CONSOB
COMMISSIONE NAZIONALE PER LE SOCIETÀ E LA BORSA

Comprehension and comparison of structured products: simplicity or yield?

Marcello Minenna
Head of the Quantitative Analysis and Financial Innovation Unit
CONSOB

Structured products EUROPE

Syllabus

- Shapes and patterns of probability distributions for different structured products
- Recognizing and assessing the main drivers of complexity
- Significance Tests
- Unveiling Risk and Opportunities to Investors
  - Proposal 1
  - Proposal 2
  - Proposal 3
- Conclusions

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Shapes and patterns of probability distributions

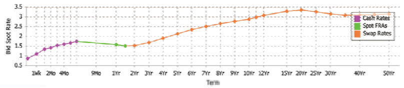
The analysis of implied probability distributions requires the estimate of all the relevant risk factors connected with the financial structure of each product

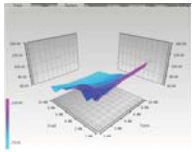
DEFAULTABLE BOND	LOW-RISK BOND	VPPI PRODUCT	INDEX-LINKED CERTIFICATE
Interest Rate Volatility	Interest Rate Volatility	Interest Rate Volatility	Interest Rate Volatility
Significant exposure to credit risk	Limited exposure to credit risk	Limited exposure to Market risk	Significant exposure to Market risk

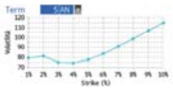
Shapes and patterns of probability distributions

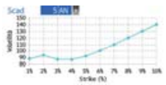
DEFAULTABLE BOND

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product







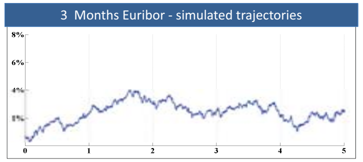


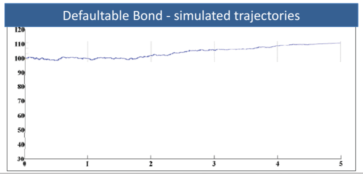
Interest Rate Volatility

Significant exposure to credit risk

Shapes and patterns of probability distributions

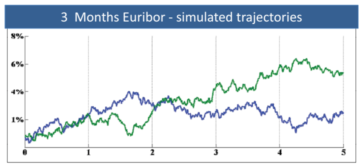
DEFAULTABLE BOND

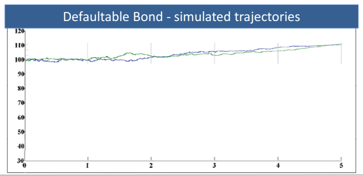




Shapes and patterns of probability distributions

DEFAULTABLE BOND





Shapes and patterns of probability distributions

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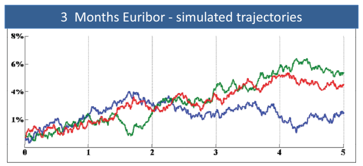

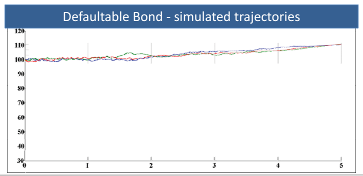

Shapes and patterns of probability distributions

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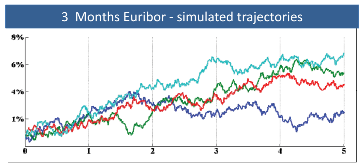
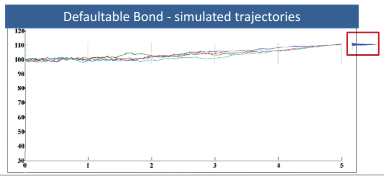
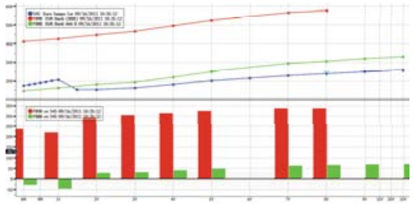




Shapes and patterns of probability distributions

DEFAULTABLE BOND

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product



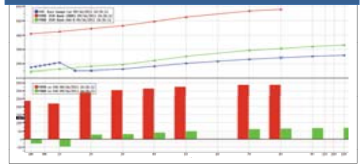
Interest
Rate
Volatility

Significant exposure to credit risk

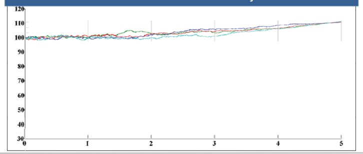
Shapes and patterns of probability distributions

DEFAULTABLE BOND

Yield curve for different issuers



Defaultable Bond - simulated trajectories



Shapes and patterns of probability distributions

DEFAULTABLE BOND

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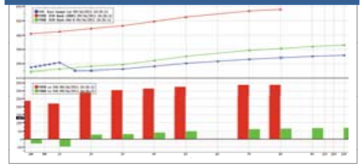

