



A quantitative methodology for risk assessment in financial products

Marcello Minenna

Opinions expressed in this work are exclusively of the author

Syllabus

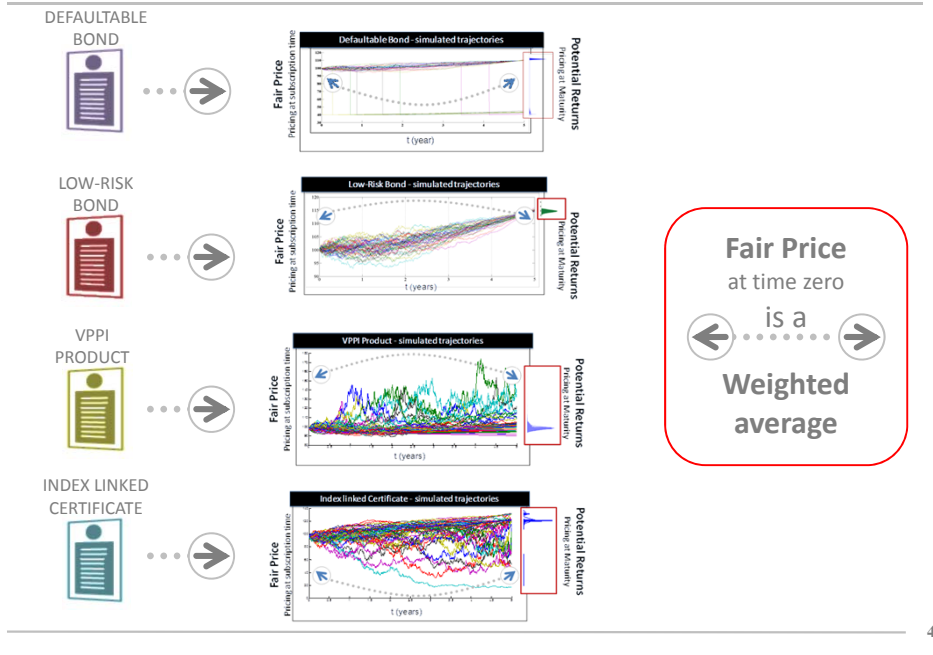
- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon

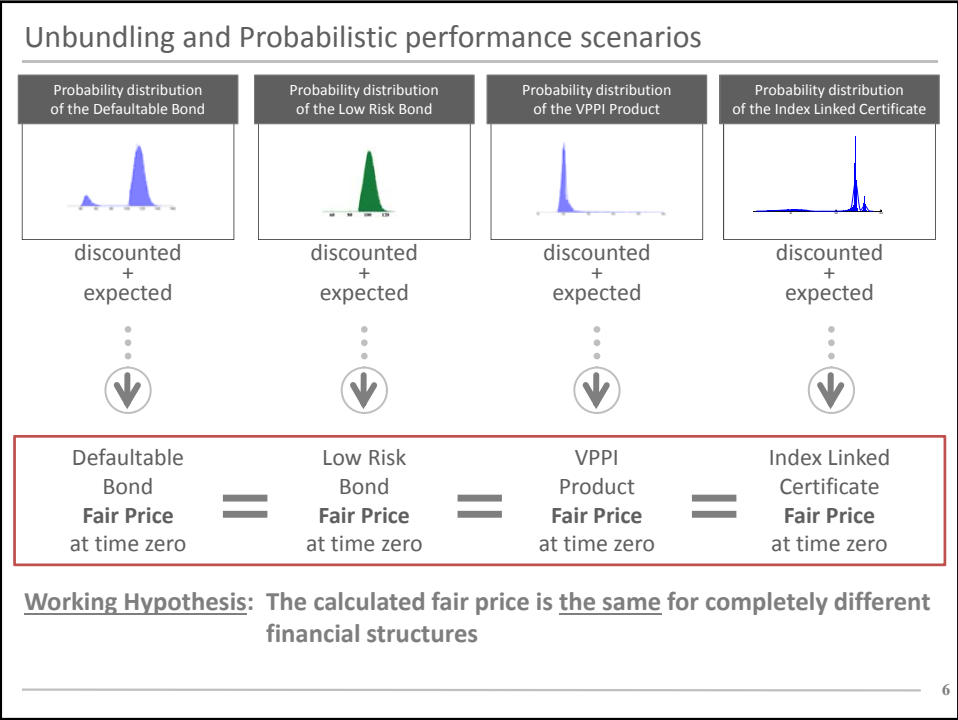
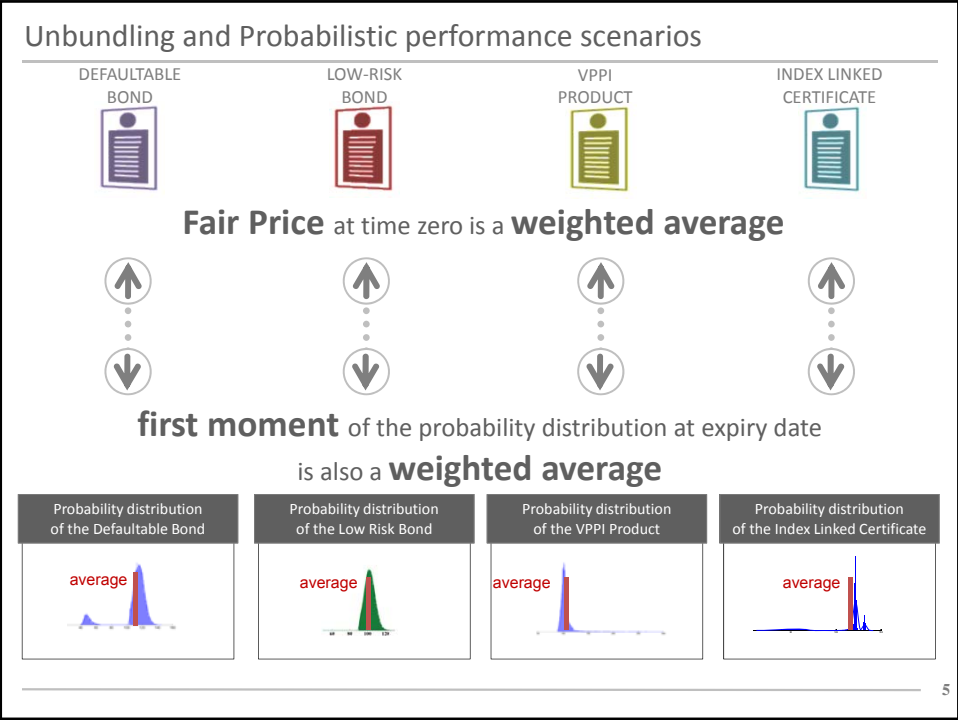
Unbundling and Probabilistic performance scenarios

The returns evaluation requires the estimate of all the relevant risk factors connected with the financial structure of each product

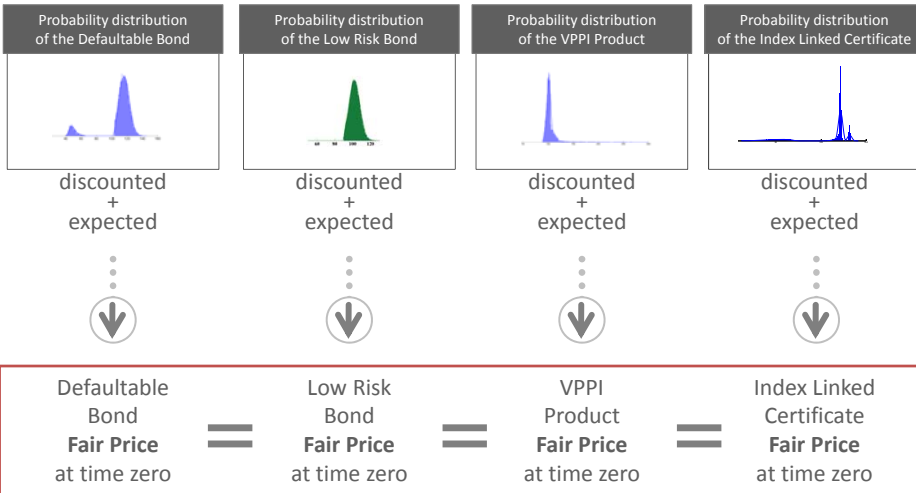


Unbundling and Probabilistic performance scenarios





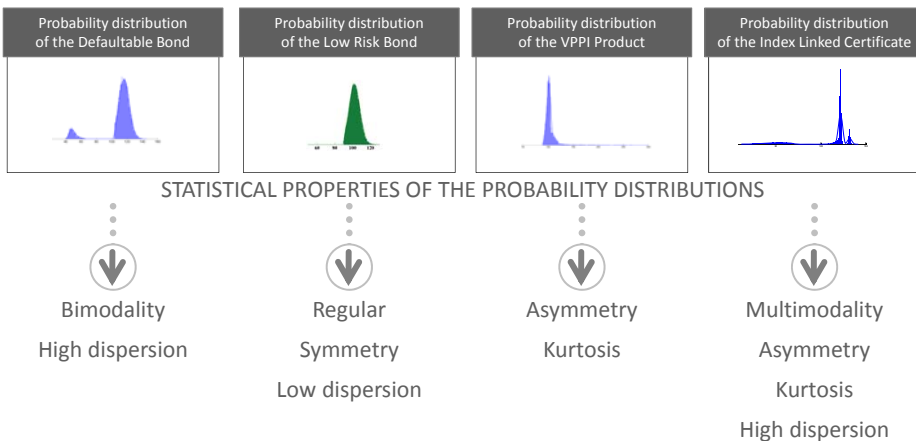
Unbundling and Probabilistic performance scenarios



Question: How much information about the original probability distribution the price will convey in each case analyzed?

