

Risk based approach to regulating structured products

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Syllabus

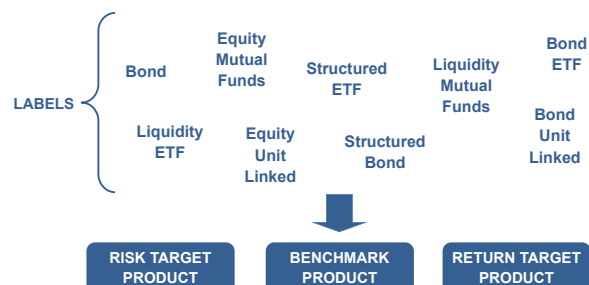
- Preliminaries: closing the gap between risk representation inside prospectus and banks' mark to market valuations
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon

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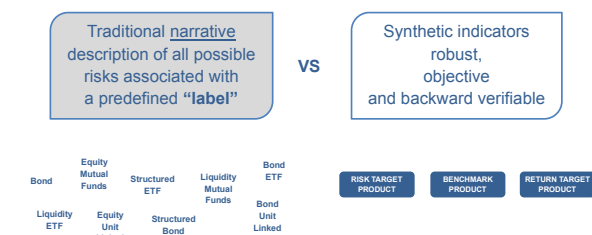
Preliminaries

Non-equity Investment products should be classified according to their financial characteristics and not by "labels" assigned by the issuer or by the regulatory framework.



Preliminaries

Consob transparency regulation on the risk profile of non-equity products is based on synthetic indicators – defined through specific quantitative methods – in order to allow investors to take informed investment decisions.



Preliminaries

The transparency approach which is developing at the level of the European Community, through the revision of the reference Directives (UCITS, Prospectus, MiFID, PRIIPs), seems to drift again towards a logic based on form ("label") as opposed to substance, as regards the risks which characterize a given product.

Non-simple products, for which an enhanced transparency supervision is viewed as necessary, are identified among different working groups by means of terms which often display a lack coherence, e.g.:



Preliminaries

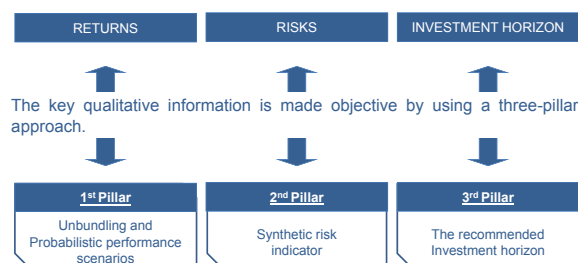
CONSOB – STRATEGIC PLAN 2010-2012

CONSOB aims at «promoting an enhancement of the transparency levels on non-equity products, particularly on the most complex ones which often incorporate components of derivative nature (also implicitly) linked to market and/or credit risk, on the basis of the so-called "three pillars approach" beyond a narrative approach.



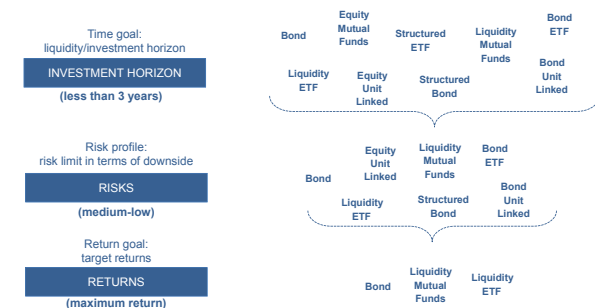
The risk-based transparency approach adopted by CONSOB, by privileging substance over form ("labels") when dealing with risks, represents an opportunity also for issuers, which can take advantage of the best opportunities in the market (even though complex in their structure) in order to offer added value to investors.

Preliminaries



Preliminaries

These metrics provide a guide to investors in the interpretation of complex information conveyed in the offering document, supporting the decision process by means of a sequential filtering procedure:



Preliminaries

The UCITS IV Directive (completely revised) has adopted in the KID (document containing the key Investor Information) only one of the three indicators promoted by Consob's approach (degree of risk), even though with a different specification.

The other two indicators of the *risk-based* approach (*unbundling*/probabilistic scenarios and time horizon) do not find a direct match. In particular:

- CESR has proposed the use of deterministic approaches of the *what-if* kind, in order to implement *performance scenarios*, despite much perplexity has been raised about them;
- the recommended time horizon represents a piece of information which the issuer is free to provide on a discretionary basis.

Introduction

Recent EC works about PRIIPs have highlighted, among other things, the following main orientations (even though not definitive for the lack of a shared vision) about pre-contractual information:

- the principle of comparability has been reaffirmed;
- the KID must be used as a reference
(for those PRIIPs characterized by a given maturity date, the information provided through the synthetic risk indicator and the narrative description could be supplemented by an additional indicator related to the time horizon);
- there exists the opportunity of including information about the expected performance of the PRIIP (an issue which raises the concerns of many subjects about the fact that introducing performance scenarios could confuse investors).

Several countries (the Netherlands, Portugal, Spain, France) have taken part in the discussion with initiatives of various nature (regulatory and not), by supporting approaches of quantitative type.

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1st Pillar: Unbundling and Probabilistic performance scenarios

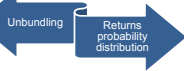
RISK TARGET PRODUCT

BENCHMARK PRODUCT

RETURN TARGET PRODUCT

In "return target" products (e.g. corporate bonds) the connection between the pricing at time zero and the pricing at maturity is evident, as the probability table is a necessary step to obtain the unbundling of the product's price at time 0.

Fair Value
Pricing at time zero



Possible Outcomes
Pricing at maturity

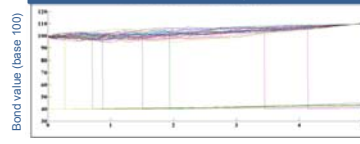
1st Pillar: Unbundling and Probabilistic performance scenarios

5 year fixed-rate bond

Euribor's simulated patterns

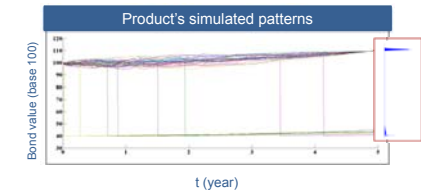


Product's simulated patterns



1st Pillar: Unbundling and Probabilistic performance scenarios

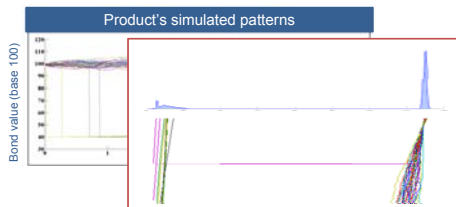
The final values of the bond at the end of the 5th year provide the probability distribution of potential returns (so-called *pricing at maturity*).



