CONSOB
COMMISSIONE NAZIONALE PER LE SOCIETÀ E LA BORSA

Rebuilding the investors' confidence through risks disclosure

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Structured products ASIA

20 September 2010, Hong Kong

Syllabus

Preliminaries

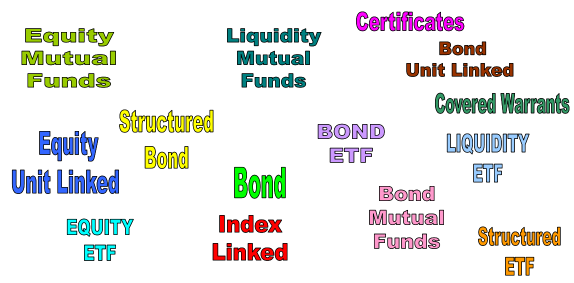
Three-pillars approach:

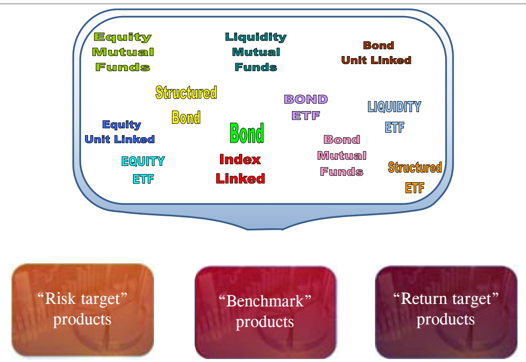
1st Pillar: unbundling and performance scenarios

2nd Pillar: the degree of risk

3rd Pillar: recommended investment time horizon

Non-equity Investment products: definition





FINANCIAL REGULATORY REFORM: A NEW FOUNDATION

Transparency.

We propose a new proactive approach to disclosure.

[...] all disclosures and other communications with consumers be reasonable: balanced in their presentation of benefits, and clear and conspicuous in their identification of costs, penalties, and risks.

Mandatory disclosure forms should be clear, simple, and concise.

Moreover, reasonableness does not mean a litany of every conceivable risk, which effectively obscures significant risks. It means identifying conspicuously the more significant risks. It means providing consumers with disclosures that help them to understand the consequences of their financial decisions.

The transparency on the risk profile of non-equity investment products is based on three synthetic indicators (three pillars) – defined through the development of specific quantitative methods – in order to allow investors to take informed investment decisions.

Traditional narrative description of all possible risks associated with a financial product	Synthetic indicators robust, objective and backward verifiable

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RETURNS	RISKS	INVESTMENT HORIZON

The key qualitative information is made objective by using a three-pillars approach based on quantitative measures.

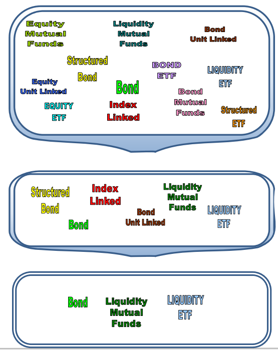
- 1st Pillar: Unbundling and Probabilistic performance scenarios
- 2nd Pillar: Synthetic risk indicator
- 3rd Pillar: The recommended investment horizon

Investor decisions as a sequential filtering problem:

Time goal: liquidity/investment horizon
INVESTMENT HORIZON
(less than 3 years)

Risk profile: risk limit in terms of downside
RISKS
(medium-low)

Return goal: target returns
RETURNS
(maximum return)



Syllabus

Preliminaries

Three-pillars approach:

1st Pillar: unbundling and performance scenarios

2nd Pillar: the degree of risk

3rd Pillar: recommended investment time horizon

1st Pillar

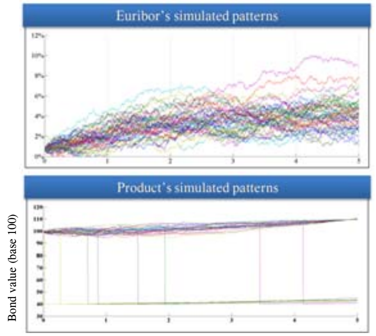
1st Pillar *Unbundling* and Probabilistic performance scenarios

In "return target" products (e.g. corporate bonds) the connection between the pricing at time zero and the pricing at maturity is evident, as the probability table is a necessary step to obtain the unbundling of the price of the product at time 0.

1st Pillar

1st Pillar *Unbundling* and Probabilistic performance scenarios

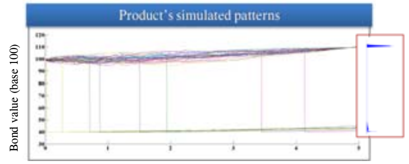
5 year fixed-rate bond



1st Pillar

1st Pillar *Unbundling* and Probabilistic performance scenarios

The final values of the bond at the end of the 5th year provide the probability distribution of potential returns (so-called *pricing* at maturity).



Possible outcomes
Pricing at maturity

1st Pillar

1st Pillar *Unbundling* and Probabilistic performance scenarios

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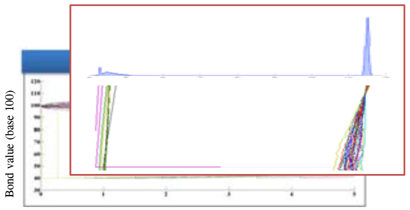

Possible outcomes
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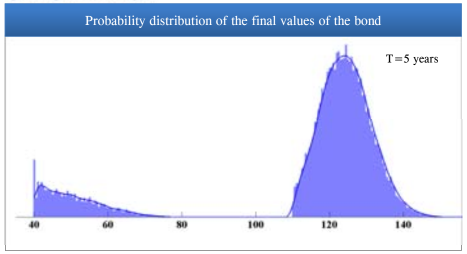
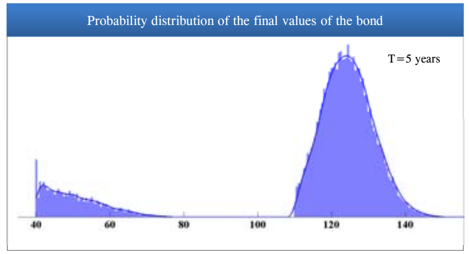


Possible outcomes
Pricing at maturity

1st Pillar

1st Pillar *Unbundling* and Probabilistic performance scenarios

The informative content of the entire probability distribution is very complex to handle for the average retail investor.

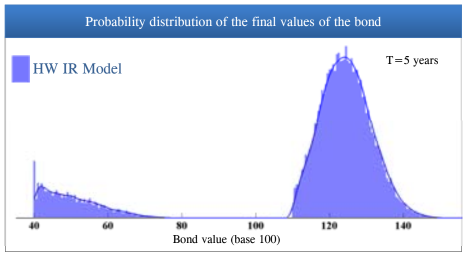


Possible outcomes
Pricing at maturity

1st Pillar

1st Pillar *Unbundling* and Probabilistic performance scenarios

The shape of the probability distribution of potential returns is obviously dependent from the model's assumption.



