Detecting market abuse

Financial regulators need a way to detect market abuse in real time. Marcello Minenna has developed such a procedure that can detect, for each quoted stock and on a daily basis, the presence of market abuse phenomena by means of a set of tripwires that analyses the flows of elementary information on trading in securities available to the financial market authorities

Security markets' history is filled with trading abuses that undermine their strength. To deter the abuse, regulators must first develop ways to detect it. A market abuse detection (MAD) procedure identifies, on a daily basis, the securities affected by abnormalities – in terms of the transactions – that could hide illicit behaviour in the form of market manipulation or insider trading. This abnormality, called stock failure, will be the starting point for analysis by regulatory officers – who exert their legal power of data requests, trade analysis, interviews, etc – that may lead to a market abuse case.

Therefore, since a failure is a necessary condition for a market abuse phenomenon, even if not a sufficient one, its identification is the aim of a MAD procedure. The procedure highlights the possible presence of market abuse phenomena by examining the behaviour of financial variables that correspond to the flows of elementary information on trading available to the regulator. Examining financial variables' behaviour requires the definition of a reference model for each of them. These models have been developed for the first time in this field by using the results of stochastic limit theory, instead of the traditional econometric methods. In particular, this new theoretical approach allows the identification of adaptive dynamic thresholds for each financial variable, the crossing of which signals an anomalous movement in the variable in question (so-called alerts). The financial variable and the related model thus become the tripwire for any abnormality in the security. Having identified the tripwires, the calibration of a MAD procedure consists of specifying their parameters and identifying an algorithm permitting several alerts to be interpreted jointly. The signals generated by this algorithm thus represent the potential market abuse phenomena to be investigated.

The tripwires

An examination of the literature and supervisory experience (Minenna, 2003a) has provided the following indications as to how transaction prices, the quantities traded and the names of the traders who carried out the transactions (that is, the flows of elementary information on trading available to the supervisory authority) must be analysed to construct financial variables whose behaviour can reveal market abuses:

□ transaction prices are analysed in terms of returns by studying the movements in the log of the price;

□ returns generally undergo sharp changes (for example, at the time privileged information is disclosed) or follow paths that are not of the mean-reverting type (for example, in the presence of manipulative phenomena);

□ the presence of abnormal returns is detected by estimating returns using diffusion processes;

□ autoregressive models can capture both the discrete mean-reversion and momentum-effect components of returns;

□ the quantities traded by individual agents are examined in terms of daily trading volumes according to an autocorrelated process;

□ the agents are analysed in relation to the quantities they have traded in a day, taking into account the depth of the market, the presence of dominant traders and the composition of the various intermediaries/traders;

□ the composition of the market is evaluated using a two-stage process: the level of concentration of the intermediaries, that is, the number of intermediaries and their shares of trading volumes (so-called static concentration); and the evolution of the concentration of the intermediaries, that is, the movement in each intermediary's share of the volume of trading in a given security (so-called dynamic concentration).

On the basis of these indications, four financial variables have been constructed that represent the behaviour of: the volumes of trading in the security; the returns on the security; the static market concentration; and the dynamic market concentration.

The definition of a MAD procedure tripwire requires the examination of these variables to be based on a reference model; appropriately calibrated, this defines the dynamic thresholds that trigger the procedure's alerts. In particular, the construction of the tripwires must guarantee the detection, in real time, of the securities that may be the subject of market abuse. The examination of the different cases of market abuse found by Consob, the Italian financial services regulator, has provided important basis criteria input in defining the diffusion processes that describe the behaviour of the financial variables and thus characterise the reference models. A description follows of the construction and functioning of the tripwires based on the four financial variables considered, with reference to the flows of elementary information on the trading in any security listed on the share market. To this end, P_t and Q_t are used to denote respectively the official price and the volume of trading observed in the financial markets for a generic security on day t.

□ **The analysis of trading volumes.** It is possible to assume that volumes are governed in discrete time by the following autoregressive process, which, by construction, shows autocorrelation:

$$Q_k - Q_{k-1} = -\gamma Q_{k-1} + \hat{\sigma} Z_k \qquad (1)$$

where $Z_1, \dots, Z_k$ is a sequence of independent random variables distributed as a standard normal and k is the indicator of discrete time. The properties of the reference model (1) do not permit the behaviour of volumes to be forecasted using a number of daily observations that refer to a time horizon of a month or less, if the statistical significance of the analysis is not to be lost or numerous procedural complications are not to be encountered. To construct a tripwire that responds to the objectives of the procedure, attention is focused on the distributive characteristics of the corresponding continuous-time version of (1), that is, the following stochastic differential equation:

$$dQ_t = -\theta Q_t dt + \sigma dW_t \qquad (2)$$

where θ and σ are deterministic time functions and W_t is a standard uni-dimensional Brownian motion.

This result derives from the application of a convergency theorem described in Minenna (2003a), Ethier & Kurtz (1986) and Stroock & Varadhan (1979). In particular, for the application of this theorem in the demonstration of the weak convergence of (1) to (2), see Minenna (2003a), Nelson (1990) and Duan (1997).