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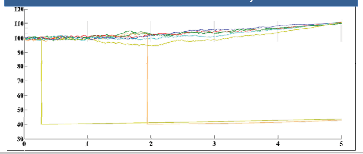

Shapes and patterns of probability distributions

DEFAULTABLE BOND

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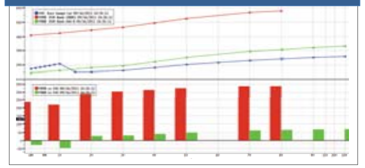

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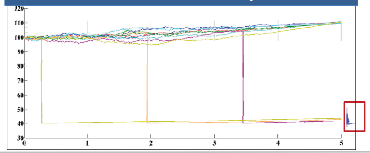
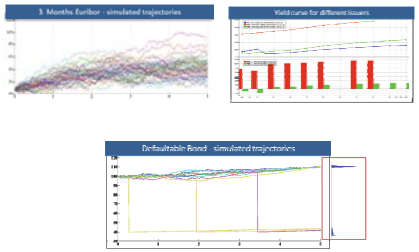


Shapes and patterns of probability distributions

DEFAULTABLE BOND

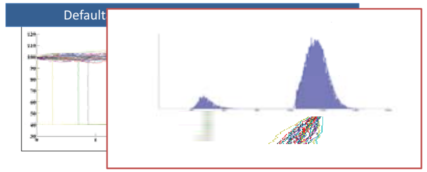
The risk factors define the product values over time and at expiry date



Shapes and patterns of probability distributions

DEFAULTABLE BOND

The final values of the product provide the probability distribution of the potential returns (so-called *pricing* at maturity)...



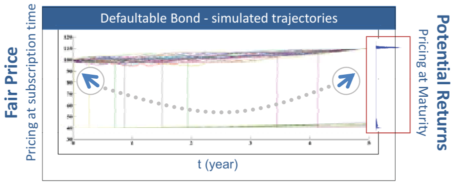
Possible Outcomes

Pricing at maturity

Shapes and patterns of probability distributions

DEFAULTABLE BOND

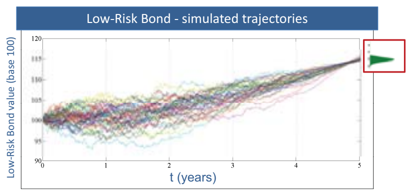
... the "*fair value*" of the product at the issue date is obtained, like in the *best practice* of the pricing procedures of intermediaries, by evaluating the expected discounted value of this distribution.



Shapes and patterns of probability distributions

LOW-RISK BOND

Limited exposure to credit risk corresponds to a lower (or zero) number of trajectories incurring in a *default event*.



Shapes and patterns of probability distributions

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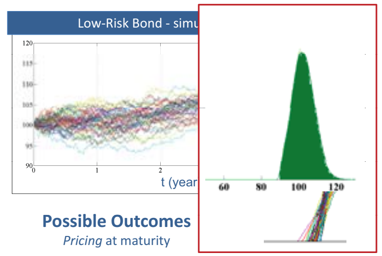


Possible Outcomes

Pricing at maturity

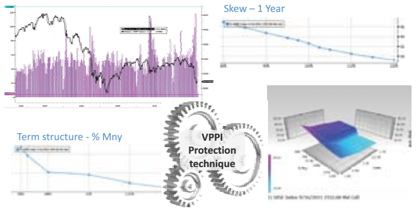
Shapes and patterns of probability distributions

VPPI PRODUCT

Interest
Rate
Volatility

Limited exposure to Market risk

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product



VPPI technique is aimed at protecting the initial value of the financial investment over a specified time horizon and obtaining possible gains by limited exposure to the equity markets.

Shapes and patterns of probability distributions

VPPI PRODUCT

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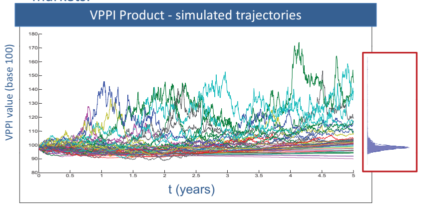

Shapes and patterns of probability distributions

VPPI PRODUCT

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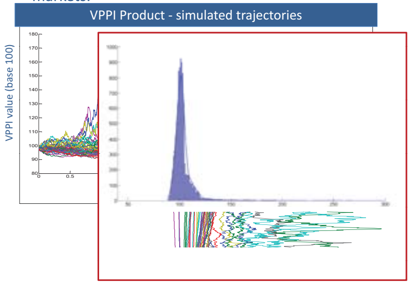
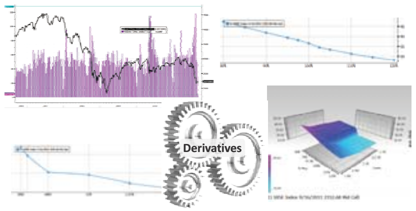


Shapes and patterns of probability distributions

INDEX LINKED CERTIFICATE

Significant exposure to Market risk

Markets data are used to estimate the relevant risk factors connected with the financial structure of the product



The index-linked certificate is characterised by a complex financial engineering that makes intensive use of different derivatives components. These derivatives link the performances of the product to the variability of an equity index.

