



Rebuilding investor confidence through risk disclosure Risk-based transparency on structured products through synthetic indicators

Marcello Minenna – Head of Quantitative Analysis Unit, ConsoB



Syllabus

- Preliminaries: closing the gap between risk representation inside prospectus and banks' mark to market valuations
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon



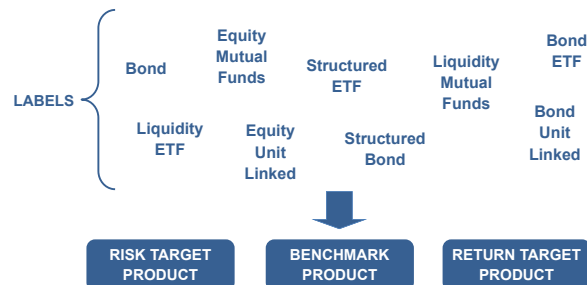
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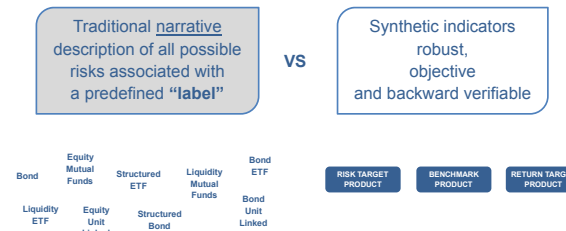
Preliminaries

Non-equity Investment products should be classified according to their financial characteristics and not by "labels" that are assigned by the issuer and/or by the European regulatory framework.

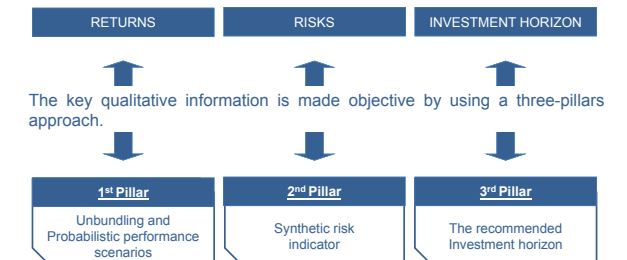


Preliminaries

ConsoB regulation on transparency on the risk profile of non-equity investment products is based on synthetic indicators – defined through the development of specific quantitative methods – in order to allow investors to take informed investment decisions.

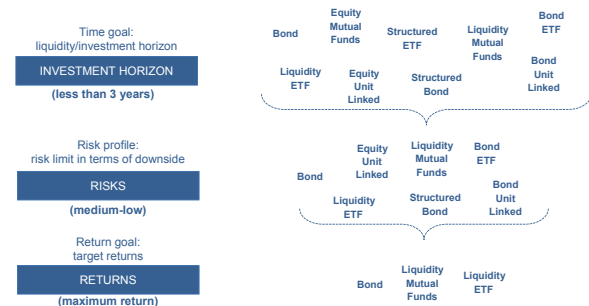


Preliminaries



Preliminaries

These metrics provide a guide to investors in the interpretation of the complex information conveyed in the offering document, supporting the decision process of the investor by using a sequential filtering procedure:



Preliminaries

CONSOB aims at «promoting an enhancement of the transparency levels on non-equity products, particularly on the most complex ones which often incorporate components of derivative nature (also implicitly) linked to market and/or credit risk, on the basis of the so-called "three pillars approach"» beyond a narrative approach.

The risk-based transparency approach adopted by CONSOB, by privileging substance over form ("labels") when dealing with risks, represents an opportunity also for issuers, which can take advantage of the best opportunities in the market (even though complex in their structure) in order to offer added value to investors.



Preliminaries

The transparency approach which is developing at the level of the European Community, through the revision of the reference Directives (UCITS, Prospectus, MiFID, PRIps), seems to drift again towards a logic based on form ("label") as opposed to substance, as regards the risks which characterize a given product.

Non-simple products, for which an enhanced transparency supervision is viewed as necessary, are identified among different working groups by means of terms which often display a lack coherence, e.g.:



Preliminaries

The UCITS IV Directive (almost completely revised) has adopted in the KID (document containing the key Investor Information) only one of the three indicators promoted by Consob's approach (degree of risk), even though with a different specification.

The other two indicators of the *risk-based* approach (*unbundling*/probabilistic scenarios and time horizon) do not find a direct match. In particular:

- CESR has proposed the use of deterministic approaches of the *what-if* kind, in order to implement *performance scenarios*, despite much perplexity has been raised about them;
- the recommended time horizon represents a piece of information which the issuer is free to provide on a discretionary basis.

Introduction

Recent EC works about PRIPs have highlighted, among other things, the following main orientations (even though not definitive for the lack of a shared vision) about pre-contractual information:

- the principle of comparability has been reaffirmed;
- the KID must be used as a reference
(for those PRIPs characterized by a given maturity date, the information provided through the synthetic risk indicator and the narrative description could be supplemented by an additional indicator related to the time horizon);
- there exists the opportunity of including information about the expected performance of the PRIIP (an issue which raises the concerns of many subjects about the fact that introducing performance scenarios could confound investors).

Introduction

At the EC level, the debate about the employment of quantitative metrics as opposed to a narrative description is still open.



Numerous countries (the Netherlands, Portugal, Spain, France) have taken part in the discussion with works of various nature (regulatory and not), by supporting approaches of quantitative kind.

Syllabus

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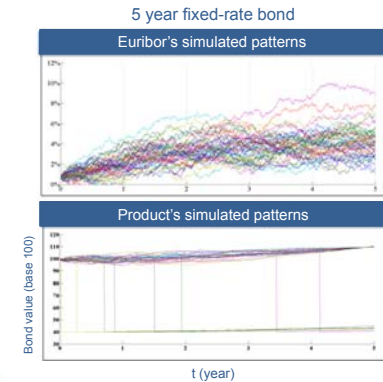
1st Pillar: Unbundling and Probabilistic performance scenarios



In "return target" products (e.g. corporate bonds) the connection between the pricing at time zero and the pricing at maturity is evident, as the probability table is a necessary step to obtain the unbundling of the price of the product at time 0.

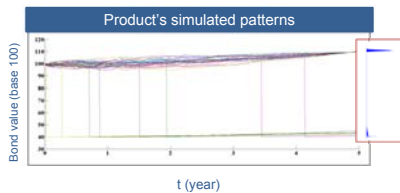


1st Pillar: Unbundling and Probabilistic performance scenarios



1st Pillar: Unbundling and Probabilistic performance scenarios

The final values of the bond at the end of the 5th year provide the probability distribution of potential returns (so-called *pricing at maturity*).