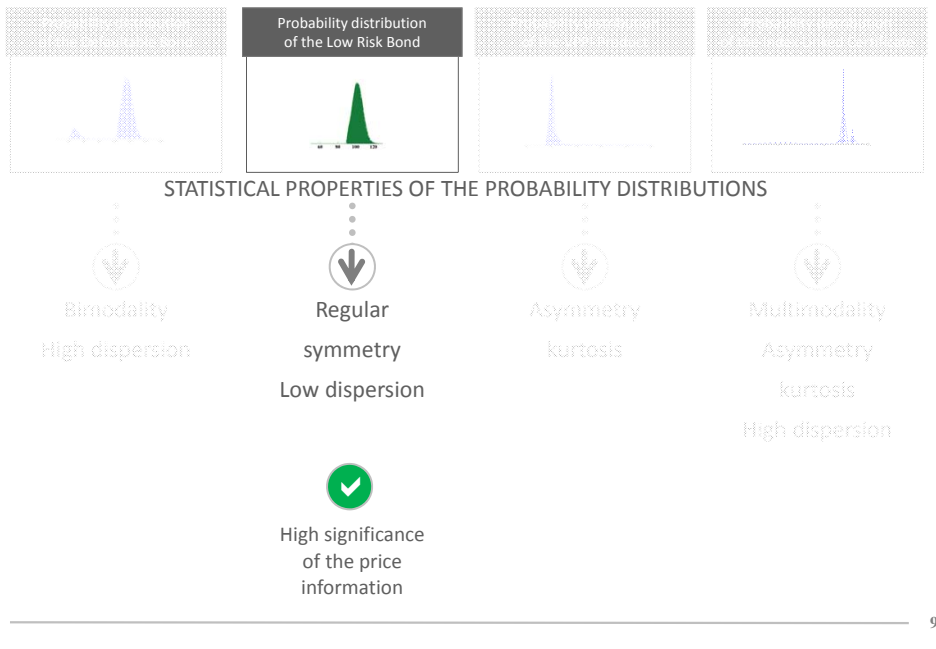
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Unbundling and Probabilistic performance scenarios

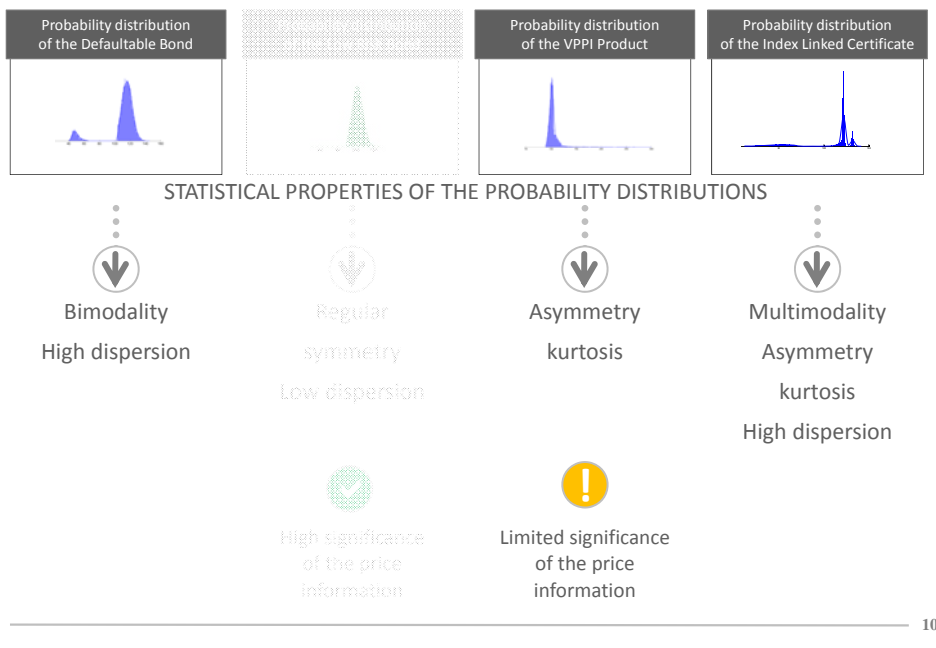


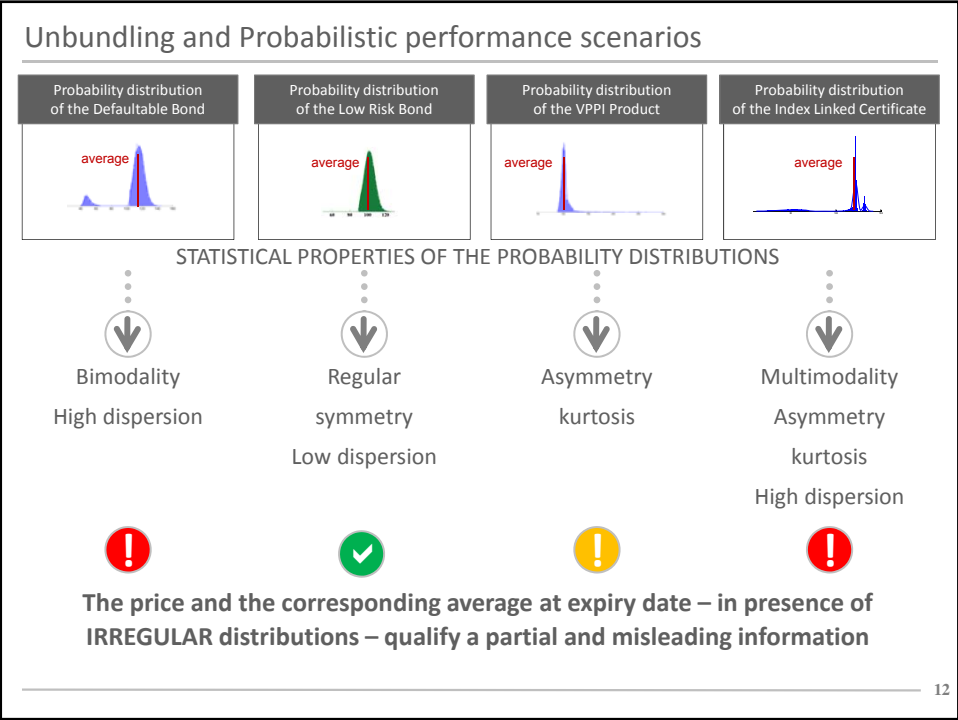
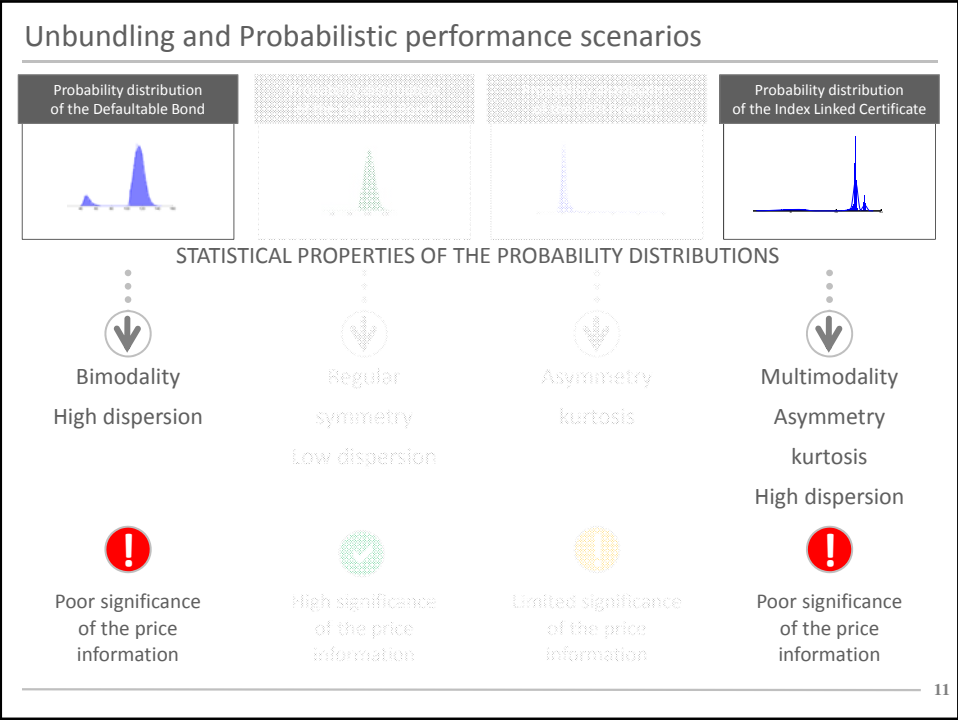
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Unbundling and Probabilistic performance scenarios



Unbundling and Probabilistic performance scenarios





Unbundling and Probabilistic performance scenarios



Significance
of the price
information



As a weighted average, the price is strictly connected with the first moment of the probability distribution

As the literature suggests, in presence of multimodality and irregular shapes for the probability distributions, the number of moments necessary to properly describe the probability distribution increases dramatically.