Possible Outcomes
Pricing at maturity

1st Pillar: Unbundling and Probabilistic performance scenarios

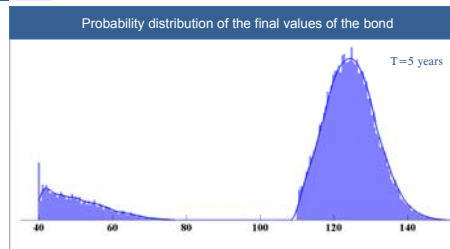
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Possible Outcomes
Pricing at maturity

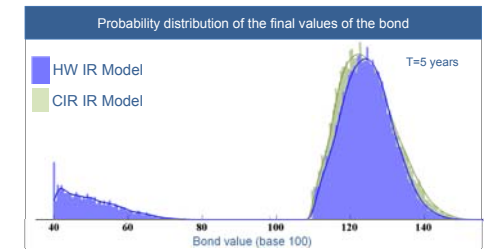
1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: The informative content of the entire probability distribution is very complex to handle for the average retail investor.



1st Pillar: Unbundling and Probabilistic performance scenarios

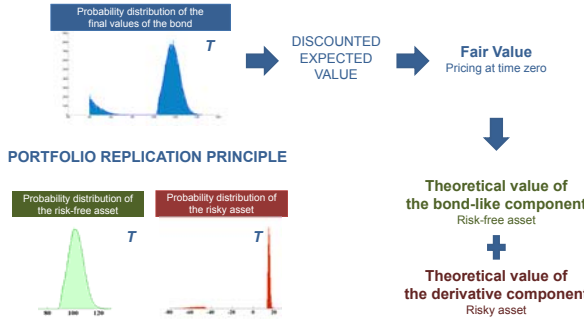
MODEL RISK: The shape of the probability distribution of potential returns is obviously dependent on the model's assumption.



1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: STANDARD SOLUTION



1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1)

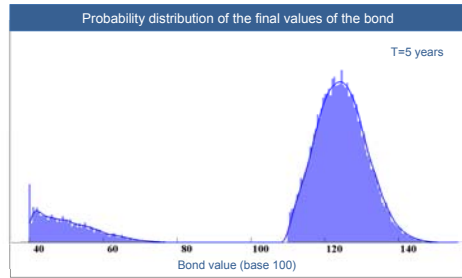


1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1)

Financial investment table
(Unbundling)

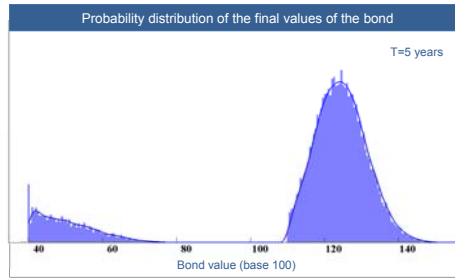
A	Theoretical value of the bond-like component	...
B	Theoretical value of the derivative component	...
C = A + B	Fair value	...
D	Explicit costs	...
E	Implicit costs	...
F = C + D + E	Issue price	100

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



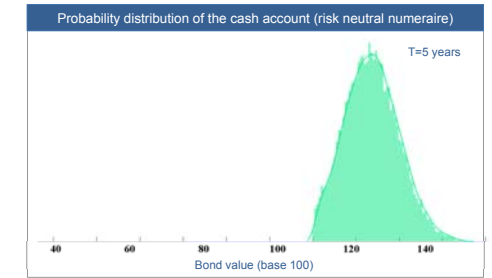
It's interesting to explore a different representation of the information contained in the probability distribution which could be useful for the average investor

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



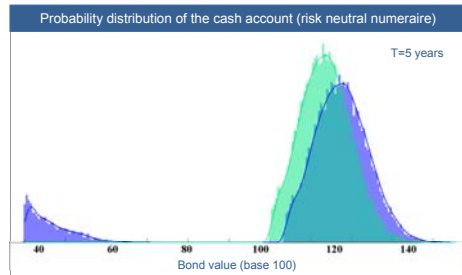
In order to provide the investor with a representation fair, easy to understand and resilient to the model's risk, a simple rescaling with respect to the risk-neutral measure numeraire is presented

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



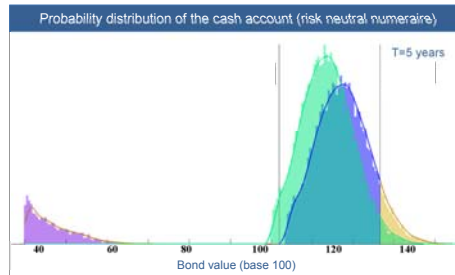
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COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



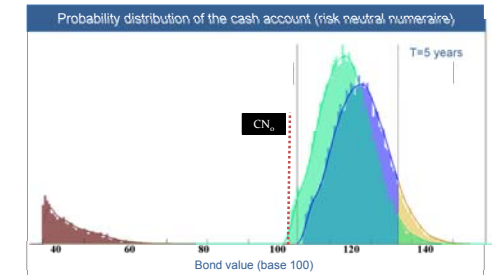
The superimposition of the product's probability distribution with the cash account naturally defines three different events which are effectively meaningful for the investor.

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



- The performance is lower than the risk-free asset
- The performance is in line with the risk-free asset
- The performance is higher than the risk-free asset

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



- The performance is negative
- The performance is positive and lower than the risk-free asset
- The performance is positive and in line with the risk-free asset
- The performance is positive and higher than the risk-free asset

Probabilistic performance scenario table

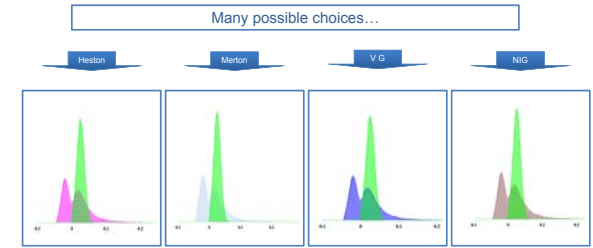
SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is <u>negative</u>	%	€
The performance is <u>positive but lower</u> than the risk-free asset	%	€
The performance is <u>positive and in line</u> with the risk-free asset	%	€
The performance is <u>positive and higher</u> than the risk-free asset	%	€

Connection between the pricing at time zero and the pricing at the end of recommended investment horizon

Time Zero			End of the recommended investment horizon		
Financial investment table			Table of probabilistic performance scenarios		
A	Residual value of the bond-like component	...	SCENARIOS	PROBABILITY	MEDIAN VALUES
B	Theoretical value of the derivative component	...	The performance is <u>negative</u>	%	€
C = A + B	Fair value	...	The performance is <u>positive but lower</u> than the risk-free asset	%	€
D	Explicit costs	...	The performance is <u>positive and in line</u> with the risk-free asset	%	€
E	Implicit costs	...	The performance is <u>positive and higher</u> than the risk-free asset	%	€
F = C + D + E	Issue price	100			

1:1 Relationship

The model risk arising from the right to freely use the proprietary models is solved with the reduction in granularity of events