Possible Outcomes
Pricing at maturity

1st Pillar: Unbundling and Probabilistic performance scenarios

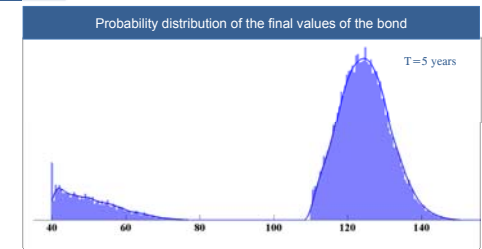
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Possible Outcomes
Pricing at maturity

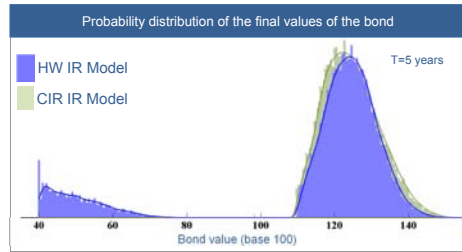
1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: The informative content of the entire probability distribution is very complex to handle for the average retail investor.



1st Pillar: Unbundling and Probabilistic performance scenarios

MODEL RISK: The shape of the probability distribution of potential returns is obviously dependent from the model's assumption.



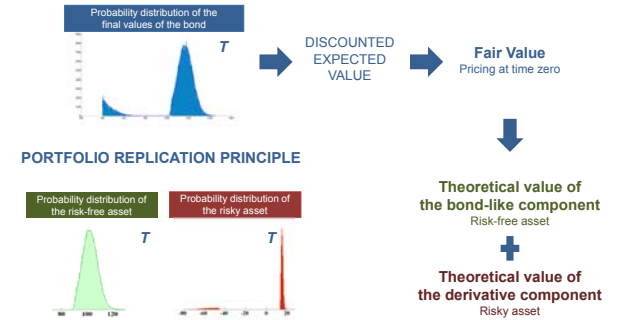
1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: STANDARD SOLUTION



1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1)



1st Pillar: Unbundling and Probabilistic performance scenarios

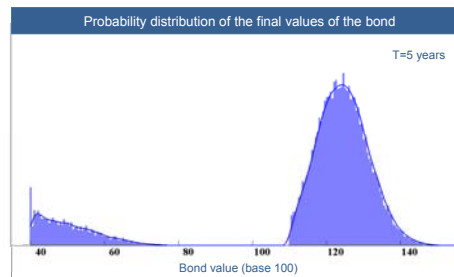
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1)

Financial investment table (Unbundling)

A	Theoretical value of the bond-like component	...
B	Theoretical value of the derivative component	...
C = A + B	Fair value	...
D	Explicit costs	...
E	Implicit costs	...
F = C + D + E	Issue price	100

1st Pillar: Unbundling and Probabilistic performance scenarios

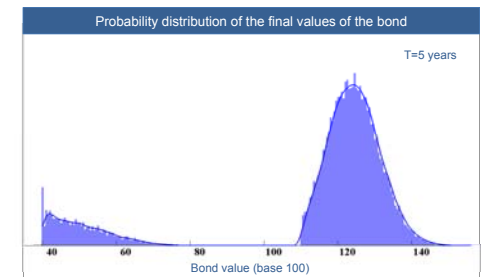
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



It's interesting to explore a different representation of the information contained in the probability distribution which could be useful for the average investor

1st Pillar: Unbundling and Probabilistic performance scenarios

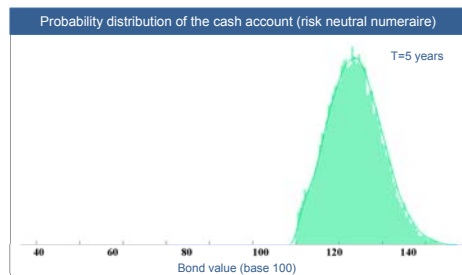
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



In order to provide the investor with a representation fair, easy to understand and resilient to the model's risk, a simple rescaling with respect to the risk-neutral measure numeraire is presented

1st Pillar: Unbundling and Probabilistic performance scenarios

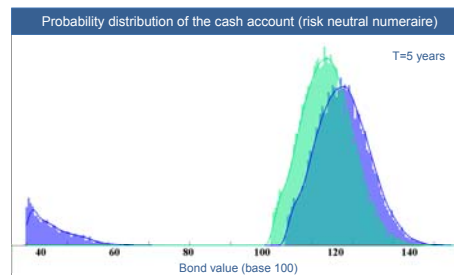
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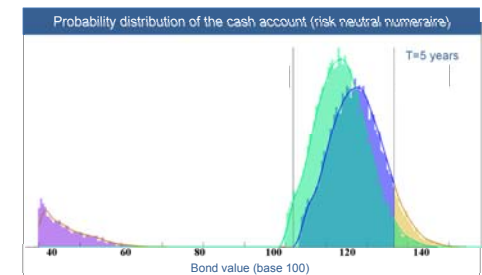
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



The superimposition of the product's probability distribution with the cash account naturally defines three different events which are effectively meaningful for the investor.

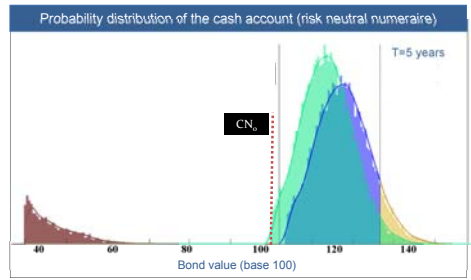
1st Pillar: Unbundling and Probabilistic performance scenarios

COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



- The performance is lower than the risk-free asset
- The performance is in line with the risk-free asset
- The performance is higher than the risk-free asset

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)



The performance is negative
 The performance is positive and lower than the risk-free asset
 The performance is positive and in line with the risk-free asset
 The performance is positive and higher than the risk-free asset

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (2)

Probabilistic performance scenario table

SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is <u>negative</u>	%	€
The performance is <u>positive but lower than the risk-free asset</u>	%	€
The performance is <u>positive and in line with the risk-free asset</u>	%	€
The performance is <u>positive and higher than the risk-free asset</u>	%	€

1st Pillar: Unbundling and Probabilistic performance scenarios
COMPLEXITY FOR RETAIL INVESTORS: CONSOB REGULATION (1) e (2)

Connection between the pricing at time zero and the pricing at the end of recommended investment horizon

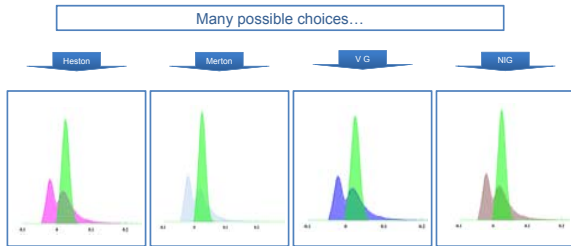
Time Zero		End of the recommended investment horizon	
Financial investment table			
A	Discounted value of the bond-like component
B	Theoretical value of the derivative component
C = A + B	Fair value
D	Explicit costs
E	Implicit costs
F = C + D + E	Issue price	100	...