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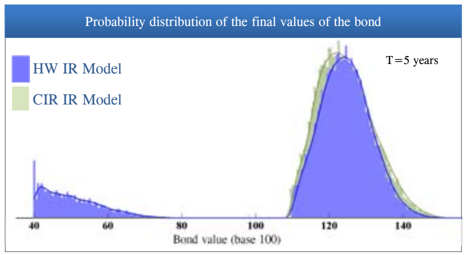
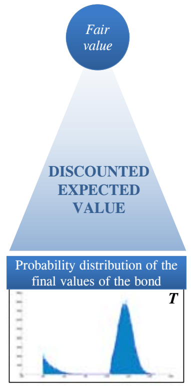


1st Pillar

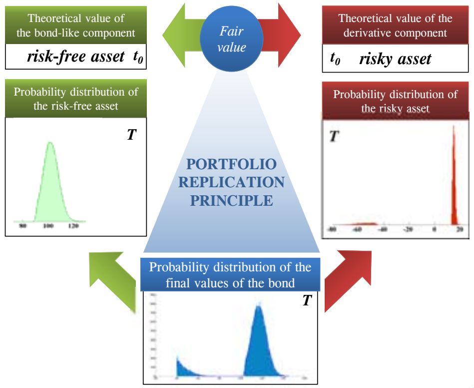
1st Pillar *Unbundling* and Probabilistic performance scenarios

STANDARD SOLUTION



1st Pillar

CONSOB REGULATION



1st Pillar

CONSOB REGULATION

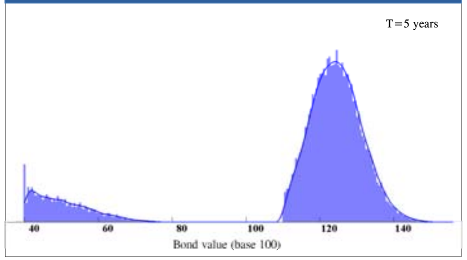
Theoretical value of the bond-like component: risk-free asset t_0	*Fair value*	Theoretical value of the derivative component: t_0 risky asset

A	Theoretical value of the bond-like component	…
B	Theoretical value of the derivative component	…
C = A + B	*Fair value*	…
D	Explicit costs	…
E	Implicit costs	…
F = C + D + E	Issue price	100

1st Pillar

CONSOB REGULATION

Probability distribution of the final values of the bond

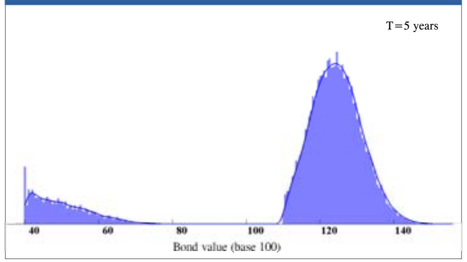


It's useful to explore a different representation of the information contained in the probability distribution which could be useful for the average investor

1st Pillar

CONSOB REGULATION

Probability distribution of the final values of the bond

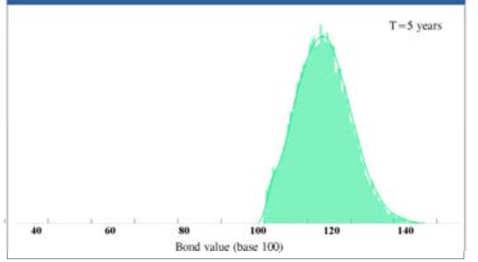


In order to provide the investor with a representation fair, easy to understand and resilient to the model's risk, a simple rescaling with respect to the risk-neutral measure numeraire is presented

1st Pillar

CONSOB REGULATION

Probability distribution of the cash account (risk neutral numeraire)

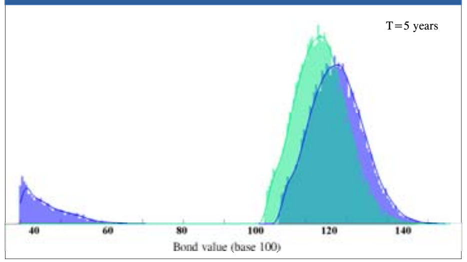


In order to provide the investor with a representation fair, easy to understand and resilient to the model's risk, a simple rescaling with respect to the risk-neutral measure numeraire is presented

1st Pillar

CONSOB REGULATION

Probability distribution of the final values of the bond and the cash account

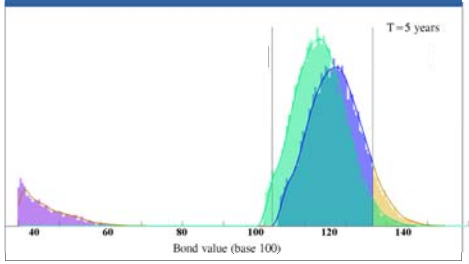


The superimposition of the product's probability distribution with the cash account naturally defines three different events which are effectively meaningful for the investor.

1st Pillar

CONSOB REGULATION

Probability distribution of the final values of the bond and the cash account



The performance is positive and lower than the risk-free asset

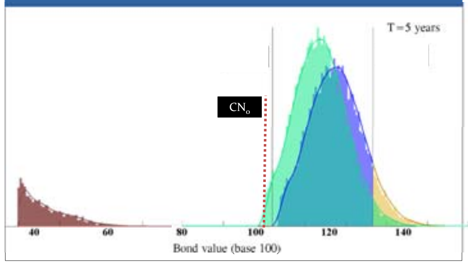
The performance is positive and in line with the risk-free asset

The performance is positive and higher than the risk-free asset

1st Pillar

CONSOB REGULATION

Probability distribution of the final values of the bond and the cash account



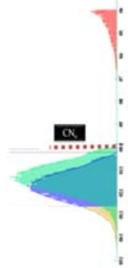
The performance is negative

The performance is positive and lower than the risk-free asset

The performance is positive and in line with the risk-free asset

The performance is positive and higher than the risk-free asset

1st Pillar



SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
The performance is positive and in line with the risk-free asset	%	€
The performance is positive and higher than the risk-free asset	%	€

1st Pillar
Unbundling
and Probabilistic
performance
scenarios

Connection between the pricing at time zero and the pricing at the end of recommended investment horizon

Time Zero

Financial investment table

A	Theoretical value of the bond-like component	…
B	Theoretical value of the derivative component	…
C = A + B	*Fair value*	…
D	Explicit costs	…
E	Implicit costs	…
F = C + D + E	Issue price	100

End of the recommended investment horizon

Table of probabilistic performance scenarios

SCENARIOS	PROBABILITY	MEDIAN VALUES
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
The performance is positive and in line with the risk-free asset	%	€
The performance is positive and higher than the risk-free asset	%	€

1:1 Relationship

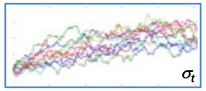
1st Pillar
Unbundling
and Probabilistic
performance
scenarios

Model Risk Assessment

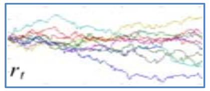
The recommended time horizon has a significant influence on the choice of the model

For Time Horizons greater than 1 year…..

I



II



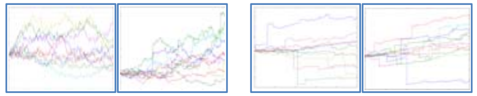
1st Pillar
Unbundling
and Probabilistic
performance
scenarios

Model Risk Assessment

The recommended time horizon has a significant influence on the choice of the model

Many possible choices…

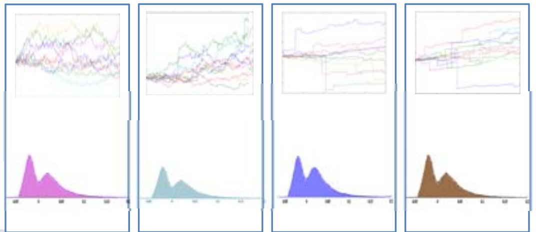
I	II	III	IV
HESTON Stochastic Volatility Model	MERTON Jump Diffusion Model	CARR MADAN CHANG Variance Gamma Model	BARNDORFF NIELSEN NIG Model



1st Pillar
Unbundling
and Probabilistic
performance
scenarios

Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

Heston | Merton | V G | NIG



1st Pillar
Unbundling
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performance
scenarios

Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

1st Pillar
Unbundling
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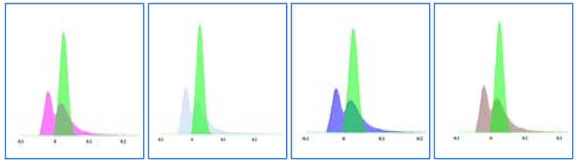
performance
scenarios

Step 3: Probabilistic comparison with the Risk-Free Asset

Analysing the probability distributions…

Heston | Merton | V G | NIG



1st Pillar
Unbundling
and Probabilistic
performance
scenarios

Step 3: Probabilistic comparison with the Risk-Free Asset

… the following output is obtained:

Heston

Scenarios	Probability	Median Values
The performance is negative	46.61 %	€ 90.50
The performance is positive but lower than the risk-free asset	3.39%	€ 101.26
The performance is positive and in line with the risk-free asset	33.28 %	€ 112.19
The performance is positive and higher than the risk-free asset	16.72 %	€ 139.93

Merton

Scenarios	Probability	Median Values
The performance is negative	42.695 %	€ 89.26
The performance is positive but lower than the risk-free asset	4.74%	€ 102.54
The performance is positive and in line with the risk-free asset	35.7%	€ 110.09
The performance is positive and higher than the risk-free asset	16.86 %	€ 142.65

V G

Scenarios	Probability	Median Values
The performance is negative	43.91 %	€ 91.25
The performance is positive but lower than the risk-free asset	5.23%	€ 102.1
The performance is positive and in line with the risk-free asset	36.8%	€ 109.24
The performance is positive and higher than the risk-free asset	14.06 %	€ 141.77

NIG

Scenarios	Probability	Median Values
The performance is negative	48.1%	€ 93.40
The performance is positive but lower than the risk-free asset	2.6%	€ 101.91
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1st Pillar
Unbundling
and Probabilistic
performance
scenarios

Step 3: Probabilistic comparison with the Risk-Free Asset

Assessing the model risk: |Δ| < 4.7%

Heston

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1st Pillar
Unbundling
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performance
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Assessing the model risk: |Δ| < 2.7%

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1st Pillar *Unbundling* and Probabilistic performance scenarios

Step 3: Probabilistic comparison with the Risk-Free Asset

Assessing the model risk: $|\Delta| < 3.7\%$

Heston

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1st Pillar *Unbundling* and Probabilistic performance scenarios

Step 3: Probabilistic comparison with the Risk-Free Asset

Assessing the model risk: $|\Delta| < 1.2\%$

Heston

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Syllabus

Preliminaries

Three-pillars approach:

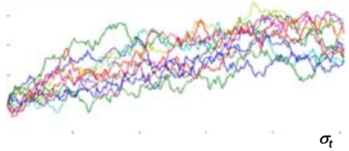
1st Pillar: unbundling and performance scenarios

2nd Pillar: the degree of risk

3rd Pillar: recommended investment time horizon

2nd Pillar Synthetic risk indicator

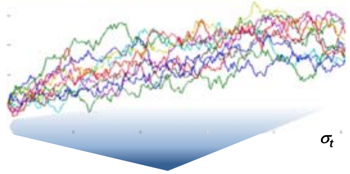
Volatility of the product's simulated returns



Volatility is the most immediate risk measure and
it has a one-to-one relationship with whatever loss measure
(VaR, ES, etc.)

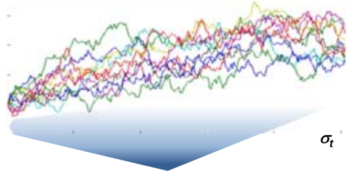
2nd Pillar Synthetic risk indicator

Volatility of the product's simulated returns



2nd Pillar Synthetic risk indicator

Volatility of the product's simulated returns



MEASUREMENT:
product's positioning inside
a grid of *n* volatility intervals

REPRESENTATION:
mapping of any volatility interval into a
corresponding qualitative risk class

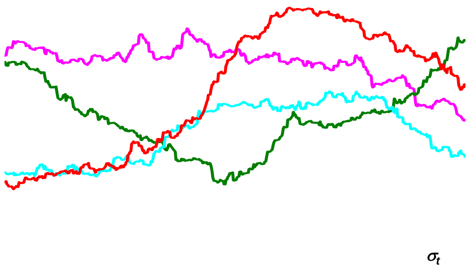
RISK

Risk Classes	Volatility Intervals σ_{min}	σ_{max}
low	$\sigma_{1,min}$	$\sigma_{1,max}$
medium-low	$\sigma_{2,min}$	$\sigma_{2,max}$
medium	$\sigma_{3,min}$	$\sigma_{3,max}$
medium -high	$\sigma_{4,min}$	$\sigma_{4,max}$
high	$\sigma_{5,min}$	$\sigma_{5,max}$
very high	$\sigma_{6,min}$	$\sigma_{6,max}$

2nd Pillar Synthetic risk indicator

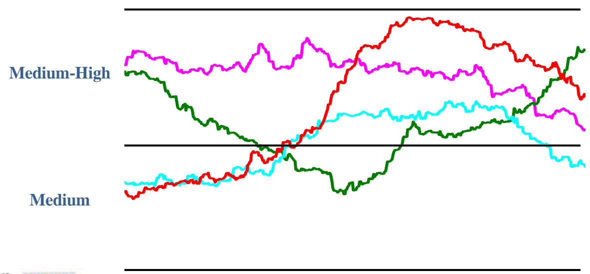
"Risk target" products | "Benchmark" products | "Return target" products

Products with the same risk budget
must have the same degree of risk



2nd Pillar Synthetic risk indicator

Volatility intervals have to be suitably calibrated
in order to avoid wrong risk representations



2nd Pillar Synthetic risk indicator

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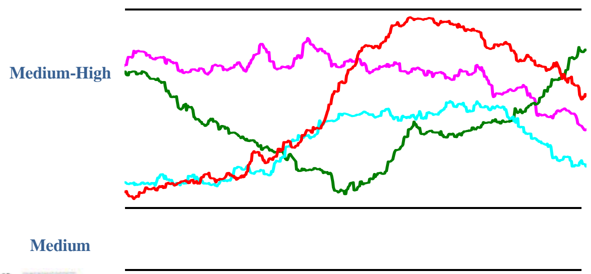


Definition of a suitable volatility grid

Looking for the number of intervals
(so-called "*n-tuple of risk classes*")
allowing the best compromise between
investors' comprehension and
detail of the information conveyed

Hypothesis
NUMBER OF INTERVALS SPANNED:
5, 6 or 7

Hypothesis
NUMBER OF INTERVALS SPANNED:

5 risk classes

Low
Medium-Low
Medium
Medium-High
High

6 risk classes

Low
Medium-Low
Medium
Medium-High
High
Very High

7 risk classes

Very Low
Low
Medium-Low
Medium
Medium-High
High
Very High

Definition of a suitable volatility grid

Minimizing the chance for an asset manager
of overcoming not intentionally
its risk budget, i.e. the volatility interval
(so-called "*management failure*")

Definition of a suitable volatility grid

ASSUMPTION

25% AS THE LOWER BOUND
OF THE LAST VOLATILITY INTERVAL

…corresponding to a percentage loss of about 50%
of the invested capital over a 1-year time horizon

Definition of a suitable volatility grid

1st INTUITION

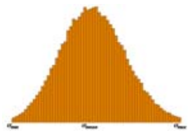
The optimal set of volatility intervals for a given *n*-tuple
of risk classes requires to solve a stochastic non linear programming
problem (i.e. minimize the chance of a "*management failure*")

In order to analise the *management failures* it has to be studied the
behavior of an automatic asset manager that has a specific risk budget,
identified by a given volatility interval

Automatic Asset Manager

Hypothesis:

Stochastic volatility model where the automatic
asset manager is "mean-reverting":