See:

- (1) Shohat, Tamarkin, 1943 - American Mathematical Survey
- (2) Szego, 1959 - American Mathematical Society
- (3) Totik, 2000 – Journal of Analytical Mathematics
- (4) Gavriiliadis, Athanassoulis, 2009 – Journal of Computational and Applied Mathematics

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Unbundling and Probabilistic performance scenarios



Significance
of the price
information



Mathematical Basis to test the significance of the price information

Given a finite number of moments $2k$, it's possible to derive the following approximate relationship between the probability function $f(x)$ and its Christoffel function of degree k :


$$f(x) \approx f_{AP,k}(x) = \frac{k}{c_0 \pi \sqrt{(x-a)(b-x)}} \lambda_k(x)$$

with $x \in [a, b]$. c_0 is a normalization factor

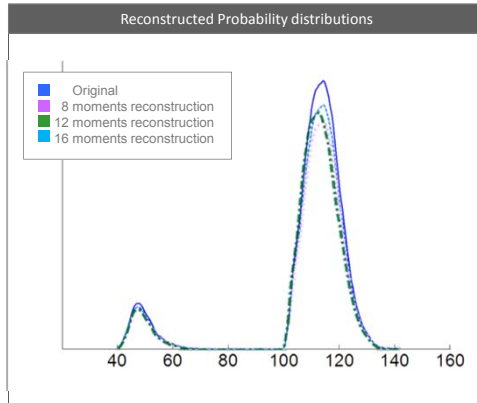


It's then immediate to apply the approximating formula for different values of k in order to test the accuracy of the approximation for the probability distributions corresponding to our different financial products


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 Bimodality
High dispersion

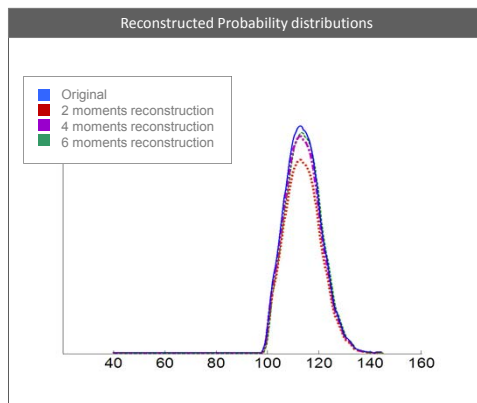
Significance test of the price information



At least 16 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

 Regular
symmetry
Low dispersion

Significance test of the price information

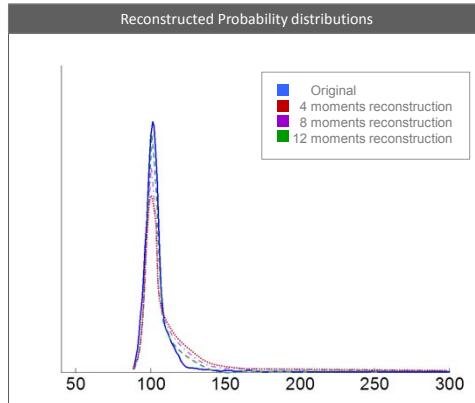


Only 4 moments are sufficient in order to describe properly the original distribution. The information content of the first moment can be considered adequate.

Unbundling and Probabilistic performance scenarios

Asymmetry
kurtosis

Significance test of the price information

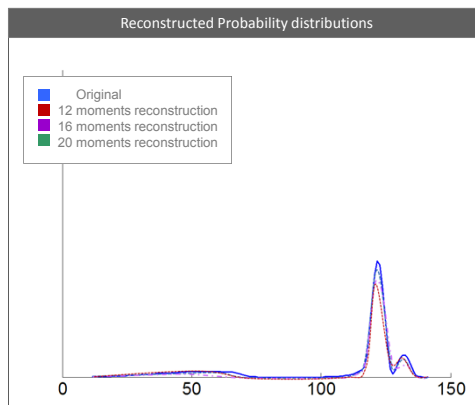


12 moments describe correctly the pattern of the original distribution. The information content of the first moment needs to be integrated.

Unbundling and Probabilistic performance scenarios

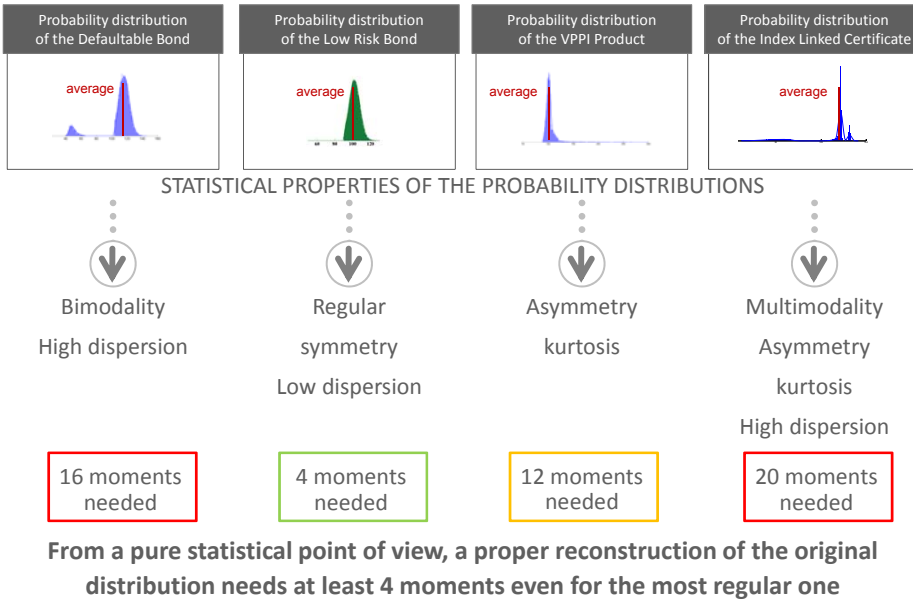
Multimodality
Asymmetry
kurtosis
High dispersion

Significance test of the price information



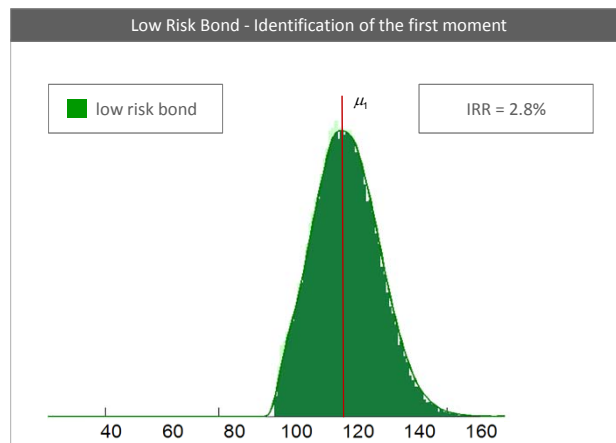
At least 20 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

Unbundling and Probabilistic performance scenarios



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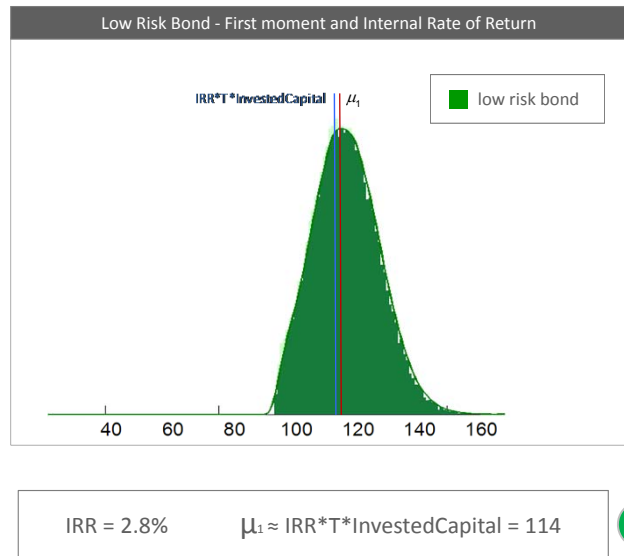
Unbundling and Probabilistic performance scenarios



IRR
← … →
First moment of the probability distribution

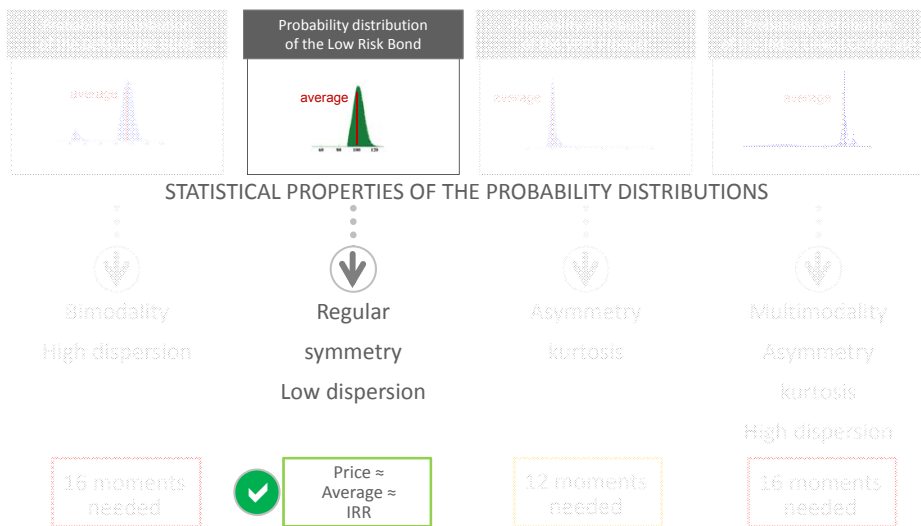
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Unbundling and Probabilistic performance scenarios



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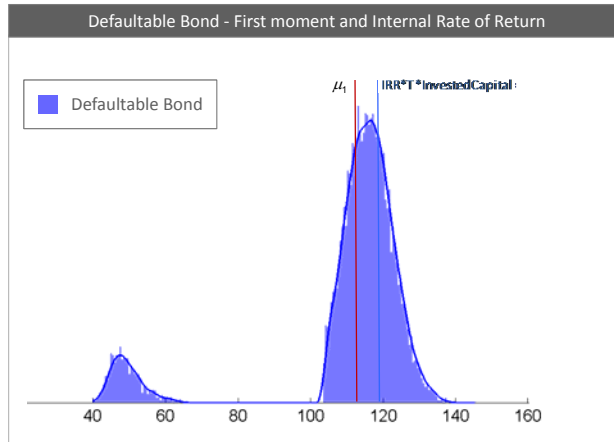
Unbundling and Probabilistic performance scenarios



Even if 4 moments are needed for a proper reconstruction of the probability distribution, the average and its related measures (IRR and price), convey sufficient information for the investor decision process

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Unbundling and Probabilistic performance scenarios



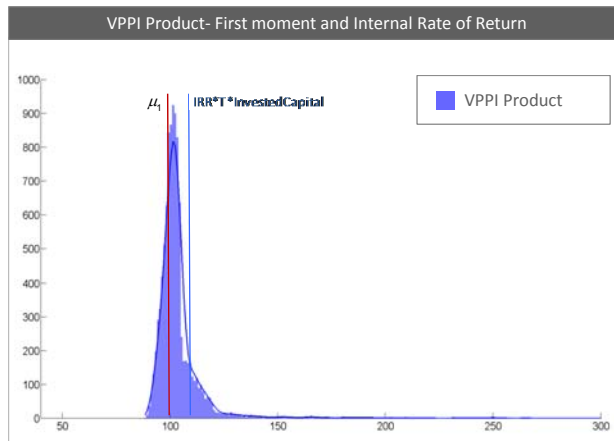
IRR = 3.85%

$\mu_1 \neq \text{IRR} * T * \text{InvestedCapital} = 119.25$



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Unbundling and Probabilistic performance scenarios