Shapes and patterns of probability distributions

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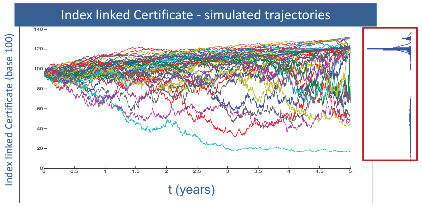

Shapes and patterns of probability distributions

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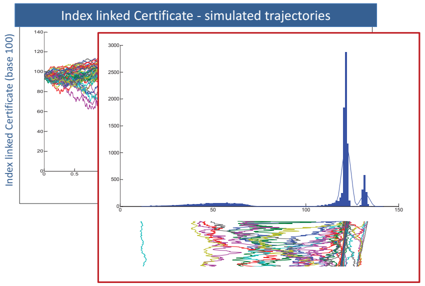


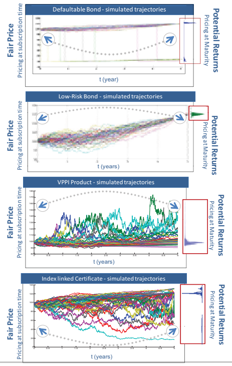
Shapes and patterns of probability distributions

DEFAULTABLE BOND

LOW-RISK BOND

VPPI PRODUCT

INDEX LINKED CERTIFICATE



Fair Price at time zero is a **Weighted average**

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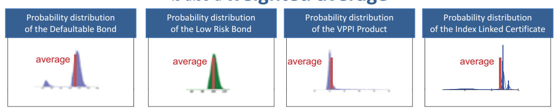
Recognizing and assessing the main drivers of complexity: Intuition

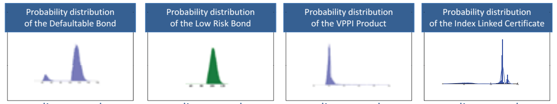
DEFAULTABLE BOND | LOW-RISK BOND | VPPI PRODUCT | INDEX LINKED CERTIFICATE

Fair Price at time zero is a **weighted average**

first moment of the probability distribution at expiry date is also a **weighted average**



Recognizing and assessing the main drivers of complexity: Intuition

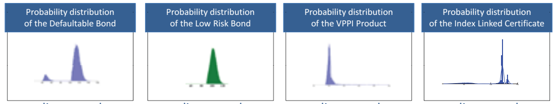


discounted + expected	discounted + expected	discounted + expected	discounted + expected

Defaultable Bond **Fair Price** at time zero = Low Risk Bond **Fair Price** at time zero = VPPI Product **Fair Price** at time zero = Index Linked Certificate **Fair Price** at time zero

Working Hypothesis: The calculated fair price is the same for completely different financial structures

Recognizing and assessing the main drivers of complexity: Intuition

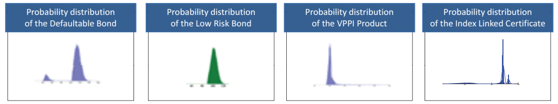


discounted + expected	discounted + expected	discounted + expected	discounted + expected

Defaultable Bond **Fair Price** at time zero = Low Risk Bond **Fair Price** at time zero = VPPI Product **Fair Price** at time zero = Index Linked Certificate **Fair Price** at time zero

Question: How much information about the original probability distribution the price will convey in each case analyzed?

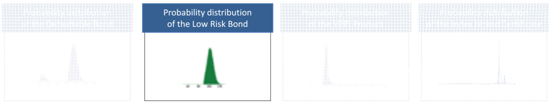
Recognizing and assessing the main drivers of complexity: Intuition



STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular Symmetry Low dispersion	Asymmetry Kurtosis	Multimodality Asymmetry Kurtosis High dispersion

Recognizing and assessing the main drivers of complexity: Intuition

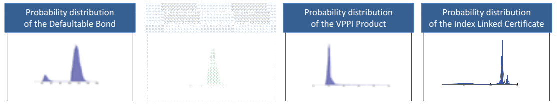


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Regular
symmetry
Low dispersion

High significance of the price information

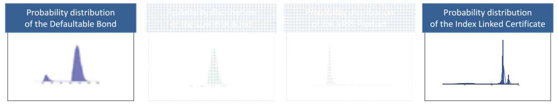
Recognizing and assessing the main drivers of complexity: Intuition



STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
	Limited significance of the price information	

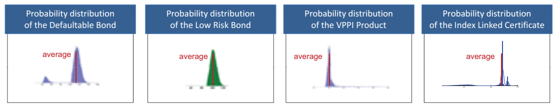
Recognizing and assessing the main drivers of complexity: Intuition



STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Multimodality Asymmetry kurtosis High dispersion
Poor significance of the price information	Poor significance of the price information

Recognizing and assessing the main drivers of complexity: Intuition



STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion

The price and the corresponding average at expiry date – in presence of IRREGULAR distributions – qualify a partial and misleading information

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Significance tests

Significance of the price information

As a weighted average, the price is strictly connected with the first moment of the probability distribution

As the literature suggests, in presence of multimodality and irregular shapes for the probability distributions, the number of moments necessary to properly describe the probability distribution increases drammatically.