Table of probabilistic performance scenarios		
SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is <u>negative</u>	%	€
The performance is <u>positive but lower than the risk-free asset</u>	%	€
The performance is <u>positive and in line with the risk-free asset</u>	%	€
The performance is <u>positive and higher than the risk-free asset</u>	%	€

1:1 Relationship

1st Pillar: Unbundling and Probabilistic performance scenarios
MODEL RISK: CONSOB REGULATION

The model risk arising from the right to freely use the proprietary models is solved with the reduction in granularity of events



1st Pillar: Unbundling and Probabilistic performance scenarios
MODEL RISK: CONSOB REGULATION

The results of the various models show differences between each box of less than 5%

... the following output is obtained:

Heston			Merton			VG			NIG		
Scenario	Prob. (Std)	Median Value	Scenario	Prob. (Std)	Median Value	Scenario	Prob. (Std)	Median Value	Scenario	Prob. (Std)	Median Value
The performance is <u>negative</u>	86.42 % (91.91)	€	The performance is <u>negative</u>	42.68 % (91.23)	€	The performance is <u>negative</u>	42.91 % (91.23)	€	The performance is <u>negative</u>	48.24 % (91.91)	€
The performance is <u>positive but lower than the risk-free asset</u>	1.38 % (101.26)	€	The performance is <u>positive but lower than the risk-free asset</u>	4.74 % (102.54)	€	The performance is <u>positive but lower than the risk-free asset</u>	8.24 % (102.1)	€	The performance is <u>positive but lower than the risk-free asset</u>	2.4 % (100.16)	€
The performance is <u>positive and in line with the risk-free asset</u>	13.28 % (112.07)	€	The performance is <u>positive and in line with the risk-free asset</u>	35.7 % (110.09)	€	The performance is <u>positive and in line with the risk-free asset</u>	36.8 % (109.24)	€	The performance is <u>positive and in line with the risk-free asset</u>	34.28 % (114.21)	€
The performance is <u>positive and higher than the risk-free asset</u>	14.72 % (119.13)	€	The performance is <u>positive and higher than the risk-free asset</u>	14.86 % (142.05)	€	The performance is <u>positive and higher than the risk-free asset</u>	14.04 % (144.77)	€	The performance is <u>positive and higher than the risk-free asset</u>	14.02 % (142.11)	€

1st Pillar: Unbundling and Probabilistic performance scenarios

Probabilistic Performance Scenarios vs What-if

1° Pilastro: Unbundling e scenari probabilistici di rendimento

Probabilistic Performance Scenarios vs What-if

Example:

Narrative description of the product's features.

The structured product, whose maturity is 7 years, presents returns which are linked to the Dow Jones Eurostoxx Index.

The fund gives annual coupons, equal to 3% of the initial invested capital, but:

- if, at any time in the fund life, the reference index falls below 50% of its initial value:
 - the payment of coupons is interrupted;
 - at the end of the 7th year the fund will pay back the value of the initial invested capital increased or reduced on the basis of the index performance;
- if the index never falls below 50% of its initial value, at the end of the 7th year the fund will pay:
 - the initial value of the investment;
 - moreover, if at the maturity date the index value is greater or equal to twice its initial value, the fund will pay an additional coupon equal to the initial value of the investment.

1° Pilastro: Unbundling e scenari probabilistici di rendimento

Probabilistic Performance Scenarios vs What-if

Unfavourable scenario	"What-if" representation	Favourable scenario
The Dow Jones Eurostoxx value falls below 50% during the first year of the fund and at the end of the 7 th year the performance of the Dow Jones Eurostoxx index is equal to 55%. The fund does not pay any coupon and at maturity, it pays 45 on an initial investment of 100.	Neutral scenario The Dow Jones Eurostoxx value never falls below 50% during the life of the fund and at the end of the 7 th year the value of the Dow Jones Eurostoxx index is less than twice its initial value. The fund pays every year a 3% coupon and at maturity it pays the initial value of the investment.	The Dow Jones Eurostoxx value never falls below 50% during the life of the fund and at the end of the 7 th year the performance of the Dow Jones Eurostoxx index is equal to 130%. The fund pays every year a 3% coupon and at maturity it pays twice the initial value of the investment.

1° Pilastro: Unbundling e scenari probabilistici di rendimento

Probabilistic Performance Scenarios vs What-if

Representation through the probabilistic performance scenarios table at the end of the 7th year

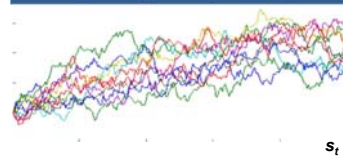
SCENARIOS	PROBABILITY	MEDIAN VALUES	YIELD
The performance is <u>negative</u>	38.71%	55.52	-8.06%
The performance is <u>positive but lower than the risk-free asset</u>	8.45%	110.58	1.45%
The performance is <u>positive and in line with the risk-free asset</u>	36.09%	123.13	3.02%
The performance is <u>positive and higher than the risk-free asset</u>	16.75%	223.27	12.16%

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2nd Pillar: Synthetic risk indicator

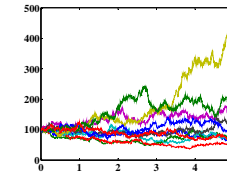
Volatility of the product's simulated returns



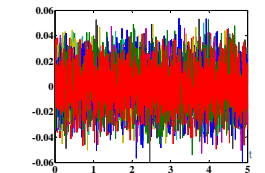
Volatility is the most immediate risk measure and it has a one-to-one relationship with whatever loss measure (VaR, ES, etc.)

2nd Pillar: Synthetic risk indicator

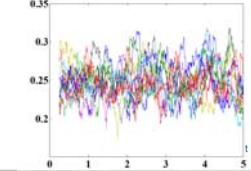
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



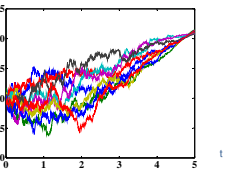
Simulation of the trajectories (Volatility)



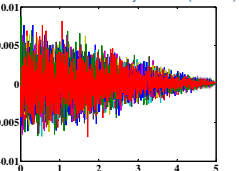
Non-equity product: Equity like

2nd Pillar: Synthetic risk indicator

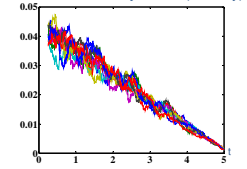
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



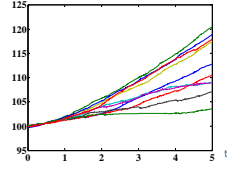
Simulation of the trajectories (Volatility)



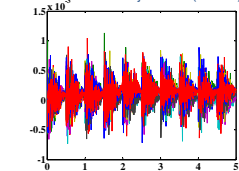
Non-equity product: Fixed bond like

2nd Pillar: Synthetic risk indicator

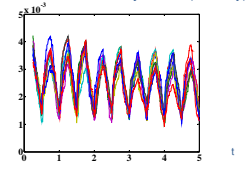
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