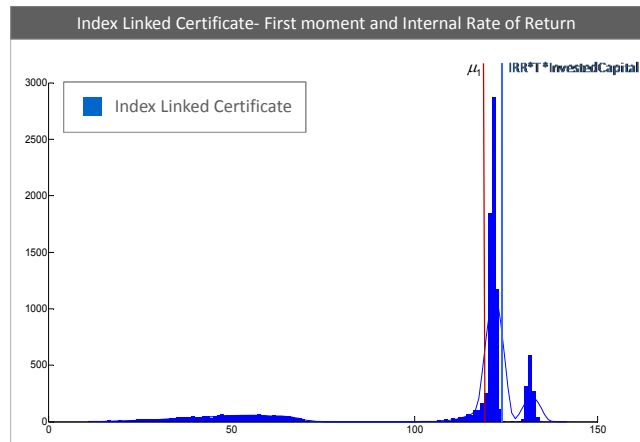
IRR = 2.53%

$\mu_1 \neq \text{IRR} * T * \text{InvestedCapital} = 112.65$



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Unbundling and Probabilistic performance scenarios



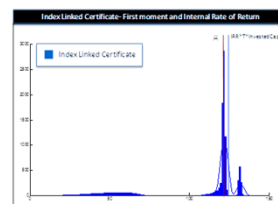
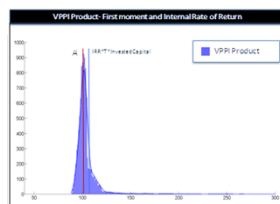
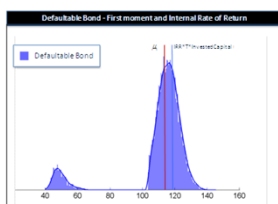
IRR = 5.91%

$\mu_1 \neq \text{IRR} * T * \text{InvestedCapital} = 129.55$



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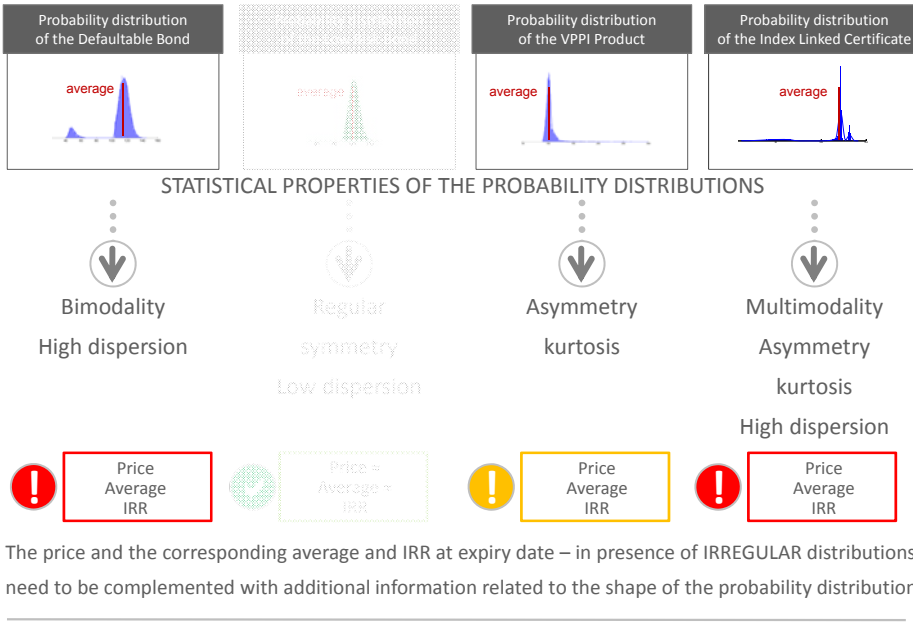
Unbundling and Probabilistic performance scenarios



For more complex financial structures, the average progressively loses its connection with the internal rate of return of the investment, so reducing its usefulness as an effective tool for the decision process

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Unbundling and Probabilistic performance scenarios



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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



be easy to understand



capture efficiently all the main statistical characteristics of the probability distribution of the product

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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



be easy to understand



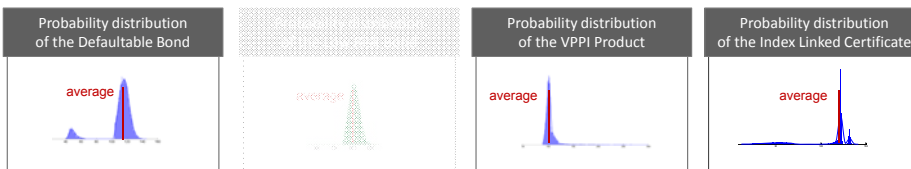
capture efficiently all the main statistical characteristics of the probability distribution of the product



Proposal 1: Convey the entire probability distribution

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Unbundling and Probabilistic performance scenarios



MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



2 Factor Short Interest Rate Hull-White Model

Short Interest Rate Cox Ingersoll Ross Model



Heston Stochastic Volatility Model for the Equity component

Barndorff Nielsen Normal Inverse Gaussian Model for the Equity component



Merton Jump Diffusion Model for the Equity component

Variance Gamma Model for the Equity component

The shape of the probability distribution of the potential returns is obviously dependent on the modelling assumptions.

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Unbundling and Probabilistic performance scenarios

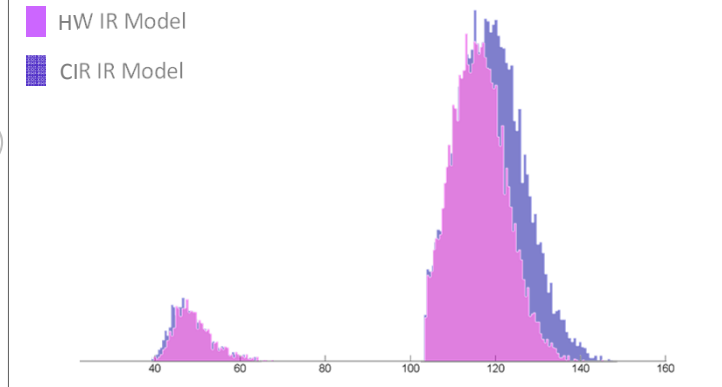
DEFAULTABLE
BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Probability distributions of the final values of the defaultable bond

- HW IR Model
- CIR IR Model



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Unbundling and Probabilistic performance scenarios

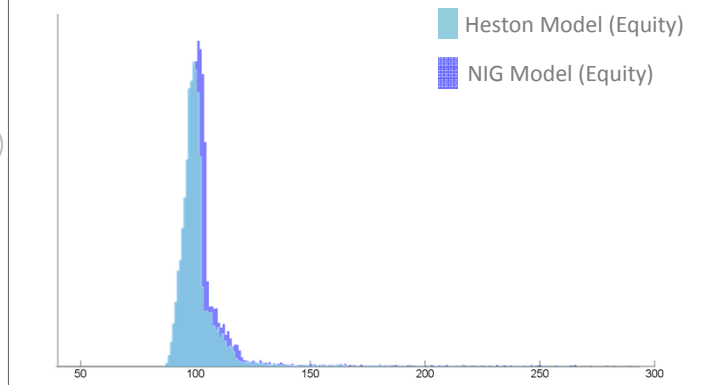
VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Probability distributions of the final values of the VPPI product

- Heston Model (Equity)
- NIG Model (Equity)



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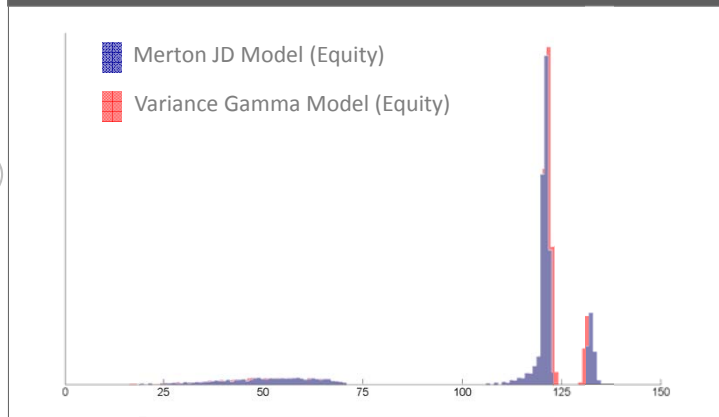
Unbundling and Probabilistic performance scenarios

INDEX LINKED
CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Probability distributions of the final values of the Index Linked Certificate



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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



be easy to understand



the probability distribution is an abstract object not easy to handle



capture efficiently all the main statistical characteristics of the probability distribution of the product



the shape of the probability distribution is dependent on the modelling assumptions



Proposal 1: Convey the entire probability distribution

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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



be easy to understand



capture efficiently all the main statistical characteristics of the probability distribution of the product



Proposal 2: Unbundling the information content of the price

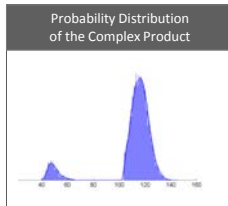
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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT



Unbundling the information content of the price



DISCOUNTED
EXPECTED
VALUE



Fair Value
(Complex Product)

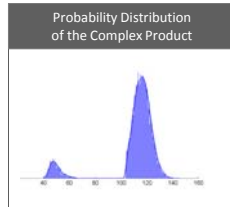
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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Unbundling the information content of the price

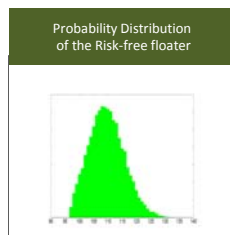


DISCOUNTED EXPECTED VALUE

Fair Value (Complex Product)

A risk-free floater with same fair value and coupon payment dates of the complex product is defined