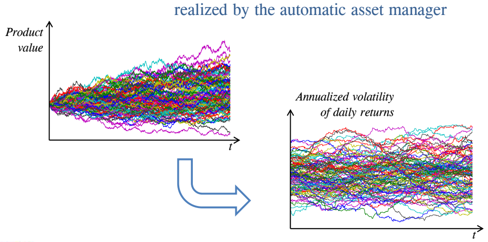


The automatic asset manager:

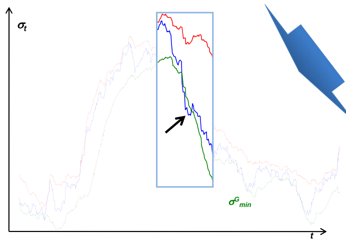
- has no systematic preference for upwards or downwards deviations from the mean ➔ symmetric distribution for the volatility
- in order to minimize the migration risk, keeps the product volatility far from the bounds of the interval ➔ probability decay over the tails

Automatic Asset Manager

Simulating the trajectories of the volatility
realized by the automatic asset manager



Definition of a suitable volatility grid



2nd INTUITION

In order to analise the *management failures* volatility prediction intervals have to be determined.
In this way, the ability of the automatic asset manager to remain within his risk budget can be measured

Automatic Asset Manager

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \tilde{Z}_k \\ \text{and} \\ \ln\sigma^2_{k+1} - \ln\sigma^2_k = \beta_0^{(k)} + (\beta_1^{(k)} - 1)\ln\sigma^2_k + \beta_1^{(k)}\ln Z^2_k \\ \text{or, equivalently} \\ \ln\sigma^2_{k+1} - \ln\sigma^2_k = \beta_0^{(k)} + (\beta_1^{(k)} - 1)\ln\sigma^2_k + 2\beta_1^{(k)}\ln|Z_k| \end{cases}$$

$\tilde{Z}_k$ and Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem of Discrete Markov Chains to Diffusions

$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d\ln\sigma^2_t = \left(\beta_0 + 2\beta_1 E(\ln|Z_t|) + (\beta_1 - 1)\ln\sigma^2_t\right)dt + 2|\beta_1|\sqrt{Var(\ln|Z_t|)}dW^*_t$$

Z_t is $N(0,1)$

Hypothesis:

GARCH diffusive models
to measure the ability of
the automatic asset
manager to remain within
his risk budget

2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

GARCH Diffusive Models: the Volatility Prediction Interval

distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d\ln\sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}(\ln|Z_t|) + (\beta_1 - 1)\ln\sigma_t^2\right]dt + 2|\beta_1|\sqrt{Var(\ln|Z_t|)}\,dW_t^*$$

O.U. process

$$\ln\sigma_t^2 \sim N\left(\left(\ln\sigma_s^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)(t-s)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)};\ \sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)(t-s)}-1\right)}{2(\beta_1-1)}}\right)$$

2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

GARCH Diffusive Models: the Volatility Prediction Interval

matching of the first two conditional moments

the discrete process becomes:

$$\ln\sigma_k^2 - \ln\sigma_{k-1}^2 = \frac{[\beta_0+2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1-1} - 2|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}}E(\ln|Z_{k-1}|) + \left(e^{(\beta_1-1)}-1\right)\ln\sigma_{k-1}^2 + 2|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}}\ln|Z_{k-1}|$$

2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

GARCH Diffusive Models: the Volatility Prediction Interval

maximum likelihood estimation

likelihood function:

$$L(w;\underline{\beta}) = \prod_{k=2}^{K}\left[\frac{1}{|\beta_1|\sqrt{2\pi}}\sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}}\cdot e^{\left(\frac{1}{2|\beta_1|}\sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}}\cdot w_k\right)}\cdot e^{\left(-\frac{1}{2}\exp\left(\frac{1}{|\beta_1|}\sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}}\cdot w_k\right)\right)}\right]$$

$\underline{\beta} := (\beta_0, \beta_1)$

where: $w_k := y_k - \frac{(\beta_0 - 1.27\beta_1)(e^{(\beta_1-1)}-1)}{\beta_1-1} - 1.27|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} - (e^{(\beta_1-1)}-1)\ln\sigma_{k-1}^2$

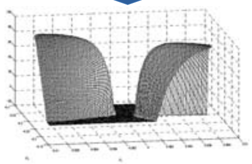
2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

GARCH Diffusive Models: the Volatility Prediction Interval

maximum likelihood estimation

shape of the associated log-likelihood



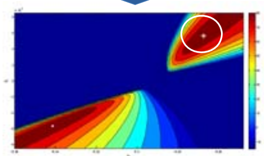
2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

GARCH Diffusive Models: the Volatility Prediction Interval

maximum likelihood estimation

β_0 and β_1 estimates



2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

GARCH Diffusive Models: the continuous limit of the M-GARCH(1,1)

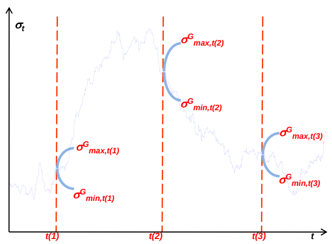
the estimated parameters enter in the bounds of the volatility prediction interval

$$\sigma^G_{t,\min} = e^{\frac{-z_{\frac{1+\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}{2}}$$

$$\sigma^G_{t,\max} = e^{\frac{z_{\frac{1+\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}{2}}$$

2nd Pillar
Synthetic risk indicator

Automatic Asset Manager

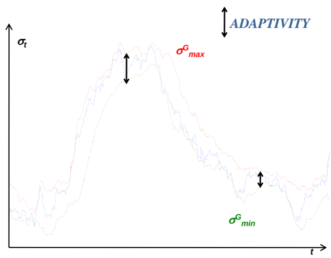


Hypothesis:

GARCH diffusive models to measure the ability of the automatic asset manager to remain within his risk budget

2nd Pillar
Synthetic risk indicator

Automatic Asset Manager



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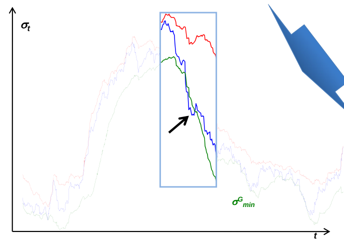
GARCH diffusive models to measure the ability of the automatic asset manager to remain within his risk budget

2nd Pillar
Synthetic risk indicator

Automatic Asset Manager



GARCH-based volatility prediction intervals to identify the "*management failures*" of the automatic asset manager

Definition of a suitable volatility grid

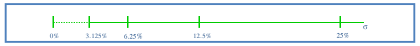
3rd INTUITION

The optimal set of volatility intervals must allow a similar number of "*management failures*" to the automatic asset managers belonging to different risk classes of a given *n*-tuple:

NO INCENTIVE TO PICK ANY SPECIFIC CLASS

NO INCENTIVE TO PICK ANY SPECIFIC CLASS

$$m = \frac{\sigma_i}{\sigma_{i-1}}$$



RESCALING LEMMA for volatility intervals

*Let $[\sigma_1,\sigma_2]$ and $[\sigma_3,\sigma_4]$ be two volatility intervals having the same multiplier **m**, i.e.:*

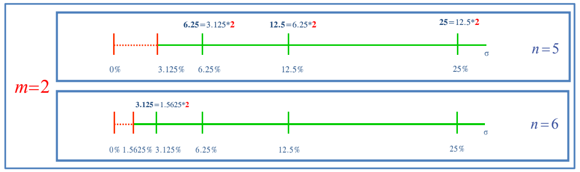
$$m = \frac{\sigma_2}{\sigma_1} = \frac{\sigma_4}{\sigma_3}$$

then, the two intervals have the same number of "management failures", that is the same number of breaches of the GARCH-based volatility prediction intervals.