The results of the various models show differences between each box of less than 5%

... the following output is obtained:

Heston			Merton			VG			NIG		
Scenario	Prob. lity	Median Value	Scenario	Prob. lity	Median Value	Scenario	Prob. lity	Median Value	Scenario	Prob. lity	Median Value
The performance is <u>negative</u>	46.42 %	€ 99.59	The performance is <u>negative</u>	45.69 %	€ 99.28	The performance is <u>negative</u>	43.93 %	€ 99.27	The performance is <u>negative</u>	45.51 %	€ 99.40
The performance is <u>positive but lower</u> than the risk-free asset	3.39 %	€ 103.26	The performance is <u>positive but lower</u> than the risk-free asset	4.74 %	€ 102.54	The performance is <u>positive but lower</u> than the risk-free asset	8.29 %	€ 102.3	The performance is <u>positive but lower</u> than the risk-free asset	2.44 %	€ 103.19
The performance is <u>positive and in line</u> with the risk-free asset	33.28 %	€ 112.39	The performance is <u>positive and in line</u> with the risk-free asset	35.7 %	€ 110.09	The performance is <u>positive and in line</u> with the risk-free asset	36.8 %	€ 109.24	The performance is <u>positive and in line</u> with the risk-free asset	34.28 %	€ 114.23
The performance is <u>positive and higher</u> than the risk-free asset	16.91 %	€ 139.33	The performance is <u>positive and higher</u> than the risk-free asset	14.86 %	€ 142.03	The performance is <u>positive and higher</u> than the risk-free asset	14.16 %	€ 141.77	The performance is <u>positive and higher</u> than the risk-free asset	15.92 %	€ 142.13

Probabilistic Performance Scenarios vs What-if

Probabilistic Performance Scenarios vs What-if

Example:

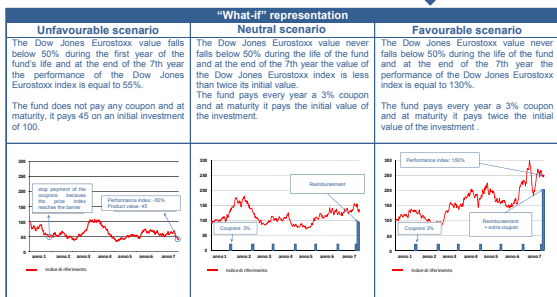
Narrative description of the product's features.

The structured product, whose maturity is 7 years, presents returns which are linked to the Dow Jones Eurostoxx Index.

The fund gives annual coupons, equal to 3% of the initial invested capital, but:

- if, at any time in the fund life, the reference index falls below 50% of its initial value:
 - the payment of coupons is interrupted;
 - at the end of the 7th year the fund will pay back the value of the initial invested capital increased or reduced on the basis of the index performance;
- if the index never falls below 50% of its initial value, at the end of the 7th year the fund will pay:
 - the initial value of the investment;
 - moreover, if at the maturity date the index value is greater or equal to twice its initial value, the fund will pay an additional coupon equal to the initial value of the investment.

Probabilistic Performance Scenarios vs What-if



Probabilistic Performance Scenarios vs What-if

Representation through the probabilistic performance scenarios table at the end of the 7 th year			
SCENARIOS	PROBABILITY	MEDIAN VALUES	YIELD
The performance is <u>negative</u>	38.71%	55.52	-8.06%
The performance is <u>positive but lower</u> than the risk-free asset	8.45%	110.58	1.45%
The performance is <u>positive and in line</u> with the risk-free asset	36.09%	123.13	3.02%
The performance is <u>positive and higher</u> than the risk-free asset	16.75%	223.27	12.16%

Probabilistic Performance Scenarios vs What-if

Interim Research Report
Research on KII Disclosures for UCITS Products

8.6 Performance scenarios relate to a particular form of communicating risk and reward for structured funds. The key considerations for performance scenarios concern the relative effectiveness of communicating risk through (i) a table showing the likelihood of achieving different rates of return; (ii) graphs to show the possible return of the fund under favourable and less favourable conditions; and (iii) a graph displaying backtesting data showing how the fund would have performed under historic market conditions. The following are the key recommendations that we believe would help improve the performance scenarios section:

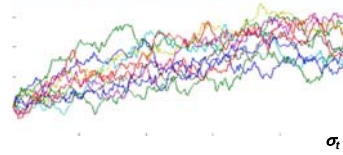
- On the test of clarity the evidence strongly supports the use of a table. Investors suggest this form of communication could be improved by defining technical terms and explaining probability in more detail. In relation to comprehension, improvements need to be made to the wording of messages about product guarantees which are set out in the initial strategy section. These are poorly understood by investors.
- In terms of understanding, a graph displaying back testing data was broadly misunderstood and we do not feel this approach facilitates consumer comprehension. The key issue concern the table showing different rates of return versus graphs to show possible returns under different conditions. On the basis of the research, investors' responses indicate that investors are more likely to understand a table showing the likelihood of achieving different rates of return than a graph showing the possible return of the fund under favourable and less favourable conditions. However, given that there are mixed consumer preferences for both tabular and graphical formats, the latter approach is to be used in testing the clarity of the information presented as a tie-break. This improvement would show probabilities in the form of a percentage and more prominently that the table provides a clearer view of the data than the graph.

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2nd Pillar: Synthetic risk indicator

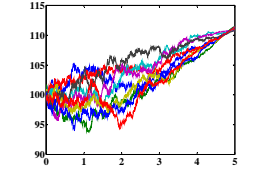
Volatility of the product's potential returns



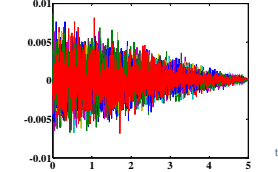
Volatility is the most immediate risk measure and it has a one-to-one relationship with whatever loss measure (VaR, ES, etc.)