See:

(1) Shohat, Tamarkin, 1943 - American Mathematical Survey
(2) Szego, 1959 - American Mathematical Society
(3) Totik, 2000 – Journal of Analytical Mathematics
(4) Gavriliadis, Athanassoulis, 2009 – Journal of Computational and Applied Mathematics

Significance tests

Significance of the price information

Mathematical Basis to test the significance of the price information

In fact, having defined the following quantities:

$(\mu_1,\mu_2,\dots,\mu_{2k})$ Vector of 2k moments for the probability distribution f(x)

$$P_k(x)=\frac{1}{\sqrt{H_{2k}H_{2k-2}}}D_k(x)$$ **Christoffel Basis Polynomials**

where

$$D_k(x)=\det\begin{bmatrix}\mu_0 & \mu_1 & \dots & \mu_k\\ \dots & \dots & & \dots\\ \mu_{k-1} & \mu_k & \dots & \mu_{2k-1}\\ 1 & x & \dots & x^k\end{bmatrix}\qquad H_{2k}=\begin{vmatrix}\mu_0 & \dots & \mu_k\\ \dots & \dots & \dots\\ \mu_k & \dots & \mu_{2k}\end{vmatrix}$$

$$H_{-2}=H_0=1$$

Significance tests

Significance of the price information

Mathematical Basis to test the significance of the price information

It's possible then to define the **Christoffel function** in the form below:

$$\lambda_k(x)=\left[\sum_{n=0}^{k}\left|P_n(x)\right|^2\right]^{-1}$$

Provided that a closed interval [a,b] for the probability density support can be identified and that in the interval [a,b] the function f(x) is bounded, the following limit condition holds:

$$\lim_{k\to\infty} k\lambda_k(x)=\pi\sqrt{(x-a)(b-x)}\cdot f(x)$$

Significance tests

Significance of the price information

Mathematical Basis to test the significance of the price information

For k finite, the limit condition implies that the probability function f(x) can be approximated by the following functional:

$$f(x)\approx f_{AP,k}(x)=\frac{k}{c_0\pi\sqrt{(x-a)(b-x)}}\lambda_k(x)$$

Gavriliadis, Athanassoulis, 2009 – Journal of Computational and Applied Mathematics

with $x\in[a,b]$. c_0 is a normalizing factor.

Significance tests

Significance of the price information

Mathematical Basis to test the significance of the price information

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with $x\in[a,b]$. c_0 is a normalizing factor.

It' s then immediate to apply the approximating formula for different values of k in order to test the accuracy of the approximation for the probability distributions corresponding to our different financial products

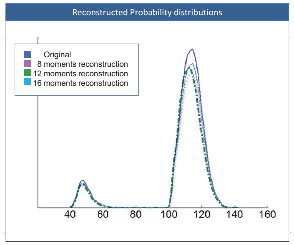
Significance tests

Bimodality

High dispersion

Significance test of the price information

DEFAULTABLE BOND



At least 16 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

Significance tests

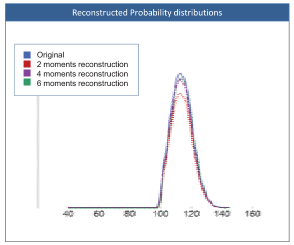
Regular

symmetry

Low dispersion

Significance test of the price information

LOW-RISK BOND



Only 4 moments are sufficient in order to describe properly the original distribution. The information content of the first moment can be considered adequate.

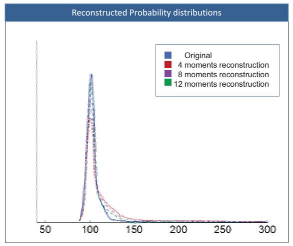
Significance tests

Asymmetry

kurtosis

Significance test of the price information

VPPI PRODUCT



12 moments describe correctly the pattern of the original distribution. The information content of the first moment needs to be integrated.

Significance tests

Multimodality

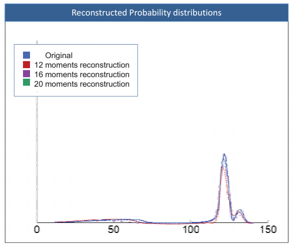
Asymmetry

kurtosis

High dispersion

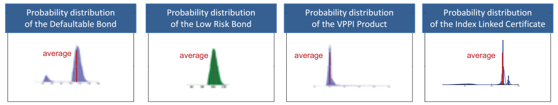
Significance test of the price information

INDEX LINKED CERTIFICATE



At least 20 moments are needed in order to obtain a satisfactory approximation of the original distribution. The information content of the first moment seems very limited.