Volatility Intervals Multiplier

Remark

For whatever *n*-tuple of classes the 1st interval cannot respect the multiplier:



Definition of a suitable volatility grid

4th INTUITION

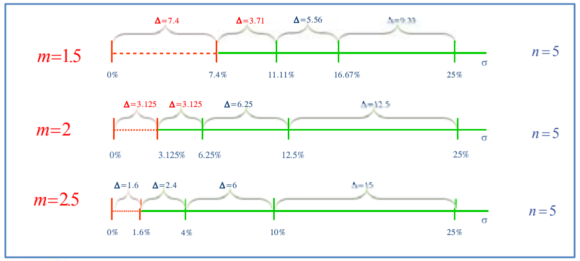
The optimal set of volatility intervals must be associated with increasing levels of losses

+ RISK + LOSSES

VOLATILITY INTERVALS MUST HAVE AN INCREASING WIDTH

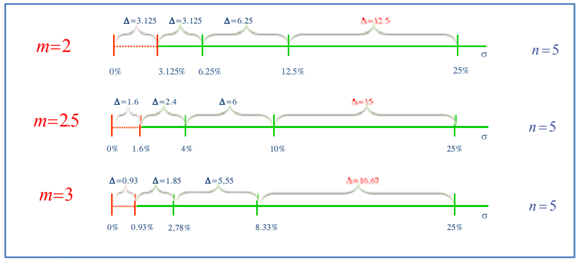
Definition of a suitable volatility grid

The requirement of increasing width holds for all the intervals (hence including the 1st) if and only if $m > 2$



Definition of a suitable volatility grid

The higher is the multiplier
the wider is the subinterval that ends with 25%
and the narrower is the first class



Definition of a suitable volatility grid

When *m* is too high the meaningfulness of the set of volatility intervals is compromised:

Hypothesis:
$2 < m < 3$

5 risk classes

					m
0%	2,70%	5,67%	11,90%	25%	2,1
0%	2,35%	5,17%	11,36%	25%	2,2
0%	2,05%	4,73%	10,87%	25%	2,3
0%	1,81%	4,34%	10,42%	25%	2,4
0%	1,60%	4,00%	10,00%	25%	2,5
0%	1,42%	3,70%	9,62%	25%	2,6
0%	1,27%	3,43%	9,26%	25%	2,7
0%	1,14%	3,19%	8,93%	25%	2,8
0%	1,03%	2,97%	8,62%	25%	2,9
0%	0,93%	2,78%	8,33%	25%	3,0

6 risk classes

						m
0%	1,29%	2,70%	5,67%	11,90%	25%	2,1
0%	1,07%	2,35%	5,17%	11,36%	25%	2,2
0%	0,89%	2,05%	4,73%	10,87%	25%	2,3
0%	0,75%	1,81%	4,34%	10,42%	25%	2,4
0%	0,64%	1,60%	4,00%	10,00%	25%	2,5
0%	0,55%	1,42%	3,70%	9,62%	25%	2,6
0%	0,47%	1,27%	3,43%	9,26%	25%	2,7
0%	0,41%	1,14%	3,19%	8,93%	25%	2,8
0%	0,35%	1,03%	2,97%	8,62%	25%	2,9
0%	0,31%	0,93%	2,78%	8,33%	25%	3,0

7 risk classes

							m
0%	0,61%	1,29%	2,70%	5,67%	11,90%	25%	2,1
0%	0,49%	1,07%	2,35%	5,17%	11,36%	25%	2,2
0%	0,39%	0,89%	2,05%	4,73%	10,87%	25%	2,3
0%	0,31%	0,75%	1,81%	4,34%	10,42%	25%	2,4
0%	0,26%	0,64%	1,60%	4,00%	10,00%	25%	2,5
0%	0,21%	0,55%	1,42%	3,70%	9,62%	25%	2,6
0%	0,17%	0,47%	1,27%	3,43%	9,26%	25%	2,7
0%	0,15%	0,41%	1,14%	3,19%	8,93%	25%	2,8
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0%	0,10%	0,31%	0,93%	2,78%	8,33%	25%	3,0

Definition of a suitable volatility grid

It is now possible to perform the minimization process of the management failures.

Main results: For whatever *n*-tuple of classes there is a **trade-off**

as *m* increases the number of management failures of

the 1st class decreases	the other classes class increases

$m^*=2.5$ achieves the best balance of this trade-off

$m^*=2.5$ identifies the optimal set of volatility intervals

Definition of a suitable volatility grid

5 risk classes

					m
0%	2,70%	5,67%	11,90%	25%	2,1
0%	2,35%	5,17%	11,36%	25%	2,2
0%	2,05%	4,73%	10,87%	25%	2,3
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0%	1,60%	4,00%	10,00%	25%	2,5
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6 risk classes

						m
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7 risk classes

							m
0%	0,61%	1,29%	2,70%	5,67%	11,90%	25%	2,1
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0%	0,12%	0,35%	1,03%	2,97%	8,62%	25%	2,9

Definition of a suitable volatility grid

The choice between the different n-tuples depends on the regulator assessment about the best compromise between investors' comprehension and detail of the information:

Consob choice:

$n=6$

Definition of a suitable volatility grid

OUTPUT

$m^*=2.5$

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Low	0.01%	0.49%
Medium-Low	0.64%	1.59%
Medium	1.60%	3.99%
Medium-High	4.00%	9.99%
High	10.00%	24.99%
Very High	25.00%	>25.00%

Syllabus

Preliminaries

Three-pillars approach:

1st Pillar: unbundling and performance scenarios

2nd Pillar: the degree of risk

3rd Pillar: recommended investment time horizon

"Risk target" products | "Benchmark" products | "Return target" products

The recommended investment time horizon

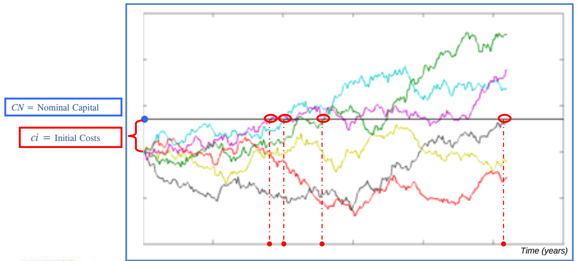
In analytical terms, the probability of the event:

The investment recovers the initial costs and off-sets the running costs at least once

can be calculated through the concept of

First Passage Time

First Passage Time:
First time (expressed in years) such that the value of the Invested Capital (CI) recovers the initial costs and off-sets the running costs.