=



DISCOUNTED EXPECTED VALUE

Fair Value (Risk-free floater)

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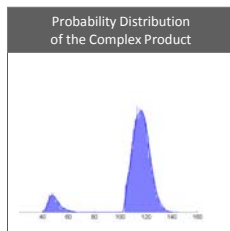
Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



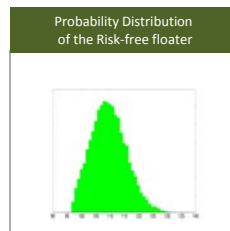
Unbundling the information content of the price

Any non-elementary return-target product can be replicated by a portfolio composed of the associated risk-free floater and of a zero-value swap which transforms the cash flow structure of the risk-free security into the cash flow structure of the product itself, ie, denoting by $\{swap_t\}_{t \in [0, T]}$ the value process of the swap



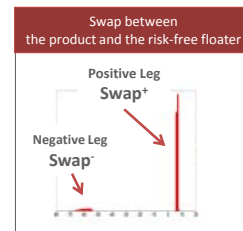
Fair Value (Complex Product)

=



Fair Value (Risk-free floater)

+



Fair Value (Swap = 0)

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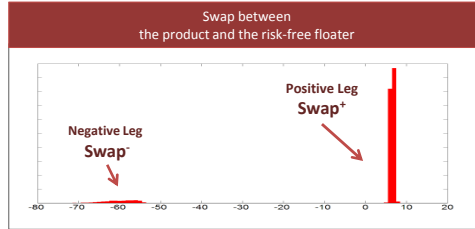
Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Fair Value
(Swap = 0)

Unbundling the information content of the price



$$|FV(\text{Swap}^-)| = |FV(\text{Swap}^+)|$$



Theoretical value of the Risky component

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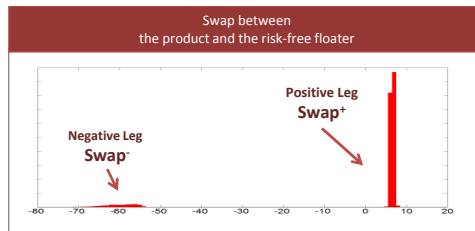
Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Fair Value
(Swap = 0)

Unbundling the information content of the price



$$|FV(\text{Swap}^-)| = |FV(\text{Swap}^+)|$$



Theoretical value of the Risky component



C	Fair value
B	Theoretical value of the Risky component
A=C-B	Theoretical value of the Risk-Free component

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Unbundling and Probabilistic performance scenarios

Financial investment table (Price Unbundling)

DEFAULTABLE BOND



A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	<i>Fair value</i>	96.3
D	Costs	3.7
E = C + D	Issue price	100

VPPI PRODUCT



A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	<i>Fair value</i>	96.5
D	Costs	3.5
E = C + D	Issue price	100

INDEX LINKED CERTIFICATE



A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	<i>Fair value</i>	96.1
D	Costs	3.9
E = C + D	Issue price	100

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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



✓ be easy to understand



capture efficiently ~~all~~ the main statistical characteristics of the probability distribution of the product

the unbundling represented by using a table is first level tool useful to appreciate the impact of the costs and the riskiness of the product

The unbundling exploits only the information contained in the first order moment of the probability distribution



Proposal 2: Unbundling the information content of the price

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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



be easy to understand



capture efficiently all the main statistical characteristics of the probability distribution of the product



Proposal 3: Perform a reduction in granularity by implementing a partition of the probability distribution

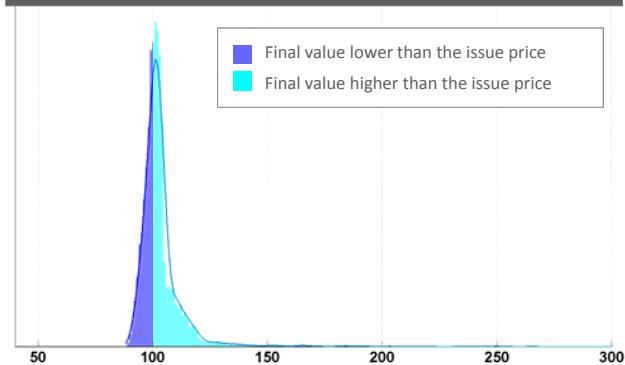
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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT



Partition of the Probability distribution of the Complex Product with respect to the point of zero return



The assessment of the probability of recovering at least the amount paid for the product is of great significance for the investor.

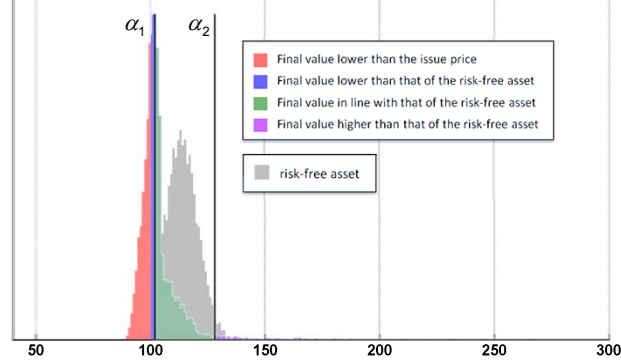
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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



It is appropriate to explore further partitions of the macro-event “the final value of the investment is higher than the issue price” by performing a direct comparison with the final values of the risk-free asset.

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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2

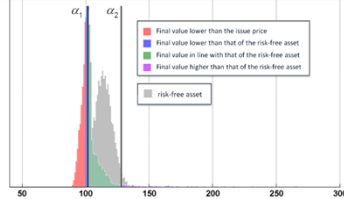


Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>
The performance is <u>positive but lower than the risk-free asset</u>
The performance is <u>positive and in line with the risk-free asset</u>
The performance is <u>positive and higher than the risk-free asset</u>

MEAN VALUES



$$E^P(S_T | S_T < 100) = \frac{1}{P(S_T < 100)} \int_{-\infty}^{100} x f_{S_T}(x) dx$$

$$E^P(S_T | 100 \leq S_T < \alpha_1) = \frac{1}{P(100 \leq S_T < \alpha_1)} \int_{100}^{\alpha_1} x f_{S_T}(x) dx$$

$$E^P(S_T | \alpha_1 \leq S_T < \alpha_2) = \frac{1}{P(\alpha_1 \leq S_T < \alpha_2)} \int_{\alpha_1}^{\alpha_2} x f_{S_T}(x) dx$$

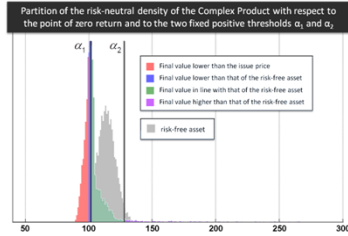


$$E^P(S_T | S_T \geq \alpha_2) = \frac{1}{P(S_T \geq \alpha_2)} \int_{\alpha_2}^{+\infty} x f_{S_T}(x) dx$$

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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Benefits of this solution:

1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information and the table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;

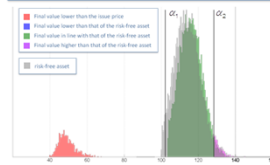
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Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND



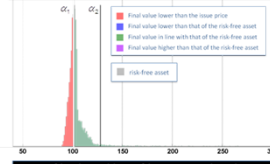
Partition of the risk-neutral density of the non-equity product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



VPPI PRODUCT



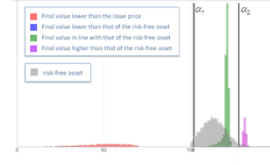
Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



INDEX LINKED CERTIFICATE



Partition of the risk-neutral density of the non-equity product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	9.5%	49.3
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	87.4%	115.6
The performance is <u>positive and higher</u> than the risk-free asset	3.1%	131.1

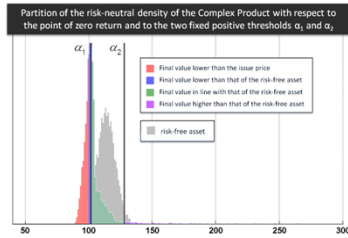
SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	36.9%	96.9
The performance is <u>positive but lower</u> than the risk-free asset	18.5%	101
The performance is <u>positive and in line</u> with the risk-free asset	39.9%	107.1
The performance is <u>positive and higher</u> than the risk-free asset	4.7%	195.5

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	18.9%	49.1
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	68.9%	120.9
The performance is <u>positive and higher</u> than the risk-free asset	12.2%	131.6

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Unbundling and Probabilistic performance scenarios

COMPLEX PRODUCT



Benefits of this solution:

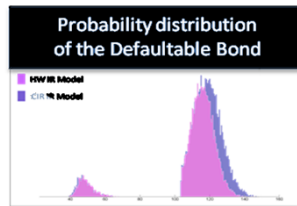
1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information; The table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;
2. The model risk arising from the different proprietary models of the issuers has a limited impact.