Significance tests

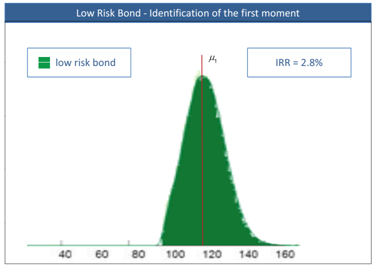


STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Bimodality High dispersion	Regular symmetry Low dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
16 moments needed	4 moments needed	12 moments needed	20 moments needed

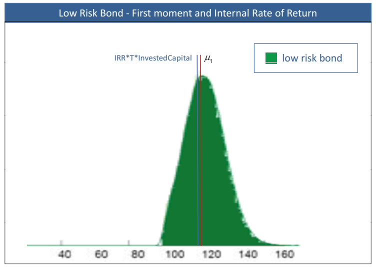
From a pure statistical point of view, a proper reconstruction of the original distribution needs at least 4 moments even for the most regular one

Significance tests: a step beyond – the investor point of view



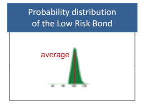
IRR ← ··· → First moment of the probability distribution

Significance tests



IRR = 2.8% $\mu_1 \approx$ IRR*T*InvestedCapital = 114

Significance tests



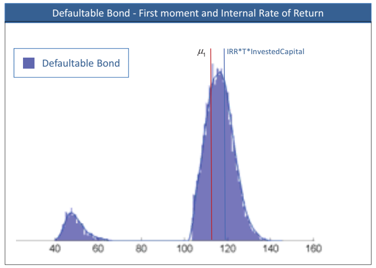
STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Regular
symmetry
Low dispersion

Price ≈ Average ≈ IRR

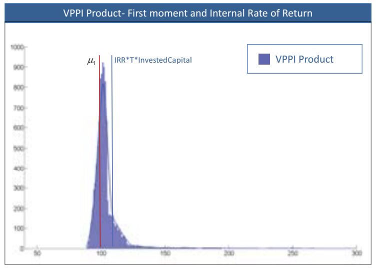
Even if 4 moments are needed for a proper reconstruction of the probability distribution, the average and its related measures (IRR and price), convey sufficient information for the investor decision process

Significance tests



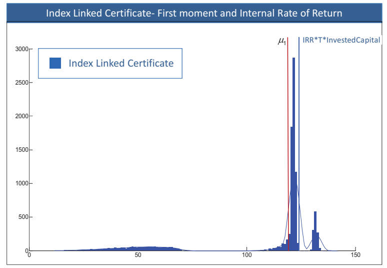
IRR = 3.85% $\mu_1 \neq$ IRR*T*InvestedCapital = 119.25

Significance tests



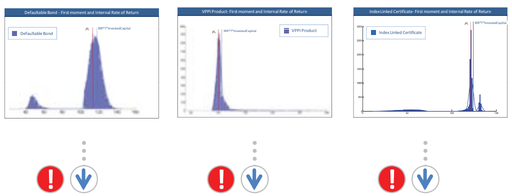
IRR = 2.53% $\mu_1 \neq$ IRR*T*InvestedCapital = 112.65

Significance tests



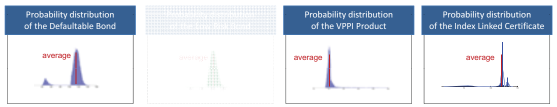
IRR = 5.91% $\mu_1 \neq$ IRR*T*InvestedCapital = 129.55

Significance tests



For more complex financial structures, the average progressively looses its connection with the internal rate of return of the investment, so reducing its usefulness as an effective tool for the decision process

Significance tests



STATISTICAL PROPERTIES OF THE PROBABILITY DISTRIBUTIONS

Defaultable Bond	VPPI Product	Index Linked Certificate
Bimodality High dispersion	Asymmetry kurtosis	Multimodality Asymmetry kurtosis High dispersion
Price Average IRR	Price Average IRR	Price Average IRR

The price and the corresponding average and IRR at expiry date – in presence of IRREGULAR distributions – need to be complemented with additional information related to the shape of the probability distribution

Syllabus

- Shapes and patterns of probability distributions for different structured products
- Recognizing and assessing the main drivers of complexity
- Significance Tests
- Unveiling Risk and Opportunities to Investors
  - Proposal 1
  - Proposal 2
  - Proposal 3
- Conclusions

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand for the average investor

capture efficiently all the main statistical characteristics of the probability distribution of the product

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand for the average investor

capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 1: Convey to the average investor the entire probability distribution

Probability distribution of the Defaultable Bond | Probability distribution of the VPPI Product | Probability distribution of the Index Linked Certificate

