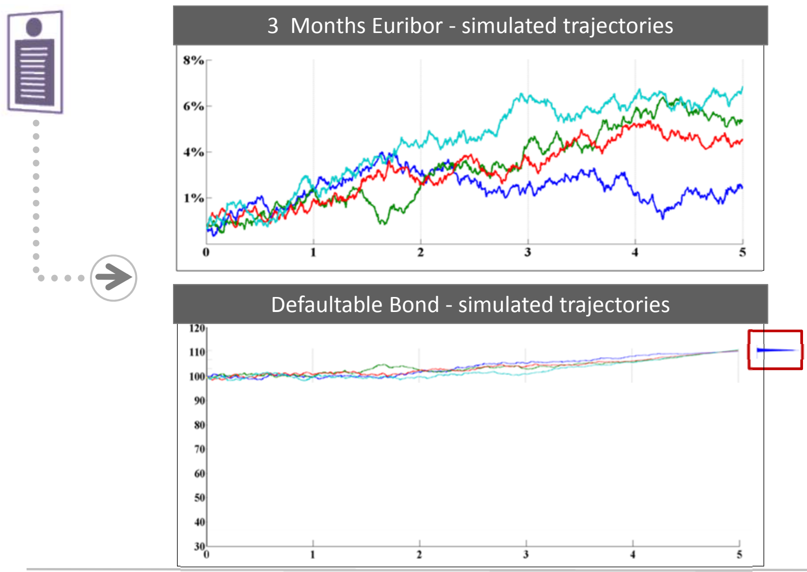
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND



Unbundling and Probabilistic performance scenarios

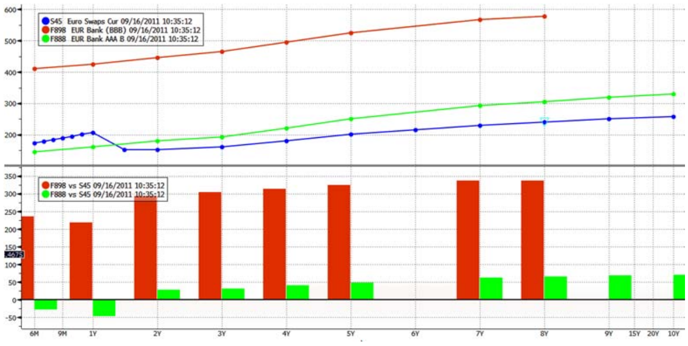
DEFAULTABLE BOND



Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product



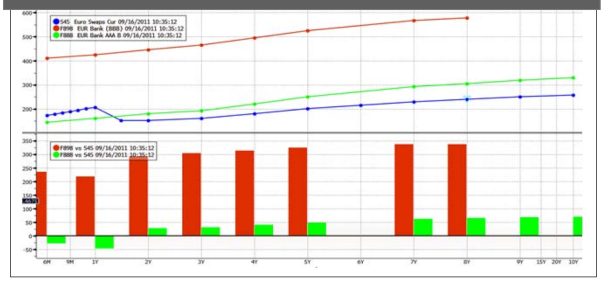
Interest Rate Volatility

Significant exposure to credit risk

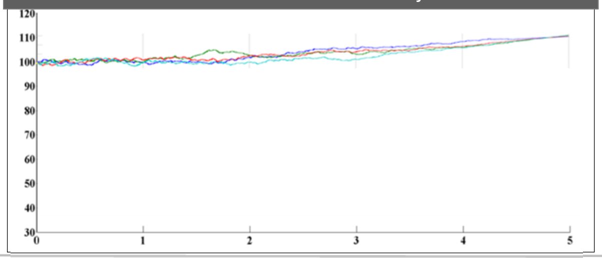
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

Yield curve for different issuers



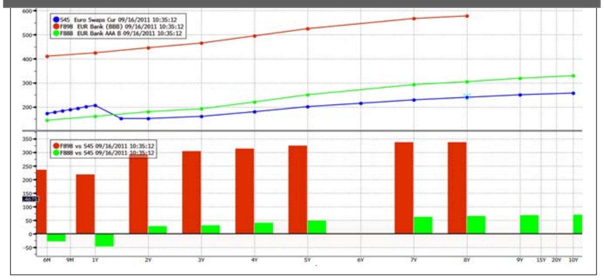
Defaultable Bond - simulated trajectories



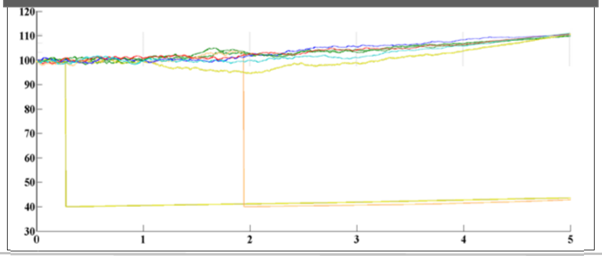
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

Yield curve for different issuers



Defaultable Bond - simulated trajectories



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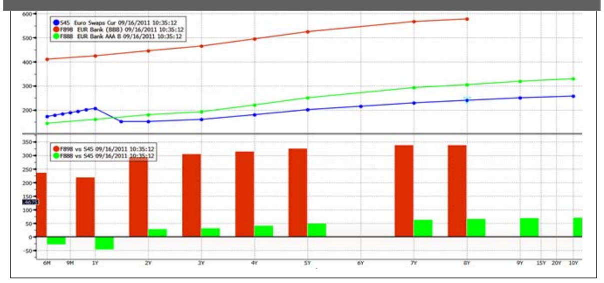
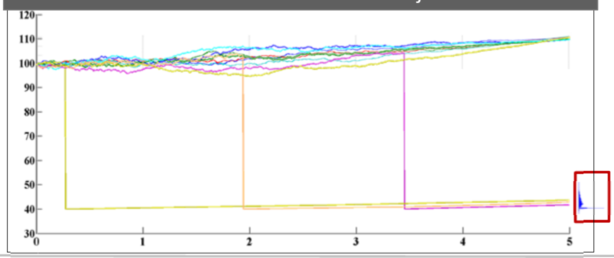
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

Yield curve for different issuers



Defaultable Bond - simulated trajectories



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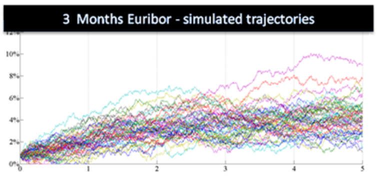
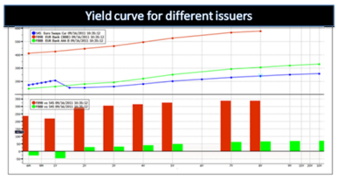
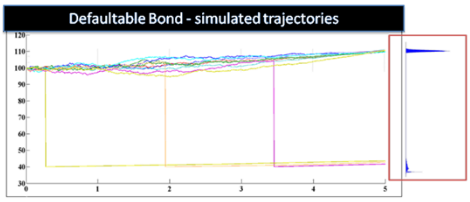
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

The risk factors define the product values over time and at expiry date







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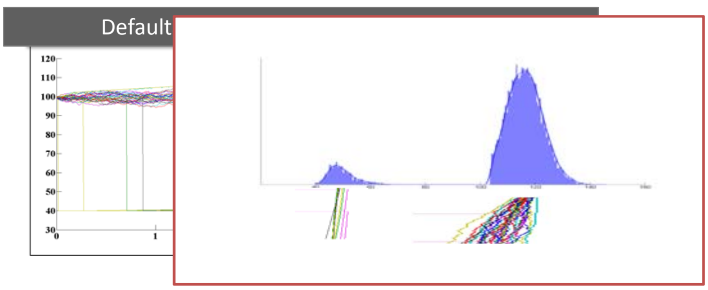
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

The final values of the product provide the probability distribution of the potential returns (so-called *pricing* at maturity)...



Possible Outcomes

Pricing at maturity

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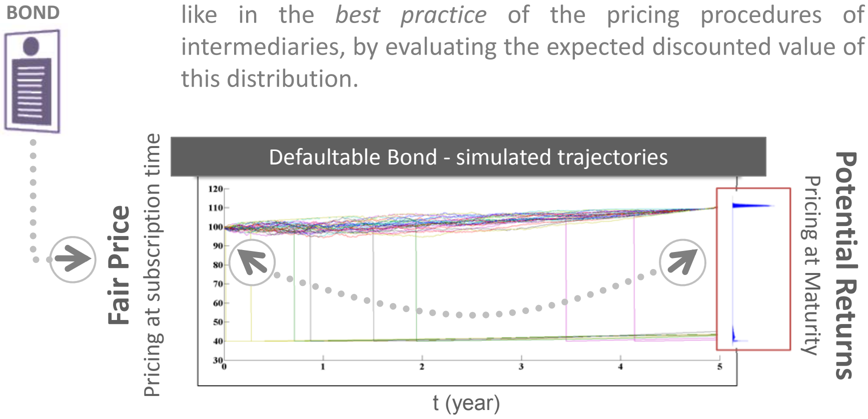
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

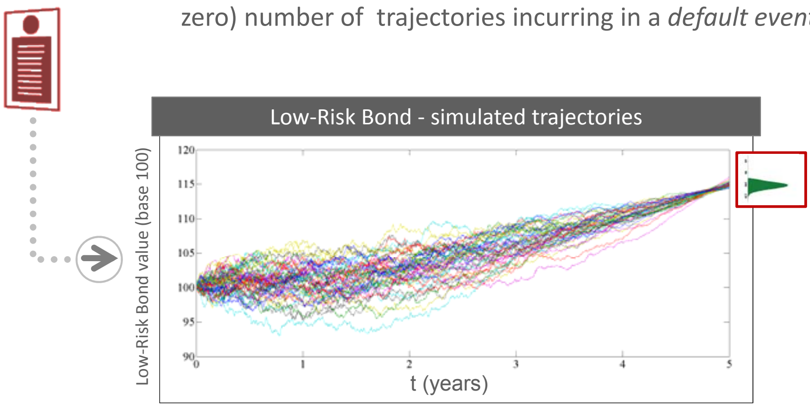
… the "*fair value*" of the product at the issue date is obtained, like in the *best practice* of the pricing procedures of intermediaries, by evaluating the expected discounted value of this distribution.



Unbundling and Probabilistic performance scenarios

LOW-RISK BOND

Limited exposure to credit risk corresponds to a lower (or zero) number of trajectories incurring in a *default event*.



Unbundling and Probabilistic performance scenarios

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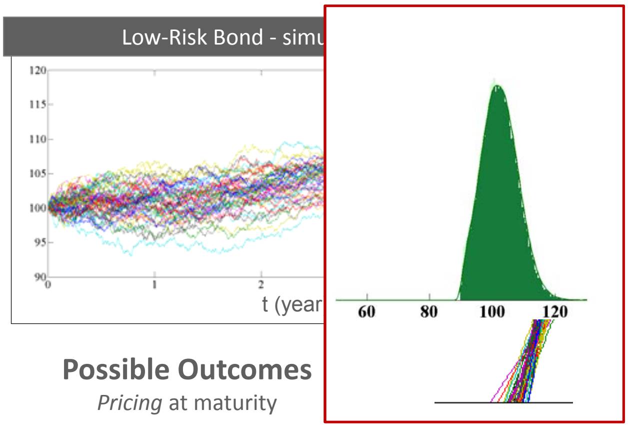


Possible Outcomes

Pricing at maturity

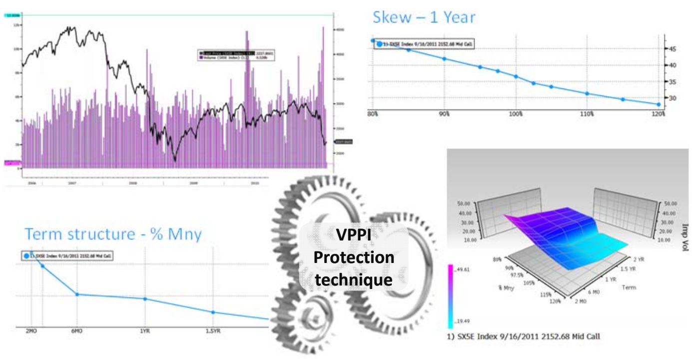
Unbundling and Probabilistic performance scenarios

VPPI PRODUCT

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product

Interest
Rate
Volatility

Limited exposure to Market risk



VPPI technique is aimed at protecting the initial value of the financial investment over a specified time horizon and obtaining possible gains by limited exposure to the equity markets.

Unbundling and Probabilistic performance scenarios

VPPI PRODUCT

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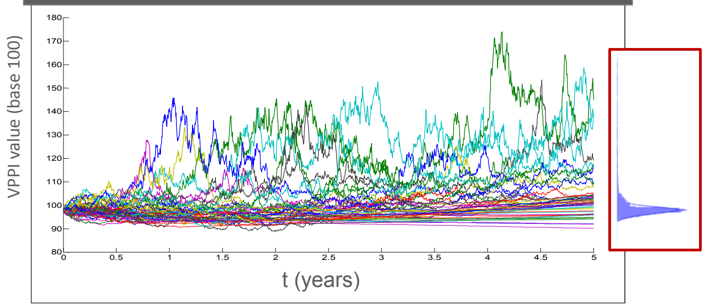
VPPI Product - simulated trajectories



Unbundling and Probabilistic performance scenarios

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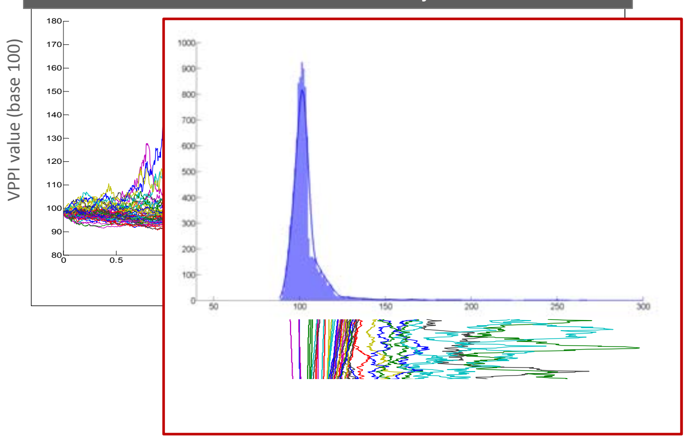
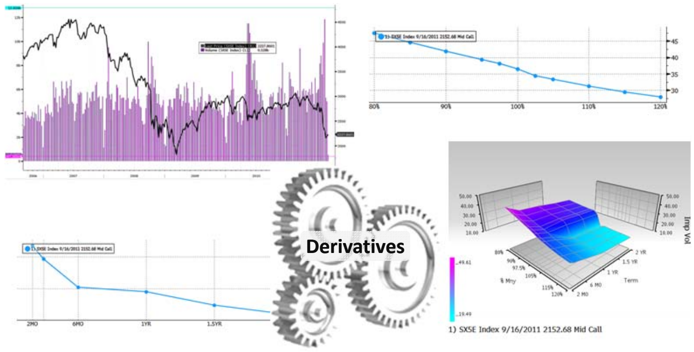


Unbundling and Probabilistic performance scenarios

INDEX LINKED CERTIFICATE

Significant exposure to Market risk

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product



The index-linked certificate is characterised by a complex financial engineering that makes intensive use of different derivatives components. These derivatives link the performances of the product to the variability of an equity index.

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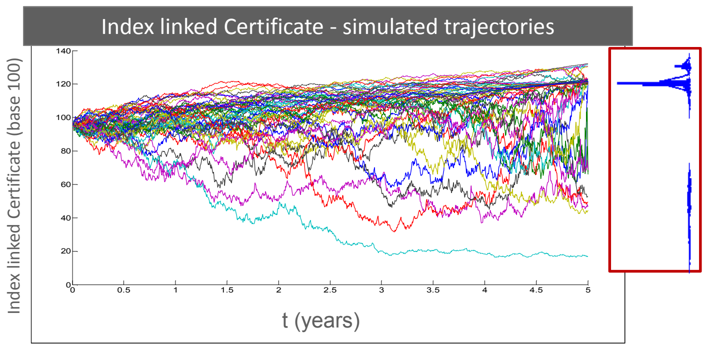
Unbundling and Probabilistic performance scenarios

INDEX LINKED CERTIFICATE

The index-linked certificate is characterised by a complex financial engineering that makes intensive use of different derivatives components. These derivatives link the performances of the product to the variability of an equity index.