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Unbundling and Probabilistic performance scenarios

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Difference less than 2%

HW IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	9.5%	49.3
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	87.4%	115.6
The performance is <u>positive and higher</u> than the risk-free asset	3.1%	131.1

CIR IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	8.3%	49.9
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	86.8%	117.9
The performance is <u>positive and higher</u> than the risk-free asset	4.9%	135.4

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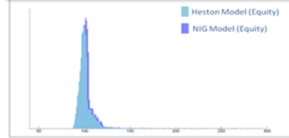
Unbundling and Probabilistic performance scenarios

VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Probability distribution of the VPPI Product



Difference less than 2%

HESTON MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	38.9%	95.5
The performance is <u>positive but lower</u> than the risk-free asset	18.9%	100.2
The performance is <u>positive and in line</u> with the risk-free asset	38.4%	106.3
The performance is <u>positive and higher</u> than the risk-free asset	3.8%	182.5

NIG MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	36.9%	96.9
The performance is <u>positive but lower</u> than the risk-free asset	18.5%	101
The performance is <u>positive and in line</u> with the risk-free asset	39.9%	107.1
The performance is <u>positive and higher</u> than the risk-free asset	4.7%	195.5

51

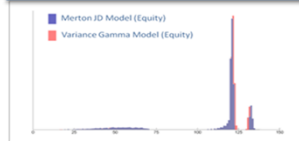
Unbundling and Probabilistic performance scenarios

INDEX LINKED CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS



Probability distribution of the Index Linked Certificate



Difference less than 4%

MERTON JD MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	18.9%	48.2
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	65.8%	117.6
The performance is <u>positive and higher</u> than the risk-free asset	15.3%	132.7

VARIANCE GAMMA MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	18.9%	49.1
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	68.9%	120.9
The performance is <u>positive and higher</u> than the risk-free asset	12.2%	131.6

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Unbundling and Probabilistic performance scenarios

! COMPLEX PRODUCT

The additional information to be supplemented must



✓ be easy to understand



✓ capture efficiently all the main statistical characteristics of the probability distribution of the product

the partition should be done by choosing events that have a strong financial meaning

the reduction in granularity mitigates in a significant way the model risk



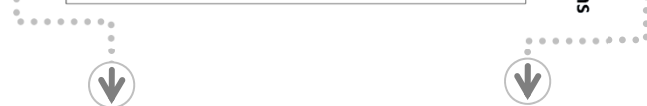
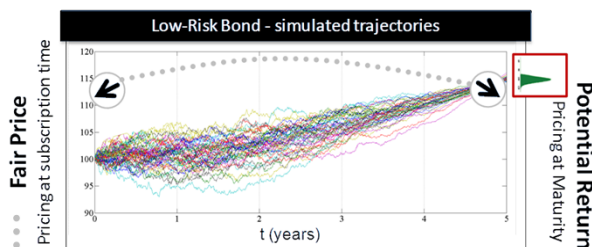
Proposal 3: Perform a reduction in granularity by implementing a partition of the probability distribution

53

Unbundling and Probabilistic performance scenarios

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

COMPLEX PRODUCT



Financial investment table
(Price Unbundling)

A	Theoretical value of the Risk-Free component	
B	Theoretical value of the Risky component	
C = A + B	Fair value	
D	Costs	
E = C + D	Issue price	

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <i>negative</i>
The performance is <i>positive but lower</i> than the risk-free asset
The performance is <i>positive and in line</i> with the risk-free asset
The performance is <i>positive and higher</i> than the risk-free asset

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Unbundling and Probabilistic performance scenarios

This approach allows to explain the “CONVERTIBILITY RISK” that actually affects the pricing of European sovereign bond.

Market quotes the event of the breaking of the Eurozone.



PIIGS countries suffer
DEVALUATION RISK



CORE countries take advantage of
APPRECIATION RISK

55

Unbundling and Probabilistic performance scenarios

This approach allows to explain the “CONVERTIBILITY RISK” that actually affects the pricing of European sovereign bond.

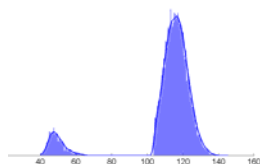
Market quotes the event of the breaking of the Eurozone.



PIIGS countries suffer
DEVALUATION RISK



CORE countries take advantage of
APPRECIATION RISK



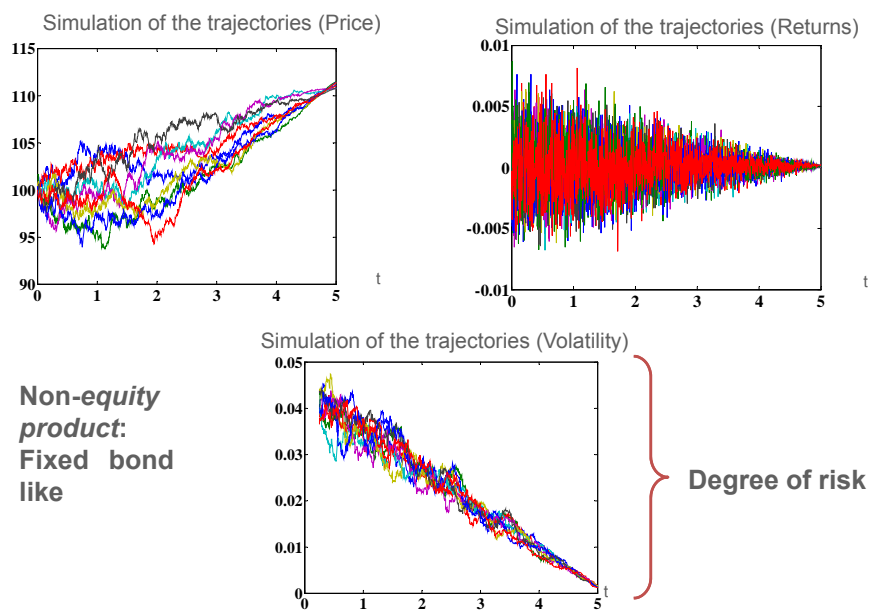
56

Syllabus

- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon

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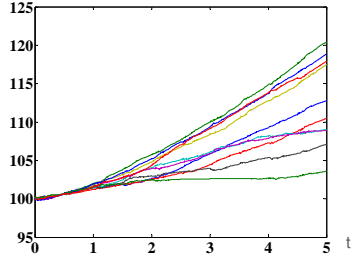
Synthetic risk indicator



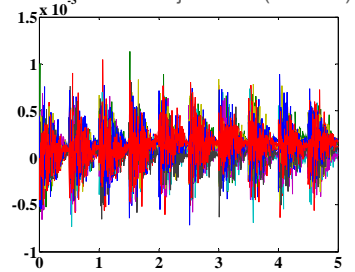
58

Synthetic risk indicator

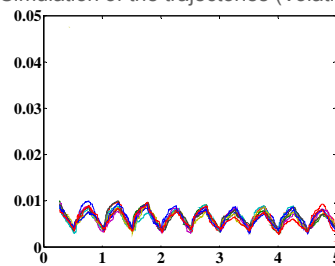
Simulation of the trajectories (Price)



Simulation of the trajectories (Returns)



Simulation of the trajectories (Volatility)



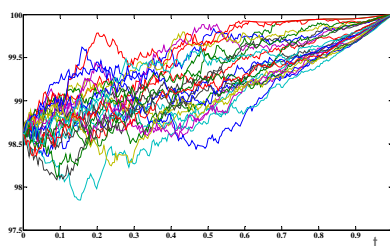
**Non-equity
product:
Floater
bond like**

Degree of risk

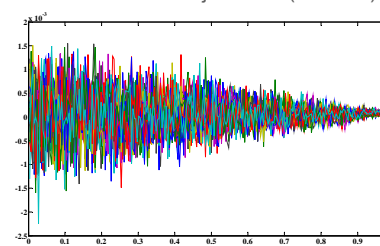
59

Synthetic risk indicator

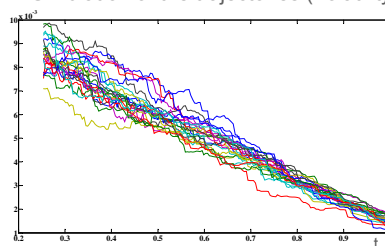
Simulation of the trajectories (Price)



Simulation of the trajectories (Returns)



Simulation of the trajectories (Volatility)



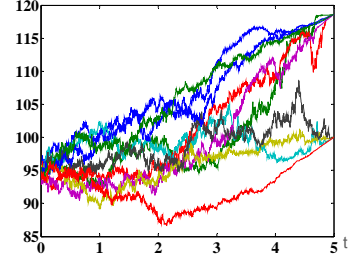
**Non-equity
product:
Zero
Coupon
Bond**

Degree of risk

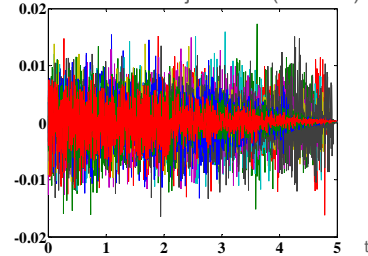
60

Synthetic risk indicator

Simulation of the trajectories (Price)

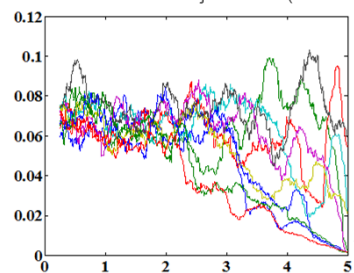


Simulation of the trajectories (Returns)



**Non-equity
product:
Structured
Bond**

Simulation of the trajectories (Volatility)



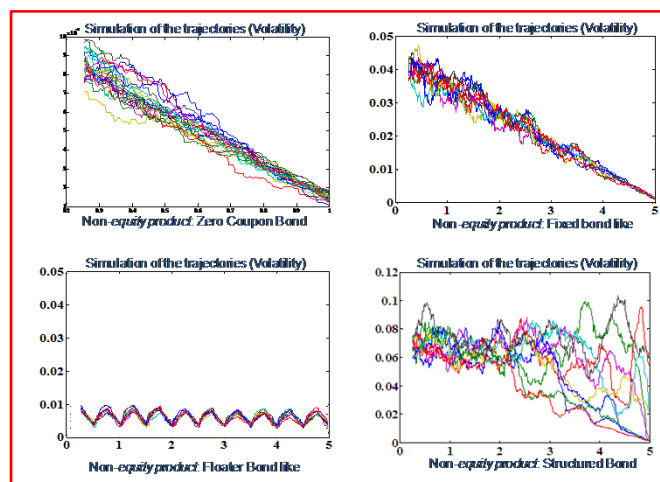
Degree of risk

61

Synthetic risk indicator

COMPLEXITY FOR RETAIL INVESTORS

The volatility patterns are abstract objects that an average investor cannot handle.