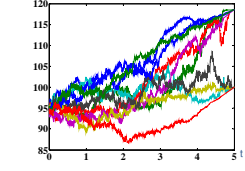
Simulation of the trajectories (Volatility)



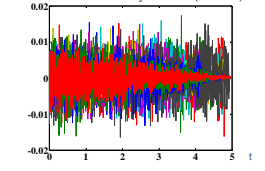
Non-equity product: Floater bond like

2nd Pillar: Synthetic risk indicator

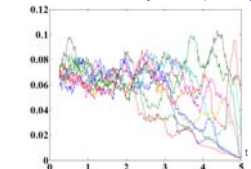
Simulation of the trajectories (Price)



Simulation of the trajectories (Return)



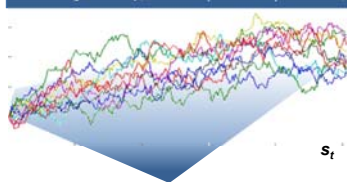
Simulation of the trajectories (Volatility)



Non-equity product: ZCB

2nd Pillar: Synthetic risk indicator

Volatility of the product's simulated returns



MEASUREMENT:
product's positioning inside a grid of n volatility intervals

REPRESENTATION:
mapping of any volatility interval into a corresponding qualitative risk class



Risk Classes	Volatility Intervals
Very Low	$\sigma_{1, \min}$ $\sigma_{1, \max}$
Low	$\sigma_{2, \min}$ $\sigma_{2, \max}$
Medium-Low	$\sigma_{3, \min}$ $\sigma_{3, \max}$
Medium	$\sigma_{4, \min}$ $\sigma_{4, \max}$
Medium-High	$\sigma_{5, \min}$ $\sigma_{5, \max}$
High	$\sigma_{6, \min}$ $\sigma_{6, \max}$
Very High	$\sigma_{7, \min}$ $\sigma_{7, \max}$

2nd Pillar: Synthetic risk indicator

MEASUREMENT:
product's positioning inside a grid of n volatility intervals

REPRESENTATION:
mapping of any volatility interval into a corresponding qualitative risk class

Looking for the number of intervals (so-called " n -tuple of risk classes") allowing the **best compromise** between investors' comprehension and detail of the information conveyed

Hypothesis
NUMBER OF INTERVALS SPANNED:
5, 6 or 7

2nd Pillar: Synthetic risk indicator

Hypothesis
NUMBER OF INTERVALS SPANNED:

5 risk classes

Low
Medium-Low
Medium
Medium-High
High

6 risk classes

Low
Medium-Low
Medium
Medium-High
High
Very High

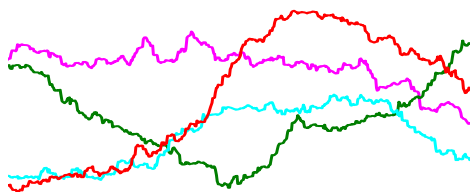
7 risk classes

Very Low
Low
Medium-Low
Medium
Medium-High
High
Very High

2nd Pillar: Synthetic risk indicator

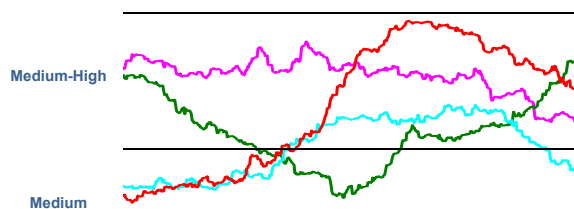


Products with the same risk budget must have the same degree of risk



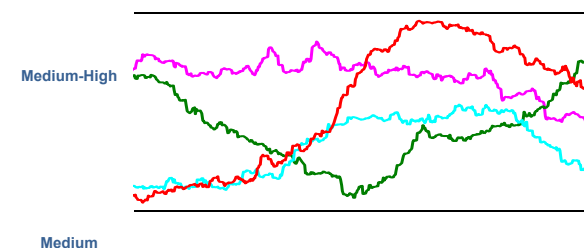
2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations



2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations



2nd Pillar: Synthetic risk indicator

Volatility intervals have to be suitably calibrated in order to avoid wrong risk representations

Moreover, the optimal set of volatility intervals has to be consistent with the principle:

+ RISK + LOSSES



VOLATILITY INTERVALS MUST HAVE AN INCREASING WIDTH

2nd Pillar: Synthetic risk indicator

How to define a suitable volatility grid



Minimizing the chance for an asset manager of overcoming not intentionally his risk budget, i.e. the volatility interval (so-called "management failure")

2nd Pillar: Synthetic risk indicator

How to define a suitable volatility grid



Minimizing the chance for an asset manager of overcoming not intentionally his risk budget, i.e. the volatility interval (so-called "management failure")



The optimal set of volatility intervals for a given n-tuple of risk classes requires to solve a stochastic NLP problem (i.e. minimize the chance of a "management failure")

2nd Pillar: Synthetic risk indicator

How to define a suitable volatility grid



In order to analyze the management failures, (i.e.: to specify and solve the SNLP problem)....

1st INTUITION

it has to be studied the behavior of an automatic asset manager that has a specific risk budget, identified by a given volatility interval

2nd Pillar: Synthetic risk indicator

How to define a suitable volatility grid



In order to analyze the management failures, (i.e.: to specify and solve the SNLP problem)....

2nd INTUITION

volatility prediction intervals have to be determined, in order to measure the ability of the automatic asset manager to remain within his risk budget

2nd Pillar: Synthetic risk indicator

How to define a suitable volatility grid



In order to analyze the management failures, (i.e.: to specify and solve the SNLP problem)....

3rd INTUITION

the optimal set of volatility intervals must allow a similar number of "management failures" to any automatic asset managers (despite his belonging to different risk classes)



NO INCENTIVE TO CHOOSE A SPECIFIC CLASS

The stochastic non linear programming problem

optimal set of volatility intervals

Let $n \in \mathbb{N}$ be the number of volatility intervals (so-called "n-tuple of risk classes")

Then, the optimization problem is twofold:

1. find the optimal number of intervals: n^*
2. for $n=n^*$ minimize the management failures as defined below:

$$\min_{\sigma_1 < \sigma_2 < \dots < \sigma_n} \left(\max_{i=1, \dots, n} mf_i \right)$$

$$s.t. \quad mf_i \approx mf_{i-1}$$

The stochastic non linear programming problem

optimal set of volatility intervals

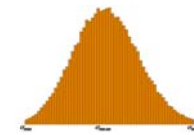
1st INTUITION 2nd INTUITION 3rd INTUITION

1st INTUITION

Automatic Asset Manager

Hypothesis:

Stochastic volatility model where the automatic asset manager is "mean-reverting":



The automatic asset manager:

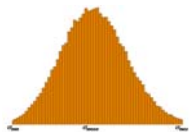
- has no systematic preference for upwards or downwards deviations from the mean → symmetric distribution for the volatility
- in order to minimize the migration risk, keeps the product volatility far from the bounds of the interval → probability decay over the tails

1st INTUITION

Automatic Asset Manager

Hypothesis:

Stochastic volatility model where the automatic asset manager is "mean-reverting":



A proper definition of the parameters for the following pair of SDEs:

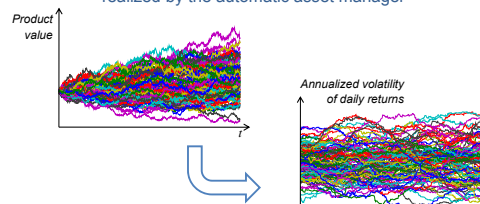
$$dS_t = rS_t dt + \sigma_t S_t dW_t^{(1)}$$

$$d\sigma_t^2 = \kappa(\theta - \sigma_t^2) dt + v_t dW_t^{(2)}$$

1st INTUITION

Automatic Asset Manager

Simulation of the trajectories of the volatility realized by the automatic asset manager



2nd INTUITION

Management Failures

Volatility Prediction Intervals

Definition 1

An a-confident volatility prediction interval is defined by the pair $[\sigma_{min}, \sigma_{max}]$ s.t.:

$$P(\sigma_{min} \leq \sigma_t^{AM} \leq \sigma_{max}) = \alpha$$

where σ_t^{AM} is the annualized daily returns volatility realized by the automatic asset manager at day t based on the last 252 product's daily returns.

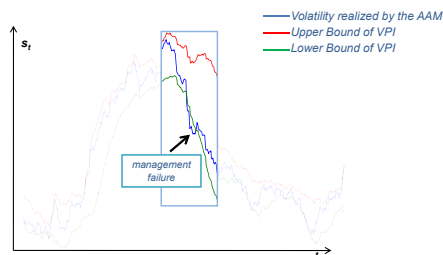
Definition 2

A "management failure" is said to occur at day t if either $\sigma_t^{AM} > \sigma_{max}$ OR $\sigma_t^{AM} < \sigma_{min}$

2nd INTUITION

Management Failures

Volatility Prediction Intervals



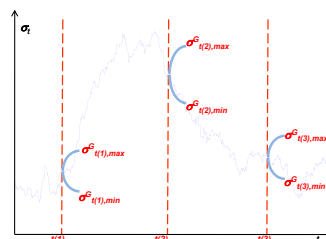
2nd INTUITION

Management Failures

Volatility Prediction Intervals

Hypothesis:

Adaptive GARCH diffusive models to measure the ability of the automatic asset manager to remain within his risk budget



2nd INTUITION

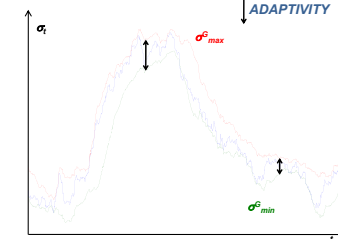
Management Failures

Volatility Prediction Intervals

ADAPTIVITY

Hypothesis:

Adaptive GARCH diffusive models to measure the ability of the automatic asset manager to remain within his risk budget



Management Failures

Diffusive GARCH Implementation

from the M-GARCH(LI)

$$\begin{cases} X_t - X_{t-1} = \gamma \cdot (y - X_{t-1}) + \sigma_t Z_t \\ \text{and} \\ \ln \sigma_{t+1}^2 - \ln \sigma_t^2 = \beta_1^2 + (\beta_2^2 - 1) \ln \sigma_t^2 + \beta_3^2 \ln Z_t^2 \\ \text{or, equivalently} \\ \ln \sigma_{t+1}^2 - \ln \sigma_t^2 = \beta_1^2 + (\beta_2^2 - 1) \ln \sigma_t^2 + 2\beta_3^2 \ln |Z_t| \end{cases}$$

Z_t and Z_t are iid N(0,1)

With Convolution Theorem of Double Markov Chains or otherwise

$$dX_t = \gamma(y - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = (\beta_1 + 2\beta_2 E(\ln |Z_t|) + (\beta_2 - 1) \ln \sigma_t^2) dt + 2|\beta_3| \sqrt{V} \arctan(|Z_t|) dW_t^*$$

$Z_t \sim N(0,1)$

O.U. process

$$\ln \sigma_t^2 \sim N \left(\frac{(\ln \sigma_0^2 + \frac{\beta_1 + 2\beta_2 E(\ln |Z_t|)}{(\beta_2 - 1)} + \beta_3^2 - 1)(t-1) - \frac{\beta_1 + 2\beta_2 E(\ln |Z_t|)}{(\beta_2 - 1)}}{2(\beta_2 - 1)}, \frac{\beta_3^2 (1 - e^{-2(\beta_2 - 1)t})}{2(\beta_2 - 1)} \right)$$

Management Failures

Diffusive GARCH Implementation

maximum likelihood estimation

the likelihood function

$$L(w; \beta) = \prod_{k=2}^K \left[\frac{1}{\sigma_k} \sqrt{\frac{2\beta_2 - 1}{2\pi(2\beta_2 - 1)}} e^{-\frac{1}{2\sigma_k} \sqrt{\frac{2\beta_2 - 1}{2\pi(2\beta_2 - 1)}}} \right]$$

where: $\beta = (\beta_1, \beta_2, \beta_3)$

$$\sigma_k = \frac{1}{\beta_2 - 1} \left[\ln \sigma_{k-1}^2 + \beta_1 + (\beta_2 - 1) \ln \sigma_{k-1}^2 + 2|\beta_3| \sqrt{V} \arctan(|Z_{k-1}|) \right]$$

K = number of observations of annualized daily volatility used to estimate the parameters

REMARK

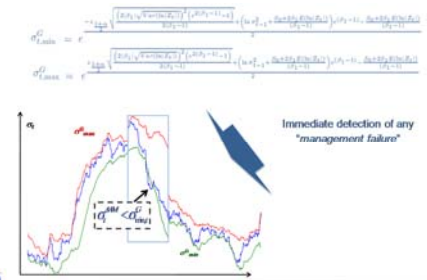
Adaptivity of Diffusive GARCH allows to work with poorer filtrations:

K reasonably small (around 60)

Management Failures

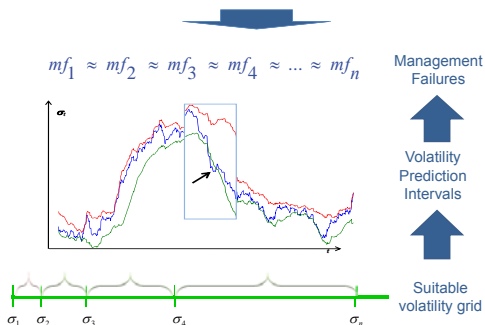
Diffusive GARCH Implementation

the estimated parameters enter in the bounds of the volatility prediction interval