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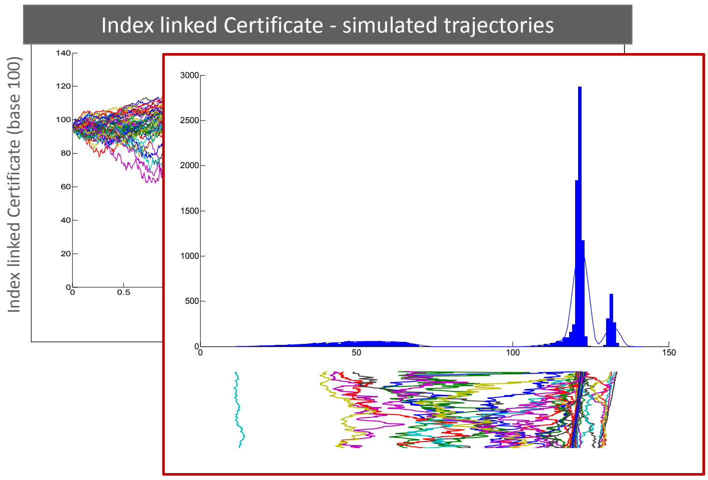
Unbundling and Probabilistic performance scenarios

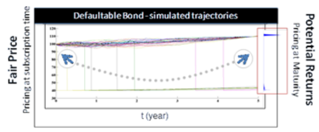
INDEX LINKED CERTIFICATE

The index-linked certificate is characterised by a complex financial engineering that makes intensive use of diverse derivatives components. These derivatives link the performances of the product to the variability of an equity index.

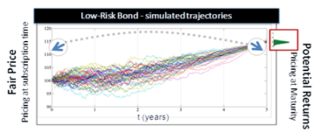


Unbundling and Probabilistic performance scenarios

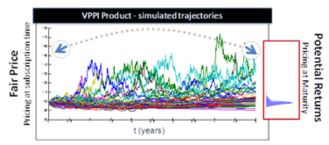
DEFAULTABLE BOND



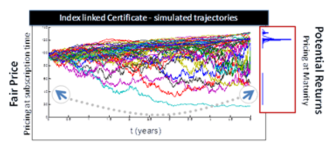
LOW-RISK BOND



VPPI PRODUCT



INDEX LINKED CERTIFICATE



Fair Price
at time zero
is a
Weighted average

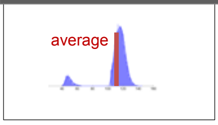
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND	LOW-RISK BOND	VPPI PRODUCT	INDEX LINKED CERTIFICATE

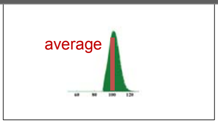
Fair Price at time zero is a **weighted average**

first moment of the probability distribution at expiry date is also a **weighted average**

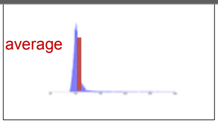
Probability distribution of the Defaultable Bond



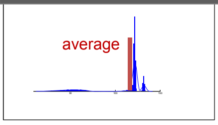
Probability distribution of the Low Risk Bond



Probability distribution of the VPPI Product

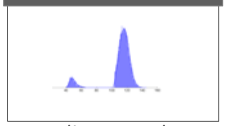


Probability distribution of the Index Linked Certificate



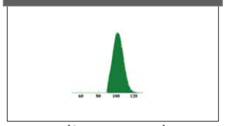
Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond



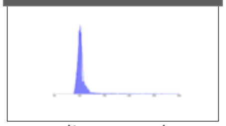
discounted + expected

Probability distribution of the Low Risk Bond



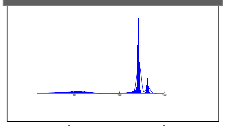
discounted + expected

Probability distribution of the VPPI Product



discounted + expected

Probability distribution of the Index Linked Certificate

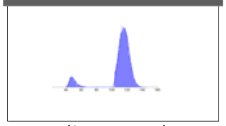
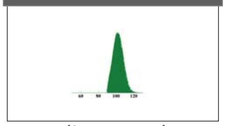
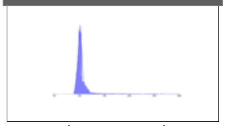
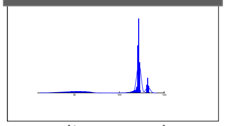


discounted + expected

Defaultable Bond **Fair Price** at time zero = Low Risk Bond **Fair Price** at time zero = VPPI Product **Fair Price** at time zero = Index Linked Certificate **Fair Price** at time zero

Working Hypothesis: The calculated fair price is the same for completely different financial structures

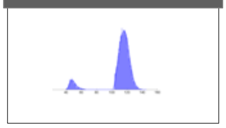
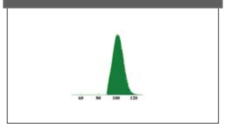
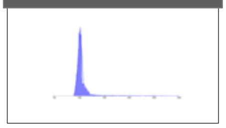
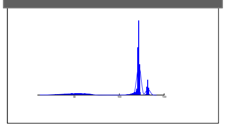
Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate
			
discounted + expected	discounted + expected	discounted + expected	discounted + expected

Defaultable Bond **Fair Price** at time zero = Low Risk Bond **Fair Price** at time zero = VPPI Product **Fair Price** at time zero = Index Linked Certificate **Fair Price** at time zero

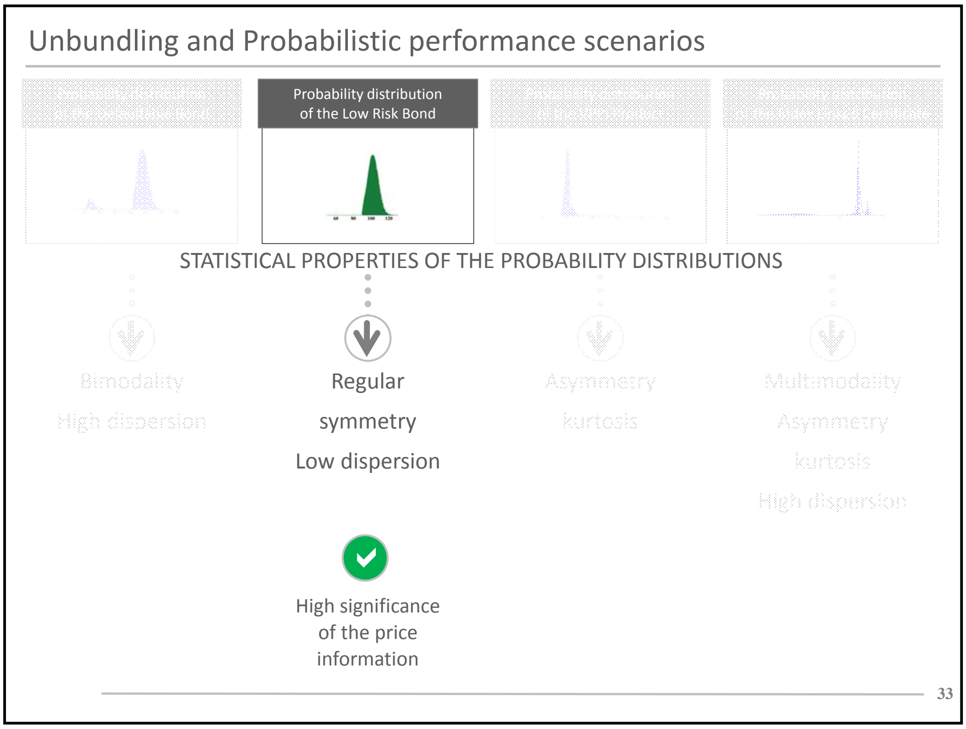
<u>Question</u>: How much information about the original probability distribution the price will convey in each case analyzed?

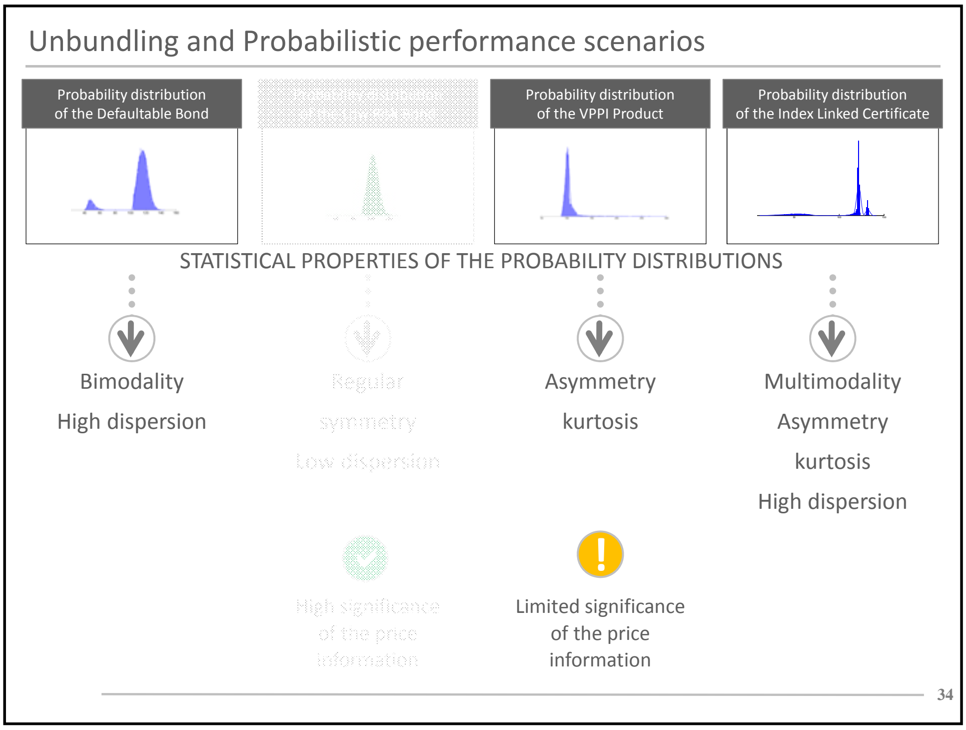
Unbundling and Probabilistic performance scenarios

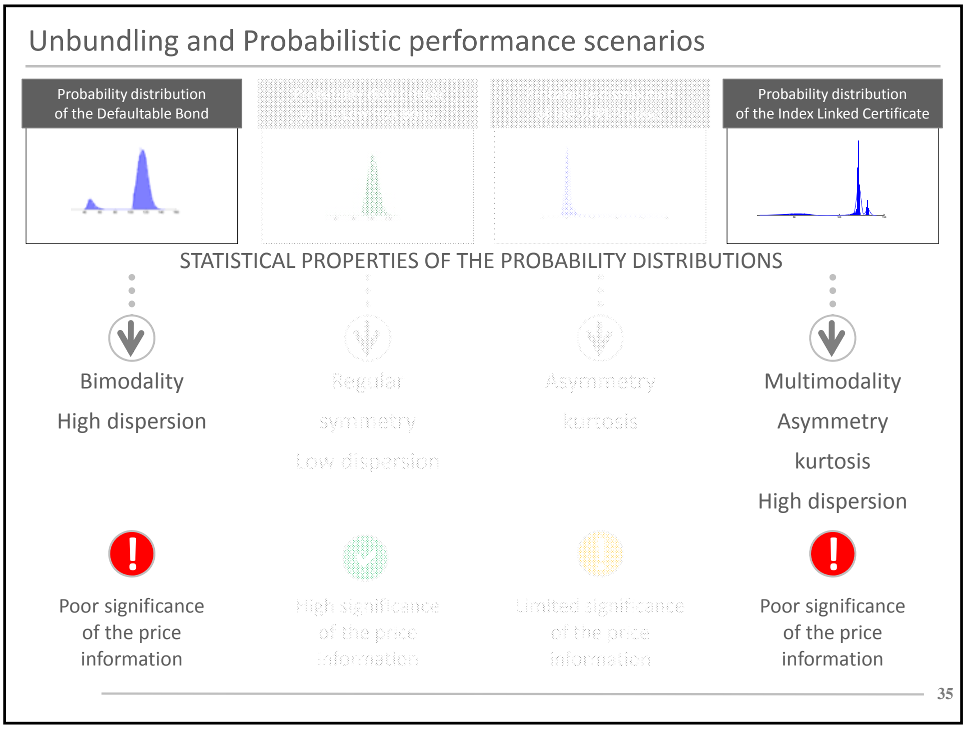
Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate
			

STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	Low Risk Bond	VPPI Product	Index Linked Certificate
Bimodality	Regular	Asymmetry	Multimodality
High dispersion	Symmetry	Kurtosis	Asymmetry
	Low dispersion		Kurtosis
			High dispersion


Unbundling and Probabilistic performance scenarios
Probability distribution
of the Low Risk Bond
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS
Regular
symmetry
Low dispersion
High significance
of the price
information
33


Unbundling and Probabilistic performance scenarios
Probability distribution
of the Defaultable Bond
Probability distribution
of the VPPI Product
Probability distribution
of the Index Linked Certificate
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS
Bimodality
High dispersion
Asymmetry
kurtosis
Multimodality
Asymmetry
kurtosis
High dispersion
Limited significance
of the price
information
34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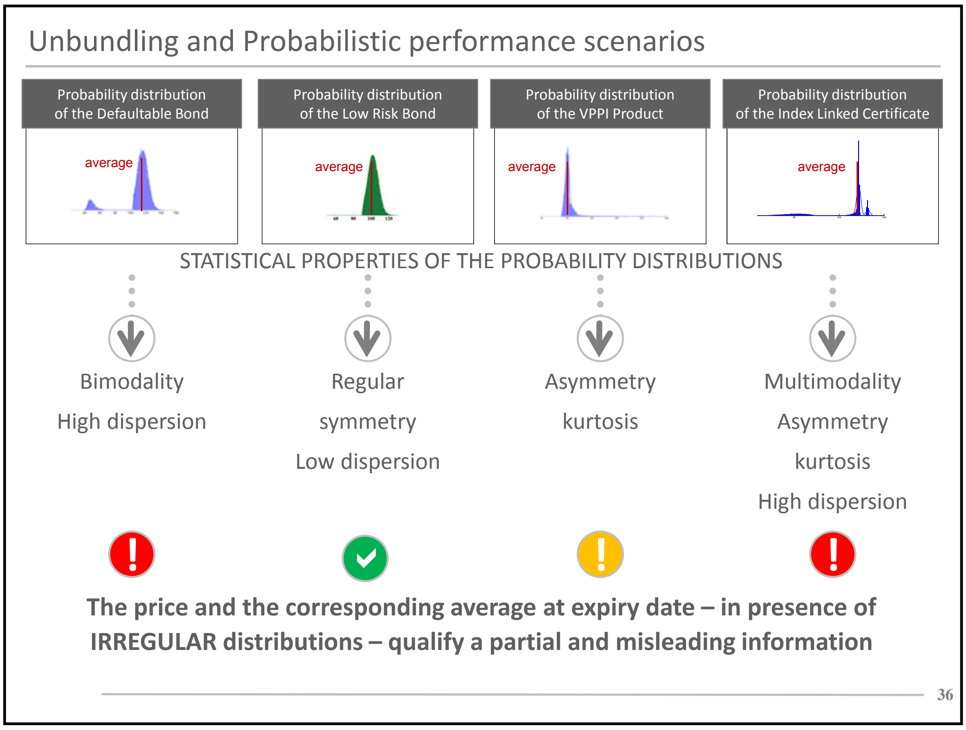
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS
Bimodality
High dispersion
Regular
symmetry
Low dispersion
Asymmetry
kurtosis
Multimodality
Asymmetry
kurtosis
High dispersion
Poor significance of the price information
High significance of the price information
Limited significance of the price information
Poor significance of the price information
35


Unbundling and Probabilistic performance scenarios
Probability distribution of the Defaultable Bond
Probability distribution of the Low Risk Bond
Probability distribution of the VPPI Product
Probability distribution of the Index Linked Certificate
average
average
average
average
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS
Bimodality
High dispersion
Regular
symmetry
Low dispersion
Asymmetry
kurtosis
Multimodality
Asymmetry
kurtosis
High dispersion
The price and the corresponding average at expiry date – in presence of IRREGULAR distributions – qualify a partial and misleading information
36

Unbundling and Probabilistic performance scenarios

Significance of the price information

As a weighted average, the price is strictly connected with the first moment of the probability distribution

As the literature suggests, in presence of multimodality and irregular shapes for the probability distributions, the number of moments necessary to properly describe the probability distribution increases drammatically.