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

2 Factor Short Interest Rate Hull-White Model

Short Interest Rate Cox Ingersoll Ross Model

Heston Stochastic Volatility Model for the Equity component

Barndorff Nielsen Normal Inverse Gaussian Model for the Equity component

Merton Jump Diffusion Model for the Equity component

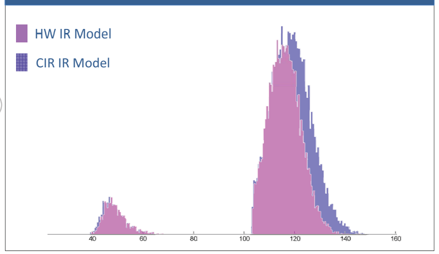
Variance Gamma Model for the Equity component

The shape of the probability distribution of the potential returns is obviously dependent on the modelling assumptions.

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

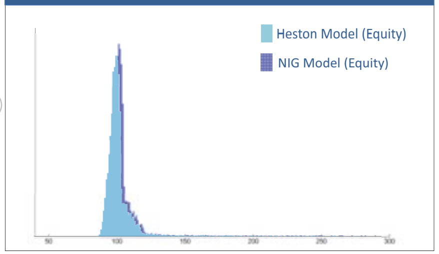
Probability distributions of the final values of the defaultable bond



VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

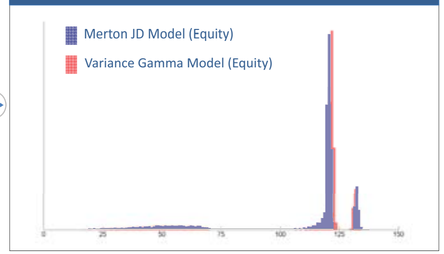
Probability distributions of the final values of the VPPI product



INDEX LINKED CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distributions of the final values of the Index Linked Certificate



COMPLEX PRODUCT

The additional information to be supplemented must

~~be easy to understand for the average investor~~

~~capture efficiently all the main statistical characteristics of the probability distribution of the product~~

the probability distribution is an abstract object not easy to handle by the average investors

the shape of the probability distribution is dependent on the modelling assumptions

Proposal 1: Convey to the average investor the entire probability distribution

Syllabus

- Shapes and patterns of probability distributions for different structured products
- Recognizing and assessing the main drivers of complexity
- Significance Tests
- Unveiling Risk and Opportunities to Investors
  - Proposal 1
  - Proposal 2
  - Proposal 3
- Conclusions

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand for the average investor

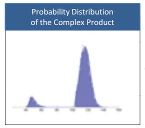
capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 2: Unbundling the information content of the price

Unveiling Risk and Opportunities to Investors: the unbundling table

COMPLEX PRODUCT

Unbundling the information content of the price



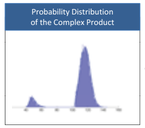
DISCOUNTED EXPECTED VALUE

Fair Value (Complex Product)

Unveiling Risk and Opportunities to Investors: the unbundling table

COMPLEX PRODUCT

Unbundling the information content of the price

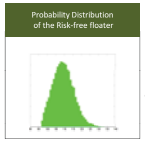


DISCOUNTED EXPECTED VALUE

Fair Value (Complex Product)

A risk-free floater with same fair value and coupon payment dates of the complex product is defined

=



DISCOUNTED EXPECTED VALUE

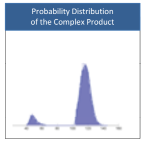
Fair Value (Risk-free floater)

Unveiling Risk and Opportunities to Investors: the unbundling table

COMPLEX PRODUCT

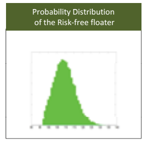
Unbundling the information content of the price

Any non-elementary return-target product can be replicated by a portfolio composed of the associated risk-free floater and of a zero-value swap which transforms the cash flow structure of the risk-free security into the cash flow structure of the product itself, ie, denoting by $\{swap_t\}_{t\in[0,T]}$ the value process of the swap



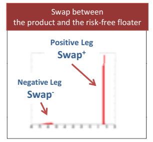
Fair Value (Complex Product)

=



Fair Value (Risk-free floater)

+



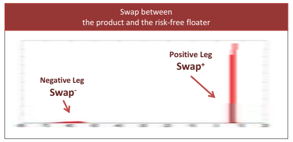
Fair Value (Swap = 0)

Unveiling Risk and Opportunities to Investors: the unbundling table

COMPLEX PRODUCT

Unbundling the information content of the price

Fair Value (Swap = 0)



$|FV(Swap^-)| = |FV(Swap^+)|$

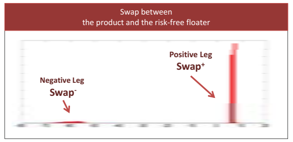
Theoretical value of the Risky component

Unveiling Risk and Opportunities to Investors: the unbundling table

COMPLEX PRODUCT

Unbundling the information content of the price

Fair Value (Swap = 0)



