

A quantitative *risk-based* approach to measure and represent the riskiness of non-equity financial products

Marcello Minenna

### Syllabus

- Preliminaries: non-equity products and their classification
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon
- Examples

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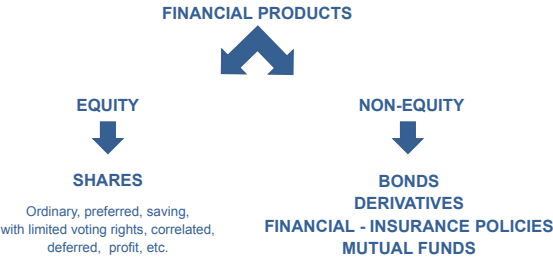
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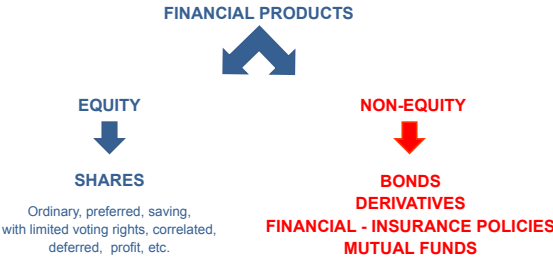
### Preliminaries



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### Preliminaries

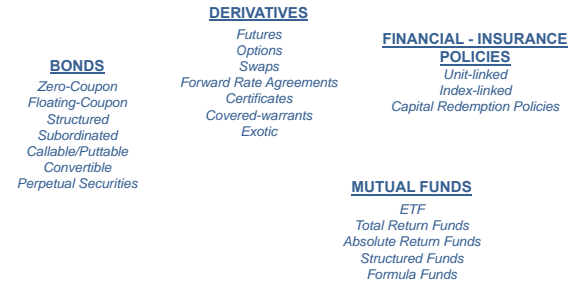


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### Preliminaries

Non-equity products are currently classified on the basis of "labels" assigned to them by the issuers or by the regulatory provisions framework.

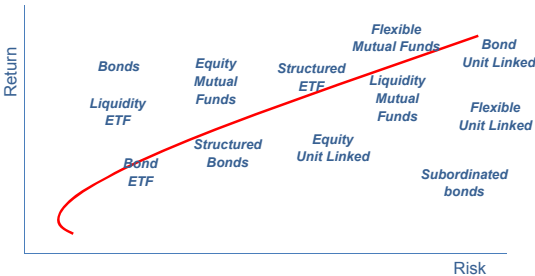


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### Preliminaries

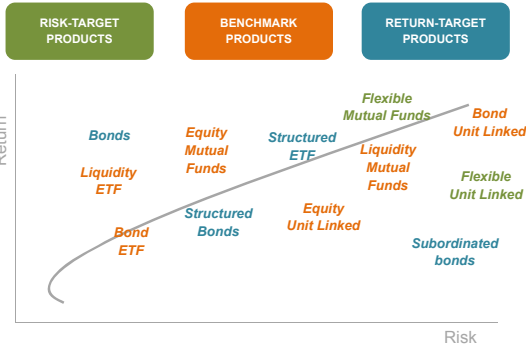
**Markowitz:** non-equity products are classified on the basis of their risk-return profile



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### Preliminaries

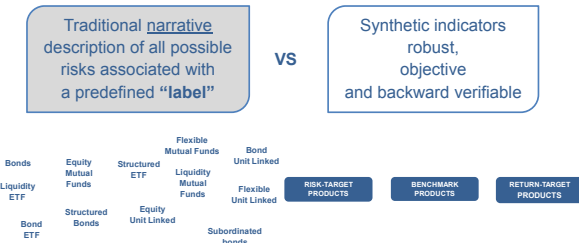


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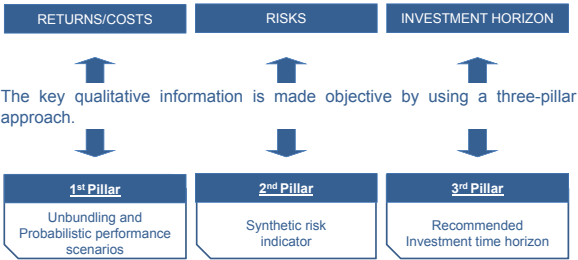
### Preliminaries

Consob transparency regulation on the risk profile of non-equity products is based on synthetic indicators – defined through specific quantitative methods – in order to allow investors to take informed investment decisions.

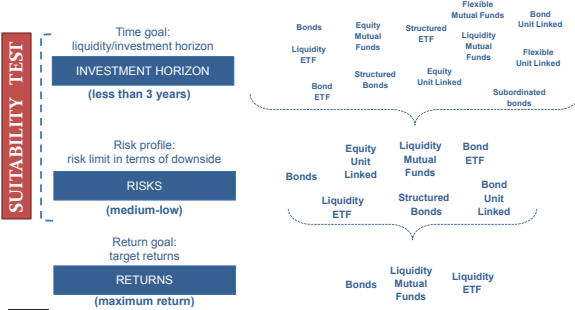


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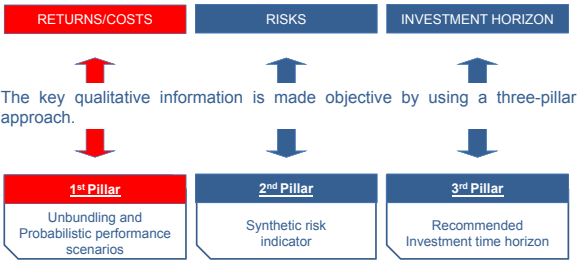


These metrics provide a guide to investors in the interpretation of complex information conveyed in the offering document, supporting the decision process by means of a sequential filtering procedure:



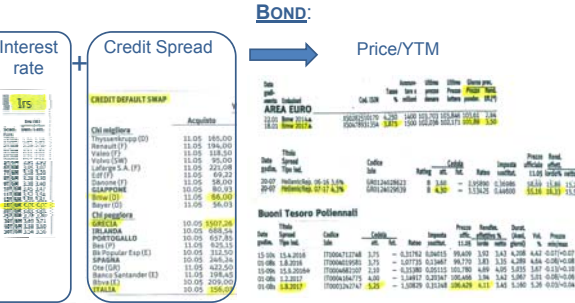
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1st Pillar: Unbundling and Probabilistic performance scenarios

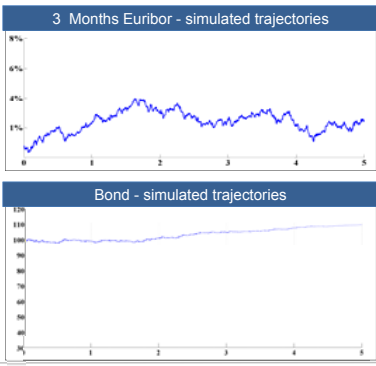


1st Pillar: Unbundling and Probabilistic performance scenarios

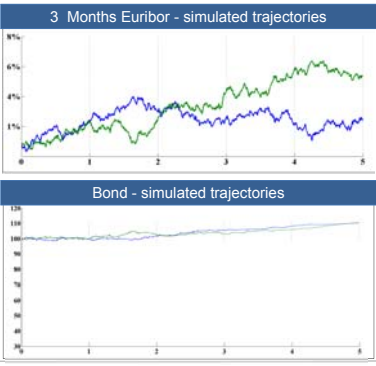
Fair evaluation principle requires the estimate of all the relevant risk factors.



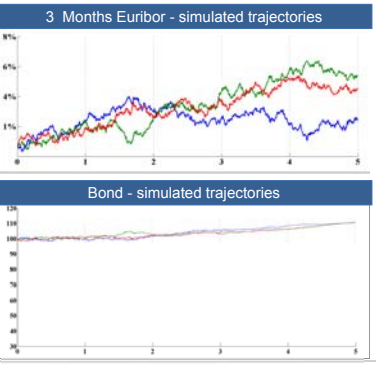
1st Pillar: Unbundling and Probabilistic performance scenarios



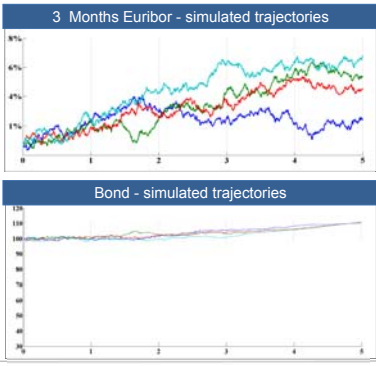
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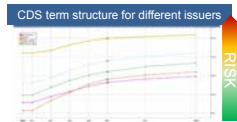
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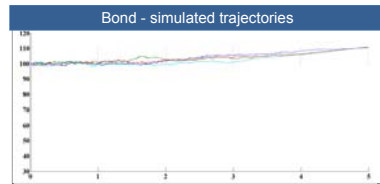
1st Pillar: Unbundling and Probabilistic performance scenarios