See:

(1) Shohat, Tamarkin, 1943 - American Mathematical Survey
(2) Szego, 1959 - American Mathematical Society
(3) Totik, 2000 – Journal of Analytical Mathematics
(4) Gavriliadis, Athanassoulis, 2009 – Journal of Computational and Applied Mathematics

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Unbundling and Probabilistic performance scenarios

Significance of the price information

Mathematical Basis to test the significance of the price information

Given a finite number of moments 2k, it's possible to derive the following approximate relationship between the probability function f (x) and its Christoffel function of degree k:

$$f(x) \approx f_{AP,k}(x) = \frac{k}{c_0 \pi \sqrt{(x-a)(b-x)}} \lambda_k(x)$$

con $x \in [a,b]$. c_0 è un fattore di normalizzazione.

It' s then immediate to apply the approximating formula for different values of k in order to test the accuracy of the approximation for the probability distributions corresponding to our different financial products

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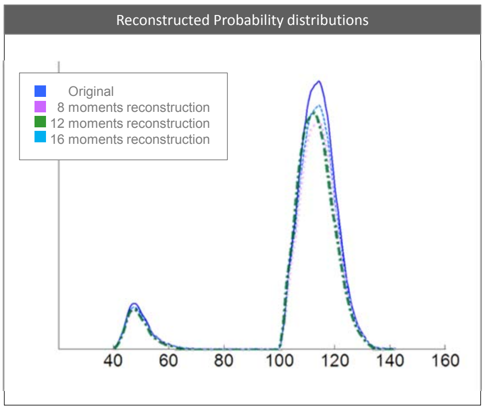
Unbundling and Probabilistic performance scenarios

Bimodality

High dispersion

Significance test of the price information

DEFAULTABLE BOND



At least 16 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

Unbundling and Probabilistic performance scenarios

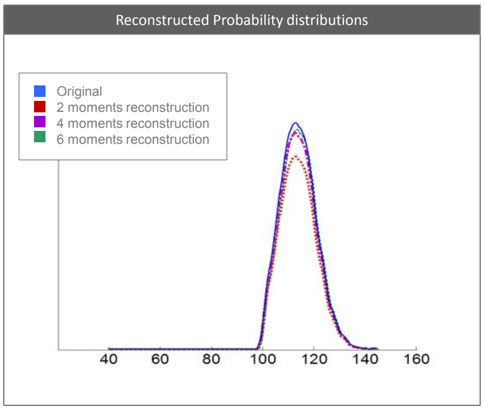
Regular

symmetry

Low dispersion

Significance test of the price information

LOW-RISK BOND



Only 4 moments are sufficient in order to describe properly the original distribution. The information content of the first moment can be considered adequate.

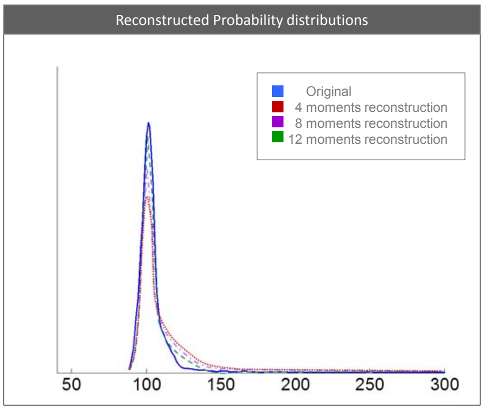
Unbundling and Probabilistic performance scenarios

Asymmetry

kurtosis

Significance test of the price information

VPPI PRODUCT



12 moments describe correctly the pattern of the original distribution. The information content of the first moment needs to be integrated.

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Unbundling and Probabilistic performance scenarios

Multimodality

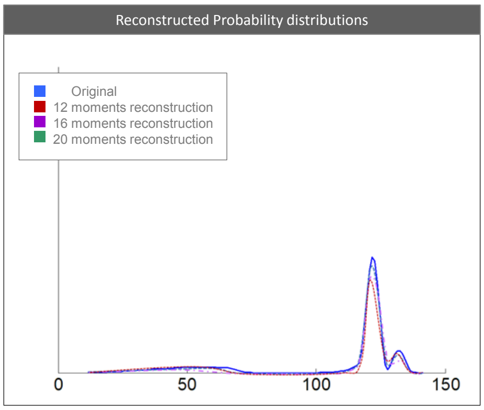
Asymmetry

kurtosis

High dispersion

Significance test of the price information

INDEX LINKED CERTIFICATE



At least 20 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

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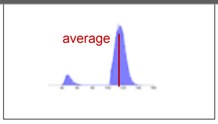
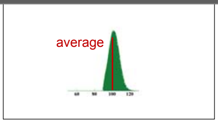
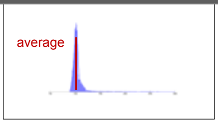
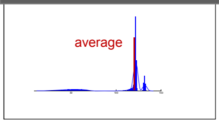
Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the Low Risk Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate
			

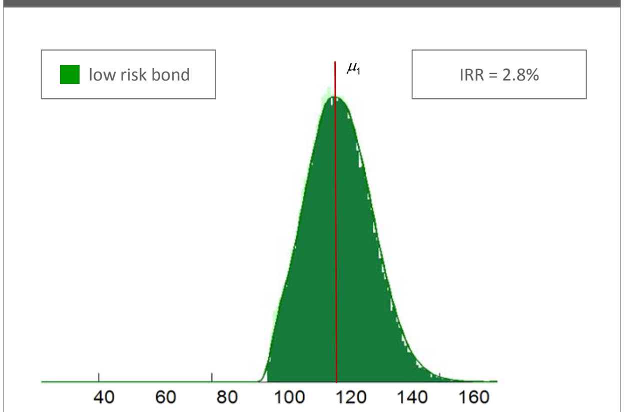
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	Low Risk Bond	VPPI Product	Index Linked Certificate
Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
16 moments needed	4 moments needed	12 moments needed	20 moments needed

From a pure statistical point of view, a proper reconstruction of the original distribution needs at least 4 moments even for the most regular one

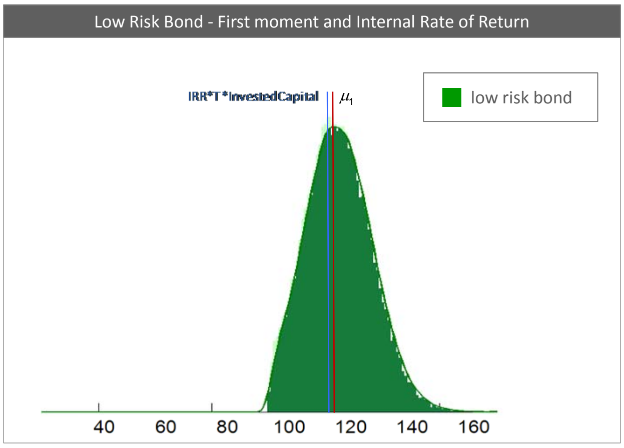
Unbundling and Probabilistic performance scenarios

Low Risk Bond - Identification of the first moment



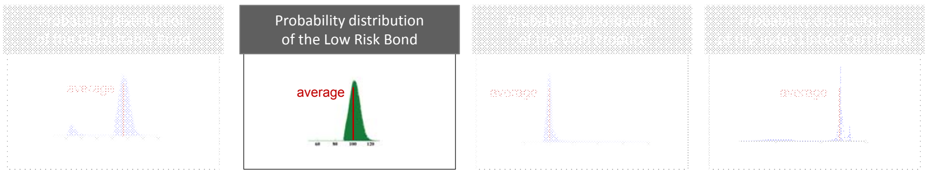
IRR ⇄ First moment of the probability distribution

Unbundling and Probabilistic performance scenarios



IRR = 2.8% $\mu_1 \approx$ IRR*T*InvestedCapital = 114

Unbundling and Probabilistic performance scenarios



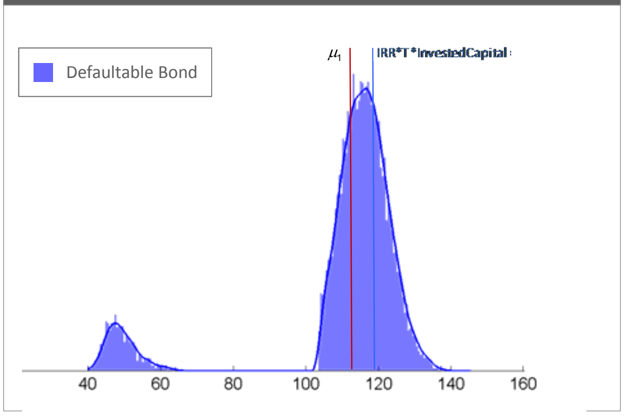
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
16 moments needed	Price ≈ Average ≈ IRR	12 moments needed	16 moments needed

Even if 4 moments are needed for a proper reconstruction of the probability distribution, the average and its related measures (IRR and price), convey sufficient information for the investor decision process

Unbundling and Probabilistic performance scenarios

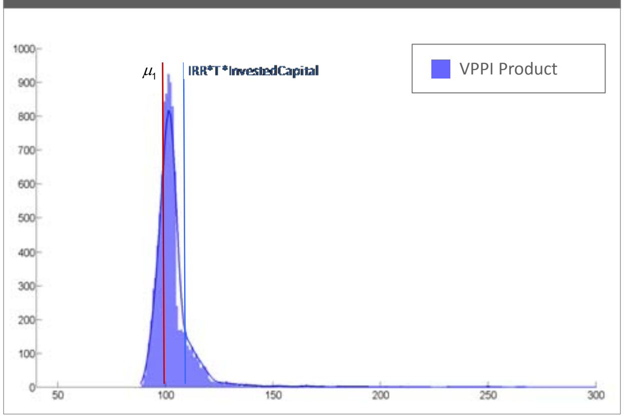
Defaultable Bond - First moment and Internal Rate of Return



IRR = 3.85% $\mu_1 \neq$ IRR*T*InvestedCapital = 119.25 !

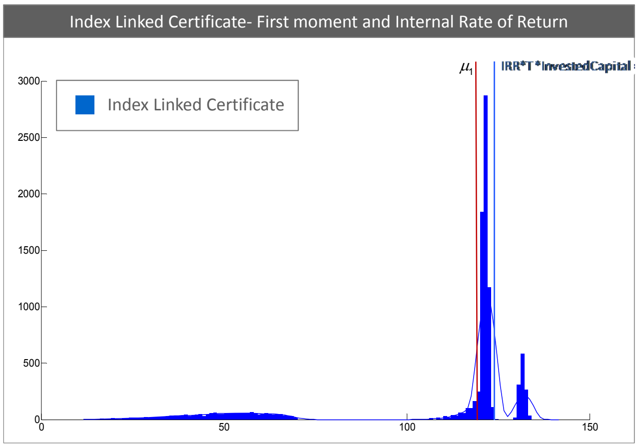
Unbundling and Probabilistic performance scenarios

VPPI Product- First moment and Internal Rate of Return



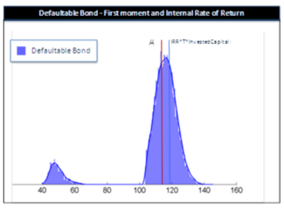
IRR = 2.53% $\mu_1 \neq$ IRR*T*InvestedCapital = 112.65 !

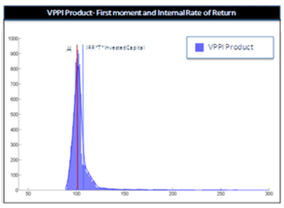
Unbundling and Probabilistic performance scenarios

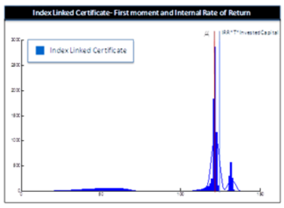


IRR = 5.91% $\mu_1 \neq$ IRR*T*InvestedCapital = 129.55

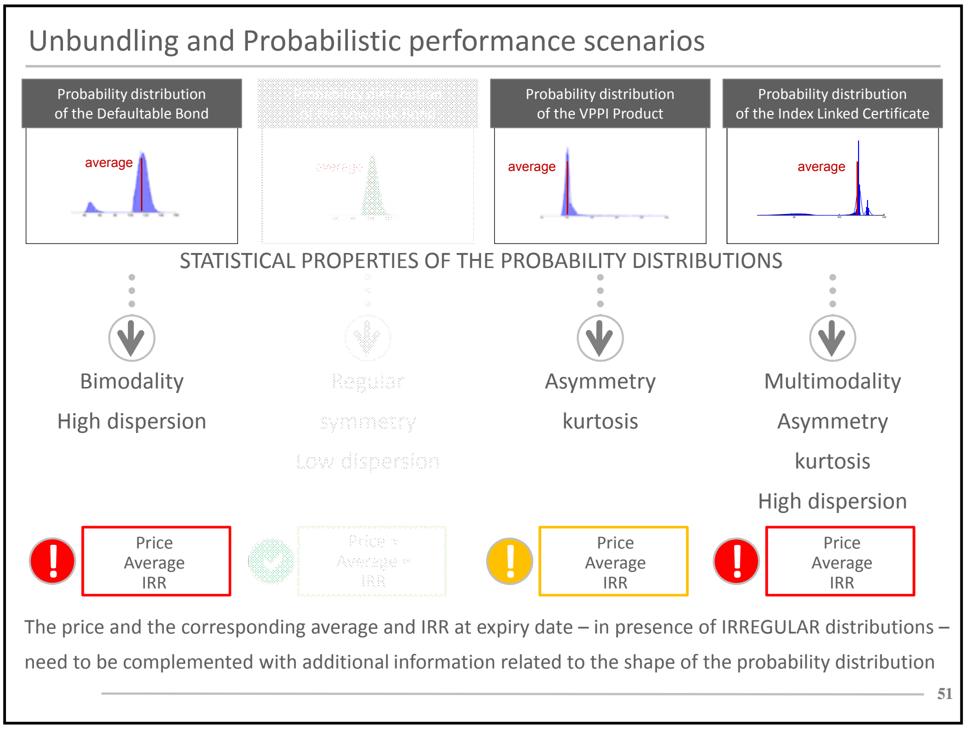
Unbundling and Probabilistic performance scenarios

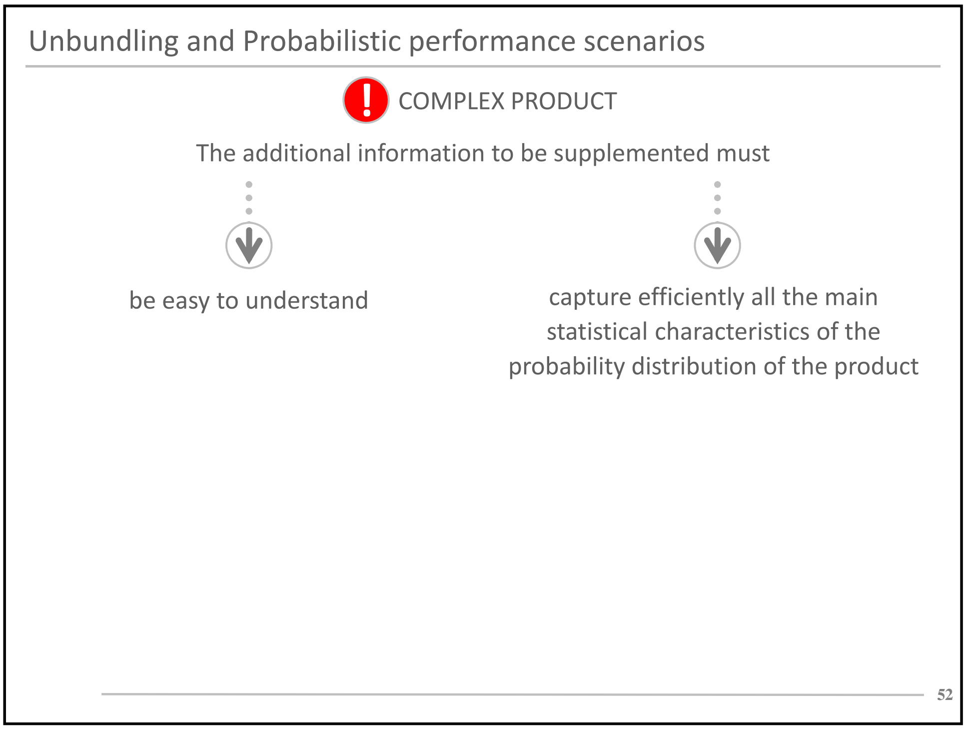






For more complex financial structures, the average progressively looses its connection with the internal rate of return of the investment, so reducing its usefulness as an effective tool for the decision process