The probability of the event:

The investment recovers the initial costs and off-sets the running costs at least once

given a confidence level α, uniquely identifies a time T^* on the cumulative distribution function of the first passage times, i.e.:

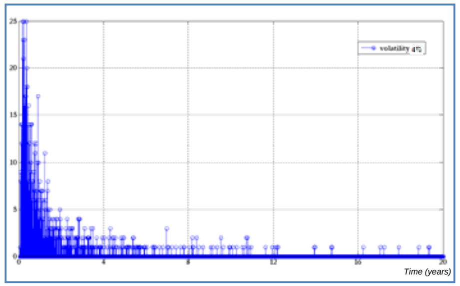
$$T^* = \left\{ T \in \Re^+ : P\left[t^* \leq T\right] = \alpha \right\}$$

where

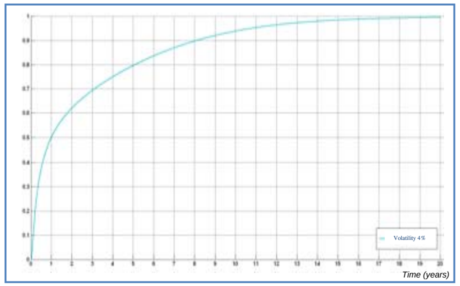
$$t^* = \inf\left[t \in \Re^+ : CI_t > CN\right]$$

is the first passage time

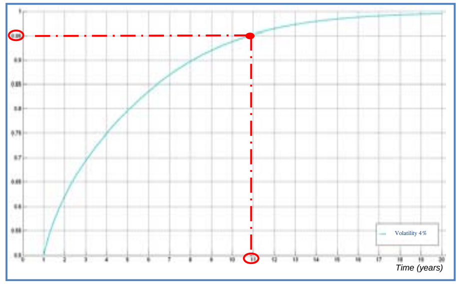
1. Calculation of the probability distribution of the first passage times:



2. Derivation of the cumulative distribution function of the first passage times:

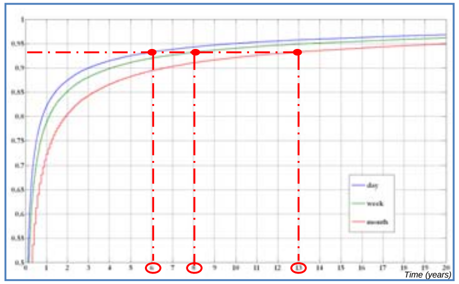


3. The confidence level α uniquely identifies T^* on the cumulative distribution function of the first passage times:



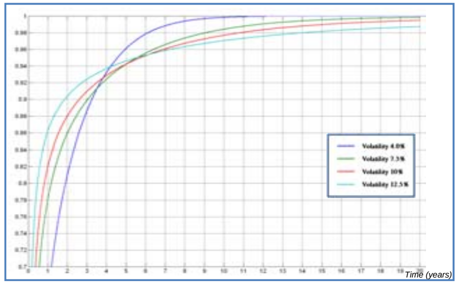
3rd Pillar

3. The discretization step is relevant in the determination of the cumulative probability function, conditioning the identification of the time horizon, given a fixed level of confidence:



3rd Pillar

When many probability distribution functions are considered, letting varying volatilities and costs, the problem of correctly identifying a set of minimum thresholds arises:



3rd Pillar

Anyway, the recommended minimum investment time horizon…

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \le T\right] = \alpha \right\}$$

…. Must be coherent with the principle

+ VOLATILITY + TIME HORIZON

3rd Pillar

Anyway, the recommended minimum investment time horizon…

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \le T\right] = \alpha \right\}$$

…. Must be coherent with the principle

+ VOLATILITY + TIME HORIZON

The correct way to solve the problem is to set up an operative procedure to select properly each threshold according to the above principle

3rd Pillar

Connection between probability, volatility and costs

First passage times for the break-even barrier are monitored at infinitesimal time intervals:

$dt \to 0$

$$T^* = \left\{ T \in \Re^+ : \mathrm{P}\left[t^* \le T\right] = \alpha \right\}$$

$$\mathrm{P}\left[t^* \le T\right] = N\left(d_2\left(\frac{CI_0}{CN}\right)\right) + \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \cdot N\left(-d_2\left(\frac{CN}{CI_0}\right)\right)$$

$$d_2(x) = \frac{\log x + \left(\bar{r} - cr - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

3rd Pillar

Connection between probability, volatility and costs

Asymptotic properties: $T \to \infty$

cr : recurrent costs as a fixed %

$dt \to 0$

$$\lim_{T\to\infty} \mathrm{P}\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

3rd Pillar

Connection between probability, volatility and costs

Under our assumptions:

$dt \to 0$

$$\lim_{T\to\infty} \mathrm{P}\left[t^* \le T\right] = \begin{cases} 1 & if \quad (\bar{r} - cr) \ge \frac{1}{2}\sigma^2 \\ \left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} & if \quad (\bar{r} - cr) < \frac{1}{2}\sigma^2 \end{cases}$$

For a given level of costs, it is possible to analytically derive the connection between volatility and time horizon

3rd Pillar

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

FIRST ORDER SENSITIVITY ANALYSIS

$$\frac{d\mathrm{P}}{d\sigma} = \left(-4\frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

FIRST ORDER ASYMPTOTIC CONDITION

3rd Pillar

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{d\mathrm{P}}{d\sigma} = \left(-4\frac{(\bar{r} - cr)}{\sigma^3} \ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1} \right)$$

1. $(\bar{r} - cr) > 0 \Leftrightarrow \frac{d\mathrm{P}}{d\sigma} < 0$
2. $(\bar{r} - cr) \le 0 \Leftrightarrow \frac{d\mathrm{P}}{d\sigma} \ge 0$

The existence of two alternative states of nature requires to verify whether both of them make sense in financial terms under the risk-neutral measure.

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1}\right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$

Being running costs a specific feature of any financial product they would interfere with the task of identifying which of the two conditions has a sound financial meaning. Therefore, they will be temporarily neglected.

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1}\right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

Since it is safe to assume a positive interest rate r in financial markets, only condition 1. correctly captures the connection between volatility and time horizon.

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1}\right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

As $T \to \infty$ condition 1. implies that the cumulative distribution function P is a strictly decreasing function of the volatility, i.e.:

$$\forall \sigma_i, \sigma_j \in \Re^+, \sigma_j > \sigma_i \Rightarrow P(\sigma_j) < P(\sigma_i)$$

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1}\right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

In other words, for a given confidence level, as the volatility grows, the recommended investment time horizon increases as well:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{\bar{r}}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2\bar{r}}{\sigma^2}-1}\right)$$

$cr = 0$

1. $\bar{r} > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. $\bar{r} \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

Furthermore, condition 1. alone is sufficient to guarantee a minimum time T^* beyond which the following strong condition holds:

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{dP}{d\sigma} = \left(-4\frac{(\bar{r}-cr)}{\sigma^3}\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1}\right)$$

$cr > 0$

1. $(\bar{r}-cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0$
2. ~~$(\bar{r}-cr) \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

Generalizing…

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

$$\frac{d^2P}{d\sigma^2} = \frac{4}{\sigma^4}(\bar{r}-cr)\ln\left(\frac{CN}{CI_0}\right)\left(\frac{CN}{CI_0}\right)^{\frac{2(\bar{r}-cr)}{\sigma^2}-1}\cdot\left[1+\frac{4(\bar{r}-cr)}{\sigma^2}\ln\left(\frac{CN}{CI_0}\right)\right]$$

$$(\bar{r}-cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0$$

→ SECOND ORDER ASYMPTOTIC CONDITION

Second Order Sensitivity Analysis

Connection between probability, volatility and costs

$T \to \infty, dt \to 0$

1.
$$\begin{cases} (\bar{r}-cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r}-cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases}$$

2. ~~$(\bar{r}-cr) \le 0 \Leftrightarrow \frac{dP}{d\sigma} \ge 0$~~

$$\exists T^* \in [0, \infty[: \frac{dP}{d\sigma} = 0$$

Summarizing the results of the asymptotic analysis in continuous time:

- As T →∞, for given a confidence level, more volatility implies a larger recommended investment time horizon
- It is always possible to find a minimum and finite time T^*, beyond which the strong condition

+VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

holds

DETERMINATION OF THE INVESTMENT TIME HORIZON

General Framework:

$$\begin{cases} T \to \infty \\ dt \to 0 \\ P(\infty, \sigma) \\ 1. \begin{cases} (\bar{r}-cr) > 0 \Leftrightarrow \frac{dP}{d\sigma} < 0 \\ (\bar{r}-cr) > 0 \Rightarrow \frac{d^2P}{d\sigma^2} > 0 \end{cases} \end{cases} \Rightarrow \begin{cases} T \ \ finite \\ dt \to 0 \\ P(T, \sigma) \\ (\bar{r}-cr) > 0 \Leftrightarrow \lim_{T\to\infty} \frac{\partial P(T,\sigma)}{\partial \sigma} < 0 \\ (\bar{r}-cr) > 0 \Rightarrow \lim_{T\to\infty} \frac{\partial^2 P(T,\sigma)}{\partial \sigma^2} > 0 \end{cases}$$

In order to determine effectively the investment time horizon, it is necessary to abandon the asymptotic environment and to shift the analysis of condition 1. in a finite time framework.