$|FV(Swap^-)| = |FV(Swap^+)|$

Theoretical value of the Risky component

C	*Fair value*
B	Theoretical value of the Risky component
A=C-B	Theoretical value of the Risk-Free component

Unveiling Risk and Opportunities to Investors: the unbundling table

Financial investment table (*Price Unbundling*)

DEFAULTABLE BOND

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	*Fair value*	96.3
D	Costs	3.7
E = C + D	Issue price	100

VPPI PRODUCT

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	*Fair value*	96.5
D	Costs	3.5
E = C + D	Issue price	100

INDEX LINKED CERTIFICATE

A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	*Fair value*	96.1
D	Costs	3.9
E = C + D	Issue price	100

Unveiling Risk and Opportunities to Investors: the unbundling table

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand for the average investor

capture efficiently all the main statistical characteristics of the probability distribution of the product

the unbundling represented by using a table is first level tool useful to appreciate the impact of the costs and the riskiness of the product

The unbundling exploits only the information contained in the first order moment of the probability distribution

Proposal 2: Unbundling the information content of the price

Syllabus

- Shapes and patterns of probability distributions for different structured products
- Recognizing and assessing the main drivers of complexity
- Significance Tests
- Unveiling Risk and Opportunities to Investors
  - Proposal 1
  - Proposal 2
  - Proposal 3
- Conclusions

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand for the average investor

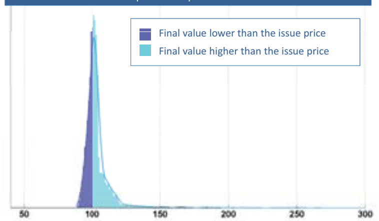
capture efficiently all the main statistical characteristics of the probability distribution of the product

Proposal 3: Perform a reduction in granularity by implementing a partition of the probability distribution

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

Partition of the Probability distribution of the Complex Product with respect to the point of zero return

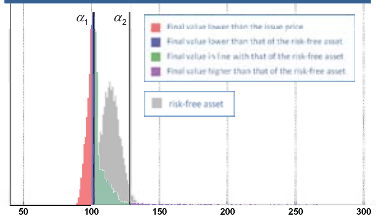


The assessment of the probability of recovering at least the amount paid for the product is of great significance for the investor.

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



It is appropriate to explore further partitions of the macro-event "the final value of the investment is higher than the issue price" by performing a direct comparison with the final values of the risk-free asset.

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2

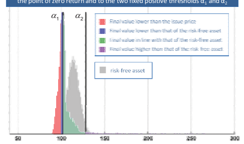


Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	…	…
The performance is positive but lower than the risk-free asset	…	…
The performance is positive and in line with the risk-free asset	…	…
The performance is positive and higher than the risk-free asset	…	…

MEAN VALUES

$$E^P(S_T \mid S_T < 100) = \frac{1}{P(S_T < 100)}\int_{-\infty}^{100} x f_{S_T}(x)\,dx$$

$$E^P(S_T \mid 100 \leq S_T < \alpha_1) = \frac{1}{P(100 \leq S_T < \alpha_1)}\int_{100}^{\alpha_1} x f_{S_T}(x)\,dx$$

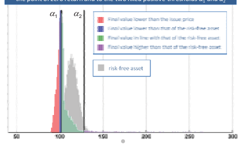
$$E^P(S_T \mid \alpha_1 \leq S_T < \alpha_2) = \frac{1}{P(\alpha_1 \leq S_T < \alpha_2)}\int_{\alpha_1}^{\alpha_2} x f_{S_T}(x)\,dx$$

$$E^P(S_T \mid S_T \geq \alpha_2) = \frac{1}{P(S_T \geq \alpha_2)}\int_{\alpha_2}^{+\infty} x f_{S_T}(x)\,dx$$

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2

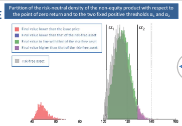


Benefits of this solution:

1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information and the table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;

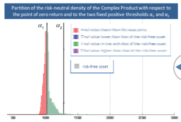
Unveiling Risk and Opportunities to Investors: the superimposition technique

DEFAULTABLE BOND



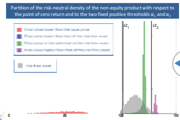
SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

VPPI PRODUCT



SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

INDEX LINKED CERTIFICATE

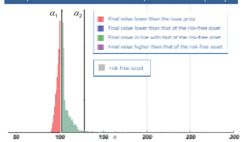


SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

Partition of the risk-neutral density of the Complex Product with respect to the point of zero return and to the two fixed positive thresholds α_1 and α_2



Benefits of this solution:

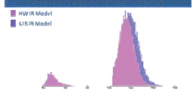
1. The reduction in granularity of the events determined by the partition involves only a very limited loss of information; The table, built by coupling for each scenario its risk-neutral probability and the associated mean value, is very easy to read;
2. The model risk arising from the different proprietary models of the issuers has a limited impact.