Unbundling and Probabilistic performance scenarios
Probability distribution of the Defaultable Bond
average
Probability distribution of the VPPI Product
average
Probability distribution of the Index Linked Certificate
average
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS
Bimodality
High dispersion
Regular
symmetry
Low dispersion
Asymmetry
kurtosis
Multimodality
Asymmetry
kurtosis
High dispersion
Price
Average
IRR
Price
Average
IRR
Price
Average
IRR
The price and the corresponding average and IRR at expiry date – in presence of IRREGULAR distributions – need to be complemented with additional information related to the shape of the probability distribution
51


Unbundling and Probabilistic performance scenarios
COMPLEX PRODUCT
The additional information to be supplemented must
be easy to understand
capture efficiently all the main statistical characteristics of the probability distribution of the product
52

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

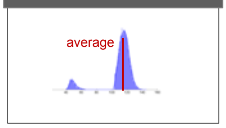
be easy to understand

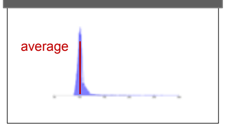
capture efficiently all the main statistical characteristics of the probability distribution of the product

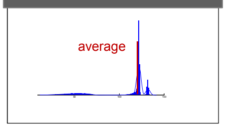
Proposal 1: Convey the entire probability distribution

Unbundling and Probabilistic performance scenarios

Probability distribution of the Defaultable Bond	Probability distribution of the VPPI Product	Probability distribution of the Index Linked Certificate







MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

2 Factor Short Interest Rate Hull-White Model

Short Interest Rate Cox Ingersoll Ross Model

Heston Stochastic Volatility Model for the Equity component

Barndorff Nielsen Normal Inverse Gaussian Model for the Equity component

Merton Jump Diffusion Model for the Equity component

Variance Gamma Model for the Equity component

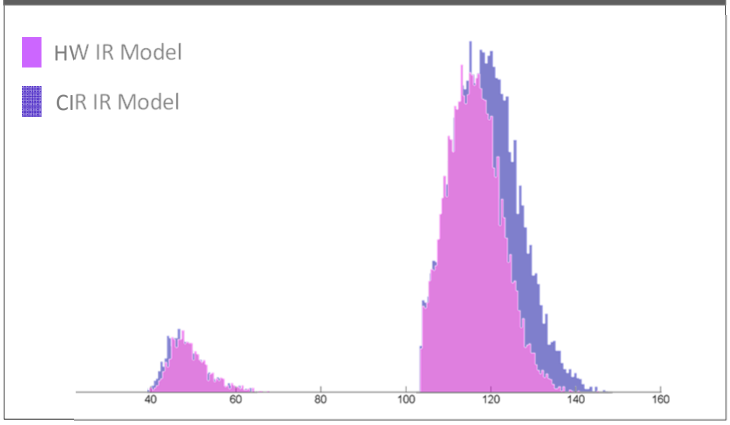
The shape of the probability distribution of the potential returns is obviously dependent on the modelling assumptions.

Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distributions of the final values of the defaultable bond



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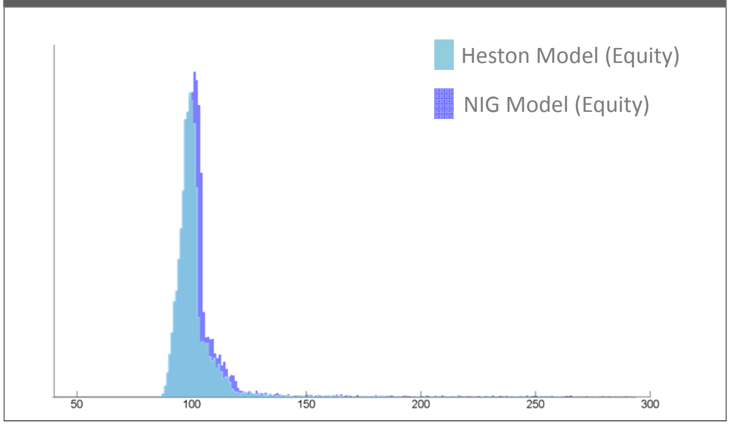
Unbundling and Probabilistic performance scenarios

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

VPPI PRODUCT

Probability distributions of the final values of the VPPI product



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Unbundling and Probabilistic performance scenarios

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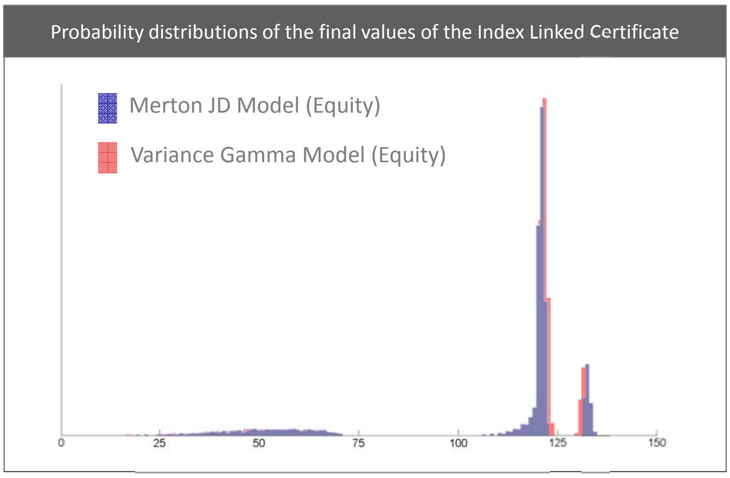
MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand

capture efficiently all the main statistical characteristics of the probability distribution of the product

the probability distribution is an abstract object not easy to handle

the shape of the probability distribution is dependent on the modelling assumptions

Proposal 1: Convey the entire probability distribution

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

- be easy to understand
- capture efficiently all the main statistical characteristics of the probability distribution of the product

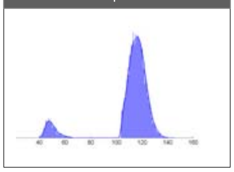
Proposal 2: Unbundling the information content of the price

Unbundling and Probabilistic performance scenarios

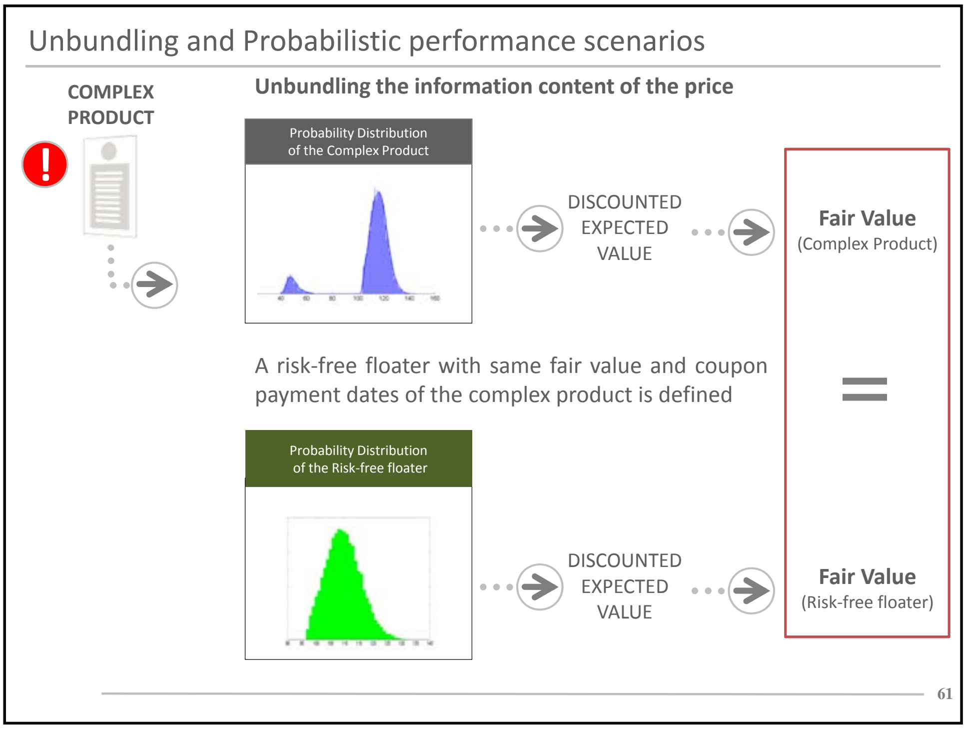
COMPLEX PRODUCT

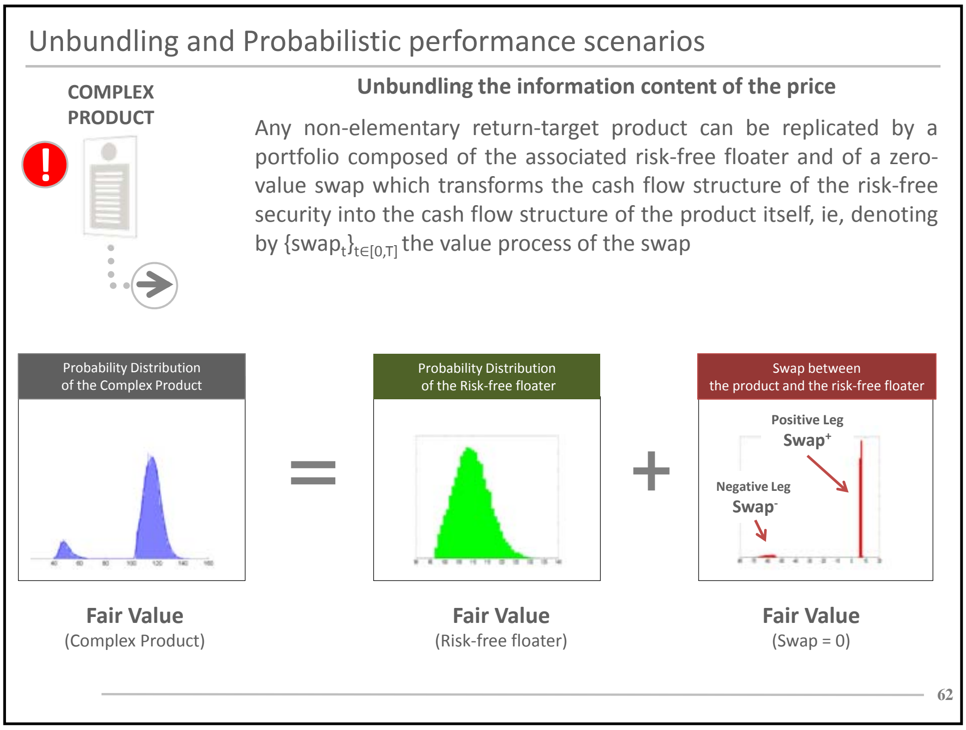
Unbundling the information content of the price

Probability Distribution of the Complex Product



DISCOUNTED EXPECTED VALUE → **Fair Value** (Complex Product)


Unbundling and Probabilistic performance scenarios
COMPLEX PRODUCT
Unbundling the information content of the price
Probability Distribution of the Complex Product
DISCOUNTED EXPECTED VALUE
Fair Value
(Complex Product)
A risk-free floater with same fair value and coupon payment dates of the complex product is defined
Probability Distribution of the Risk-free floater
DISCOUNTED EXPECTED VALUE
Fair Value
(Risk-free floater)
61


Unbundling and Probabilistic performance scenarios
COMPLEX PRODUCT
Unbundling the information content of the price
Any non-elementary return-target product can be replicated by a portfolio composed of the associated risk-free floater and of a zero-value swap which transforms the cash flow structure of the risk-free security into the cash flow structure of the product itself, ie, denoting by $\{swap_t\}_{t\in[0,T]}$ the value process of the swap
Probability Distribution of the Complex Product
Probability Distribution of the Risk-free floater
Swap between the product and the risk-free floater
Positive Leg
Swap+
Negative Leg
Swap-
Fair Value
(Complex Product)
Fair Value
(Risk-free floater)
Fair Value
(Swap = 0)
62

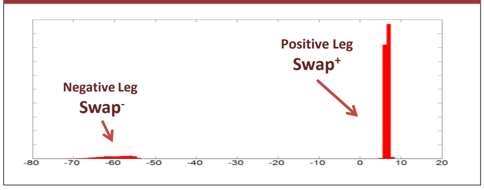
Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Fair Value
(Swap = 0)

Unbundling the information content of the price

Swap between
the product and the risk-free floater



|FV(Swap⁻)| = |FV(Swap⁺)|

Theoretical value of the Risky component

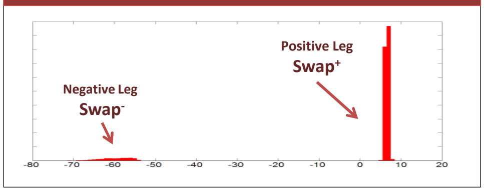
Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Fair Value
(Swap = 0)

Unbundling the information content of the price

Swap between
the product and the risk-free floater



|FV(Swap⁻)| = |FV(Swap⁺)|

Theoretical value of the Risky component

C	*Fair value*
B	Theoretical value of the Risky component
A=C-B	Theoretical value of the Risk-Free component

Unbundling and Probabilistic performance scenarios

Financial investment table
(*Price Unbundling*)

DEFAULTABLE BOND

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	*Fair value*	96.3
D	Costs	3.7
E = C + D	Issue price	100

VPPI PRODUCT

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	*Fair value*	96.5
D	Costs	3.5
E = C + D	Issue price	100

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A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	*Fair value*	96.1
D	Costs	3.9
E = C + D	Issue price	100

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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand

capture efficiently all the main statistical characteristics of the probability distribution of the product

the unbundling represented by using a table is first level tool useful to appreciate the impact of the costs and the riskiness of the product

The unbundling exploits only the information contained in the first order moment of the probability distribution

Proposal 2: Unbundling the information content of the price

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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand

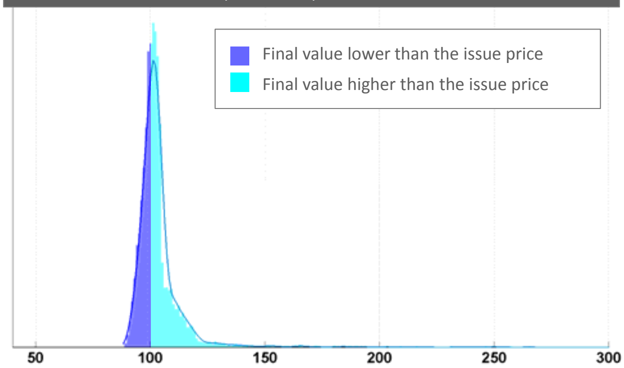
capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 3: **Perform a reduction in granularity by implementing a partition of the probability distribution**

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Partition of the Probability distribution of the Complex Product with respect to the point of zero return

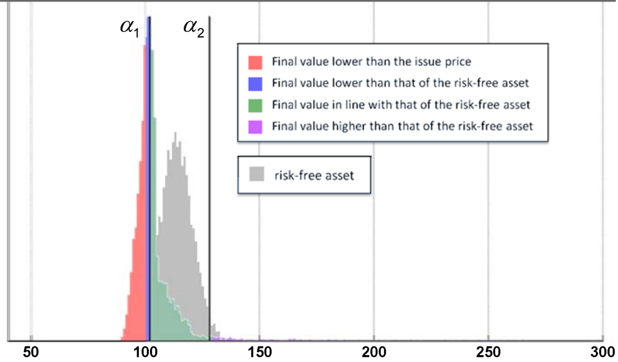


The assessment of the probability of recovering at least the amount paid for the product is of great significance for the investor.