3rd INTUITION

NO INCENTIVE TO CHOOSE A SPECIFIC CLASS



The stochastic non linear programming problem
Solution to step 1

The higher is n the smaller will be the average width of the volatility intervals and the lower is the average number of the management failures

$n^* = 7$

The stochastic non linear programming problem
Solution to step 2

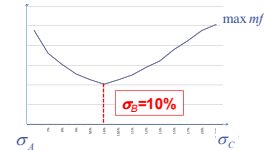
LEMMA (for two consecutive intervals)

Let σ_A and σ_C be two known volatilities with $\sigma_A < \sigma_C$. Then, the value of σ_B s.t.:

$$\min_{\sigma_B} \left(\max \{ mf_{(\sigma_A, \sigma_B)}, mf_{(\sigma_B, \sigma_C)} \} \right)$$

is: $\sigma_B = \sqrt{\sigma_A \sigma_C}$ or, equivalently: $\frac{\sigma_B}{\sigma_A} = \frac{\sigma_C}{\sigma_B} = m$

where m is called "multiplier".



The stochastic non linear programming problem

COROLLARY

Let $[\sigma_1, \sigma_2]$ and $[\sigma_3, \sigma_4]$ be two volatility intervals having the same multiplier m , i.e.:

$$m = \frac{\sigma_2}{\sigma_1} = \frac{\sigma_4}{\sigma_3}$$

then, the two intervals have the same number of "management failures", i.e.:

$$mf_1 = mf_2$$

where $mf_i, i=1,2$ is the total number of management failures occurred to the automatic asset manager of the i^{th} volatility interval.

The stochastic non linear programming problem



the 1st and the n^{th} interval cannot respect the multiplier

the 1st and the n^{th} interval must be chosen looking at exogenous information

The stochastic non linear programming problem

ASSUMPTIONS

25% AS THE LOWER BOUND OF THE LAST VOLATILITY INTERVAL

0.25% AS THE UPPER BOUND OF THE FIRST VOLATILITY INTERVAL

...corresponding to a percentage loss of about 50% of the invested capital over a 1-year time horizon

...corresponding to typical results of monetary markets instruments

2nd Pillar: Synthetic risk indicator

The stochastic non linear programming problem

the optimization problem becomes:

given $n^*=7$:

$$\min_{\sigma_2 < \sigma_3 < \dots < \sigma_7} \left(\max_{i=2, \dots, 6} mf_i \right)$$

s.t. $mf_i \approx mf_{i-1}$

with: $\sigma_2=0.25\%$ $\sigma_7=25\%$

2nd Pillar: Synthetic risk indicator

Suitable volatility grid

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Very Low	0.01%	0.24%
Low	0.25%	0.63%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

$m^* = 2.5$

The optimal set of volatility intervals is consistent with the principle: **+ RISK + LOSSES**

2nd Pillar: Synthetic risk indicator

Definition of a suitable volatility grid

summarizing:

automatic asset manager

Product

Annualized volatility

diffusive GARCH to detect management failures

given $n^*=7$:

$$\min_{\sigma_2 < \sigma_3 < \dots < \sigma_7} \left(\max_{i=2, \dots, 6} mf_i \right)$$

s.t. $mf_i \approx mf_{i-1}$

with: $\sigma_2=0.25\%$ $\sigma_7=25\%$

Risk Classes	Volatility Intervals	
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Very Low	0.01%	0.24%
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Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

$m^* = 2.5$

$\frac{\sigma_2}{\sigma_1} = \frac{\sigma_3}{\sigma_2} = \frac{\sigma_4}{\sigma_3} = \dots = \frac{\sigma_n}{\sigma_{n-1}} = m$

Syllabus

- Preliminaries: closing the gap between risk representation inside prospectus and banks' mark to market valuations
- Investment returns maximization via probabilistic scenarios
- Assessing the comfortable level of risk for the retail investor: a volatility based criterion
- Optimal exit strategies for the retail investor: the recommended investment time horizon

3rd Pillar: The recommended Investment horizon



The recommended investment time horizon

for performance target products the recommended minimum investment horizon is inherent to their financial engineering, as the recommended investment horizon is equal to the period of validity (or the time to maturity) of their target

The payoff at maturity uniquely identifies the time when the potential returns are optimized

3rd Pillar : The recommended Investment horizon



The recommended investment time horizon

The use of solutions aimed at ensuring the liquidity and/or marketability of a return target product changes its risk-return profile and its recommended investment time horizon

The event to study from a probabilistic point of view transforms into:

The investment recovers the initial costs and off-sets the running costs at least once

that can be calculated through the concept of

First Passage Time

The "minimum" recommended investment time horizon

3rd Pillar : The recommended Investment horizon



The "minimum" recommended investment time horizon

For risk target products, the natural way to define a cost recovery event is also:

The investment recovers the initial costs and off-sets the running costs at least once

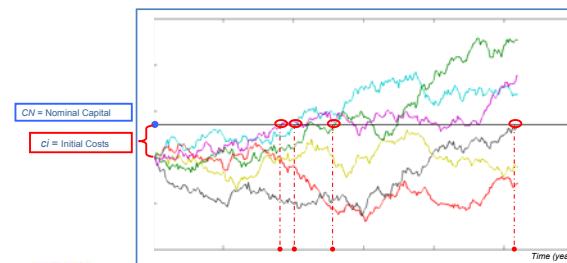
that can be calculated through the concept of

First Passage Time

3rd Pillar : The recommended Investment horizon

First Passage Time:

First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



3rd Pillar : The recommended Investment horizon

The probability of the event:

The investment recovers the initial costs and off-sets the running costs at least once

given a confidence level α , uniquely identifies a time T^* on the cumulative distribution function of the first passage times, i.e.:

$$T^* = \left\{ T \in \mathcal{R}^+ : \mathbb{P}[t^* \leq T] = \alpha \right\}$$

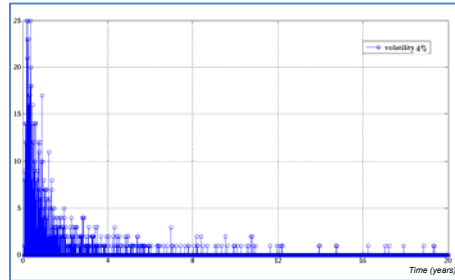
where

$$t^* = \inf \left\{ t \in \mathcal{R}^+ : CI_t > CN \right\}$$

is the first passage time

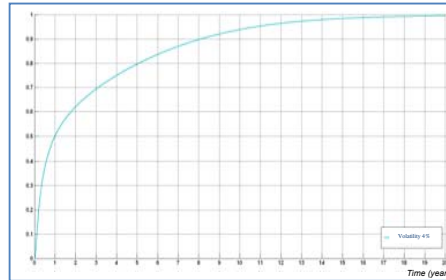
3rd Pillar : The recommended Investment horizon