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Synthetic risk indicator

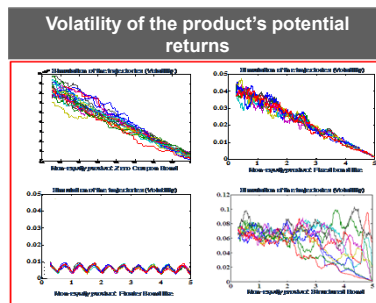
Conversely, a table with qualitative labels that characterizes the risk classes is very easy to understand

Risk Classes
Very Low
Low
Medium-Low
Medium
Medium-High
High
Very High

The assignment of the degree of risk is made according to a quantitative criterion that maps coherently any volatility interval into a corresponding qualitative risk class

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Synthetic risk indicator



DEGREE OF RISK

MEASUREMENT:
product's positioning inside
a grid of n volatility intervals

REPRESENTATION:
mapping of any volatility interval into
a corresponding qualitative risk class

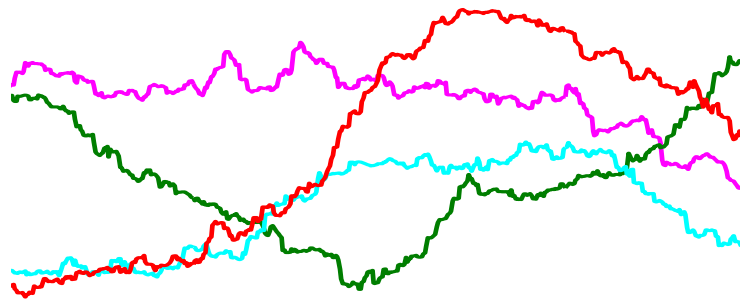


Risk Classes	Volatility Intervals	
Very Low	$\sigma_{1,min}$	$\sigma_{1,max}$
Low	$\sigma_{2,min}$	$\sigma_{2,max}$
Medium-Low	$\sigma_{3,min}$	$\sigma_{3,max}$
Medium	$\sigma_{4,min}$	$\sigma_{4,max}$
Medium-High	$\sigma_{5,min}$	$\sigma_{5,max}$
High	$\sigma_{6,min}$	$\sigma_{6,max}$
Very High	$\sigma_{7,min}$	$\sigma_{7,max}$

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Synthetic risk indicator

Products with the same risk budget
must have the same degree of risk

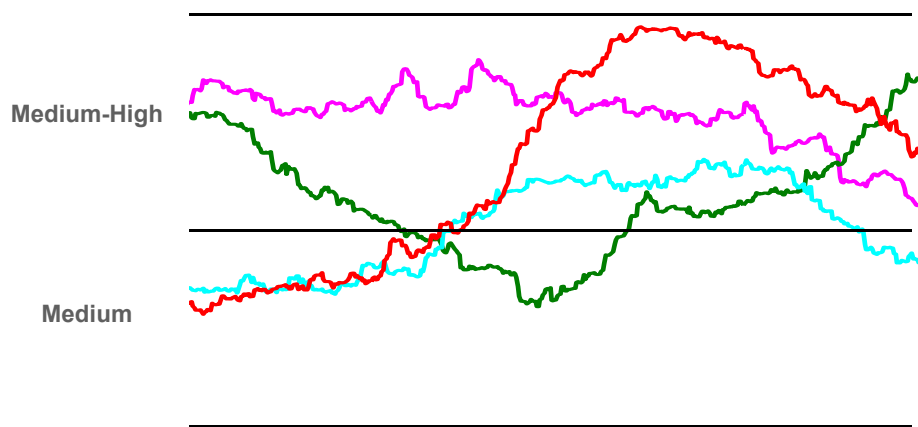


σ_t

65

Synthetic risk indicator

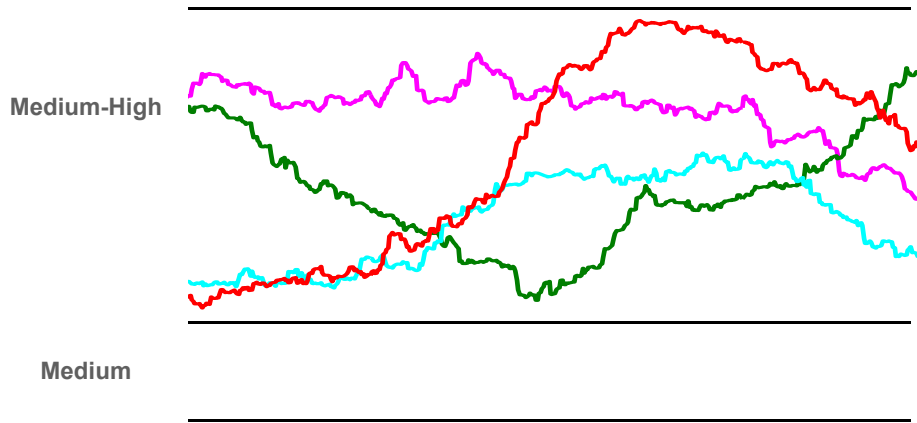
Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations



66

Synthetic risk indicator

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations



67

Synthetic risk indicator

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations

THE ISSUE

Defining suitable requirements to partition the
volatility space $[0, +\infty)$ into an optimal number n^* of
subsequent intervals with optima extrema



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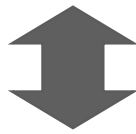
Synthetic risk indicator

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations

Requirement n.1

the **optimal grid** of volatility intervals
has to be **consistent** with the **principle**:

+ RISK + LOSSES



**VOLATILITY INTERVALS MUST HAVE
AN INCREASING WIDTH IN ABSOLUTE TERMS**

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Synthetic risk indicator

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations

Requirement n.2

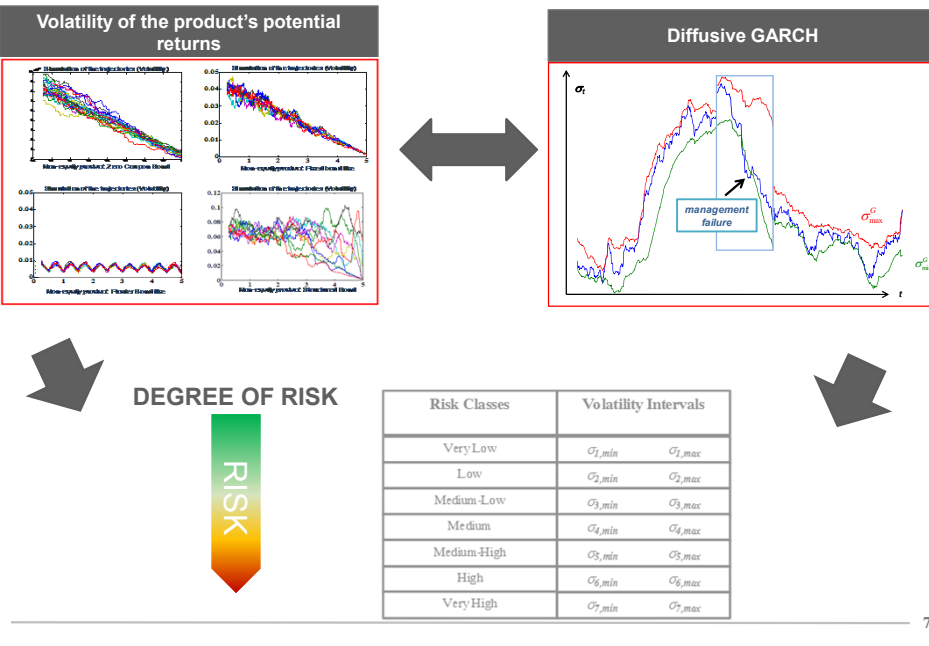
the optimal grid of volatility intervals must be
market feasible



**REALIZED VOLATILITY CONSISTENT WITH MARKET
EXPECTATIONS OF FUTURE VOLATILITY**
(UNLESS FOR SIGNIFICANT SUDDEN SHOCKS)

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Synthetic risk indicator



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Synthetic risk indicator

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

The optimal grid of volatility intervals is consistent with the 1st requirement:

+ RISK + LOSSES

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Syllabus

- Unbundling and Probabilistic performance scenarios
- Synthetic risk indicator
- The optimal time horizon

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Recommended Investment Time Horizon

The recommended investment time horizon

The event to study from a probabilistic point of view is related to possible exit strategies after having recovered all the costs of the product :

The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

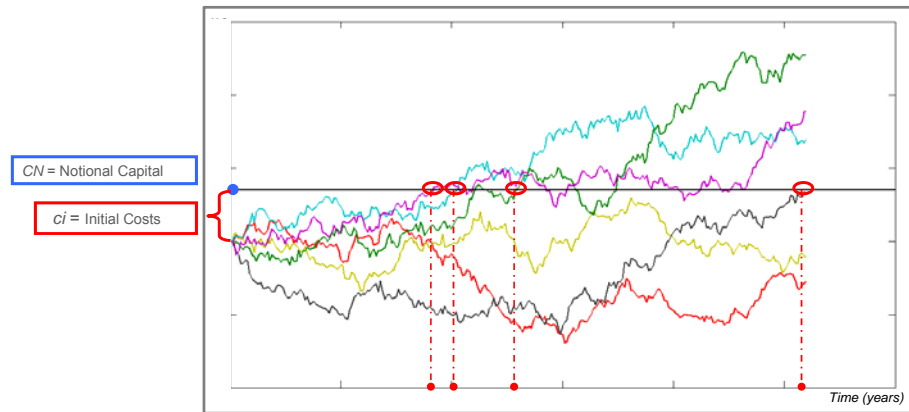
First Passage Time for the cost recovery barrier

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Recommended Investment Time Horizon

First Passage Time:

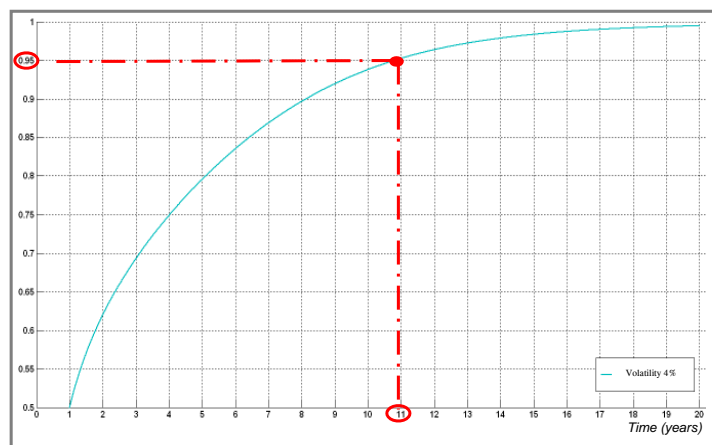
First time (expressed in years) such that the value of the Invested Capital (C) recovers the initial costs and off-sets the running costs.