## 1st Pillar: Unbundling and Probabilistic performance scenarios



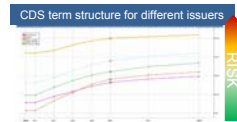
$$\begin{aligned} dB_t &= q(\mu - R_t)dt + \sigma dW_t \\ dR_t &= \gamma d\theta + \sigma_R dW_t \end{aligned}$$



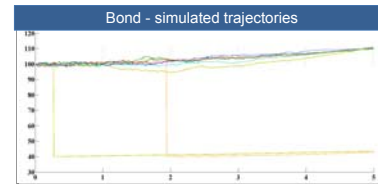
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## 1st Pillar: Unbundling and Probabilistic performance scenarios



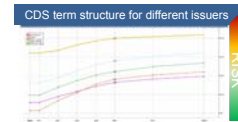
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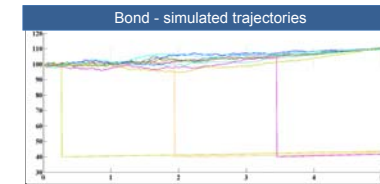
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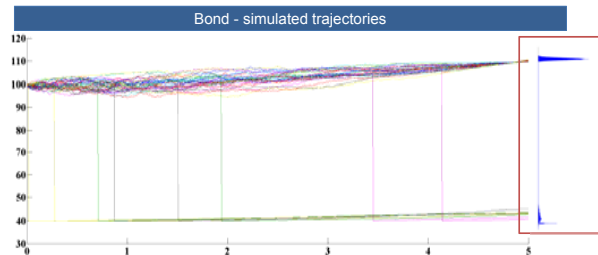


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## 1st Pillar: Unbundling and Probabilistic performance scenarios

Higher values of credit spreads of the issuer (i.e. worse market estimates about the issuer's standing) correspond to a higher number of trajectories incurring in a *default event*.

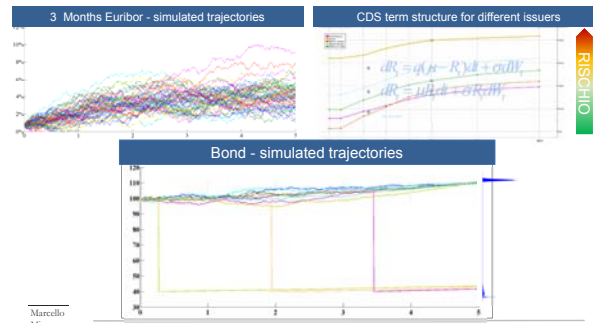


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## 1st Pillar: Unbundling and Probabilistic performance scenarios

The risk factors define the product values over time and at expiry date (hence the potential returns)

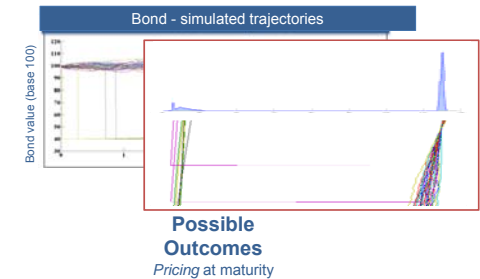


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## 1st Pillar: Unbundling and Probabilistic performance scenarios

The final values of the product provide the probability distribution of the potential returns (so-called *pricing at maturity*).



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## 1st Pillar: Unbundling and Probabilistic performance scenarios

... the "*fair value*" of the product at the issue date is obtained, like in the *best practice* of the pricing procedures of intermediaries, by evaluating the expected discounted value of this distribution.



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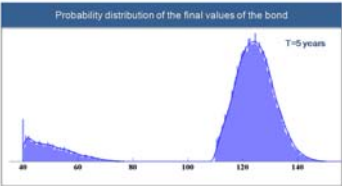
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1st Pillar: Unbundling and Probabilistic performance scenarios

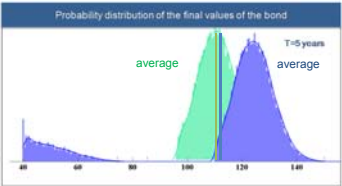


Fair Price at time zero  $\longleftrightarrow$  is a  $\longleftrightarrow$  Weighted average

1st issue

The average – in presense of irregular distribution – qualifies a partial and misleading information for the retail investor

1st Pillar: Unbundling and Probabilistic performance scenarios



Distribution with same average

1st - Example

1st Pillar: Unbundling and Probabilistic performance scenarios



Fair Price at time zero  $\longleftrightarrow$  is a  $\longleftrightarrow$  Weighted average

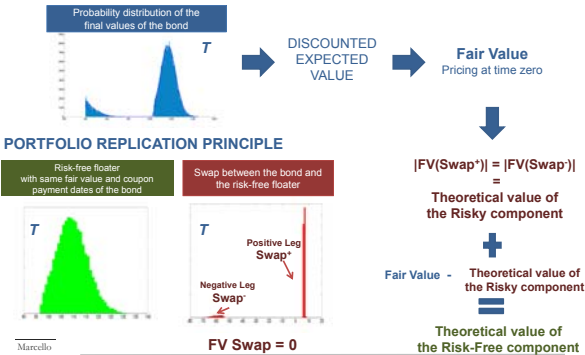
2nd issue

The probability distribution is not easy to handle by the retail investors

1st Pillar: Unbundling and Probabilistic performance scenarios  
MISLEADING RISK REPRESENTATION : STANDARD SOLUTION



1st Pillar: Unbundling and Probabilistic performance scenarios  
MISLEADING RISK REPRESENTATION : CONSOB REGULATION (1)



1st Pillar: Unbundling and Probabilistic performance scenarios  
MISLEADING RISK REPRESENTATION : CONSOB REGULATION (1)

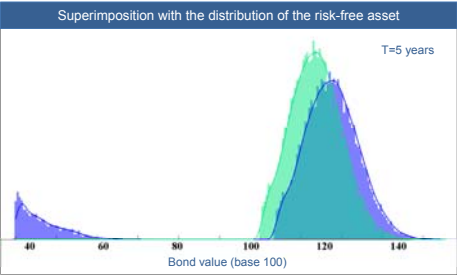
Financial investment table  
(Price Unbundling)

A	Theoretical value of the risk-free component	...
B	Theoretical value of the risky component	...
C = A + B	Fair value	...
D	Explicit costs	...
E	Implicit costs	...
F = C + D + E	Issue price	100

Example

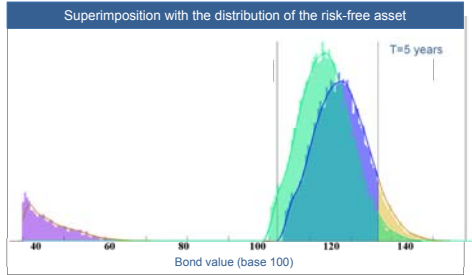
Zeta Bank BOND	
DESCRIPTION	Four-year contingent convertible bond that provides the mandatory conversion into shares of the issuer at predefined date and pricing conditions according to a basket of put and call of European and American options.
STRUCTURE	RETURN - TARGET
1st PILLAR	<b>Unbundling table</b>
	Theoretical value of the risk-free component 70.12
	Theoretical value of the risky component 25.05
	Theoretical value of the product 95.17
	Costs 4.83
	Issue price 100.00

1st Pillar: Unbundling and Probabilistic performance scenarios  
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



The superimposition of the product's probability distribution with the distribution of the risk-free asset naturally defines three different events which are effectively meaningful for the investor.