1. Calculation of the probability distribution of the first passage times:



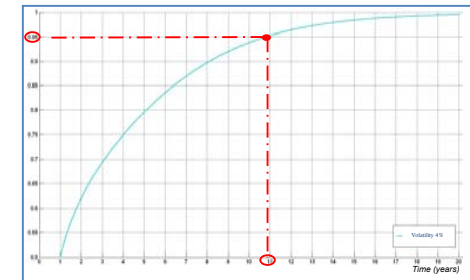
3rd Pillar : The recommended Investment horizon

2. Derivation of the cumulative distribution function of the first passage times:



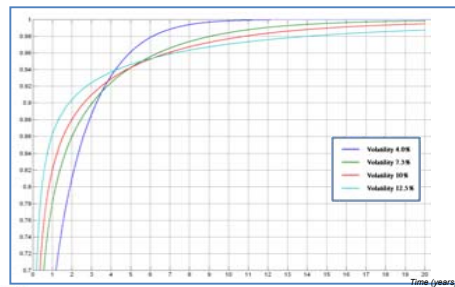
3rd Pillar : The recommended Investment horizon

3. The confidence level α uniquely identifies T on the cumulative distribution function of the first passage times:



3rd Pillar : The recommended Investment horizon

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



3rd Pillar : The recommended Investment horizon

Anyway, the recommended minimum investment time horizon...

$$T^* = \left\{ T \in \mathcal{R}^+ : \mathbb{P}[t^* \leq T] = \alpha \right\}$$

... Must be coherent with the principle

+ VOLATILITY + TIME HORIZON

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

First passage times for the break-even barrier are monitored at infinitesimal time intervals:

$dt \rightarrow 0$

$$T^* = \left\{ T \in \mathcal{R}^+ : \mathbb{P}[t^* \leq T] = \alpha \right\}$$

$$\mathbb{P}[t^* \leq T] = N\left(d_2\left(\frac{CN}{CI_0}\right)\right) + \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot N\left(-d_2\left(\frac{CN}{CI_0}\right)\right)$$

$$d_2(x) = \frac{\log x + \left(\bar{r} - cr - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$N(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

Asymptotic properties: $T \rightarrow \infty$

cr : recurrent costs as a fixed %

$$\lim_{T \rightarrow \infty} \mathbb{P}[t^* \leq T] = \begin{cases} 1 & \text{if } (\bar{r} - cr) \geq \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & \text{if } (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

Under our assumptions:

$$\lim_{T \rightarrow \infty} \mathbb{P}[t^* \leq T] = \begin{cases} 1 & \text{if } (\bar{r} - cr) \geq \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & \text{if } (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

For a given level of costs, it is possible to analytically derive the connection between volatility and time horizon

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$T \rightarrow \infty, dt \rightarrow 0$

FIRST ORDER SENSITIVITY ANALYSIS

$$\frac{dP}{d\sigma} = \left[-4 \frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right) \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right]$$

FIRST ORDER ASYMPTOTIC CONDITION

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left[-4 \frac{\bar{r} - cr}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right]$$

- $(\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
- $(\bar{r} - cr) \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

The existence of two alternative states of nature requires to verify whether both of them make sense in financial terms under the risk-neutral measure.

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left[-4 \frac{\bar{r}}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2\bar{r}}{\sigma^2}-1} \right]$$

- $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
- $\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$

$cr = 0$

Being running costs a specific feature of any financial product they would interfere with the task of identifying which of the two conditions has a sound financial meaning. Therefore, they will be temporarily neglected.

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left[-4 \frac{\bar{r}}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2\bar{r}}{\sigma^2}-1} \right]$$

- $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
- ~~$\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$~~

$cr = 0$

Since it is safe to assume a positive interest rate r in financial markets, only condition 1. correctly captures the connection between volatility and time horizon.

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left[-4 \frac{\bar{r}}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2\bar{r}}{\sigma^2}-1} \right]$$

- $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
- ~~$\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$~~

$cr = 0$

As $T \rightarrow \infty$ condition 1. implies that the cumulative distribution function P is a strictly decreasing function of the volatility, i.e.:

$$\forall \sigma_1, \sigma_2 \in \mathfrak{R}^+, \sigma_1 > \sigma_2 \Rightarrow P(\sigma_1) < P(\sigma_2)$$

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

94
97
98

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{dP}{d\sigma} = \left[-4 \frac{\bar{r}}{\sigma^3} \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2\bar{r}}{\sigma^2}-1} \right]$$

- $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
- ~~$\bar{r} \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$~~

$cr = 0$

In other words, for a given a confidence level, as the volatility grows, the recommended investment time horizon increases as well:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$$T \rightarrow \infty, dt \rightarrow 0$$

$$\frac{d^2P}{d\sigma^2} = \frac{4}{\sigma^4} (\bar{r} - cr) \ln \left(\frac{CN}{CI_0} \right) \left(\frac{CN}{CI_0} \right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot \left[1 + \frac{4(\bar{r}-cr)}{\sigma^2} \ln \left(\frac{CN}{CI_0} \right) \right]$$

$$(\bar{r} - cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0$$

SECOND ORDER ASYMPTOTIC CONDITION

Second Order Sensitivity Analysis

3rd Pillar : The recommended Investment horizon

Connection between probability, volatility and costs

$$T \rightarrow \infty, dt \rightarrow 0$$

- $(\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
 $(\bar{r} - cr) > 0 \Leftrightarrow \frac{d^2P}{d\sigma^2} > 0$
- ~~$(\bar{r} - cr) \leq 0 \Leftrightarrow \frac{dP}{d\sigma} \geq 0$~~

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

Summarizing the results of the asymptotic analysis in continuous time:

- As $T \rightarrow \infty$, for given a confidence level, more volatility implies a larger recommended investment time horizon
- It is always possible to find a minimum and finite time T^* , beyond which the strong condition

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

holds

3rd Pillar : The recommended Investment horizon

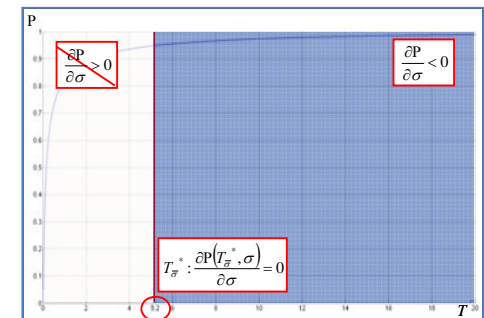
DETERMINATION OF THE INVESTMENT TIME HORIZON

$\left. \begin{array}{l} T \rightarrow \infty \\ dt \rightarrow 0 \\ P(\infty, \sigma) \\ \left\{ \begin{array}{l} (\bar{r} - cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r} - cr) > 0 \Leftrightarrow \frac{d^2P}{d\sigma^2} > 0 \end{array} \right. \end{array} \right\} \Rightarrow$	<p>General Framework:</p> $\left\{ \begin{array}{l} T \text{ finite} \\ dt \rightarrow 0 \\ P(T, \sigma) \\ \left\{ \begin{array}{l} (\bar{r} - cr) > 0 \Leftrightarrow \lim_{T \rightarrow \infty} \frac{\partial P(T, \sigma)}{\partial \sigma} < 0 \\ (\bar{r} - cr) > 0 \Leftrightarrow \lim_{T \rightarrow \infty} \frac{\partial^2 P(T, \sigma)}{\partial \sigma^2} > 0 \end{array} \right. \end{array} \right.$
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Everything shown above also holds with T finite!