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



It is appropriate to explore further partitions of the macro-event “the final value of the investment is higher than the issue price” by performing a direct comparison with the final values of the risk-free asset.

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2

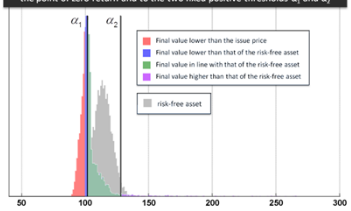


Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	…	…
The performance is positive but lower than the risk-free asset	…	…
The performance is positive and in line with the risk-free asset	…	…
The performance is positive and higher than the risk-free asset	…	…

MEAN VALUES

$$E^P(S_T \mid S_T < 100) = \frac{1}{P(S_T < 100)} \int_{-\infty}^{100} x f_{S_T}(x)\,dx$$

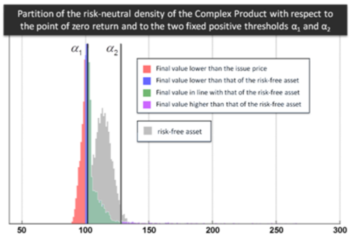
$$E^P(S_T \mid 100 \leqslant S_T < \alpha_1) = \frac{1}{P(100 \leqslant S_T < \alpha_1)} \int_{100}^{\alpha_1} x f_{S_T}(x)\,dx$$

$$E^P(S_T \mid \alpha_1 \leqslant S_T < \alpha_2) = \frac{1}{P(\alpha_1 \leqslant S_T < \alpha_2)} \int_{\alpha_1}^{\alpha_2} x f_{S_T}(x)\,dx$$

$$E^P(S_T \mid S_T \geqslant \alpha_2) = \frac{1}{P(S_T \geqslant \alpha_2)} \int_{\alpha_2}^{+\infty} x f_{S_T}(x)\,dx$$

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

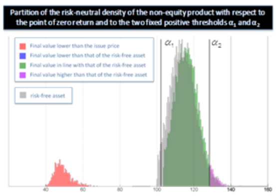


Benefits of this solution:

1. The <u>reduction in granularity</u> of the events determined by the partition involves only a very limited loss of information and <u>the table</u>, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;

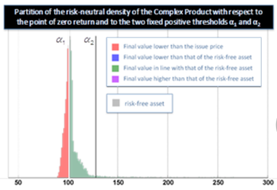
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	9.5%	49.3
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	87.4%	115.6
The performance is <u>positive and higher</u> than the risk-free asset	3.1%	131.1

VPPI PRODUCT



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	36.9%	96.9
The performance is <u>positive but lower</u> than the risk-free asset	18.5%	101
The performance is <u>positive and in line</u> with the risk-free asset	39.9%	107.1
The performance is <u>positive and higher</u> than the risk-free asset	4.7%	195.5

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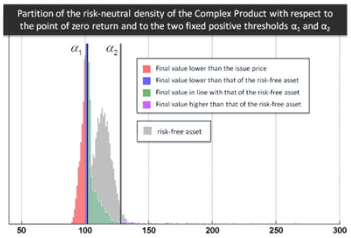
SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	18.9%	49.1
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	68.9%	120.9
The performance is <u>positive and higher</u> than the risk-free asset	12.2%	131.6

Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Benefits of this solution:

1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information; The table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;
2. The model risk arising from the different proprietary models of the issuers has a limited impact.

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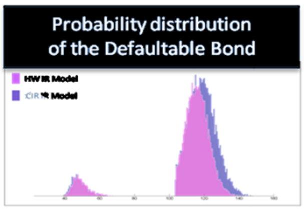
Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Difference less than 2%

HW IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

CIR IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	8.3%	49.9
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	86.8%	117.9
The performance is positive and higher than the risk-free asset	4.9%	135.4

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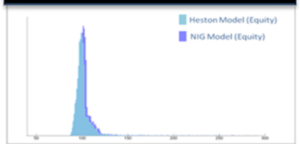
Unbundling and Probabilistic performance scenarios

VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the VPPI Product



Difference less than 2%

HESTON MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	38.9%	95.5
The performance is positive but lower than the risk-free asset	18.9%	100.2
The performance is positive and in line with the risk-free asset	38.4%	106.3
The performance is positive and higher than the risk-free asset	3.8%	182.5

NIG MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

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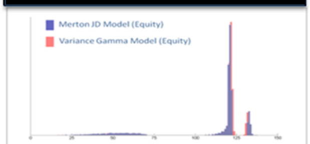
Unbundling and Probabilistic performance scenarios

INDEX LINKED CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the Index Linked Certificate



Difference less than 4%

MERTON JD MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	48.2
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	65.8%	117.6
The performance is positive and higher than the risk-free asset	15.3%	132.7

VARIANCE GAMMA MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT

The additional information to be supplemented must

- be easy to understand
- capture efficiently all the main statistical characteristics of the probability distribution of the product

the partition should be done by choosing events that have a strong financial meaning

the reduction in granularity mitigates in a significant way the model risk

Proposal 3: **Perform a reduction in granularity by implementing a partition of the probability distribution**

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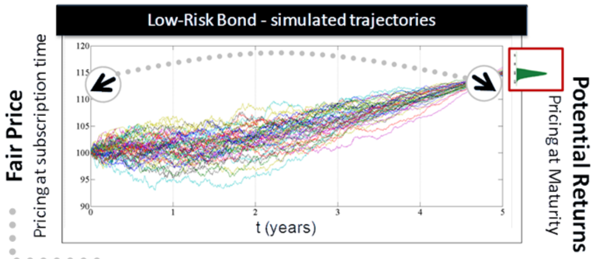
Unbundling and Probabilistic performance scenarios

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

COMPLEX PRODUCT



Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	
B	Theoretical value of the Risky component	
C = A + B	*Fair value*	
D	Costs	
E = C + D	Issue price	

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	...	...
The performance is positive but lower than the risk-free asset	...	...
The performance is positive and in line with the risk-free asset	...	...
The performance is positive and higher than the risk-free asset	...	...

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Syllabus

- Preliminaries: the three pillars
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon
- An Application of the methodology

Synthetic risk indicator

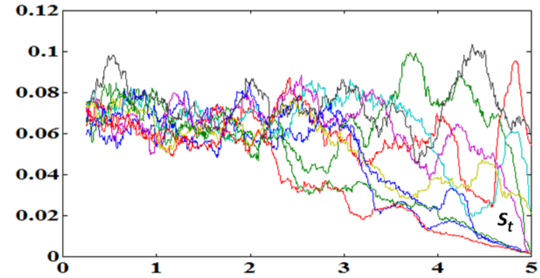
RETURNS	RISKS	INVESTMENT HORIZON

The key qualitative information is made objective by using a three-pillars approach.

1st Pillar	2nd Pillar	3rd Pillar
Unbundling and Probabilistic performance scenarios	Synthetic risk indicator	The recommended Investment horizon

Synthetic risk indicator

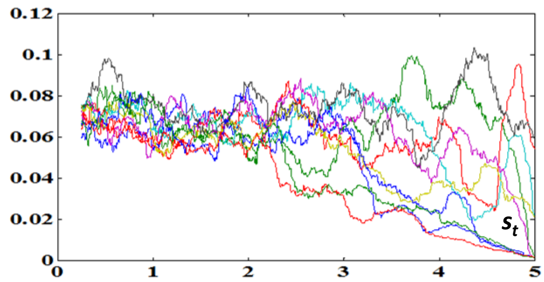
Volatility of the product's potential returns



Volatility is the most immediate risk measure and
it has a one-to-one relationship with whatever loss measure
(VaR, ES, etc.)

Synthetic risk indicator

Volatility of the product's potential returns



VOLATILITY

e.g. : geometric brownian motion

$$dS_t = rS_t dt + \sigma S_t dW_t$$

$$VaR_{\alpha,1year} = e^{\sigma \Phi^{-1}(\alpha) + \left(r - \frac{\sigma^2}{2}\right)} - 1$$

$$ES_{\alpha,1year} = \frac{1}{\alpha} e^{r} \Phi\left(\Phi^{-1}(\alpha) - \sigma\right) - 1$$

Synthetic risk indicator

EXAMPLES

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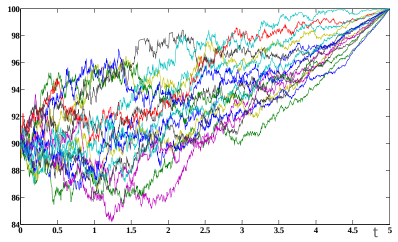
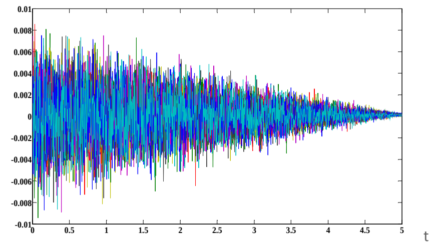
Synthetic risk indicator

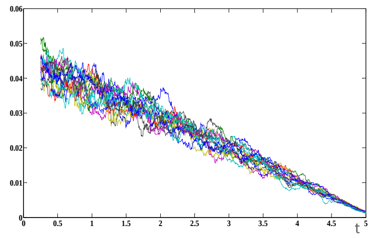
Simulation of the trajectories (Price)



Simulation of the trajectories (Returns)



Simulation of the trajectories (Volatility)



Non-*equity* *product*: Zero Coupon Bond

Degree of risk

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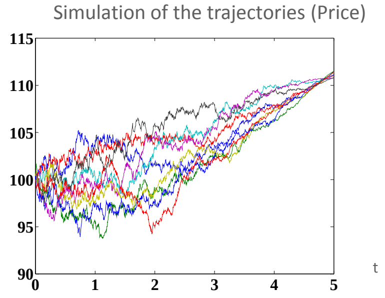
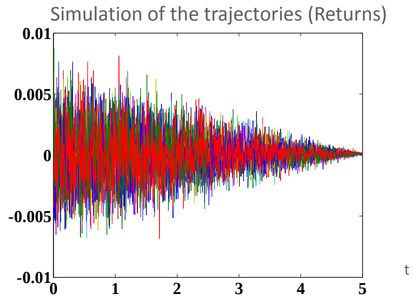
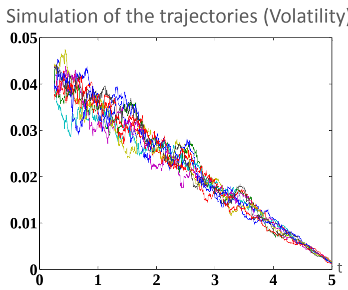
Synthetic risk indicator



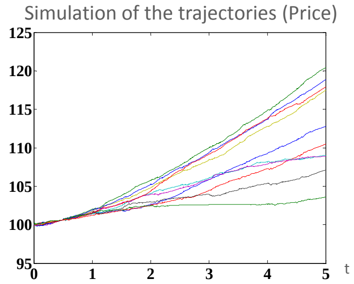


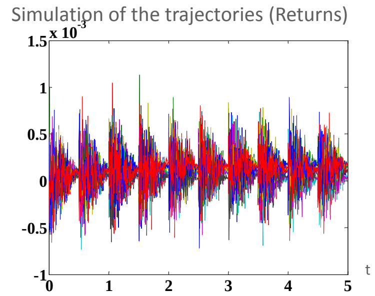
Non-*equity* *product*: Fixed bond like



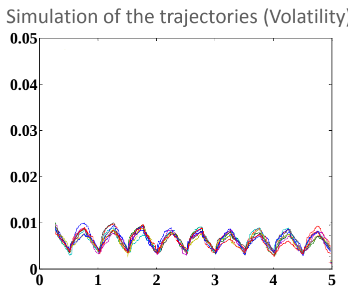
Degree of risk

Synthetic risk indicator



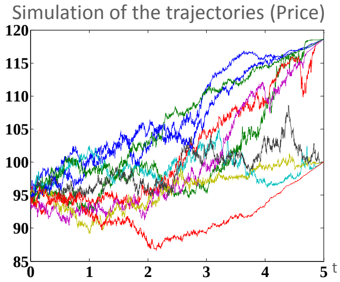


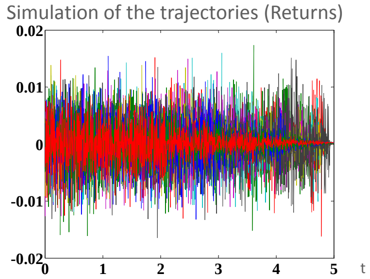
Non-*equity* *product*: Floater bond like



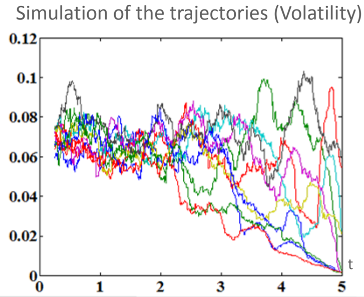
Degree of risk

Synthetic risk indicator





Non-*equity product*: Structured Bond

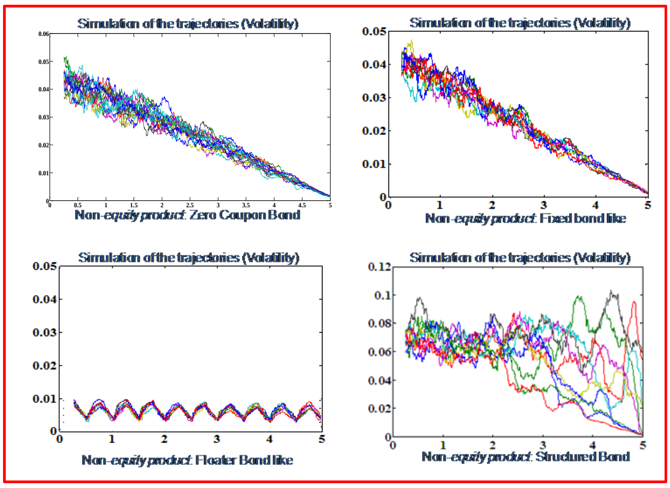


Degree of risk

Synthetic risk indicator

COMPLEXITY FOR RETAIL INVESTORS

The volatility patterns are abstract objects that an average investor cannot handle.