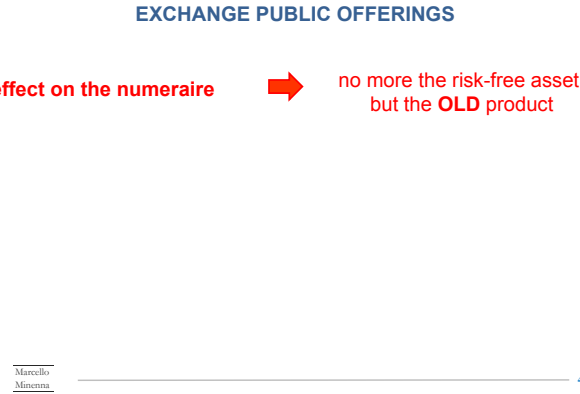
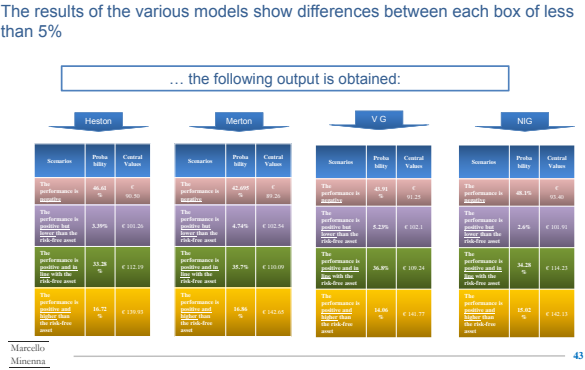
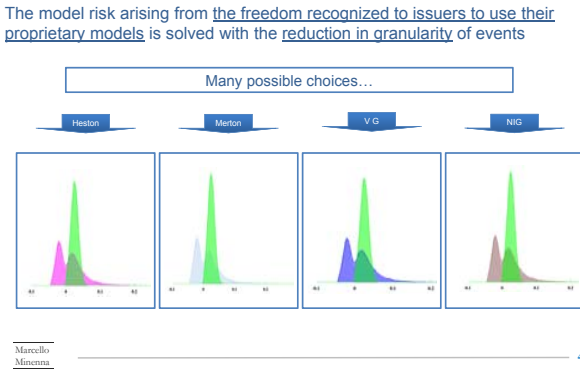
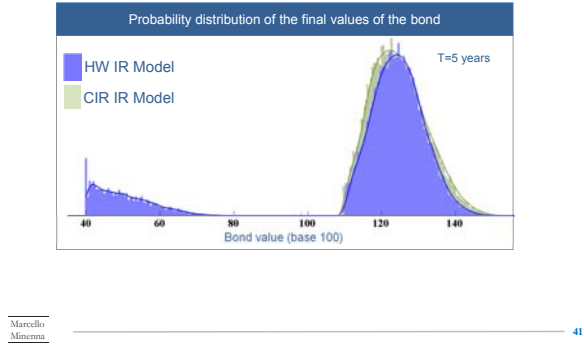
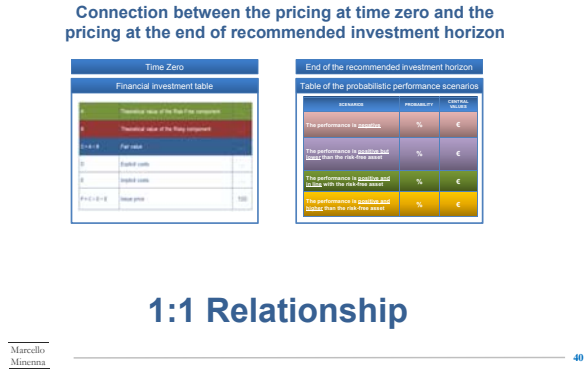
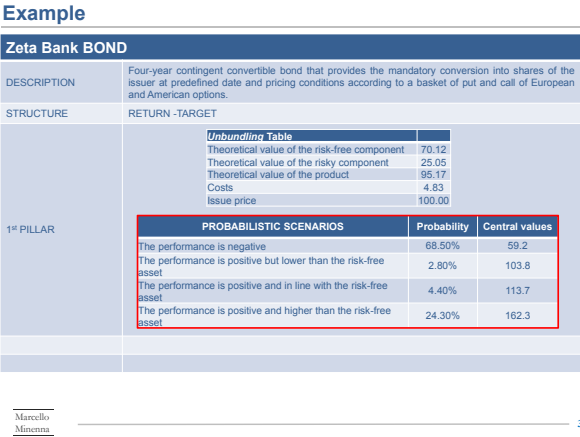
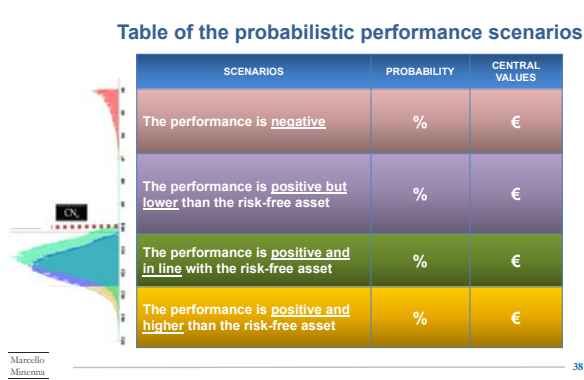
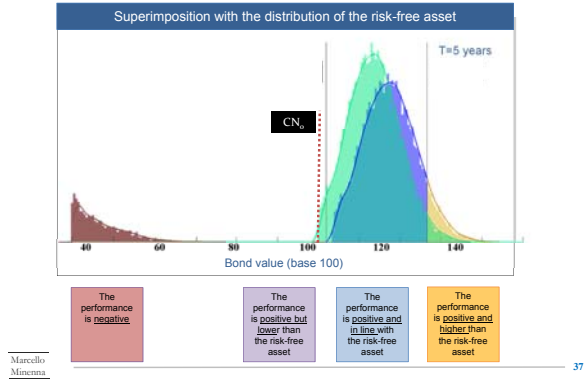
1st Pillar: Unbundling and Probabilistic performance scenarios  
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



The performance is positive but lower than the risk-free asset

The performance is positive and in line with the risk-free asset

The performance is positive and higher than the risk-free asset

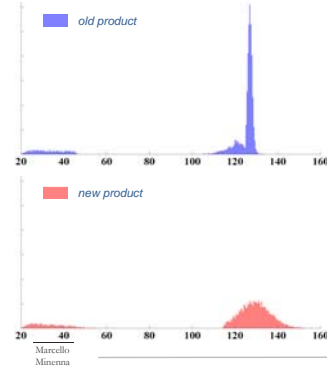


## EXCHANGE PUBLIC OFFERINGS

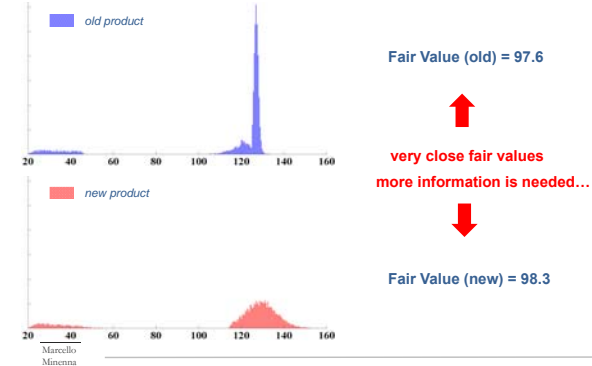
effect on the numeraire → no more the risk-free asset but the **OLD** product

effect on the methodology → no more superimposition but **trajectory-by-trajectory**

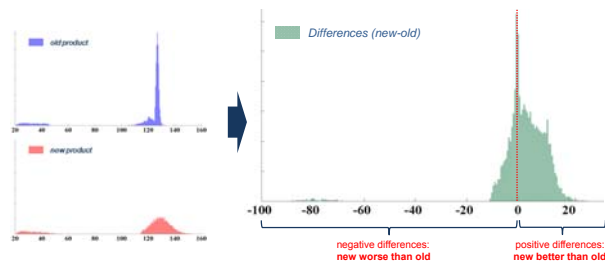
## EXCHANGE PUBLIC OFFERINGS



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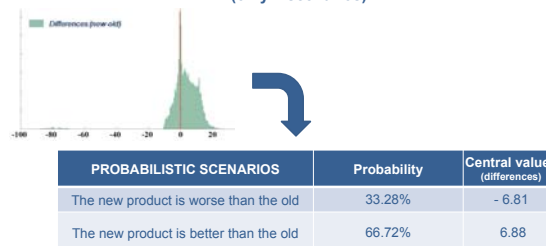


## EXCHANGE PUBLIC OFFERINGS



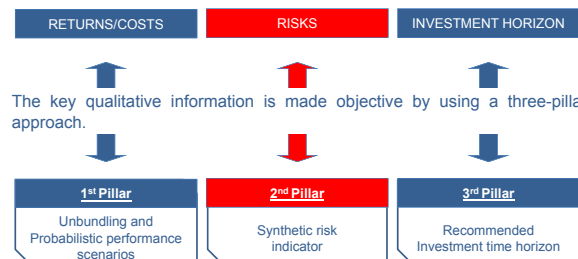
## EXCHANGE PUBLIC OFFERINGS

the **zero-point** on the distribution of the **differences** identifies the natural partition to get useful information on the fairness of the exchange (only 2 scenarios)

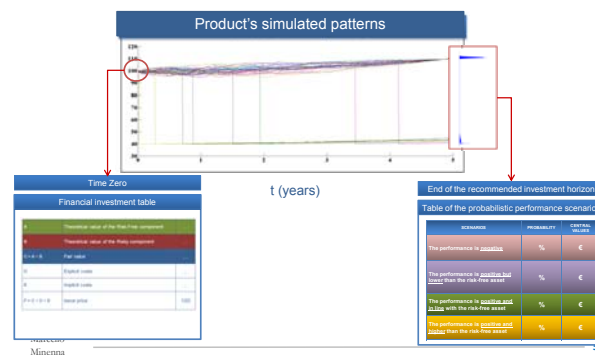


## Syllabus

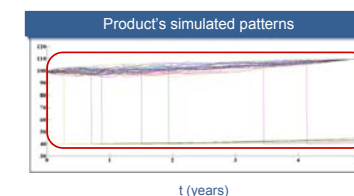
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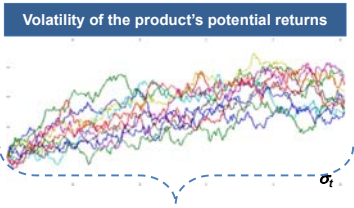
The first pillar focus on costs/returns at the issue time and at maturity.