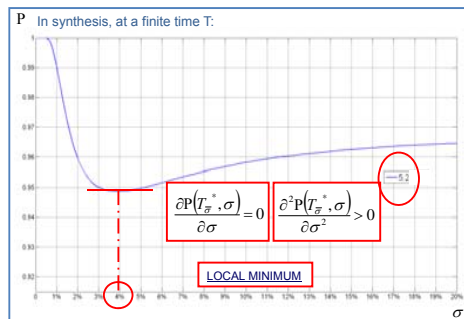
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

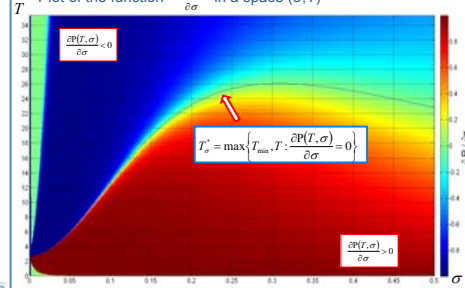


3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS

Plot of the function $\frac{\partial P(T, \sigma)}{\partial \sigma}$ in a space (σ, T)



3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON

SECOND ORDER SENSITIVITY ANALYSIS

$$\frac{\partial^2 P(T, \sigma)}{\partial \sigma^2}$$

Given the monotonicity condition of the probability distribution with respect to volatility, i.e.:

$$\forall \sigma_i, \sigma_j \in \mathfrak{N}^+, \sigma_j > \sigma_i \Rightarrow P(\infty, \sigma_j) < P(\infty, \sigma_i)$$

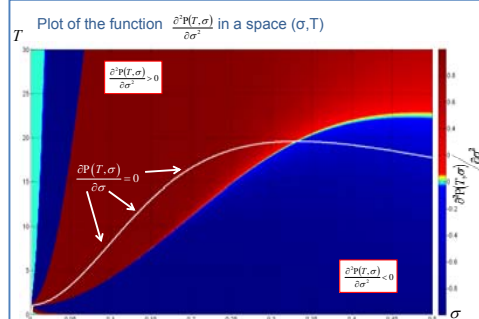
In order to fulfill this condition, it's necessary to restrict the analysis in the region where the probability function is strictly increasing, i.e.:

$$\frac{\partial^2 P(T, \sigma)}{\partial \sigma^2} > 0 \Rightarrow T_\sigma \text{ increasing}$$

$$\frac{\partial^2 P(T, \sigma)}{\partial \sigma^2} < 0 \Rightarrow T_\sigma \text{ decreasing}$$

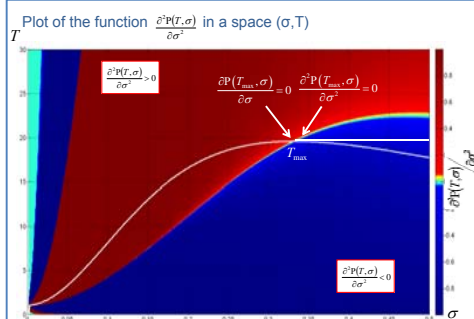
3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



3rd Pillar : The recommended Investment horizon

DETERMINATION OF THE INVESTMENT TIME HORIZON



Rebuilding investor confidence through risk disclosure
Risk-based transparency on structured products through synthetic indicators

EXAMPLES

Examples

DERIVATIVE PRODUCT

The product presents the following payoff:

- if the reference equity index remains above 50% of its initial value, the investor receives a quarterly fixed coupon equal to 1.8% of the issue price and the payment of the invested capital at maturity;
- if the index reaches 50% of its initial value the coupon flow is interrupted and at maturity the investor receives a payment for the investment equal to the performance of the Index.

Unbundling Table	
Theoretical value of the Debt component	0.00
Theoretical value of the Derivative component	88.44
Theoretical value of the product	88.44
Costs	11.56
Issue price	100.00

PROBABILISTIC SCENARIOS	Event Probability	Median Value
The performance is negative	46.160%	60.120%
The performance is positive but lower than the risk-free asset	4.860%	107.130%
The performance is positive and in line with the risk-free asset	3.430%	128.380%
The performance is positive and higher than the risk-free asset	45.550%	152.820%

2nd PILLAR Degree of Risk: Medium-High

3rd PILLAR Recommended investment time horizon: 6 years and 6 months

Examples

STRUCTURED PRODUCT

The investor receives fixed coupons with values increasing from 1% to 2.5% for the first 3 years. At maturity, she receives the payment of the issue price possibly increased by an additional bonus equal to 35% of the reference index performance (if positive) multiplied by the issue price.

Unbundling Table	
Theoretical value of the Debt component	85.62
Theoretical value of the Derivative component	7.09
Theoretical value of the product	92.71
Costs	7.29
Issue price	100.00

PROBABILISTIC SCENARIOS	Event Probability	Median Value
The performance is negative	8.72%	45.59%
The performance is positive but lower than the risk-free asset	0%	0%
The performance is positive and in line with the risk-free asset	87.10%	111.97%
The performance is positive and higher than the risk-free asset	4.18%	155.91%

2nd PILLAR Degree of Risk: Medium

3rd PILLAR Recommended investment time horizon: 6 years e 9 months

Examples

SUBORDINATED BOND

DESCRIPTION Subordinated bond with a 7 year maturity, paying bi-annual step-up coupons ranging from 4.7% to 5.30% and characterized by an amortizing plan from the 3rd to the 7th year.

STRUCTURE RETURN TARGET

Unbundling Table	
Theoretical value of the Debt component	83.361
Theoretical value of the Derivative component	11.032
Theoretical value of the product	94.393
Costs	5.607
Issue price	100.00

PROBABILISTIC SCENARIOS	Event Probability	Median Value
The performance is negative	23.51%	54.73%
The performance is positive but lower than the risk-free asset	0.55%	100.23%
The performance is positive and in line with the risk-free asset	74.48%	133.05%
The performance is positive and higher than the risk-free asset	1.46%	144.66%

2nd PILLAR Degree of Risk: Medium-High

3rd PILLAR Recommended investment time horizon: 7 years



Rebuilding investor confidence through risk disclosure
Risk-based transparency on structured products through
synthetic indicators

Marcello Minenna – Head of Quantitative Analysis Unit, Consob