Synthetic risk indicator

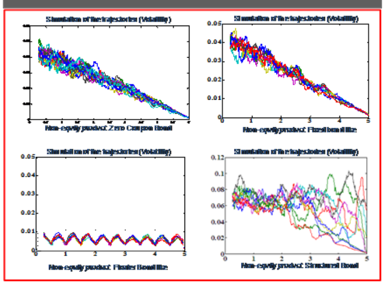
Conversely, a table with qualitative labels that characterizes the risk classes is <u>very easy to understand</u>

Risk Classes
Very Low
Low
Medium-Low
Medium
Medium-High
High
Very High

The assignment of the degree of risk is made according to a quantitative criterion that maps coherently any volatility interval into a corresponding qualitative risk class

Synthetic risk indicator

Volatility of the product's potential returns



MEASUREMENT:
product's positioning inside
a grid of *n* volatility intervals

REPRESENTATION:
mapping of any volatility interval into a corresponding qualitative risk class

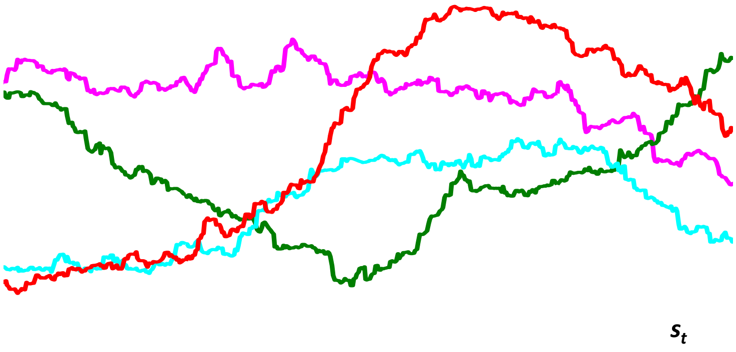
DEGREE OF RISK

Risk Classes	Volatility Intervals	
Very Low	$\sigma_{1,min}$	$\sigma_{1,max}$
Low	$\sigma_{2,min}$	$\sigma_{2,max}$
Medium-Low	$\sigma_{3,min}$	$\sigma_{3,max}$
Medium	$\sigma_{4,min}$	$\sigma_{4,max}$
Medium-High	$\sigma_{5,min}$	$\sigma_{5,max}$
High	$\sigma_{6,min}$	$\sigma_{6,max}$
Very High	$\sigma_{7,min}$	$\sigma_{7,max}$

Synthetic risk indicator

Products with the same risk budget

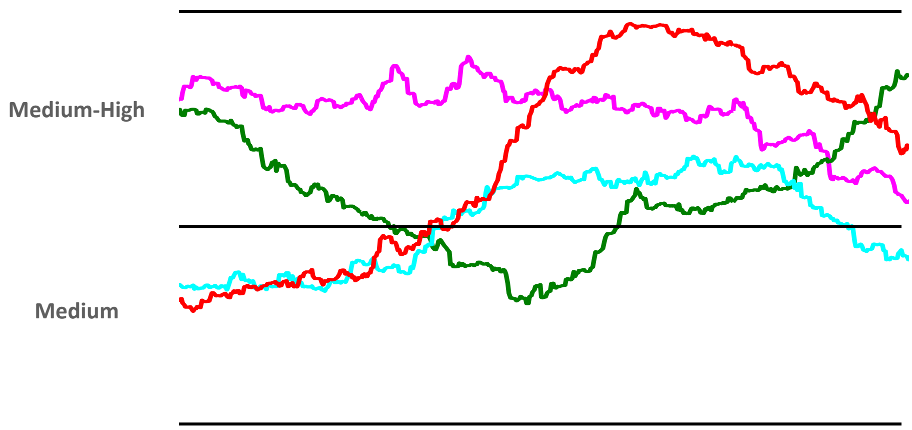
must have the same degree of risk



Synthetic risk indicator

Volatility intervals have to be suitably calibrated

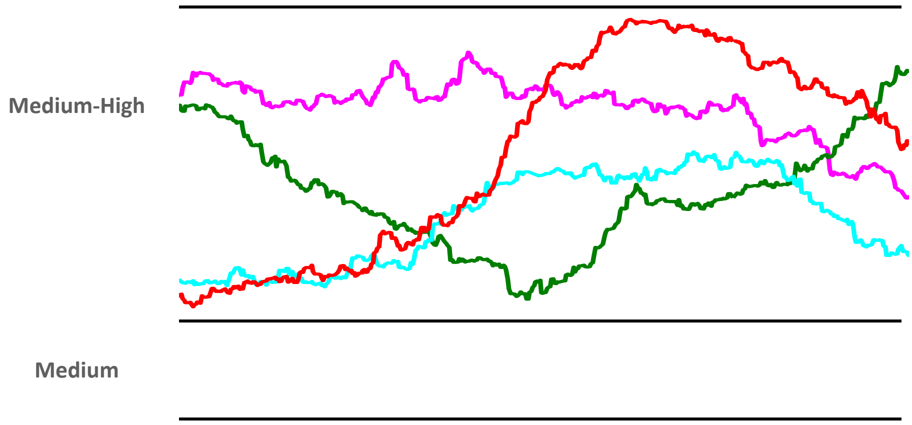
in order to avoid wrong risk representations



Synthetic risk indicator

Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations



Synthetic risk indicator

Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations

THE ISSUE

Defining suitable requirements to partition the volatility space $[0, +\infty)$ into an optimal number n^* of subsequent intervals with optima extrema

Synthetic risk indicator

Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations

Requirement n.1

the **optimal grid** of volatility intervals
has to be **consistent** with the **principle**:

+ RISK + LOSSES

**VOLATILITY INTERVALS MUST HAVE
AN INCREASING WIDTH IN ABSOLUTE TERMS**

Synthetic risk indicator

Volatility intervals have to be suitably calibrated

in order to avoid wrong risk representations

Requirement n.2

the optimal grid of volatility intervals must be

market feasible

REALIZED VOLATILITY CONSISTENT WITH MARKET EXPECTATIONS OF FUTURE VOLATILITY

(UNLESS FOR SIGNIFICANT SUDDEN SHOCKS)

Synthetic risk indicator

Realized volatility

Any product on the markets reflects specific/different asset management policies

Historical data can be "dirty"

need for comparability

1st INTUITION

It has to be studied a theoretical product
managed by an automatic asset manager
who has a specific risk budget,
identified by a given volatility interval

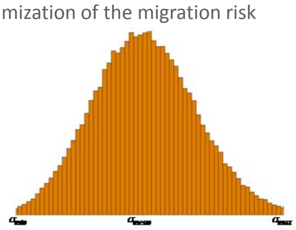
Synthetic risk indicator

1st INTUITION

AUTOMATIC ASSET MANAGER:

described by a stochastic volatility model with:

- mean reversion
- symmetry w.r.t. to a given risk budget
- ex ante minimization of the migration risk



$$dS_t = rS_t dt + \sigma_t S_t dW_t^{(1)}$$
$$d\sigma_t^2 = \kappa(\vartheta - \sigma_t^2)dt + v_t \sigma_t dW_t^{(2)}$$

Market expectations of future volatility

future volatility is predicted by exploiting
information embedded in recently observed data

2nd INTUITION

Market expectation is given by volatility prediction intervals based on proper diffusive models

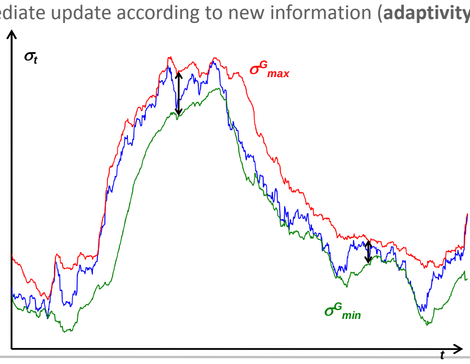
2nd INTUITION

VOLATILITY PREDICTION INTERVALS:

obtained by the diffusion limit of a multiplicative GARCH model

$$d\ln\sigma_t^2 = \left(\beta_0 + 2\beta_1 E(\ln|Z_t|) + (\beta_1 - 1)\ln\sigma_t^2\right)dt + 2|\beta_1|\sqrt{Var\left(\ln|Z_t|\right)}dW_t^*$$

- well-known distributional properties
- immediate update according to new information (**adaptivity**)



Synthetic risk indicator

Assessing ***market feasibility***

putting together the two ingredients

3rd INTUITION

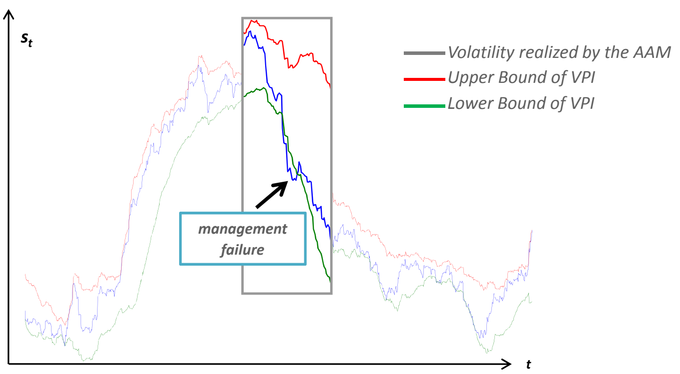
It requires to study when the realized volatility of the automatic asset manager is outside the Garch-based volatility prediction interval:

MANAGEMENT FAILURES

Synthetic risk indicator

3rd INTUITION

MANAGEMENT FAILURES:



Synthetic risk indicator

3rd INTUITION

MANAGEMENT FAILURES:

Market feasibility regards all ***n*** intervals of the optimal grid at the same time:

NOT ABNORMALITY

low number of failures

HOMOGENEITY

equal number of failures

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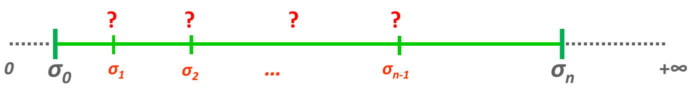
Synthetic risk indicator

Solving for the optimal grid



in a reduced volatility space $[\sigma_0, \sigma_n]$

NOT ABNORMALITY

&

HOMOGENEITY

same relative width:

$$\lambda = \left(\frac{\sigma_n}{\sigma_0}\right)^{\frac{1}{n}}$$

for all intervals

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Synthetic risk indicator

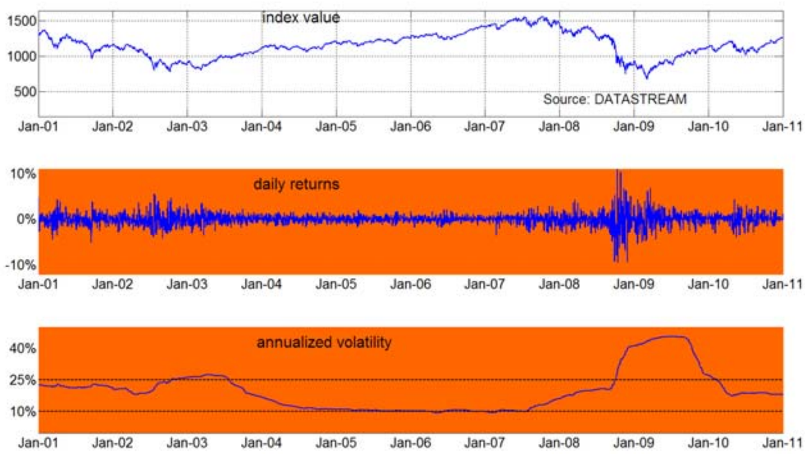
The optimal grid

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

The optimal grid is consistent with the 1st requirement: **+ RISK + LOSSES**

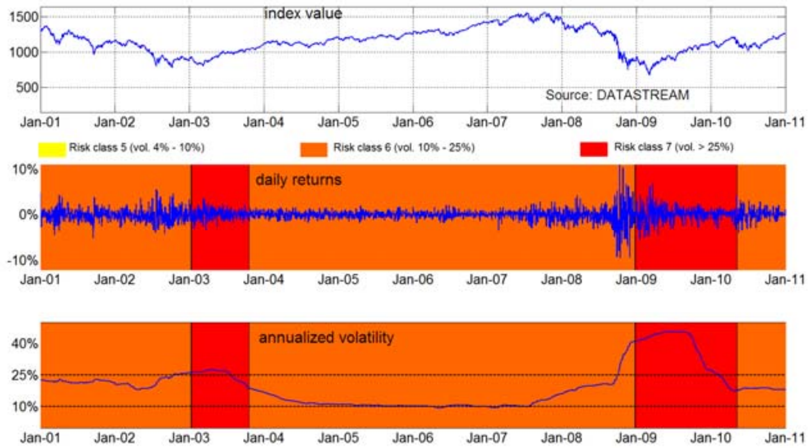
Synthetic risk indicator

Application to a market index: the optimal grid



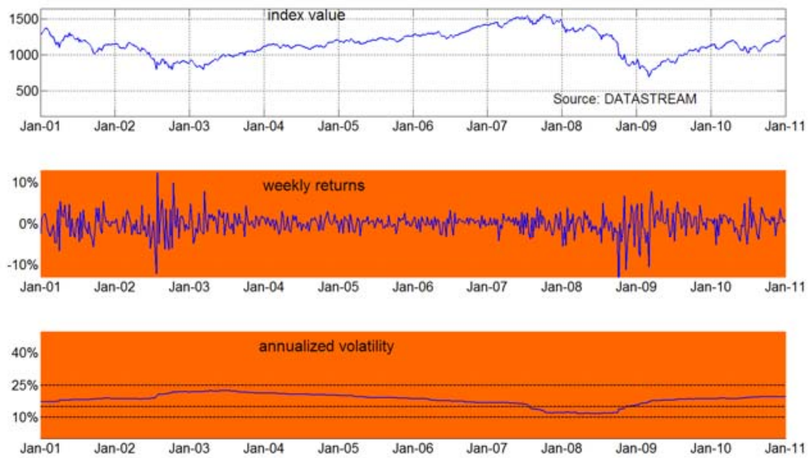
Synthetic risk indicator

Application to a market index: the optimal grid



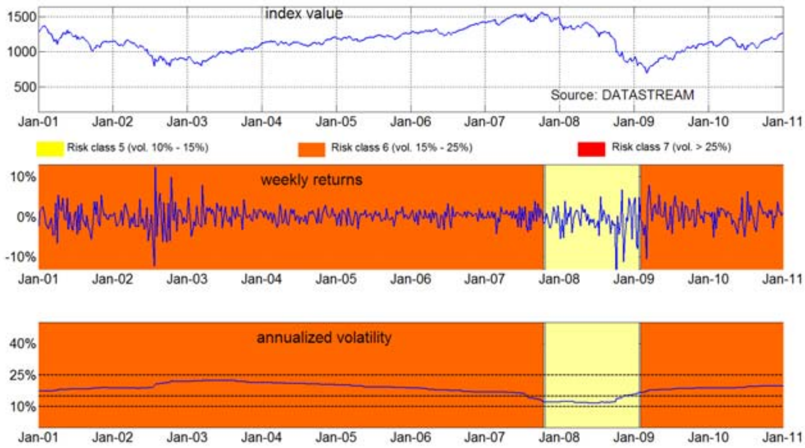
Synthetic risk indicator

Application to a market index: a non-optimal grid



Synthetic risk indicator

Application to a market index: a non-optimal grid



Syllabus

- Preliminaries: the three pillars
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon
- An Application of the methodology

The recommended Investment horizon

RETURNS | RISKS | INVESTMENT HORIZON

The key qualitative information is made objective by using a three-pillars approach.