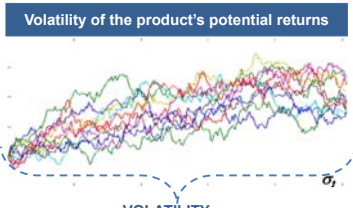
The second pillar focus on risks occurring during the life of the product.







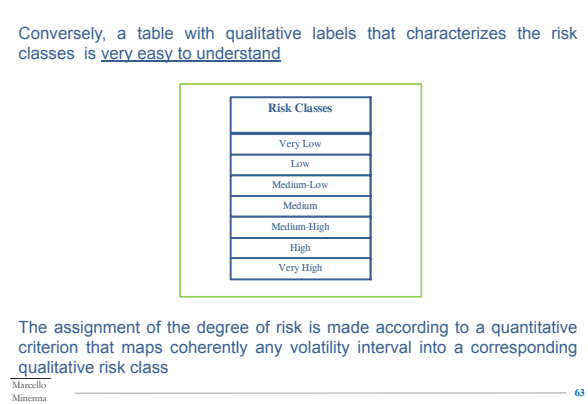
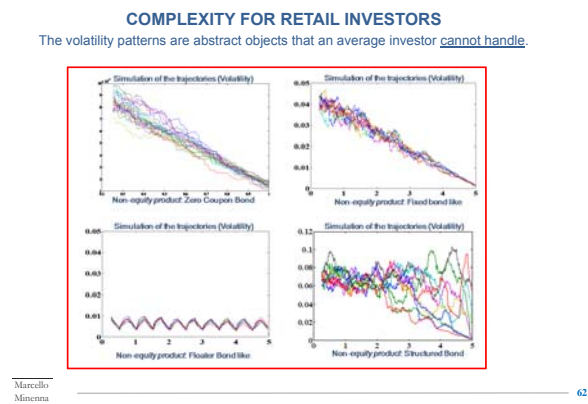
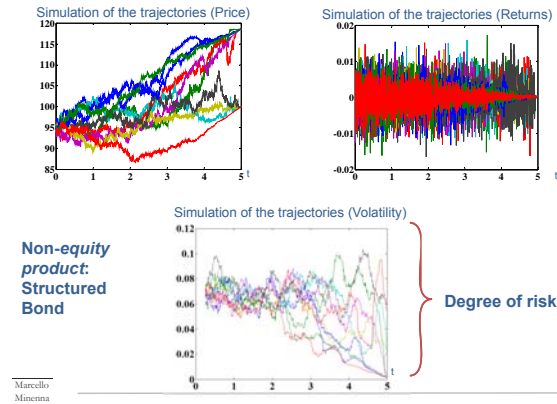
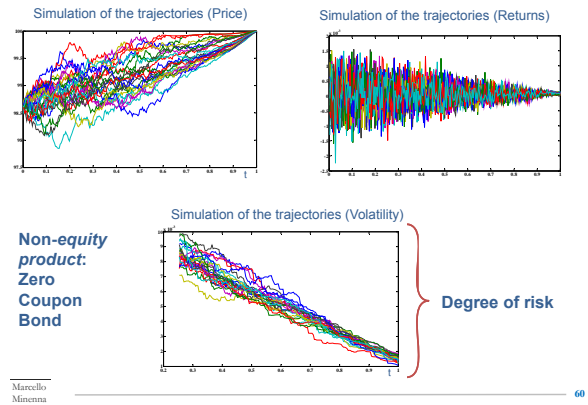
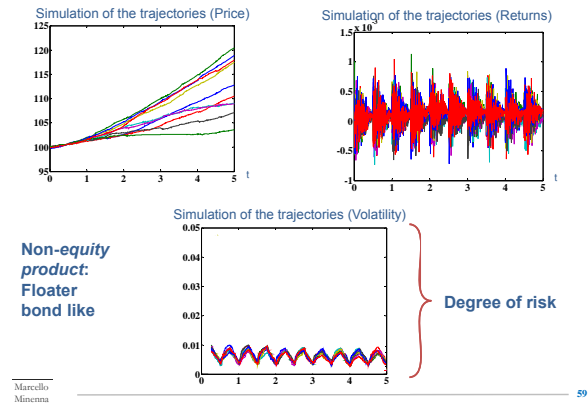
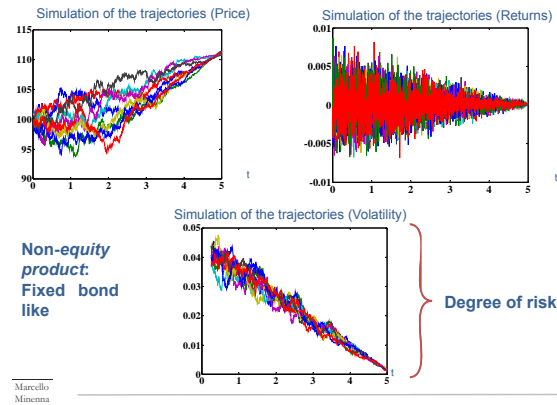
Volatility is the most immediate risk measure and it has a one-to-one relationship with whatever loss measure (VaR, ES, etc.)