2nd Pillar: Synthetic risk indicator

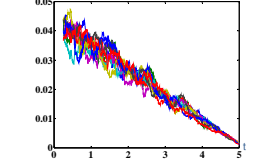
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



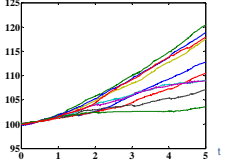
Simulation of the trajectories (Volatility)



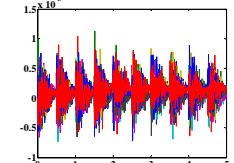
Non-equity product:
Fixed bond like

2nd Pillar: Synthetic risk indicator

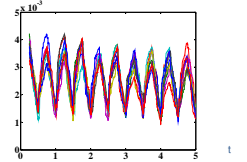
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



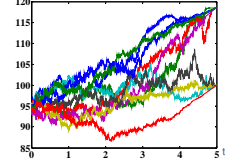
Simulation of the trajectories (Volatility)



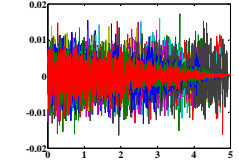
Non-equity product:
Floater bond like

2nd Pillar: Synthetic risk indicator

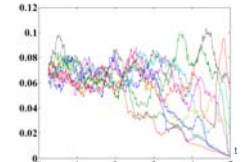
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



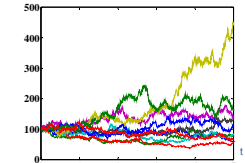
Simulation of the trajectories (Volatility)



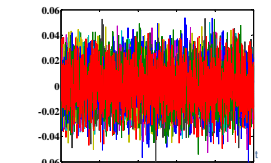
Non-equity product:
ZCB

2nd Pillar: Synthetic risk indicator

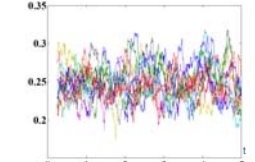
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



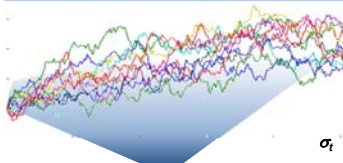
Simulation of the trajectories (Volatility)



Non-equity product:
Equity like

2nd Pillar: Synthetic risk indicator

Volatility of the product's potential returns



DEGREE OF RISK

MEASUREMENT:
product's positioning inside a grid of n volatility intervals

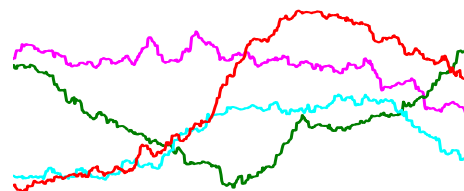
REPRESENTATION:
mapping of any volatility interval into a corresponding qualitative risk class



Risk Classes	Volatility Intervals	
Very Low	σ_{min}	σ_{max}
Low	σ_{min}	σ_{max}
Medium-Low	σ_{min}	σ_{max}
Medium	σ_{min}	σ_{max}
Medium-High	σ_{min}	σ_{max}
High	σ_{min}	σ_{max}
Very High	σ_{min}	σ_{max}

2nd Pillar: Synthetic risk indicator

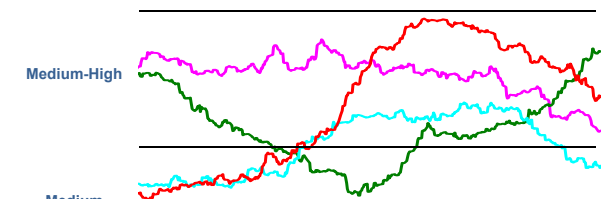
Products with the same risk budget must have the same degree of risk



σ_t

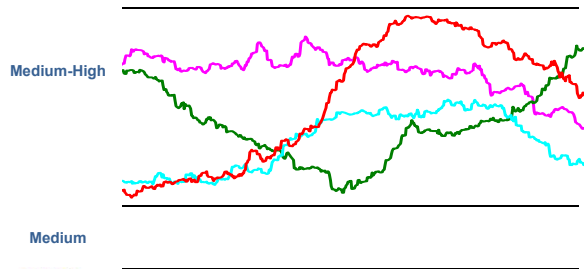
2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations



2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations



2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations

THE ISSUE

Defining suitable requirements to partition the volatility space $[0, +\infty)$ into an optimal number n^* of subsequent intervals with optima extrema



2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations

Requirement n.1 the optimal grid of volatility intervals has to be consistent with the principle:

+ RISK + LOSSES



VOLATILITY INTERVALS MUST HAVE AN INCREASING WIDTH IN ABSOLUTE TERMS

2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations

Requirement n.2

the optimal grid of volatility intervals must be market feasible



REALIZED VOLATILITY CONSISTENT WITH MARKET EXPECTATION OF FUTURE VOLATILITY (UNLESS FOR SIGNIFICANT SUDDEN SHOCKS)

2nd Pillar: Synthetic risk indicator

Realized volatility

Any product on the markets reflects specific asset management policies

Historical data can be "dirty"



1st INTUITION

It has to be studied a theoretical product managed by an automatic asset manager who has a specific risk budget, identified by a given volatility interval

2nd Pillar: Synthetic risk indicator

Market expectations of future volatility

future volatility is predicted by exploiting information embedded in recently observed data



2nd INTUITION

Market expectation is given by volatility prediction intervals based on proper diffusive models

2nd Pillar: Synthetic risk indicator

Assessing market feasibility

putting together the two ingredients



3rd INTUITION

It requires to study when the volatility realized by the automatic asset manager is outside the volatility prediction interval (so-called management failures)

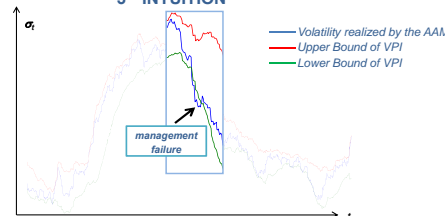
2nd Pillar: Synthetic risk indicator

Assessing market feasibility

putting together the two ingredients



3rd INTUITION



2nd Pillar: Synthetic risk indicator

The three intuitions lead to restate the requirement of market feasibility as a two-fold problem

NOT ABNORMALITY (for any interval):

none interval displays an abnormal number of management failures

HOMOGENEITY (across risk budgets):

the number of management failures is (almost) the same for all volatility intervals



SUITABLE WIDTH OF THE INTERVAL



NO INCENTIVES TO CHOOSE ANY SPECIFIC RISK BUDGET

2nd Pillar: Synthetic risk indicator

Solving for the optimal grid

ON THE FULL VOLATILITY SPACE [0, +∞)

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

The optimal grid of volatility intervals is consistent with the 1st requirement:

+ RISK + LOSSES

2nd Pillar: Synthetic risk indicator

CONSOB

vs

CESR

Volatility grid

Risk Classes	Volatility Intervals
Very Low	0.01% - 0.24%
Low	0.25% - 0.63%
Medium-Low	0.64% - 1.59%
Medium	1.60% - 3.99%
Medium-High	4.00% - 9.99%
High	10.00% - 24.99%
Very High	25.00% - >25.00%

Annualized volatility estimated on daily returns over 1 year



Migration

3 months out of the risk class indicated in the prospectus

Volatility grid

Risk Classes	Volatility Intervals
Very Low	0% - 0.5%
Low	0.5% - 2.0%
Medium-Low	2.0% - 5.0%
Medium	5.0% - 15.0%
Medium-High	15.0% - 15.0%
High	15.0% - 25.0%
Very High	25.0%