1st Pillar
Unbundling and Probabilistic performance scenarios

2nd Pillar
Synthetic risk indicator

3rd Pillar
The recommended Investment horizon

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The recommended Investment horizon

RISK TARGET PRODUCT | **BENCHMARK PRODUCT** | **RETURN TARGET PRODUCT**

The recommended investment time horizon

The event to study from a probabilistic point of view is related to possible exit strategies after having recovered all the costs of the product :

The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

First Passage Time

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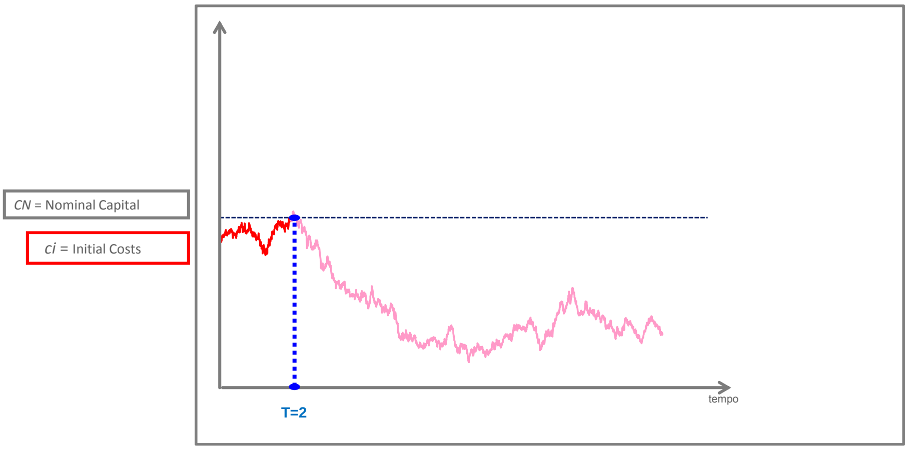
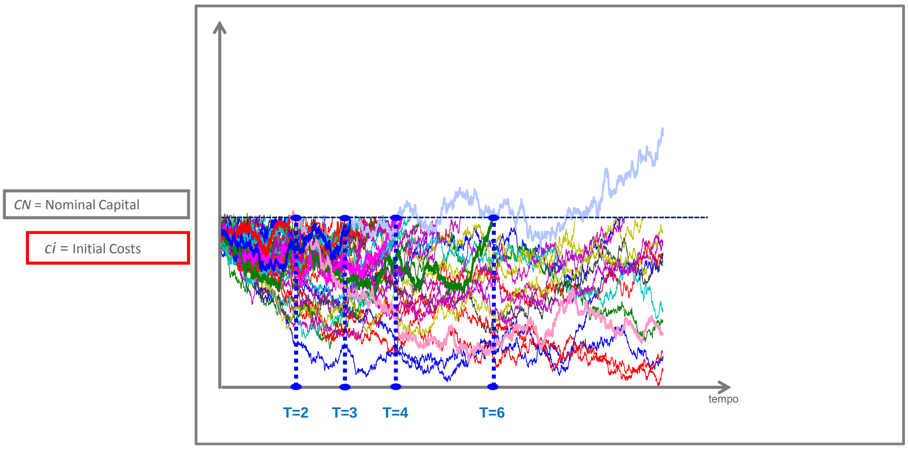
The recommended Investment horizon

First Passage Time:
First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



The recommended Investment horizon

First Passage Time:
First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



The recommended Investment horizon

The probability of the event:

> The investment recovers the initial costs and off-sets the running costs at least once

given a confidence level α, uniquely identifies a time T^* on the cumulative distribution function of the first passage times, i.e.:

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \leq T\right] = \alpha \right\}$$

where

$$t^* = \inf\left[t \in \Re^+ : CI_t > CN\right]$$

is the first passage time

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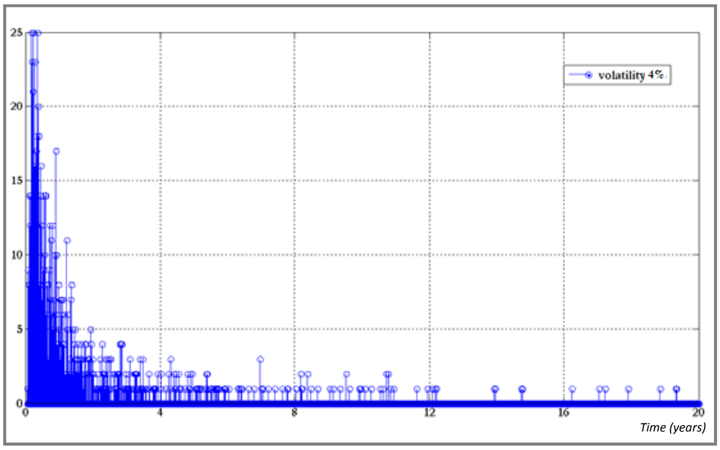
The recommended Investment horizon

1. Calculation of the probability distribution of the first passage times:



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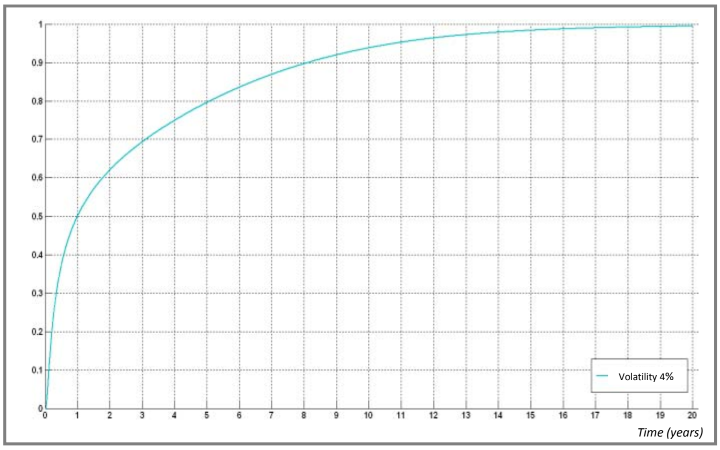
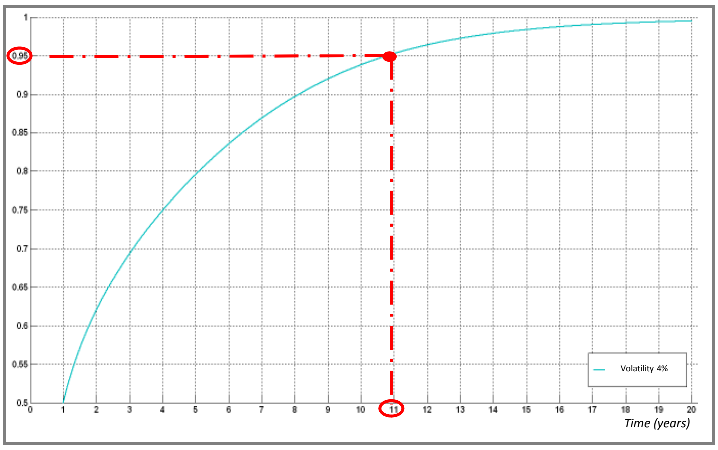
The recommended Investment horizon

2. Derivation of the cumulative distribution function of the first passage times:



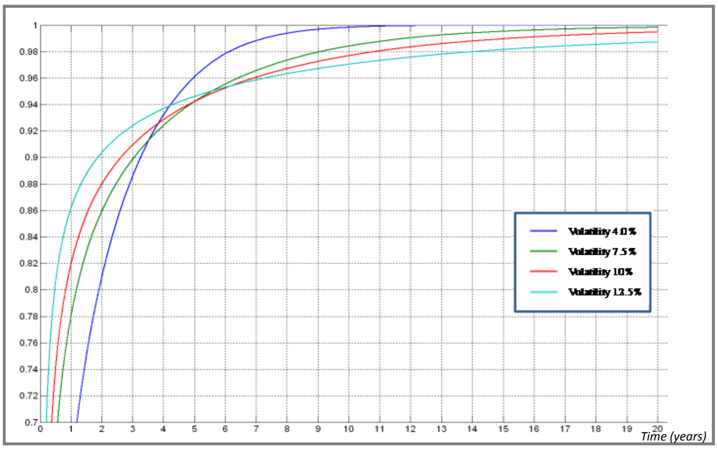
The recommended Investment horizon

3. The confidence level α uniquely identifies T^* on the cumulative distribution function of the first passage times:



The recommended Investment horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



The recommended Investment horizon

Anyway, the recommended minimum investment time horizon…

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \leq T\right] = \alpha \right\}$$

…. Must be coherent with the principle

+ VOLATILITY + TIME HORIZON

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

The recommended Investment horizon

Connection between probability, volatility and costs

First passage times for the break-even barrier are monitored at infinitesimal time intervals:

$dt \to 0$

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \le T\right] = \alpha \right\}$$

$$\mathrm{P}\left[t^* \le T\right] = N\left(d_2\left(\frac{CI_0}{CN}\right)\right) + \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot N\left(-d_2\left(\frac{CN}{CI_0}\right)\right)$$

$$d_2(x) = \frac{\log x + \left(\bar{r} - cr - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

The recommended Investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

cr : recurrent costs as a fixed %

$dt \to 0$

$$\lim_{T \to \infty} \mathrm{P}\left[t^* \le T\right] = \begin{cases} 1 & \text{if} \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & \text{if} \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

The recommended Investment horizon

Connection between probability, volatility and costs

Under our assumptions:

$dt \to 0$

$$\lim_{T\to\infty} \mathrm{P}\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

For a given level of costs, it is possible to analytically derive the connection between volatility and time horizon

The recommended Investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

FIRST ORDER SENSITIVITY ANALYSIS

$$\frac{d\mathrm{P}}{d\sigma} = \left(-4\frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right) \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

FIRST ORDER ASYMPTOTIC CONDITION

The recommended Investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

$$\frac{dP}{d\sigma} = \left(-4\frac{(\bar{r}-cr)}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

1. $(\bar{r}-cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $(\bar{r}-cr) \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

The existence of two alternative states of nature requires to verify whether both of them make sense in financial terms under the risk-neutral measure.

The recommended Investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

$cr = 0$

Being running costs a specific feature of any financial product they would interfere with the task of identifying which of the two conditions has a sound financial meaning. Therefore, they will be temporarily neglected.

The recommended Investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

$cr = 0$

Since it is safe to assume a positive interest rate *r* in financial markets, only condition 1. correctly captures the connection between volatility and time horizon.

The recommended Investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

$cr = 0$

As $T \to \infty$ condition 1. implies that the cumulative distribution function P is a strictly decreasing function of the volatility, i.e.:

$$\forall \sigma_i, \sigma_j \in \Re^+, \sigma_j > \sigma_i \Rightarrow \mathrm{P}(\sigma_j) < \mathrm{P}(\sigma_i)$$

The recommended Investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$~~

In other words, for a given a confidence level, as the volatility grows, the recommended investment time horizon increases as well:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

The recommended Investment horizon

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{d^2P}{d\sigma^2} = \frac{4}{\sigma^4}(\bar{r} - cr)\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot \left[1 + \frac{4(\bar{r}-cr)}{\sigma^2}\ln\left(\frac{CN}{CI_0}\right)\right]$$

$(\bar{r} - cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0$ → SECOND ORDER ASYMPTOTIC CONDITION

Second Order Sensitivity Analysis

The recommended Investment horizon

Connection between probability, volatility and costs

$$T \to \infty, dt \to 0$$

1.
$$\begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases}$$

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

2. $(\bar{r} - cr) \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

Summarizing the results of the asymptotic analysis in continuous time:

- As T $\to\infty$, for given a confidence level, more volatility implies a larger recommended investment time horizon
- It is always possible to find a minimum and finite time T^*, beyond which the strong condition

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

holds

The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

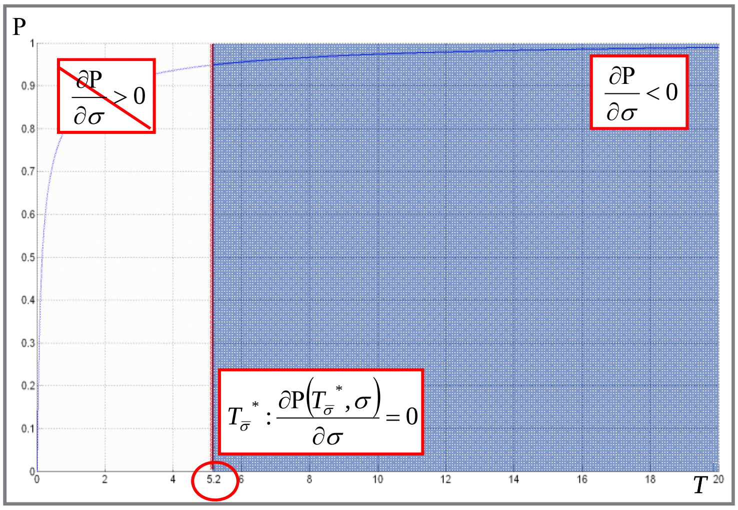
General Framework:

$$\begin{cases} T \to \infty \\ dt \to 0 \\ P(\infty, \sigma) \\ 1. \begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases} \end{cases} \Rightarrow \begin{cases} T \ \ finite \\ dt \to 0 \\ P(T, \sigma) \\ (\bar{r} - cr) > 0 \Leftrightarrow \lim_{T\to\infty} \frac{\partial P(T,\sigma)}{\partial \sigma} < 0 \\ (\bar{r} - cr) > 0 \Rightarrow \lim_{T\to\infty} \frac{\partial^2 P(T,\sigma)}{\partial \sigma^2} > 0 \end{cases}$$

Everything shown above also holds with *T* finite!

The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



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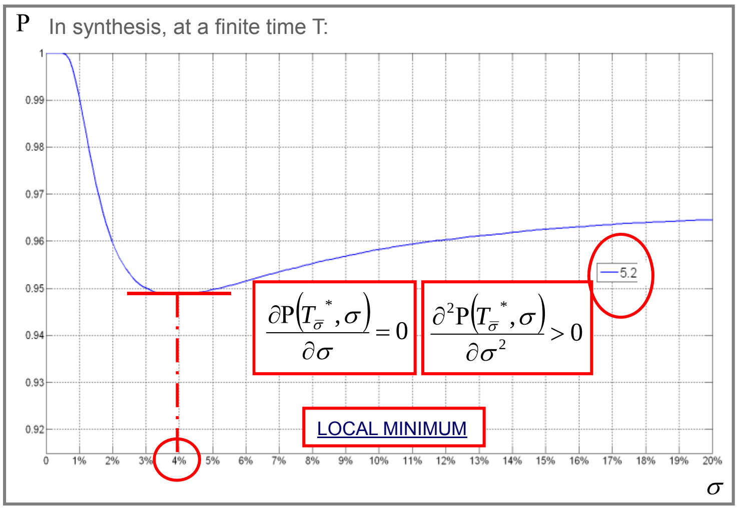
The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



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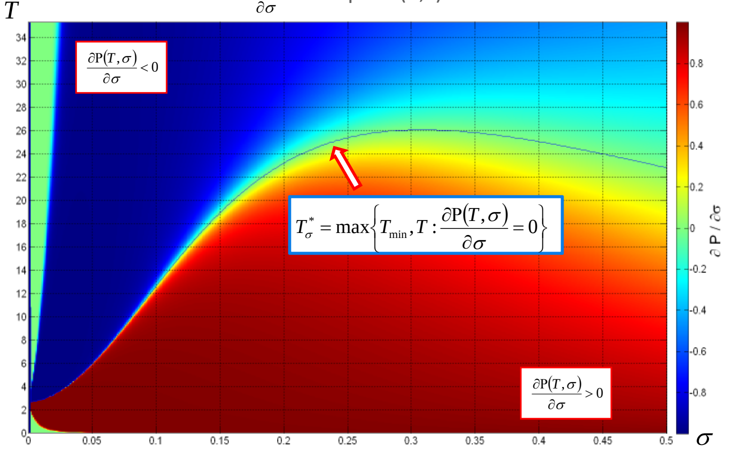
The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS

Plot of the function $\frac{\partial \mathrm{P}(T,\sigma)}{\partial \sigma}$ in a space (σ,T)



The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

SECOND ORDER SENSITIVITY ANALYSIS

$$\frac{\partial^2 \mathrm{P}(T,\sigma)}{\partial \sigma^2}$$

Given the monotonicity condition of the probability distribution with respect to volatility, i.e.:

$$\forall \sigma_i, \sigma_j \in \Re^+, \sigma_j > \sigma_i \Rightarrow \mathrm{P}(\infty, \sigma_j) < \mathrm{P}(\infty, \sigma_i)$$

In order to fulfill this condition, it's necessary to restrict the analysis in the region where the probability function is strictly increasing, i.e.:

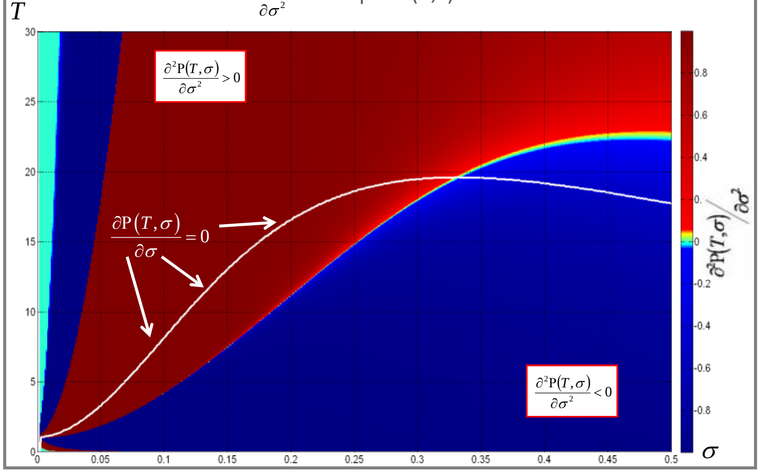
$$\left.\frac{\partial^2 \mathrm{P}(T,\sigma)}{\partial \sigma^2}\right|_{T=T_\sigma^*} > 0 \Rightarrow T_\sigma^* \text{ increasing}$$

$$\left.\frac{\partial^2 \mathrm{P}(T,\sigma)}{\partial \sigma^2}\right|_{T=T_\sigma^*} < 0 \Rightarrow T_\sigma^* \text{ decreasing}$$