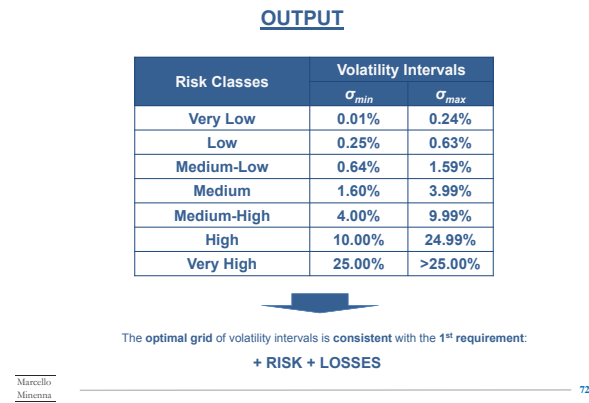
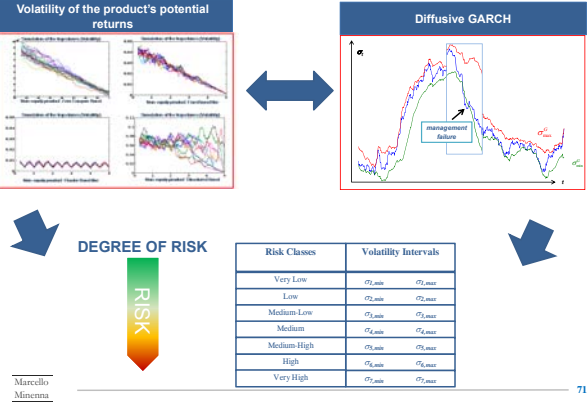
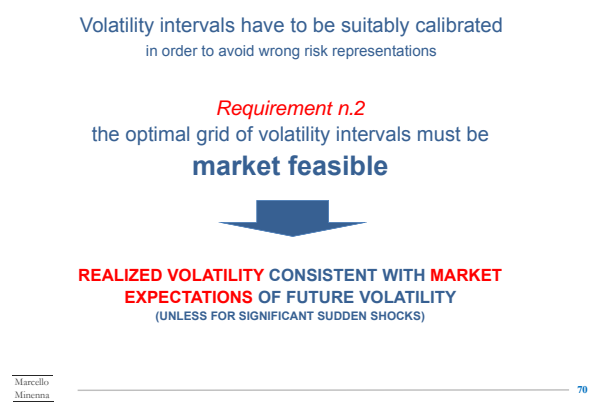
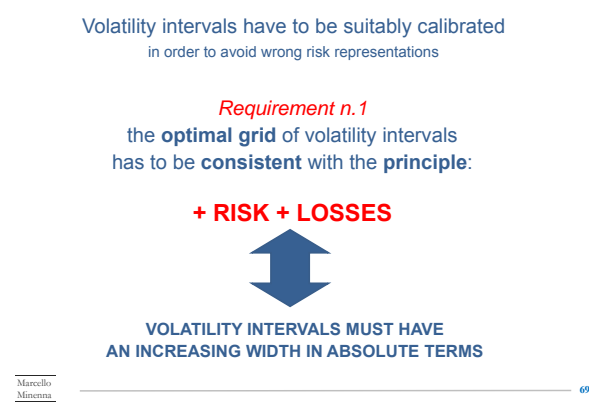
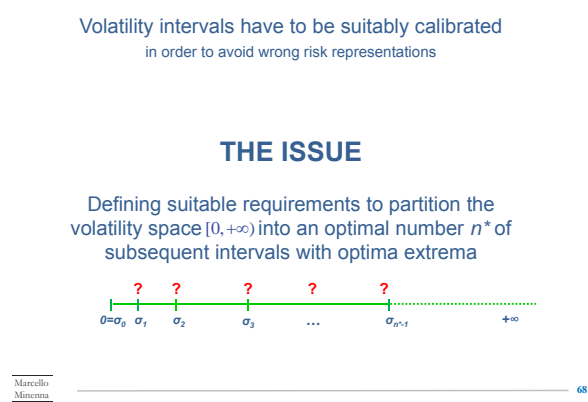
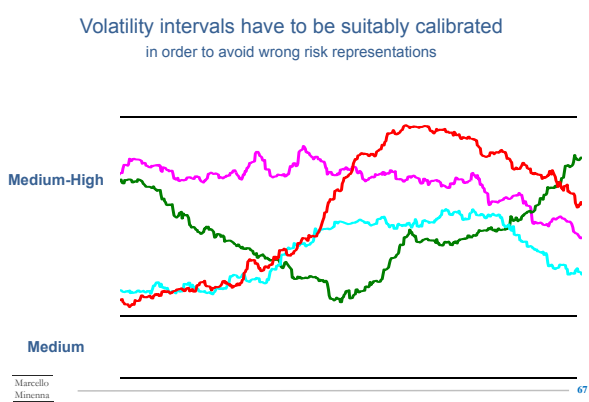
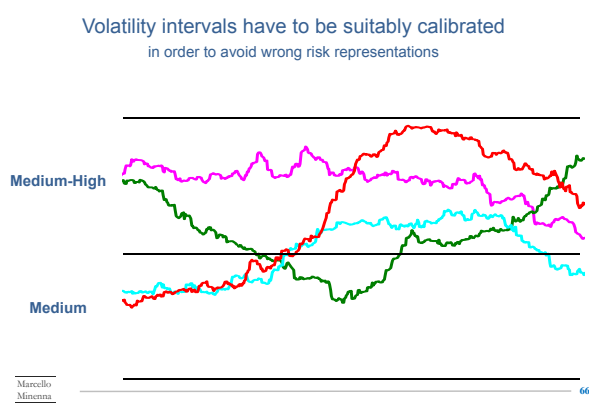
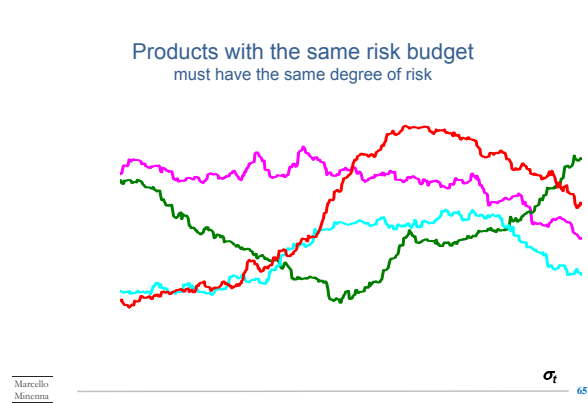
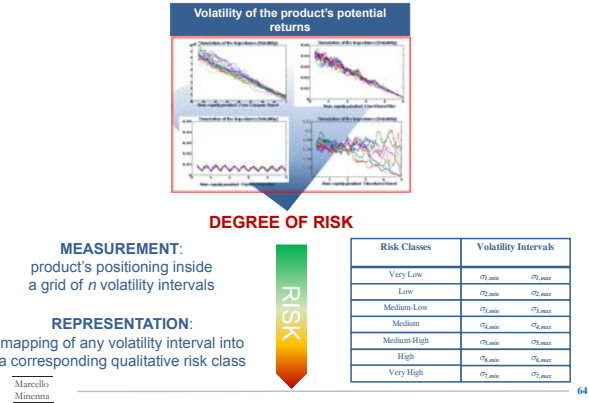


VOLATILITY  
e.g. : geometric brownian motion  
 $dS_t = rS_t dt + \sigma S_t dW_t$

$$VaR_{\alpha, 1 year} = e^{\sigma \Phi^{-1}(\alpha) \sqrt{\frac{252}{252}} \left( r - \frac{\sigma^2}{2} \right) \frac{252}{252} - 1}$$
$$ES_{\alpha, 1 year} = \frac{1}{\alpha} e^{\sigma \Phi^{-1}(\alpha) \sqrt{\frac{252}{252}} \left( r - \frac{\sigma^2}{2} \right) \frac{252}{252} - 1}$$