Annualized volatility estimated on weekly returns over 5 years

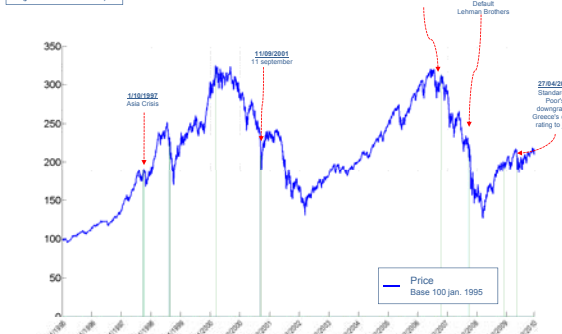


Migration

4 months out of the risk class indicated in the prospectus

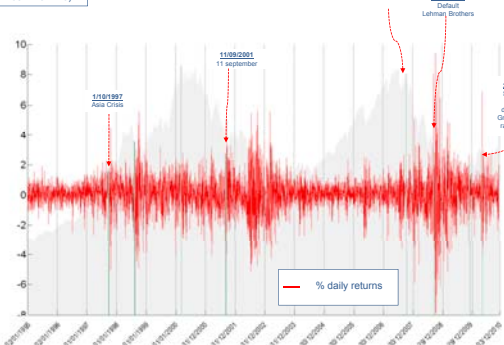
2nd Pillar: Synthetic risk indicator

STOXX EUROPE 600 (jan. '95 – nov. '10)



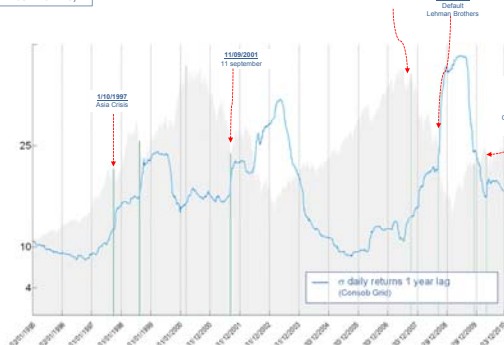
2nd Pillar: Synthetic risk indicator

STOXX EUROPE 600 (jan. '95 – nov. '10)



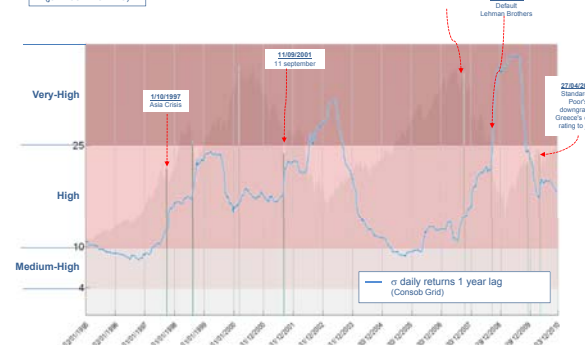
2nd Pillar: Synthetic risk indicator

STOXX EUROPE 600 (jan. '95 – nov. '10)



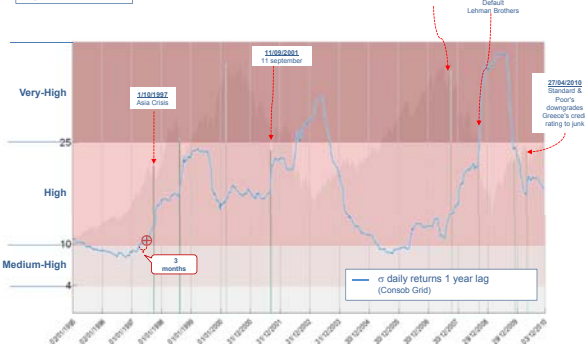
2nd Pillar: Synthetic risk indicator

STOXX EUROPE 600 (jan. '95 – nov. '10)



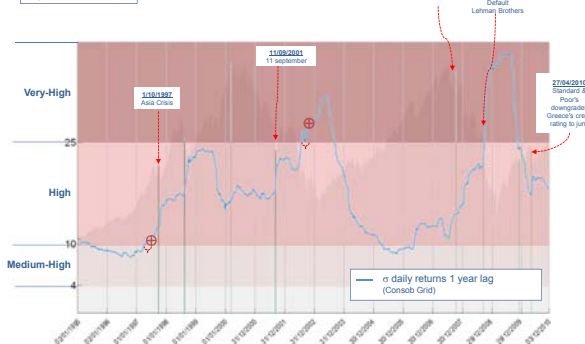
2nd Pillar: Synthetic risk indicator

STOXX EUROPE 600 (jan. '95 – nov. '10)



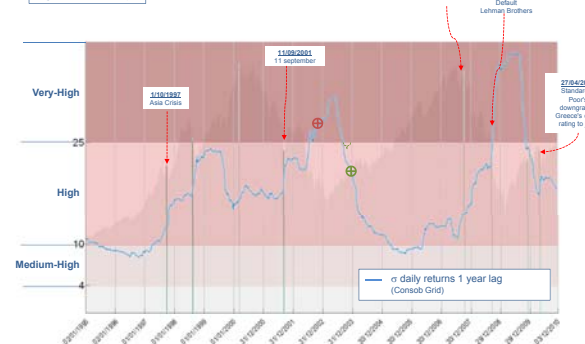
2nd Pillar: Synthetic risk indicator

STOXX EUROPE 600 (jan. '95 – nov. '10)

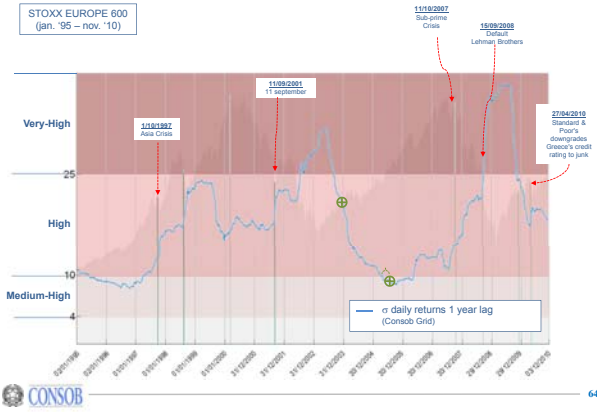


2nd Pillar: Synthetic risk indicator

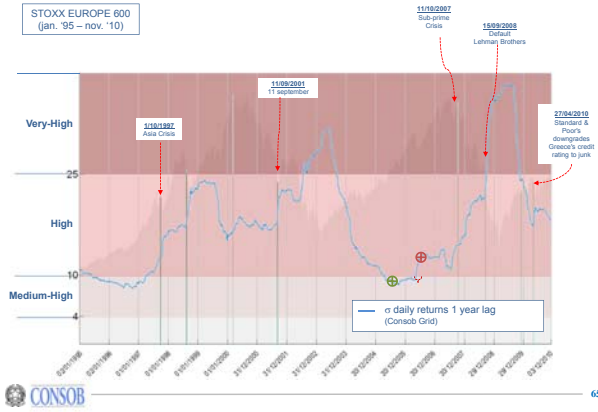
STOXX EUROPE 600 (jan. '95 – nov. '10)