Unveiling Risk and Opportunities to Investors: the superimposition technique

DEFAULTABLE BOND

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the Defaultable Bond



Difference less than 2%

HW IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

CIR IR MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	8.3%	49.9
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	86.8%	117.9
The performance is positive and higher than the risk-free asset	4.9%	135.4

Unveiling Risk and Opportunities to Investors: the superimposition technique

VPPI PRODUCT

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the VPPI Product

Difference less than 2%

HESTON MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	38.9%	95.5
The performance is positive but lower than the risk-free asset	18.9%	100.2
The performance is positive and in line with the risk-free asset	38.4%	106.3
The performance is positive and higher than the risk-free asset	3.8%	182.5

NIG MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

Unveiling Risk and Opportunities to Investors: the superimposition technique

INDEX LINKED CERTIFICATE

MODELLING CHOICES FOR THE SELECTED FINANCIAL PRODUCTS

Probability distribution of the Index Linked Certificate

Difference less than 4%

MERTON JD MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	48.2
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	65.8%	117.6
The performance is positive and higher than the risk-free asset	15.3%	132.7

VARIANCE GAMMA MODEL

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

Unveiling Risk and Opportunities to Investors: the superimposition technique

COMPLEX PRODUCT

The additional information to be supplemented must

be easy to understand for the average investor	capture efficiently all the main statistical characteristics of the probability distribution of the product
the partition should be done by choosing events that have a strong financial meaning for the investor	the reduction in granularity mitigates in a significant way the model risk

Proposal 3: **Perform a reduction in granularity by implementing a partition of the probability distribution**

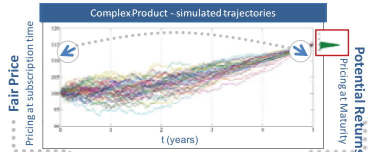
Syllabus

- Shapes and patterns of probability distributions for different structured products
- Recognizing and assessing the main drivers of complexity
- Significance Tests
- Unveiling Risk and Opportunities to Investors
    - Proposal 1
    - Proposal 2
    - Proposal 3
- Conclusions

Conclusions

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

COMPLEX PRODUCT



Financial investment table
(Price Unbundling)

A	Theoretical value of the Risk-Free component	
B	Theoretical value of the Risky component	
C = A + B	*Fair value*	
D	Costs	
E = C + D	Issue price	

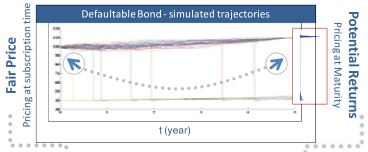
Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	...	...
The performance is positive but lower than the risk-free asset	...	...
The performance is positive and in line with the risk-free asset	...	...
The performance is positive and higher than the risk-free asset	...	...

Conclusions

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

DEFAULTABLE BOND



Financial investment table
(Price Unbundling)

A	Theoretical value of the Risk-Free component	91.3
B	Theoretical value of the Risky component	5
C = A + B	*Fair value*	96.3
D	Costs	3.7
E = C + D	Issue price	100

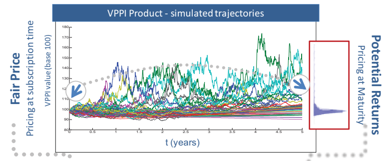
Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	9.5%	49.3
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	87.4%	115.6
The performance is positive and higher than the risk-free asset	3.1%	131.1

Conclusions

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

VPPI PRODUCT



Financial investment table
(Price Unbundling)

A	Theoretical value of the Risk-Free component	90.1
B	Theoretical value of the Risky component	6.4
C = A + B	*Fair value*	96.5
D	Costs	3.5
E = C + D	Issue price	100

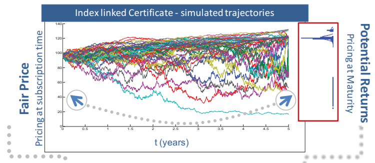
Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	36.9%	96.9
The performance is positive but lower than the risk-free asset	18.5%	101
The performance is positive and in line with the risk-free asset	39.9%	107.1
The performance is positive and higher than the risk-free asset	4.7%	195.5

Conclusions

Since there's a close one-to-one relationship between the two tables, the two sets of information can be easily coupled in an easy-to-read sheet

INDEX LINKED CERTIFICATE



Financial investment table
(Price Unbundling)

A	Theoretical value of the Risk-Free component	86.2
B	Theoretical value of the Risky component	9.9
C = A + B	*Fair value*	96.1
D	Costs	3.9
E = C + D	Issue price	100

Table of the probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEAN VALUES
The performance is negative	18.9%	49.1
The performance is positive but lower than the risk-free asset	0.0%	-
The performance is positive and in line with the risk-free asset	68.9%	120.9
The performance is positive and higher than the risk-free asset	12.2%	131.6

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Comprehension and comparison of structured products: simplicity or yield?

Marcello Minenna
Head of the Quantitative Analysis and Financial Innovation Unit
CONSOB