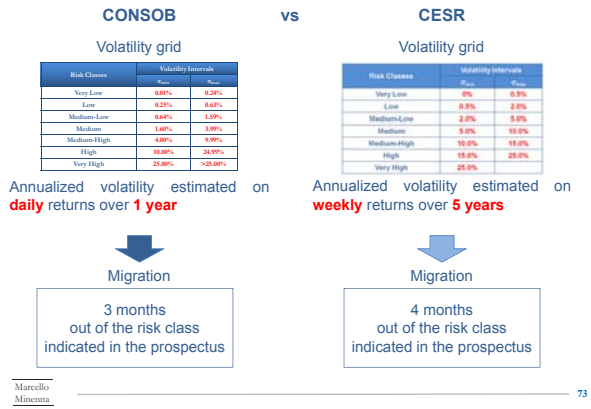
# EXAMPLES







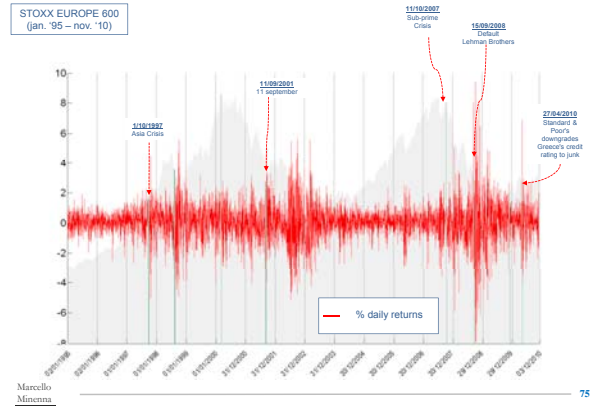
2nd Pillar: Synthetic risk indicator



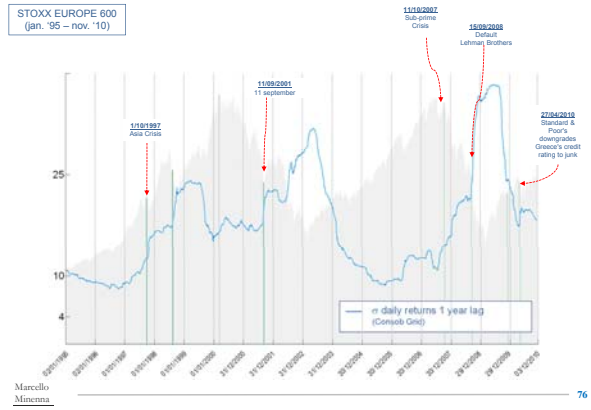
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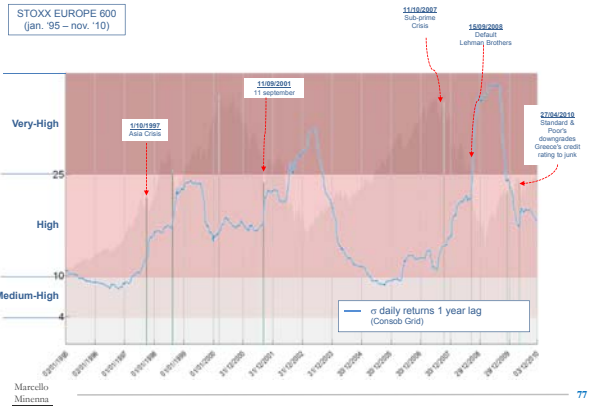
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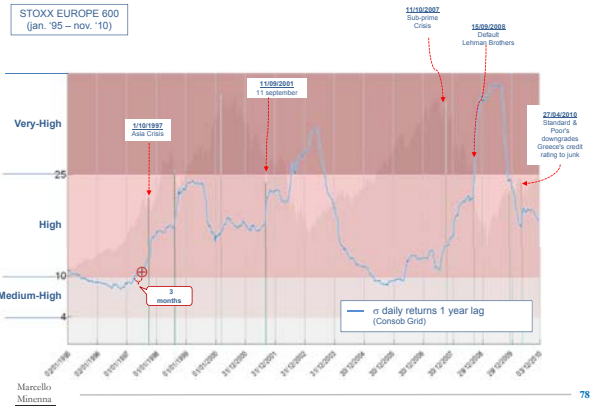
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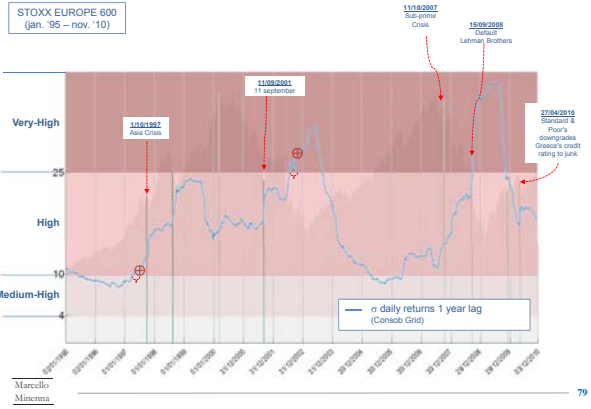
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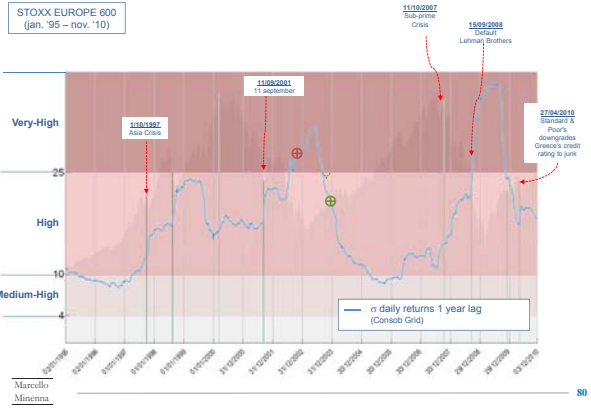
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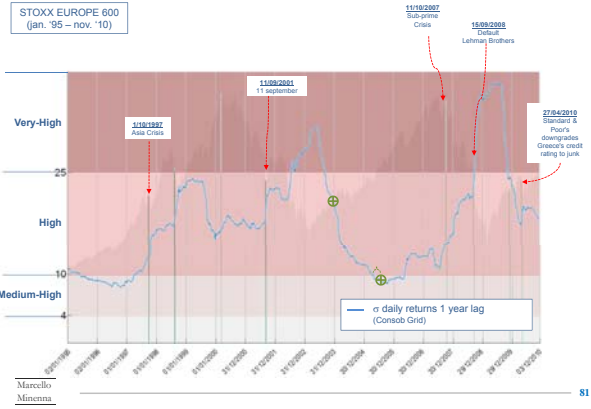
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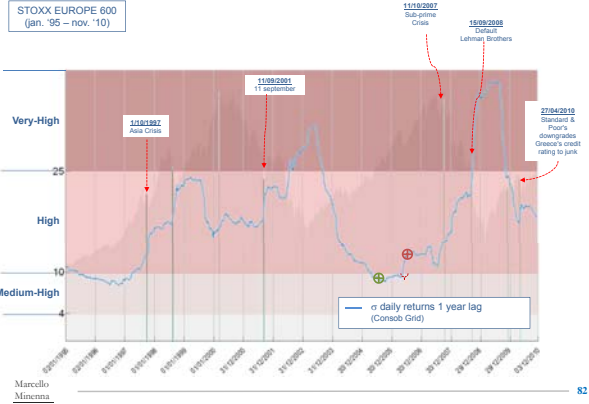
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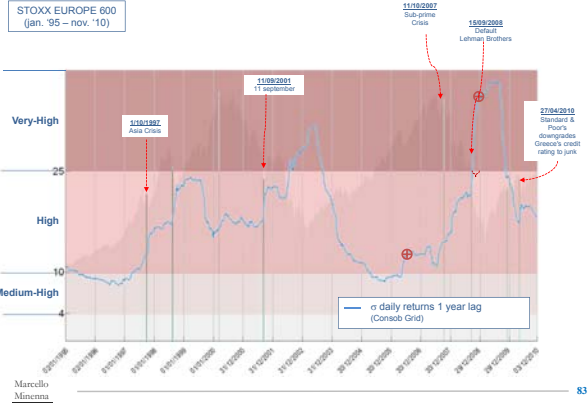
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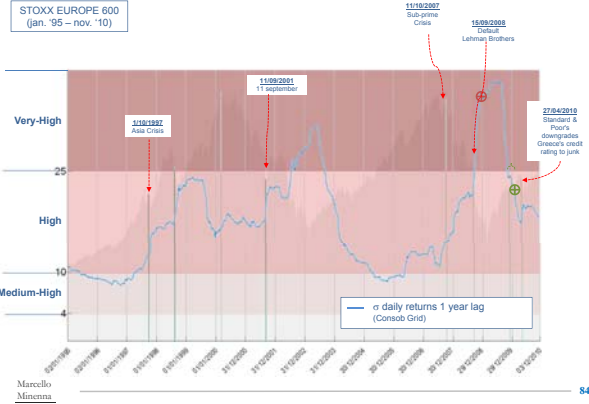
2<sup>nd</sup> Pillar: Synthetic risk indicator



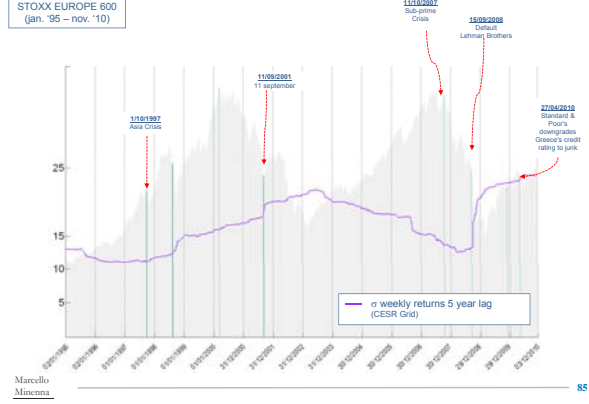
2<sup>nd</sup> Pillar: Synthetic risk indicator



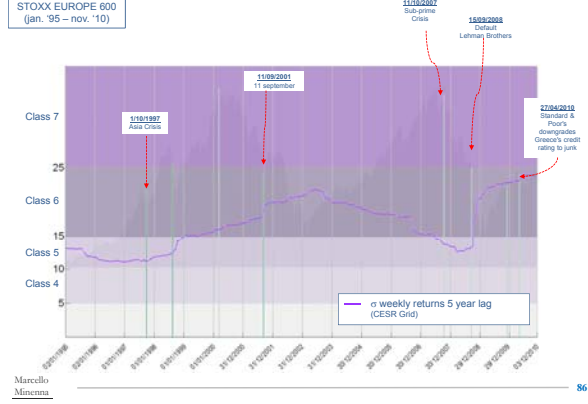
2<sup>nd</sup> Pillar: Synthetic risk indicator



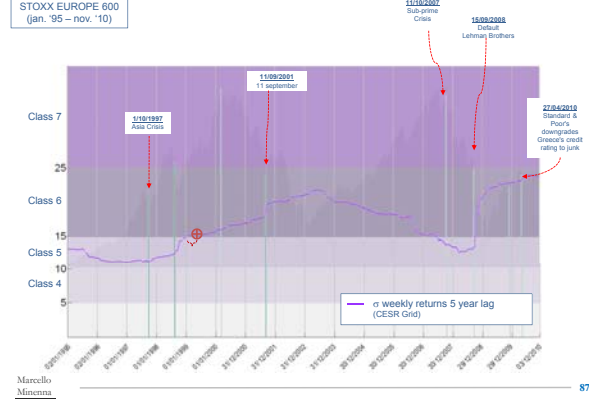
2<sup>nd</sup> Pillar: Synthetic risk indicator



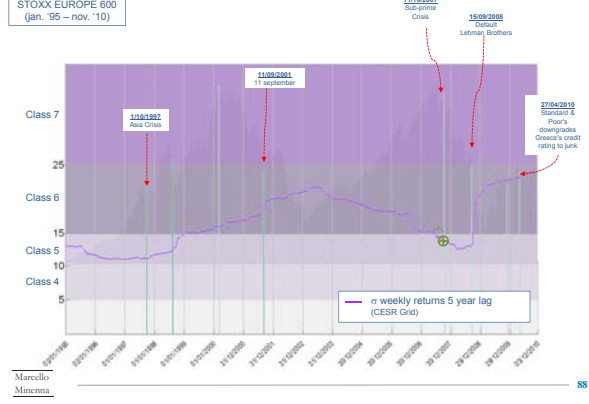
2<sup>nd</sup> Pillar: Synthetic risk indicator