The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

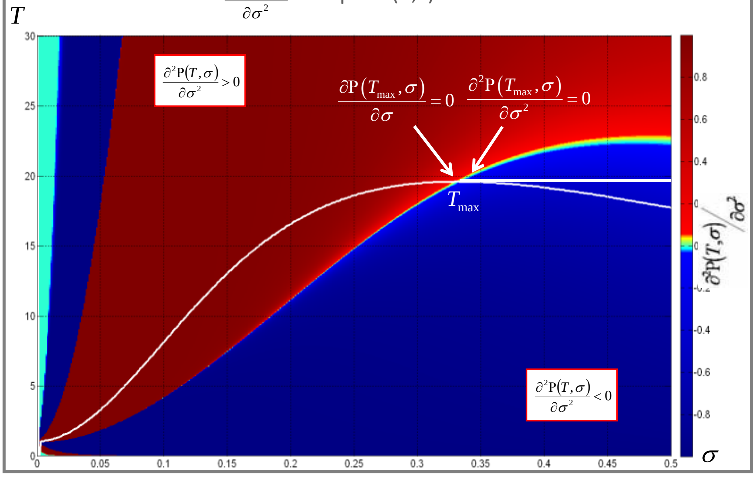
Plot of the function $\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$ in a space (σ,T)



The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

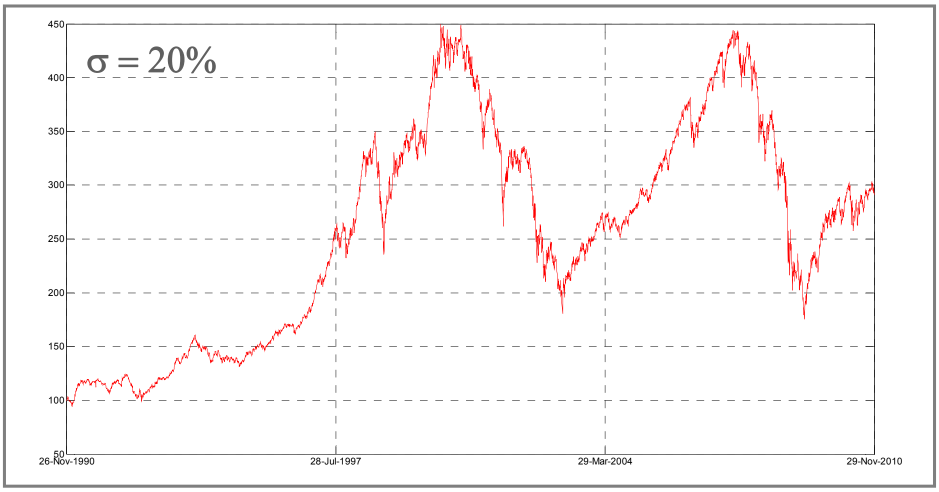
Plot of the function $\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$in a space (σ,T)



The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

STOXX EUROPE 600
(26-Nov-1990 – 26-Nov-2010, BASE 100: 1990)



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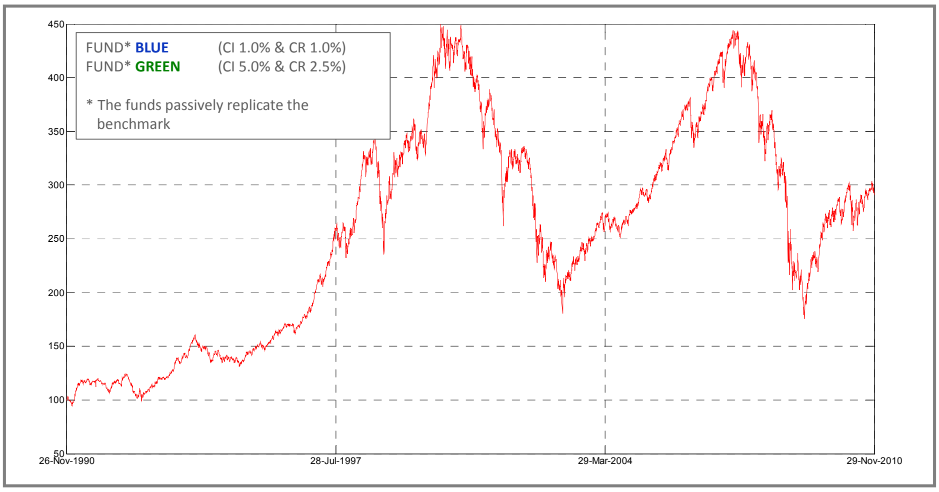
The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

STOXX EUROPE 600
(26-Nov-1990 – 26-Nov-2010, BASE 100: 1990)



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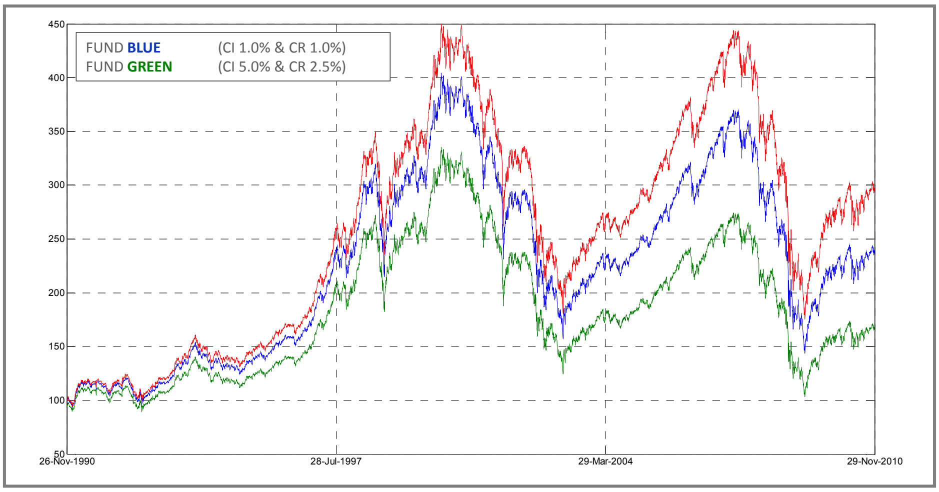
The recommended Investment horizon

DETERMINAZIONE DELL'ORIZZONTE TEMPORALE

STOXX EUROPE 600
(26/11/1990 – 26/11/2010, BASE 100: 1990)



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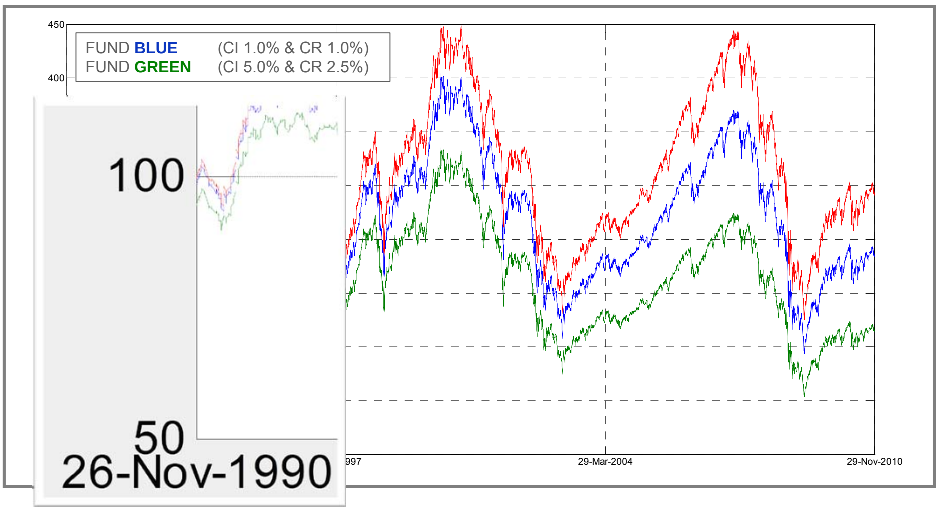
The recommended Investment horizon

DETERMINAZIONE DELL'ORIZZONTE TEMPORALE

STOXX EUROPE 600
(26/11/1990 – 26/11/2010, BASE 100: 1990)



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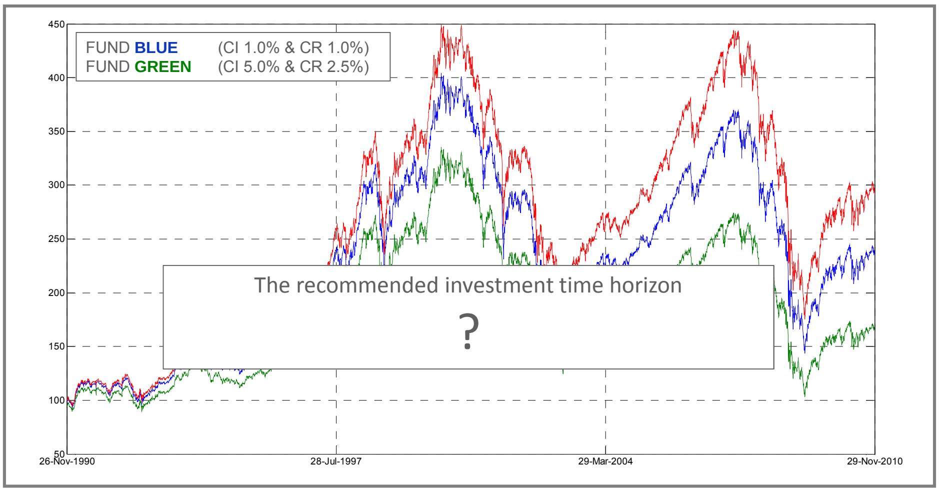
The recommended Investment horizon

DETERMINAZIONE DELL'ORIZZONTE TEMPORALE

STOXX EUROPE 600
(26/11/1990 – 26/11/2010, BASE 100: 1990)



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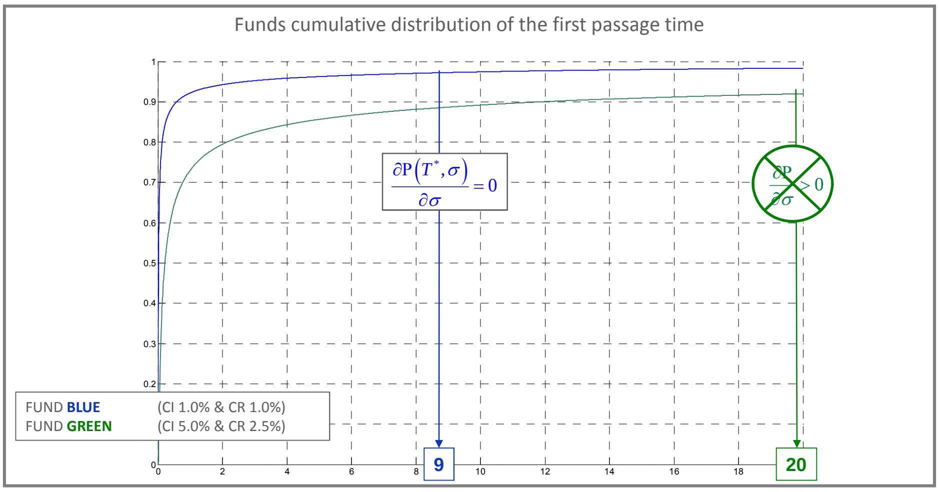
The recommended Investment horizon

DETERMINAZIONE DELL'ORIZZONTE TEMPORALE

STOXX EUROPE 600
(26/11/1990 – 26/11/2010, BASE 100: 1990)



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Syllabus

- Preliminaries: the three pillars
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon
- An Application of the methodology

Examples

DEFAULTABLE BOND

DESCRIPTION	Senior bond with a 5 year maturity, paying bi-annual step-up coupons ranging from 4.7% to 5.30%.
1st PILLAR	See tables below
2nd PILLAR	Degree of Risk: Medium-High
3rd PILLAR	Recommended investment time horizon: 5 years

1st PILLAR:

Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	*Fair value*	96.3
D	Costs	3.7
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

Examples

VPPI PRODUCT

DESCRIPTION	VPPI technique is aimed at protecting the initial value of the financial investment over a specified time horizon and obtaining possible gains by limited exposure to the equity markets.
1st PILLAR	see tables below
2nd PILLAR	Degree of Risk: Medium
3rd PILLAR	Recommended investment time horizon: 5 years

Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	*Fair value*	96.5
D	Costs	3.5
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

Examples

INDEX LINKED CERTIFICATE

DESCRIPTION	The index-linked certificate is characterised by a complex financial engineering that makes intensive use of diverse derivatives components. These derivatives link the performances of the product to the variability of an equity index.
1st PILLAR	see tables below
2nd PILLAR	Degree of Risk: High
3rd PILLAR	Recommended investment time horizon: 5 years

Financial investment table
(*Price Unbundling*)

A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	*Fair value*	96.1
D	Costs	3.9
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

Testimonials

“This book fills the gap that exists between the risk management tools available to industry insiders, and those available to investors. It is a welcome contribution that will be helpful to anyone who needs to assess the risk of non-equity products.”

Jaksa Cvitanic, Professor of Mathematical Finance, Caltech

“Rigor and clarity characterize this methodology to assess the risk of every non-equity product. Well established stochastic techniques are applied in an original way to convey the key information on the time horizon, the degree of risk, the costs and potential returns of the investment and therefore to match the investor’s preferences in terms of liquidity attitude, risk taking, desired returns and acceptable losses.”

Prof. Svetlozar Rachev, Department of Statistics and Applied Probability, University of California at Santa Barbara

“I warmly welcome the publication of this book which describes a probabilistic framework for risk evaluation. The specific aim is that of providing financial institutions and regulators with tools and techniques for an objective and clear representation of key investor information. This shall help in orientating buyers through the difficult path of non-equity products selection.”

Prof. Francesco Corielli, Department of Finance, Bocconi University

"This book constitutes an excellent collection of quantitative methods to the measurement and representation of the risks of non-equity products that comes from a simple but also winning intuition: the information needs of retail investors are not really different from those of financial institutions since they both want the upside gain by trying to contain the downside risk.“

Prof. Hélyette Geman, School of Business, Economics and Informatics, Birkbeck, University of London

"This important book establishes a benchmark for a future financial regulation based on quantitative techniques. At the same time it casts a serious challenge to the financial industry on the need of quantitative disclosure, that will be the future of the financial system worldwide. Hope the challenge will be accepted.“

Prof. Umberto Cherubini, Department of Mathematical Economics, University of Bologna

"This book contains a valid quantitative methodology to shed light on the risks embedded in any non-equity product. By answering the key questions of any investor about the potential performances, the risk rating and the optimal holding time of the product, the three “pillars” of the book are the best candidates to definitely remove the informative lack that worldwide regulators have recognized in the existing rules on risks disclosure. The adoption of these “pillars” would be the ideal completion of the regulatory reform undertaken by the European Authorities regarding the revision of the information contents for Packaged Retail Investment Products. Should the quantitative framework set forth in this work become the reference to update the regulatory framework on transparency, an authentic reversal of the traditional approaches to risks transparency would be realized with effective benefits for investors' comprehension and for allowing them to pick the product that best fits their needs.“

Prof. Riccardo Cesari, Professor of Mathematical Methods for Economic and Financial Sciences, University of Bologna

"This innovative book sheds a light on the dark path of the financial risks intrinsic to non-equity financial products, which are often underestimated, or even poorly understood, by investors seeking higher returns. Mathematical finance techniques are here applied in an original and unconventional manner for the purpose of effectively disclosing these risks and properly assessing their impact on investments' returns.“

Fabio Mercurio, Head of Quant Business Managers at Bloomberg LP and adjunct professor at NYU

http://riskbooks.com/