DETERMINATION OF THE INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS

$$\frac{\partial P(T,\sigma)}{\partial \sigma}$$

At a finite time T, the sufficient condition of the first order that allows to state the core relationship

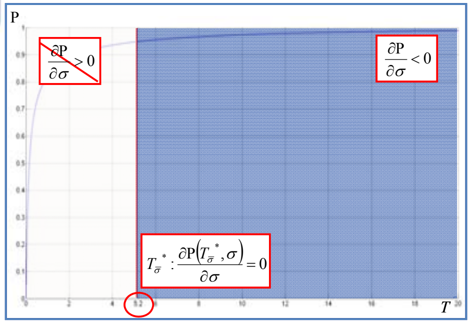
+ volatility + time horizon

is then specified in the following form:

$$\left.\frac{\partial P(T,\sigma)}{\partial \sigma}\right|_{\sigma=\bar{\sigma}} > 0 \quad if \quad 0 \le T < T^*_{\bar{\sigma}}$$

$$\left.\frac{\partial P(T,\sigma)}{\partial \sigma}\right|_{\sigma=\bar{\sigma}} \le 0 \quad if \quad T \ge T^*_{\bar{\sigma}}$$

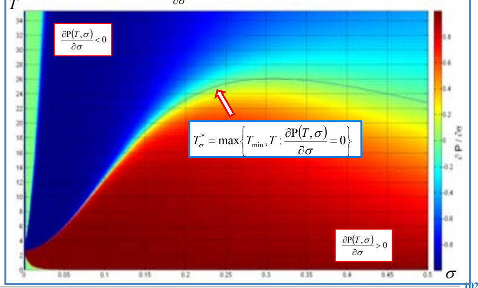
DETERMINATION OF THE INVESTMENT TIME HORIZON



DETERMINATION OF THE INVESTMENT TIME HORIZON

FIRST ORDER SENSITIVITY ANALYSIS

Plot of the function $\frac{\partial P(T,\sigma)}{\partial \sigma}$ in a space (σ,T)



DETERMINATION OF THE INVESTMENT TIME HORIZON

SECOND ORDER SENSITIVITY ANALYSIS

$$\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$$

Given the monotonicity condition of the probability distribution with respect to volatility, i.e.:

$$\forall \sigma_i, \sigma_j \in \Re^+, \sigma_j > \sigma_i \Rightarrow P(\infty, \sigma_j) < P(\infty, \sigma_i)$$

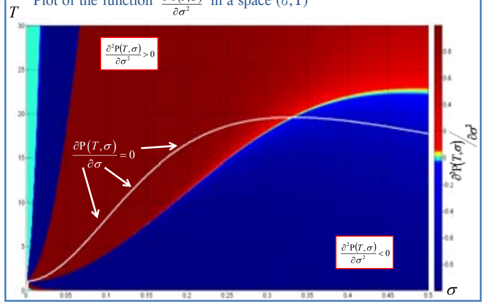
In order to fulfill this condition, it's necessary to restrict the analysis in the region where the probability function is strictly increasing, i.e.:

$$\left.\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}\right|_{T=T^*_\sigma} > 0 \Rightarrow T^*_\sigma \text{ increasing}$$

$$\left.\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}\right|_{T=T^*_\sigma} < 0 \Rightarrow T^*_\sigma \text{ decreasing}$$

DETERMINATION OF THE INVESTMENT TIME HORIZON

Plot of the function $\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$ in a space (σ,T)



DETERMINATION OF THE INVESTMENT TIME HORIZON

SECOND ORDER SENSITIVITY ANALYSIS

$$\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$$

Having defined the maximum time in the form:

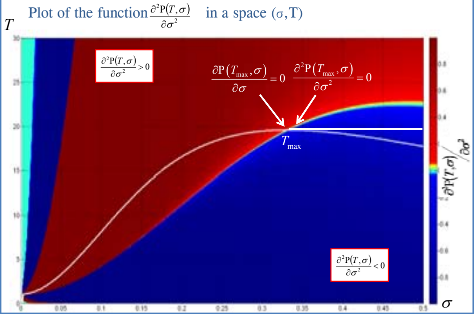
$$\begin{cases} \sigma \in \Re^+ \\ T_{max} \in T^*_\sigma \end{cases} : \frac{\partial^2 P(T_{max},\sigma)}{\partial \sigma^2} = 0$$

The sufficient condition of the 2° order is specified as:

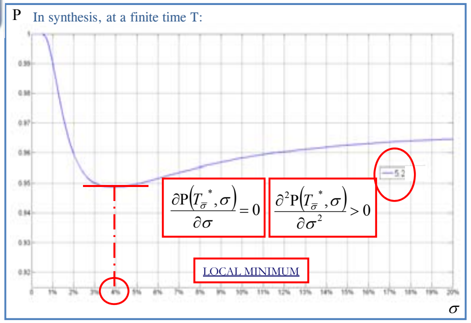
$$T^* = \begin{cases} T^*_\sigma \; if \; \left.\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}\right|_{T=T^*_\sigma} \ge 0 \\ T_{max} \; if \; \left.\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}\right|_{T=T^*_\sigma} < 0 \end{cases}$$

DETERMINATION OF THE INVESTMENT TIME HORIZON

Plot of the function $\frac{\partial^2 P(T,\sigma)}{\partial \sigma^2}$ in a space (σ,T)



DETERMINATION OF THE INVESTMENT TIME HORIZON



+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

STRONG CONVERGENCE LEMMA for times

Given a sequence of financial products F_j with volatility σ_j and recalling the first order sufficient condition:

$$T^*_\sigma = \max\left\{T_{min}, T : \frac{\partial P(T,\sigma)}{\partial \sigma} = 0\right\}, \quad \forall \sigma \in \Re^+$$

the first order sufficient condition can be specified for the class of products F_j in the following form:

$$T^{\varepsilon_j}_{\sigma_j} : P\left(T^{\varepsilon_j}_{\sigma_j}, \sigma_{j+1}\right) = P\left(T^{\varepsilon_j}_{\sigma_j}, \sigma_j\right)$$

It therefore holds the following strong convergence relation with respect to times:

$$\lim_{\sigma_{j+1} \to \sigma_j} T^{\varepsilon_j}_{\sigma_j} = T^*_{\sigma_j}$$

where $\varepsilon_j = (\sigma_{j+1} - \sigma_j) > 0.$

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

In order to have an intuitive explanation of the lemma, let's consider the following volatility levels:

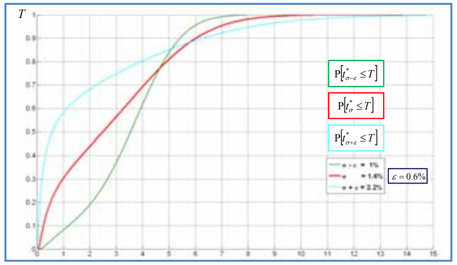
$\sigma-\varepsilon, \varepsilon \in \Re^{+}$ | σ | $\sigma+\varepsilon, \varepsilon \in \Re^{+}$

and the respective probability distribution functions, i.e.:

$P[t^{*}_{\sigma-\varepsilon} \le T]$ | $P[t^{*}_{\sigma} \le T]$ | $P[t^{*}_{\sigma+\varepsilon} \le T]$

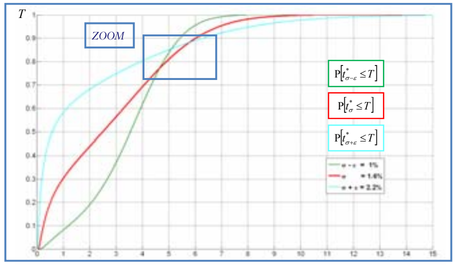
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



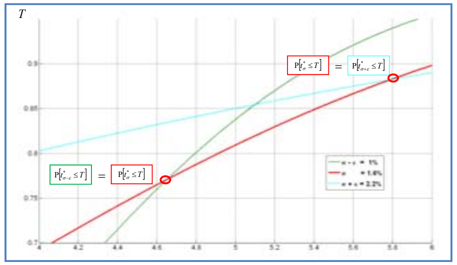
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



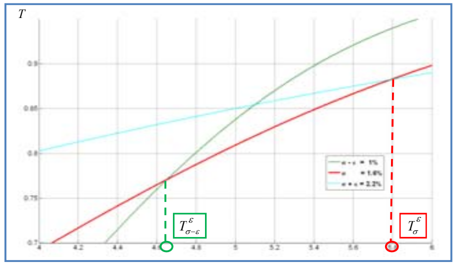
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



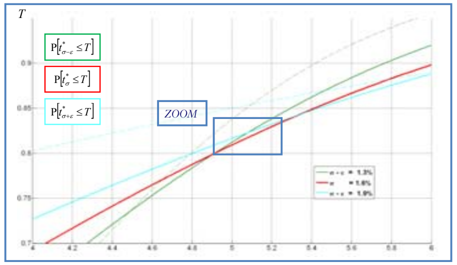
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



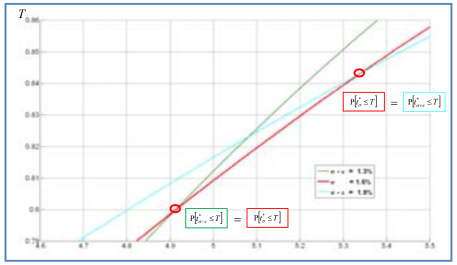
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



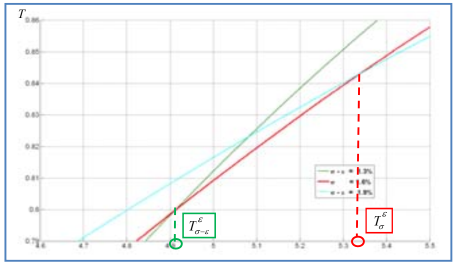
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



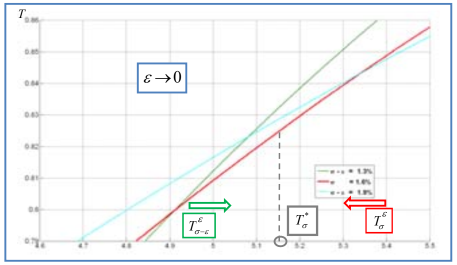
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons



+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

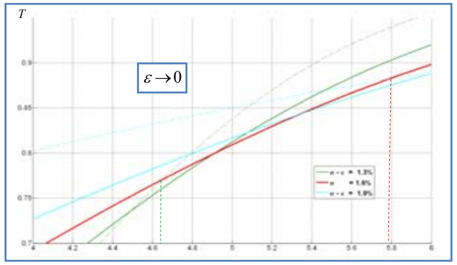


3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

In synthesis..

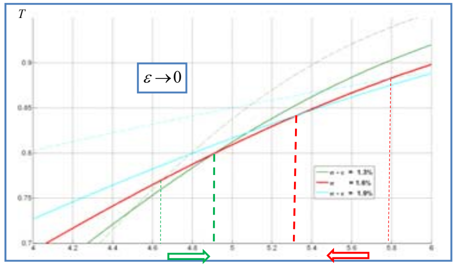


3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

In synthesis..



3rd Pillar
The recommended investment horizon

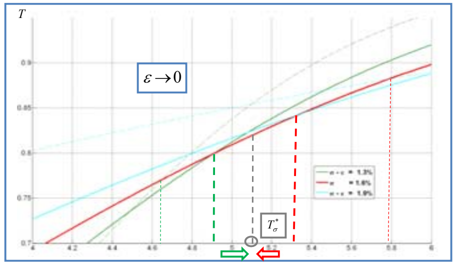
+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

In synthesis..

$\sigma = 1.6\%$

$T^*_\sigma = 5.1$ years

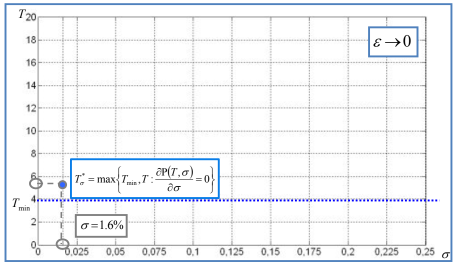


3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

The time is characterized on the curve of minimum times..



3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

Generalizing the lemma for all σ, the following characterization of the first order sufficient condition is given:

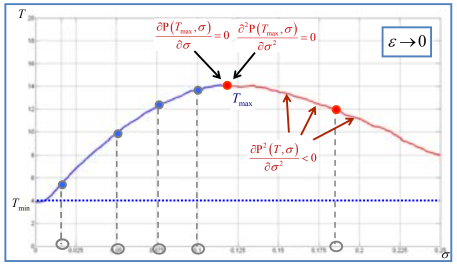
$\sigma - \varepsilon, \varepsilon \in \Re^+$	σ	$\sigma + \varepsilon, \varepsilon \in \Re^+$
$P[t^*_{\sigma-\varepsilon} \le T]$	$P[t^*_\sigma \le T]$	$P[t^*_{\sigma+\varepsilon} \le T]$

3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

The time is characterized on the curve of minimum times..

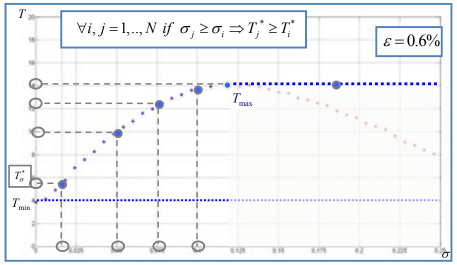


3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

SECOND ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

Weak monotonicity condition of times w.r.t. volatility



3rd Pillar
The recommended investment horizon

+ VOLATILITY + RECOMMENDED INVESTMENT TIME HORIZON

FIRST ORDER SUFFICIENT CONDITION
to determine a sequence of consistent time horizons

Formally, for any sequence of products with volatility σ_j, defined in a given class of costs (ci,cr):

Strong convergence lemma for times
First order sufficient condition

Weak monotonicity condition of times w.r.t. volatility
Second order sufficient condition

$$\forall j = 1,...N, \sigma_{j+1} > \sigma_j,$$
$$T^*_{j+1} = \max\left\{T^*_j,\; T \in [T_{min}, T_{max}]: P\left[t^*_{\sigma_{j+1}} \le T\right] = P\left[t^*_{\sigma_j} \le T\right] = \alpha^*_{j+1}\right\}$$

Opinions expressed may not reflect the ones of Consob

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Rebuilding the investors' confidence through risks disclosure

Marcello Minenna – Head of Quantitative Analysis Unit, Consob

Structured products ASIA
20 September 2010, Hong Kong