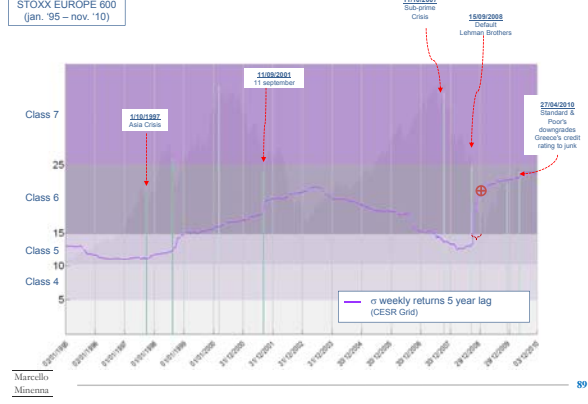
2<sup>nd</sup> Pillar: Synthetic risk indicator



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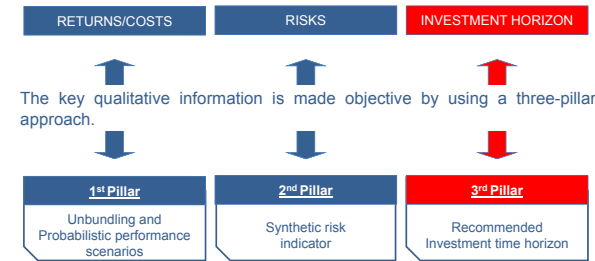
Example

Zeta Bank BOND			
DESCRIPTION	Four-year contingent convertible bond that provides the mandatory conversion into shares of the issuer at predefined date and pricing conditions according to a basket of put and call of European and American options.		
STRUCTURE	RETURN - TARGET		
1 <sup>st</sup> PILLAR	<b>Unbonding table</b>		
	Theoretical value of the risk-free component	70.12	
	Theoretical value of the risky component	25.05	
	Theoretical value of the product	95.17	
	Costs	4.83	
2 <sup>nd</sup> PILLAR	Issue price	100.00	
	<b>PROBABILISTIC SCENARIOS</b>		
	The performance is negative	68.50%	59.2
	The performance is positive but lower than the risk-free asset	2.80%	103.8
	The performance is positive and in line with the risk-free asset	4.40%	113.7
	The performance is positive and higher than the risk-free asset	24.30%	162.3
DEGREE OF RISK: VERY HIGH			

## Syllabus

- Preliminaries: non-equity products and their classification
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon
- Examples

## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon



## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

RISK-TARGET  
PRODUCTS

BENCHMARK  
PRODUCTS

RETURN-TARGET  
PRODUCTS

### The recommended investment time horizon

for return-target products the recommended investment time horizon is *implicit* in their financial engineering, as it can be identified as the period of validity (or the time to maturity) of their target

The payoff at maturity implicitly identifies the time horizon

## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

RISK-TARGET  
PRODUCTS

BENCHMARK  
PRODUCTS

RETURN-TARGET  
PRODUCTS

The use of solutions aimed at ensuring the liquidity and/or the liquidability of a return target product influences its recommended investment time horizon and allows to determine:

### The “minimum” recommended investment time horizon

The event to study from a probabilistic point of view is related to possible exit strategies after having recovered all the costs of the product :

The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

First Passage Time for the cost recovery barrier

## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

RISK-TARGET  
PRODUCTS

BENCHMARK  
PRODUCTS

RETURN-TARGET  
PRODUCTS

### The “minimum” recommended investment time horizon

For risk-target products, the natural way to define a cost recovery event is also:

The investment recovers the initial costs and off-sets the running costs at least once

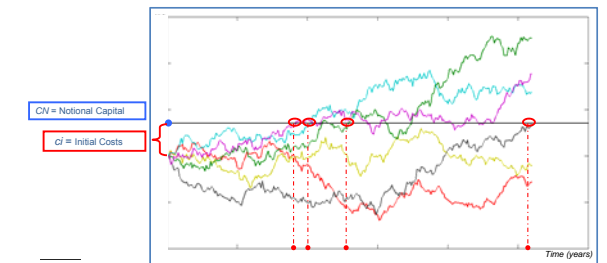
that can be calculated through the concept of

First Passage Time for the cost recovery barrier

## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

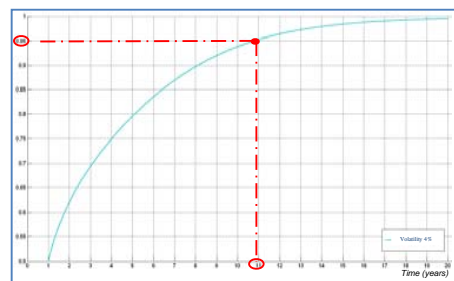
### First Passage Time:

First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



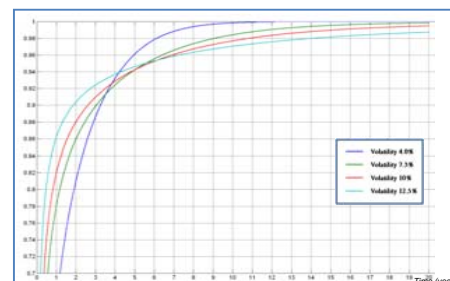
## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

The confidence level  $\alpha$  uniquely identifies  $T^*$  on the cumulative distribution function of the first passage times:



## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



## 3<sup>rd</sup> Pillar: Recommended Investment Time Horizon

minimum investment time horizon ...

$$T^* = \{ T \in \mathbb{R}^+ : P[t^* \leq T] = \alpha \}$$

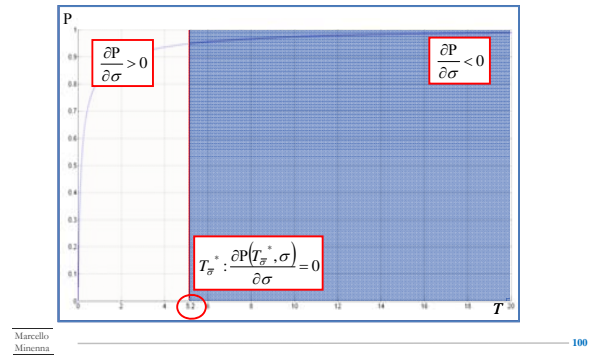
.... must be coherent with the principle

+ VOLATILITY + TIME HORIZON

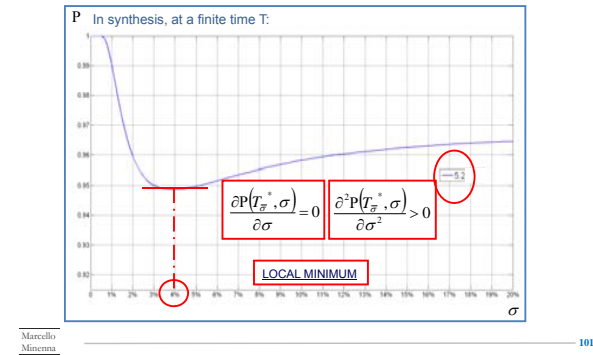
$$\exists T^* \in [0, \infty[ : \frac{dP}{d\sigma} = 0$$

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

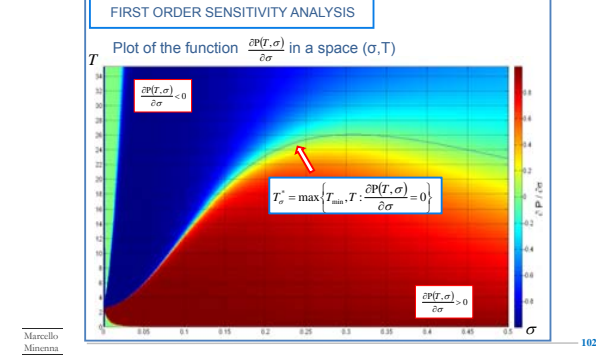
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



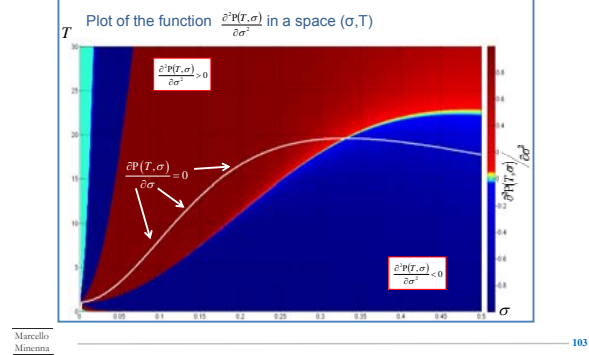
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



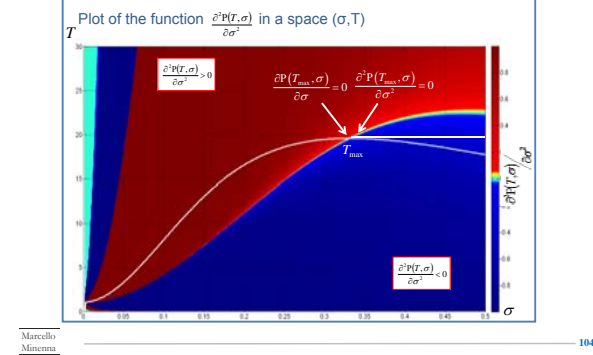
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