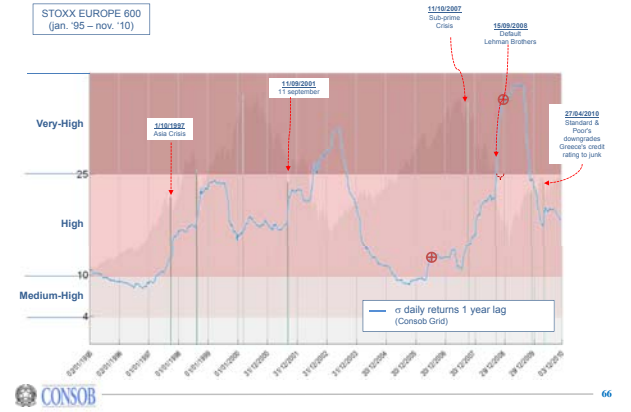
2nd Pillar: Synthetic risk indicator



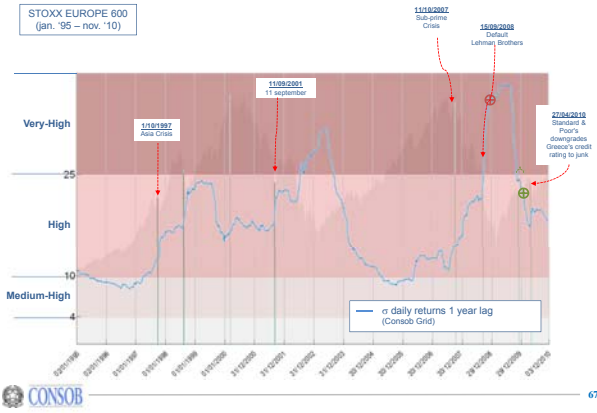
2nd Pillar: Synthetic risk indicator



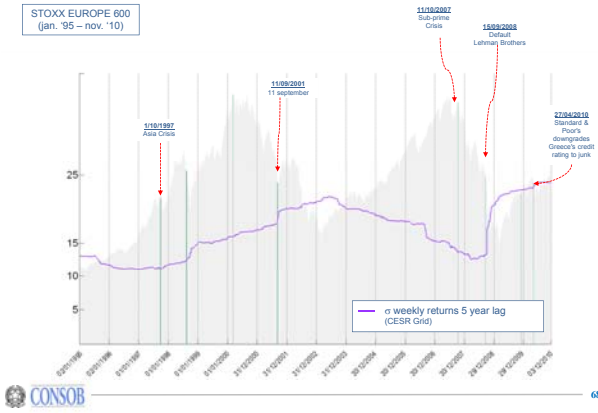
2nd Pillar: Synthetic risk indicator



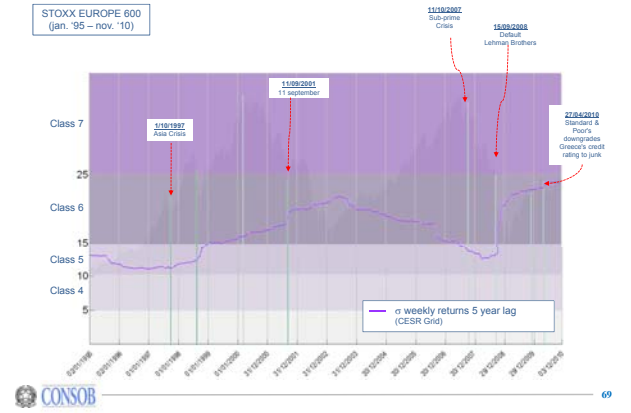
2nd Pillar: Synthetic risk indicator



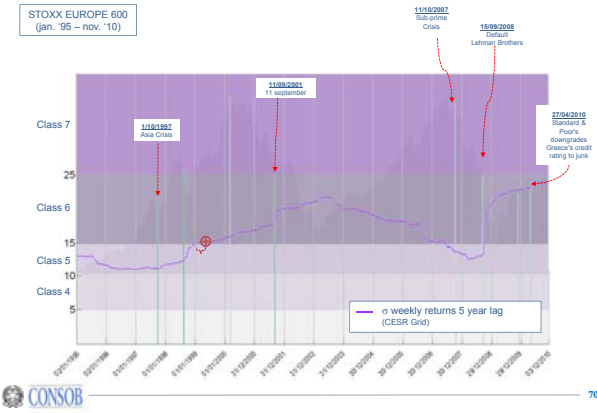
2nd Pillar: Synthetic risk indicator



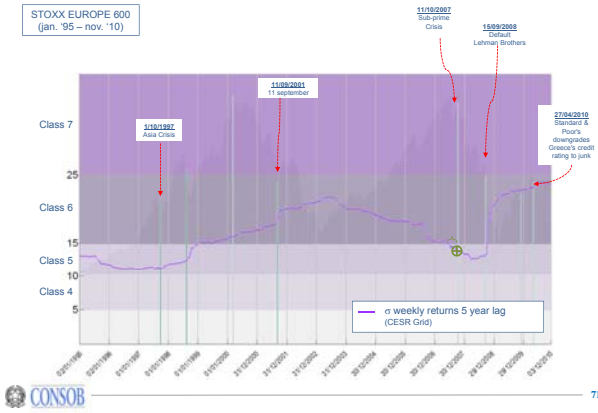
2nd Pillar: Synthetic risk indicator



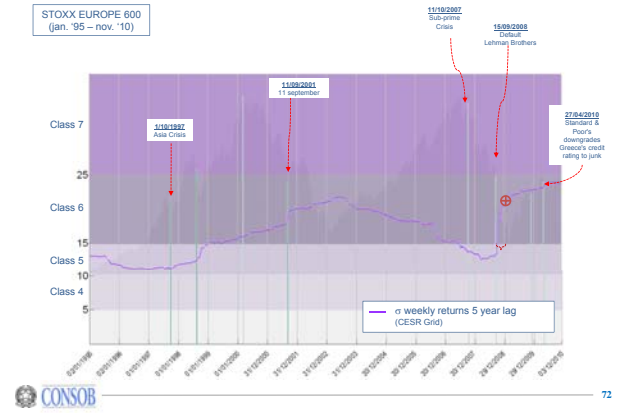
2nd Pillar: Synthetic risk indicator



2nd Pillar: Synthetic risk indicator



2nd Pillar: Synthetic risk indicator



Syllabus

- Preliminaries: closing the gap between risk representation inside prospectus and banks' mark to market valuations
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon

3rd Pillar: The recommended Investment horizon

RISK TARGET PRODUCT

BENCHMARK PRODUCT

RETURN TARGET PRODUCT

The recommended investment time horizon

for performance target products the recommended minimum investment horizon is inherent to their financial engineering, as the recommended investment horizon is equal to the period of validity (or the time to maturity) of their target

The payoff at maturity uniquely identifies the time when the potential returns are optimized