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Recommended Investment Time Horizon

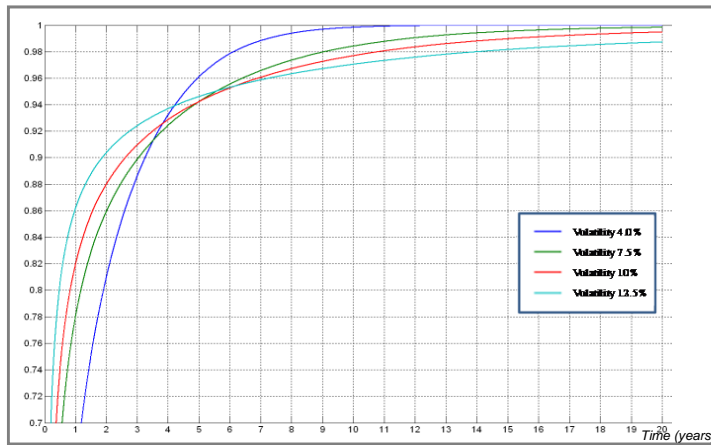
The confidence level α uniquely identifies T^* on the cumulative distribution function of the first passage times:



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Recommended Investment Time Horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



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Recommended Investment Time Horizon

minimum investment time horizon ...

$$T^* = \left\{ T \in \mathfrak{R}^+ : P[t^* \leq T] = \alpha \right\}$$

.... must be coherent with the principle

+ VOLATILITY + TIME HORIZON

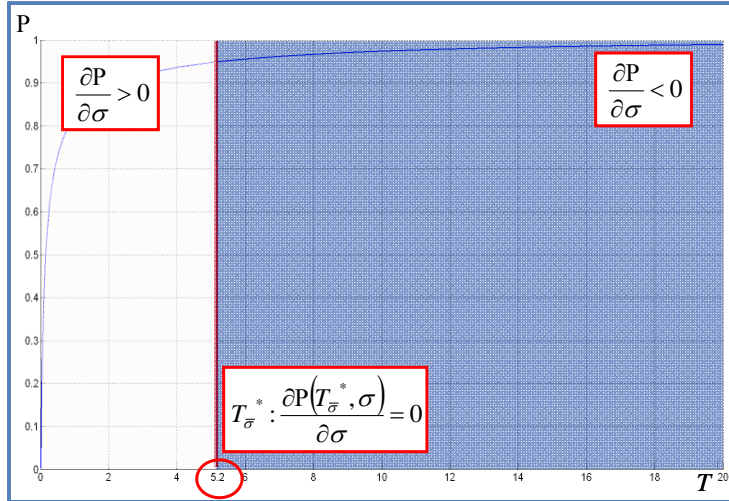
$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

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Recommended Investment Time Horizon

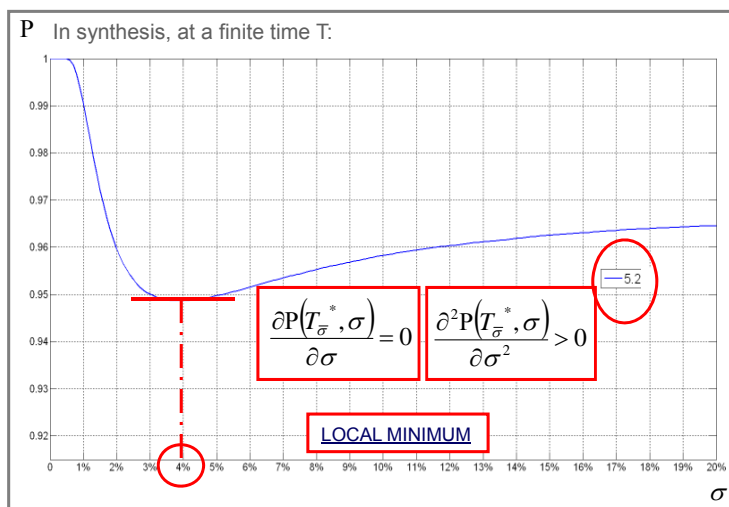
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



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Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



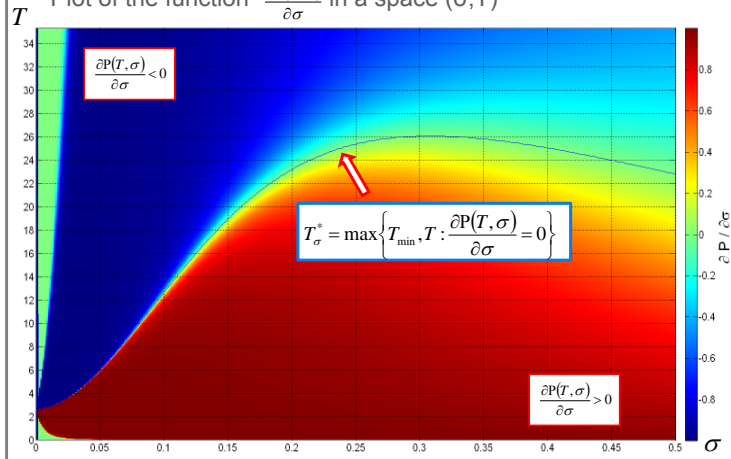
80

Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS

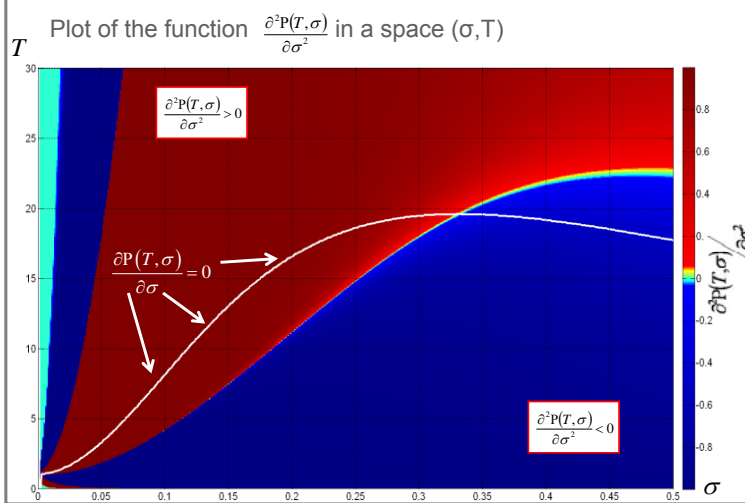
Plot of the function $\frac{\partial P(T, \sigma)}{\partial \sigma}$ in a space (σ, T)



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Recommended Investment Time Horizon

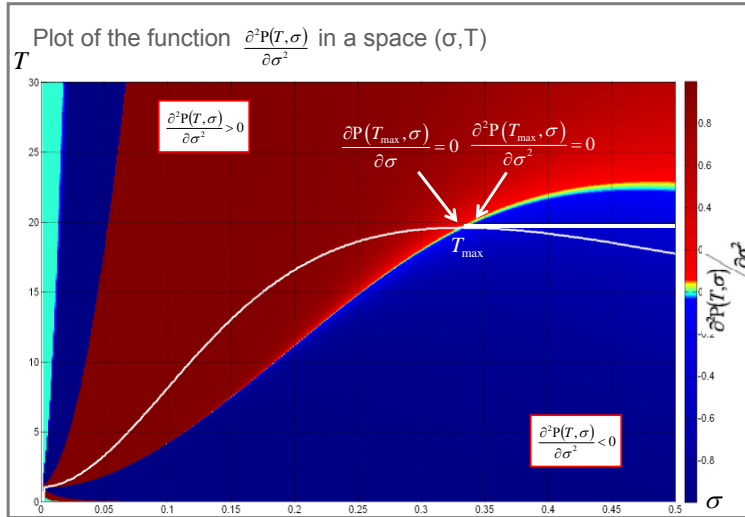
Searching the minimum: the 2nd order condition



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Recommended Investment Time Horizon

DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



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Examples

DEFAULTABLE BOND

DESCRIPTION Senior bond with a 5 year maturity, paying bi-annual step-up coupons ranging from 4.7% to 5.30%.

Financial investment table (Price Unbundling)

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	Fair value	96.3
D	Costs	3.7
E = C + D	Issue price	100

1st PILLAR

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	9.5%	49.3
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	87.4%	115.6
The performance is <u>positive and higher</u> than the risk-free asset	3.1%	131.1

2nd PILLAR Degree of Risk: Medium-High

3rd PILLAR Recommended investment time horizon: 5 years

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Examples



VPPI PRODUCT

DESCRIPTION VPPI technique is aimed at protecting the initial value of the financial investment over a specified time horizon and obtaining possible gains by limited exposure to the equity markets.

1st PILLAR

Financial investment table (Price Unbundling)

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	Fair value	96.5
D	Costs	3.5
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	36.9%	96.9
The performance is <u>positive but lower</u> than the risk-free asset	18.5%	101
The performance is <u>positive and in line</u> with the risk-free asset	39.9%	107.1
The performance is <u>positive and higher</u> than the risk-free asset	4.7%	195.5

2nd PILLAR

Degree of Risk: Medium

3rd PILLAR

Recommended investment time horizon: 5 years

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Examples



INDEX LINKED CERTIFICATE

DESCRIPTION The index-linked certificate is characterised by a complex financial engineering that makes intensive use of diverse derivatives components. These derivatives link the performances of the product to the variability of an equity index.

1st PILLAR

Financial investment table (Price Unbundling)

A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	Fair value	96.1
D	Costs	3.9
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is <u>negative</u>	18.9%	49.1
The performance is <u>positive but lower</u> than the risk-free asset	0.0%	-
The performance is <u>positive and in line</u> with the risk-free asset	68.9%	120.9
The performance is <u>positive and higher</u> than the risk-free asset	12.2%	131.6

2nd PILLAR

Degree of Risk: High

3rd PILLAR

Recommended investment time horizon: 5 years

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