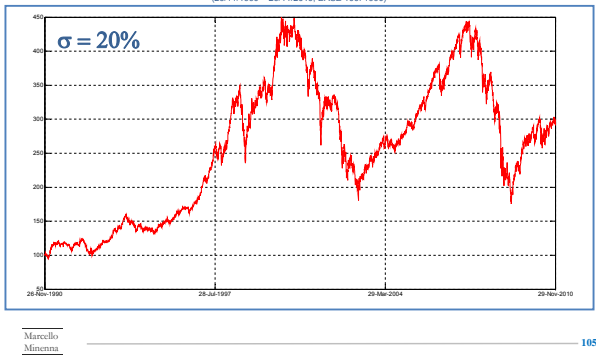
Searching the minimum: the 2<sup>nd</sup> order condition



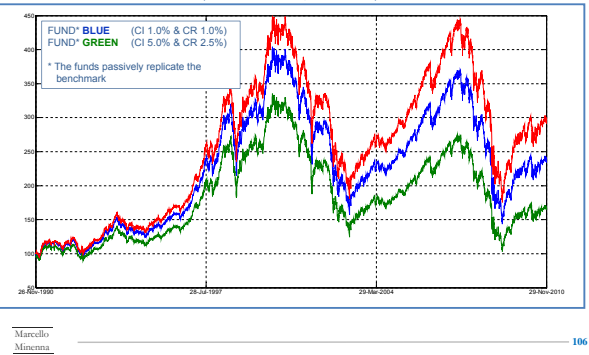
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



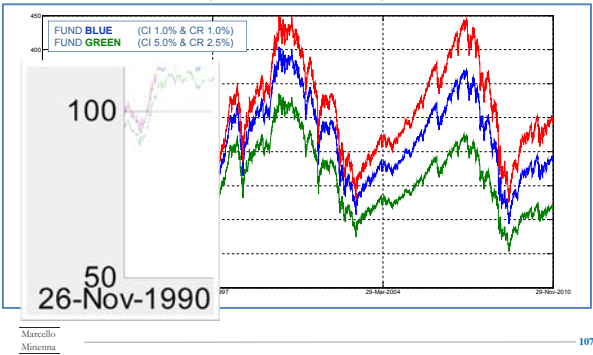
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



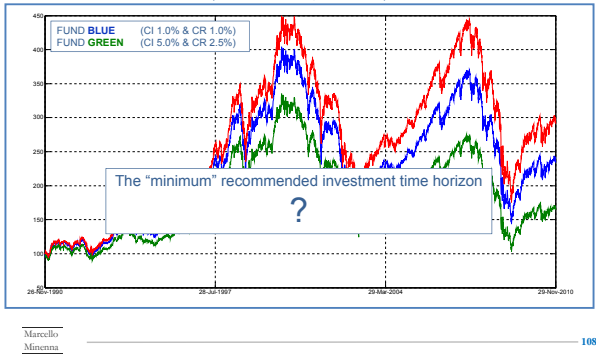
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



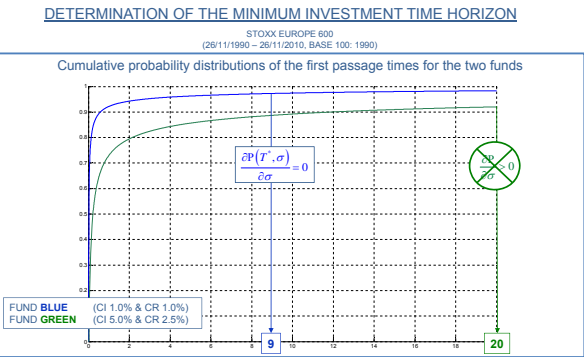
DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



DETERMINATION OF THE MINIMUM INVESTMENT TIME HORIZON



3<sup>rd</sup> Pillar: Recommended Investment Time Horizon



Example

Zeta Bank BOND		
DESCRIPTION	Four-year contingent convertible bond that provides the mandatory conversion into shares of the issuer at predefined date and pricing conditions according to a basket of put and call of European and American options.	
STRUCTURE	RETURN - TARGET	
1 <sup>st</sup> PILLAR	<b>Unbundling Table</b>	
	Theoretical value of the risk-free component	70.12
	Theoretical value of the risky component	25.05
	Theoretical value of the product	95.17
	Costs	4.83
	Issue price	100.00
	<b>PROBABILISTIC SCENARIOS</b>	<b>Probability Central values</b>
	The performance is negative	68.50% 59.2
	The performance is positive but lower than the risk-free asset	2.80% 103.8
	The performance is positive and in line with the risk-free asset	4.40% 113.7
	The performance is positive and higher than the risk-free asset	24.30% 162.3
2 <sup>nd</sup> PILLAR	DEGREE OF RISK: VERY HIGH	
3 <sup>rd</sup> PILLAR	RECOMMENDED INVESTMENT TIME HORIZON: 4 years	

Syllabus

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- Examples

Risk-based approach  
VS  
Narrative approach

Product 1

Narrative approach									
DESCRIPTION	"Formula" mutual fund with a maturity of 8 years and payment of annual random coupons and capital redemption at maturity. The performance of the fund is linked to the variation of the underlying index DJ EURO STOXX 50. Each year the issuer has the option to repay the capital in advance (callability).								
COSTS	<b>COSTS</b>								
	Initial fees	0.65% p/a							
	Management fees	1.00% p/a							
	Issue costs	0.00% p/a							
	Price	100.00							
WHAT-IF SCENARIOS	<b>Unfavorable scenario</b> If at maturity the value of the index DJ EURO STOXX 50 is less than the 20% of its value at the issue date the gross annual return of the fund would be equal to -35%.								
	<b>Neutral scenario</b> If at maturity the value of the index DJ EURO STOXX 50 is between its value at the issue date and a value lower than the 20% of this value the gross annual return of the fund would be equal to -5%.								
	<b>Favorable scenario</b> If at maturity the value of the index DJ EURO STOXX 50 is higher than its value at the issue date the gross annual return of the fund would be equal to 37%.								
SYNTHETIC RISK INDICATOR:									
INVESTMENT TIME HORIZON: ND									
<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr></table>			1	2	3	4	5	6	7
1	2	3	4	5	6	7			

Product 1

Risk-based approach		
DESCRIPTION	"Formula" mutual fund with a maturity of 8 years and payment of annual random coupons and capital redemption at maturity. The performance of the fund is linked to the variation of the underlying index DJ EURO STOXX 50. Each year the issuer has the option to repay the capital in advance (callability).	
STRUCTURE	RETURN - TARGET	
1 <sup>st</sup> PILLAR	<b>Financial investment table</b>	
	Theoretical value of the risk-free component	67.5
	Theoretical value of the risky component	12.5
	Theoretical value of the product (fair value)	80.0
	Costs	20.0
	Price	100.0
	<b>PROBABILISTIC SCENARIOS</b>	<b>Probability Central values</b>
	The performance is negative	46.57% 83.83
	The performance is positive but lower than the risk-free asset	27.93% 110.75
	The performance is positive and in line with the risk-free asset	25.50% 135.76
	The performance is positive and higher than the risk-free asset	0% --
2 <sup>nd</sup> PILLAR	DEGREE OF RISK: <span>Very low</span> <span>Low</span> <span>Medium-low</span> <span>Medium</span> <span>Medium-high</span> <span>High</span> <span>Very high</span>	
3 <sup>rd</sup> PILLAR	RECOMMENDED INVESTMENT TIME HORIZON: 8 YEARS	

Example

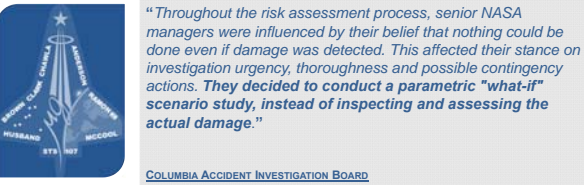
**Probabilistic Performance Scenarios** vs **What-if**

Risk is uncertainty and uncertainty is only quantified by probability.  
That's why the deterministic *what-if* approach is unsuitable to assess **whatever** risky phenomenon...

Example

**Probabilistic Performance Scenarios** vs **What-if**

Risk is uncertainty and uncertainty is only quantified by probability.  
That's why the deterministic *what-if* approach is unsuitable to assess **whatever** risky phenomenon...



References

A quantitative risk-based approach to the transparency on non-equity investment products, *Quaderno di Finanza n. 63, CONSOB* (2009)

A Quantitative Framework to Assess the Risk-Reward Profile of Non-Equity Products, *Riskbooks* (2011)