3rd Pillar : The recommended Investment horizon

RISK TARGET PRODUCT

BENCHMARK PRODUCT

RETURN TARGET PRODUCT

The recommended investment time horizon

The use of solutions aimed at ensuring the liquidity and/or marketability of a return target product changes its risk-return profile and its recommended investment time horizon

The event to study from a probabilistic point of view transforms into:

The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

First Passage Time

The "minimum" recommended investment time horizon

3rd Pillar : The recommended Investment horizon

RISK TARGET PRODUCT

BENCHMARK PRODUCT

RETURN TARGET PRODUCT

The "minimum" recommended investment time horizon

For risk target products, the natural way to define a cost recovery event is also:

The investment recovers the initial costs and off-sets the running costs at least once

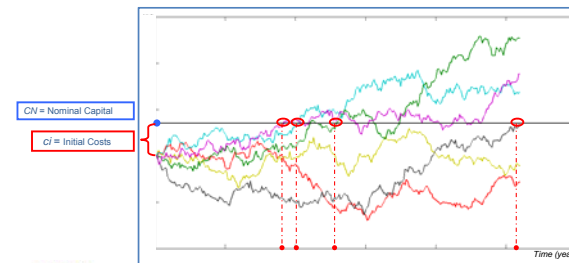
that can be calculated through the concept of

First Passage Time

3rd Pillar : The recommended Investment horizon

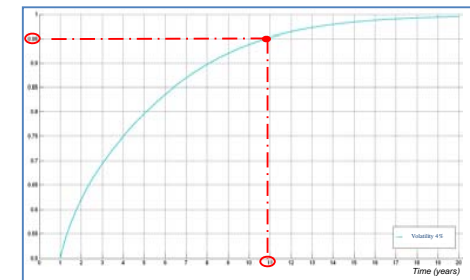
First Passage Time:

First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



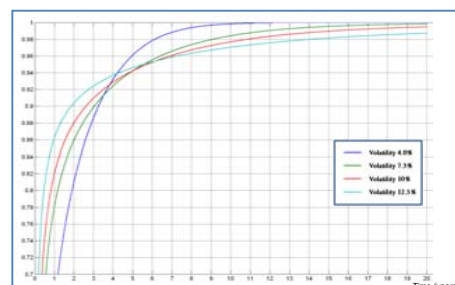
3rd Pillar : The recommended Investment horizon

The confidence level α uniquely identifies T on the cumulative distribution function of the first passage times.



3rd Pillar : The recommended Investment horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



3rd Pillar : The recommended Investment horizon

Anyway, the recommended **minimum** investment time horizon...

$$T^* = \left\{ T \in \mathbb{R}^+ : \mathbb{P}[t^* \leq T] = \alpha \right\}$$

... must be coherent with the principle

+ VOLATILITY + TIME HORIZON

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

3rd Pillar : The recommended Investment horizon

Searching the minimum: the **1st order condition**

$T \rightarrow \infty, dt \rightarrow 0$

FIRST ORDER SENSITIVITY ANALYSIS

$$\frac{dP}{d\sigma} = \left[-4 \frac{(\bar{r} - cr)}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2(T-cr)}{\sigma^2} - 1} \right]$$

FIRST ORDER ASYMPTOTIC CONDITION

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left(-4 \frac{\bar{r}}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$$1. \bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$$

$$2. \bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$$

$cr = 0$

Since it is safe to assume a positive interest rate r in financial markets, only condition 1. correctly captures the connection between volatility and time horizon.

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left(-4 \frac{\bar{r}}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2\bar{r}}{\sigma^2}-1} \right)$$

$$1. \bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$$

$$2. \bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$$

$cr = 0$

In other words, for a given confidence level, as the volatility grows, the recommended investment time horizon increases as well:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 1st order condition**

$$T \rightarrow \infty, dt \rightarrow 0$$

$$1. \begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Leftrightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases}$$

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

$$2. \bar{r} - cr \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$$

Summarizing the results of the asymptotic analysis in continuous time:

- As $T \rightarrow \infty$, for a given confidence level, more volatility implies a larger recommended investment time horizon
- It is always possible to find a minimum and finite time T , beyond which the strong condition

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

holds

3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

General Framework:

$$T \rightarrow \infty$$

$$dt \rightarrow 0$$

$$P(x, \sigma)$$

$$1. \begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Leftrightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases}$$

$$\Rightarrow P(T, \sigma)$$

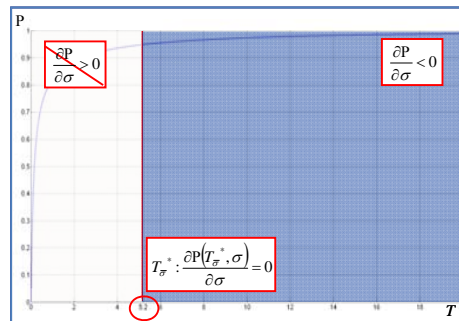
$$\begin{cases} (\bar{r} - cr) > 0 \Leftrightarrow \lim_{T \rightarrow \infty} \frac{\partial P(T, \sigma)}{\partial \sigma} < 0 \\ (\bar{r} - cr) > 0 \Leftrightarrow \lim_{T \rightarrow \infty} \frac{\partial^2 P(T, \sigma)}{\partial \sigma^2} > 0 \end{cases}$$

T finite

Everything shown above also holds with T finite!

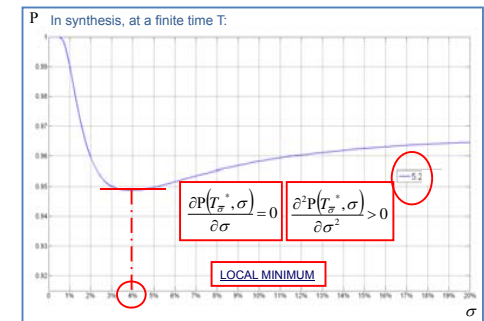
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended Investment horizon

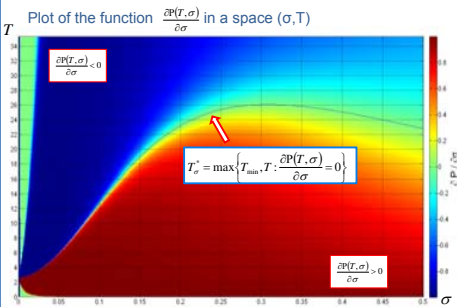
DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended Investment horizon

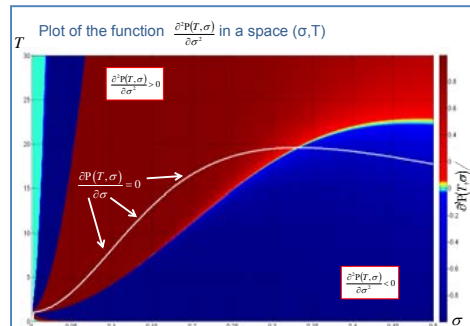
DETERMINATION OF THE INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS



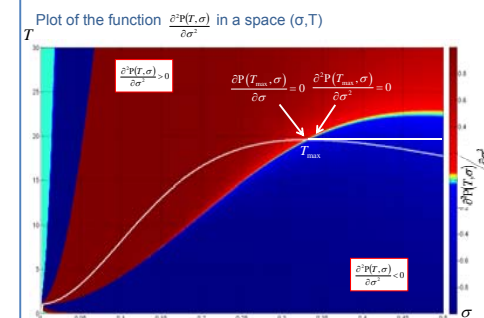
3rd Pillar : The recommended Investment horizon

Searching the minimum: **the 2nd order condition**



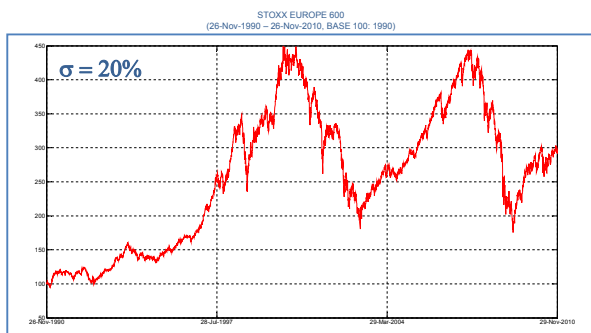
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



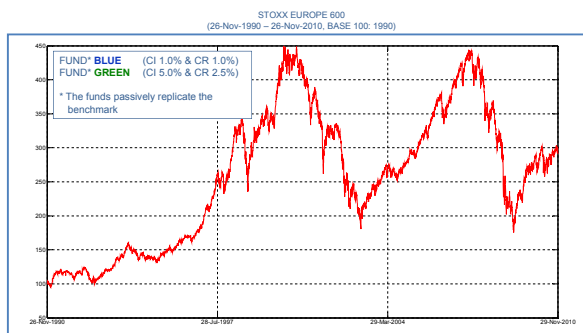
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



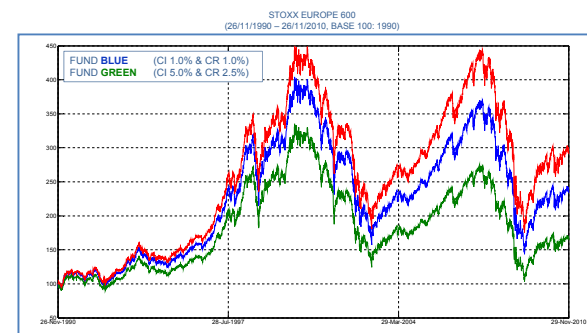
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



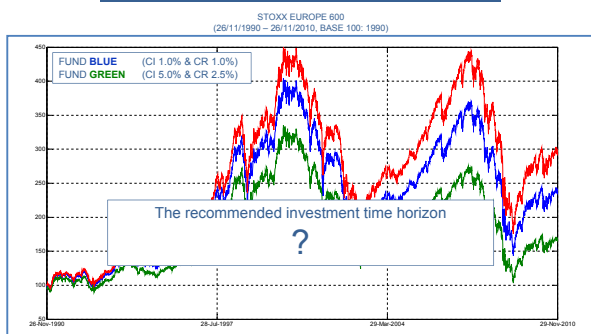
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



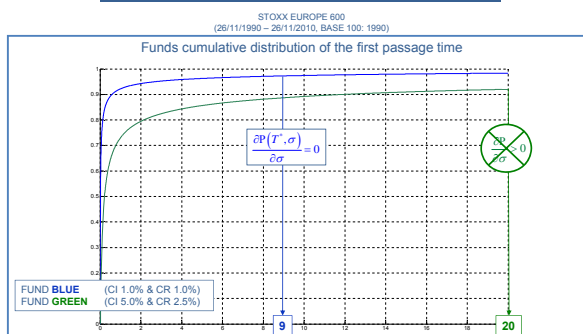
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



Risk based approach to regulating structured products

EXAMPLES

Examples

DERIVATIVE PRODUCT

The product presents the following payoff:

- if the reference equity index remains above 50% of its initial value, the investor receives a quarterly fixed coupon equal to 1.8% of the issue price and the payment of the invested capital at maturity;
- if the index reaches 50% of its initial value the coupon flow is interrupted and at maturity the investor receives a payment for the investment equal to the performance of the Index.

Unbundling Table		
Theoretical value of the Debt component		0.00
Theoretical value of the Derivative component		88.44
Theoretical value of the product		88.44
Costs		11.56
Issue price		100.00

PROBABILISTIC SCENARIOS	Event Probability	Median Value
The performance is negative	46.160%	60.120%
The performance is positive but lower than the risk-free asset	4.860%	107.130%
The performance is positive and in line with the risk-free asset	3.430%	128.380%
The performance is positive and higher than the risk-free asset	45.550%	152.820%

1st PILLAR

2nd PILLAR Degree of Risk: Medium-High

3rd PILLAR Recommended investment time horizon: 6 years and 6 months

Examples

STRUCTURED PRODUCT

The investor receives fixed coupons with values increasing from 1% to 2.5% for the first 3 years. At maturity, she receives the payment of the issue price possibly increased by an additional bonus equal to 35% of the reference index performance (if positive) multiplied by the issue price.

Unbundling Table		
Theoretical value of the Debt component		85.62
Theoretical value of the Derivative component		7.09
Theoretical value of the product		92.71
Costs		7.29
Issue price		100.00

PROBABILISTIC SCENARIOS	Event Probability	Median Value
The performance is negative	8.72%	45.59%
The performance is positive but lower than the risk-free asset	0%	0%
The performance is positive and in line with the risk-free asset	87.10%	111.97%
The performance is positive and higher than the risk-free asset	4.18%	155.91%

1st PILLAR

2nd PILLAR Degree of Risk: Medium

3rd PILLAR Recommended investment time horizon: 6 years e 9 months

Examples

SUBORDINATED BOND

Subordinated bond with a 7 year maturity, paying bi-annual step-up coupons ranging from 4.7% to 5.30% and characterized by an amortizing plan from the 3rd to the 7th year.

STRUCTURE RETURN TARGET

Unbundling Table		
Theoretical value of the Debt component		83.361
Theoretical value of the Derivative component		11.032
Theoretical value of the product		94.393
Costs		5.607
Issue price		100.00

PROBABILISTIC SCENARIOS	Event Probability	Median value
The performance is negative	23.51%	54.73%
The performance is positive but lower than the risk-free asset	0.55%	100.23%
The performance is positive and in line with the risk-free asset	74.48%	133.05%
The performance is positive and higher than the risk-free asset	1.46%	144.66%

1st PILLAR

2nd PILLAR Degree of Risk: Medium-High

3rd PILLAR Recommended investment time horizon: 7 years