The advantage of the continuous time approach is that by referring to a stochastic differential equation, if this has an integrated solution or if the distributive properties of the solution are known, it is possible to construct a confidence interval for the prediction of the variable described by the diffusion process (Stroock & Varadhan, 1979, and Minenna, 2003a). This interval defines the trading volume dynamic thresholds that identify the alerts of the MAD procedure. The logic underlying the construction of the indicator is based on Minenna (2002), which seeks to predict the behaviour of the re-

turn on a security and detect the presence of abnormal returns.

Equation (2) is the continuous-time version of (1). This stochastic differential equation is known as an Ornstein-Uhlenbeck arithmetic diffusion process, which has the following distributive properties with reference to any constant initial condition identified at time s, with $s < t$, equal to Q_s (Arnold, 1974, and Minenna, 2003a):

$$Q_t \sim N\left(Q_s e^{-\theta(t-s)}; \sqrt{\frac{\sigma^2}{2\theta}\left(1-e^{-2\theta(t-s)}\right)}\right) \qquad (3)$$

The relationship between (1) and (2) and the distributive properties of the latter stochastic differential equation (3) define the reference model to be used in examining the time series of trading volumes, and thus uniquely define the tripwire with reference to this financial variable.

□ **The analysis of returns.** The following model appropriately describes the behaviour of this variable in discrete time:

$$R_k - R_{k-1} = \gamma \times \left(\eta - R_{k-1}\right) + \hat{\sigma} Z_k \qquad (4)$$

By time rescaling, as in the above section, the discrete Markov process $\{R_k\}_{k \geq 0}$ and showing (4) converges weakly towards the diffusion process $\{R_t\}$ given by the following stochastic differential equation:

$$dR_t = q\left(\mu - R_t\right)dt + \sigma dW_t \qquad (5)$$

where q and μ are deterministic time functions and W_t is a standard unidimensional Brownian motion. Equation (5) is the continuous-time version of (4). This is again an Ornstein-Uhlenbeck arithmetic diffusion process, which has the following distributive properties with reference to any constant initial condition identified at time s, with $s < t$, equal to R_s:

$$R_t \sim N\left(\left(R_s - \mu\right)e^{-q(t-s)} + \mu; \sqrt{\frac{\sigma^2}{2q}\left(1-e^{-2q(t-s)}\right)}\right) \qquad (6)$$

□ **The analysis of static concentration.** The composition of the market intermediaries/traders needs to be studied. A first analysis of this kind can be made by examining the so-called static concentration, that is, the number of intermediaries and their shares of trading volumes. The first step in constructing a tripwire on the basis of this aggregate is to identify the related financial variable. The choice among the various indexes of market concentration put forward in Minenna (2003a), supported by Consob's experience, would be an index of entropy, that is:

$$\Theta_t = \frac{1}{n_t}\sum_{i=1}^{n_t}\left(\frac{Q_t(i)}{\mu_t}\right)^\alpha \qquad (7)$$

where n_t is the number of traders on the market at time t; $Q_t(i)$, $i = 1, \ldots, n_t$ are the quantities traded by the ith intermediary at time t; and:

$$\mu_t = \frac{\sum_{i=1}^{n_t} Q_t(i)}{n_t}$$

The reference model that gives the tripwire is based on a stochastic differential equation:

$$d\Theta_t = -\zeta\Theta_t dt + \sigma dW_t \qquad (8)$$

which we derived, similar to trading volumes, through the shift to continuous time of the corresponding discrete process (see above, *The analysis of trading volumes*), that is:

$$\Theta_k - \Theta_{k-1} = -\xi\Theta_{k-1} + \hat{\sigma}Z_k \qquad (9)$$

This model has the following distributive properties with reference to any constant initial condition identified at time s, with $s < t$, equal to Θ_s:

$$\Theta_t \sim N\left(\Theta_s e^{-\zeta(t-s)}; \sqrt{\frac{\sigma^2}{2\zeta}\left(1-e^{-2\zeta(t-s)}\right)}\right) \qquad (10)$$

The reason for this choice is that in normal trading conditions, that is, in the absence of failures that could point to possible market abuse phenomena, the index for short-time horizons should be autoregressive in discrete time and mean-reverting in continuous time. We examined the financial variable Θ_t with reference to the quantities bought, the quantities sold and the gross turnover, that is, the sum of purchases and sales, of the various intermediaries/traders. This choice was due to the need to capture not only the movement in the variable for the total turnover of the market but also the directions taken by individual intermediaries, and hence the market, since this could point to possible market abuse phenomena.

□ **The analysis of dynamic concentration.** The analysis of the market's composition must also take into account the evolution of the intermediaries' concentration, that is, the behaviour of each intermediary's share of the volume of trading in a given security (so-called dynamic concentration).

This additional analysis of concentration can reveal changes in the role played by a particular intermediary/trader in determining the total daily turnover in a security.

The first step in constructing a tripwire on the basis of this aggregate is to identify the related financial variable. The examination of the various cases of market abuse found by Consob – supported by what the literature highlights – suggested using an index of dissimilarity, that is:

$$\Psi_t = \sqrt{\frac{1}{\tilde{n}_t}\sum_{i=1}^{\tilde{n}_t}\tilde{Q}_t(i)^2} \qquad (11)$$

where $\tilde{Q}_t(i) = Q_t(i) - Q_{t-k}(i)$, $Q_t(i)$, $i = 1, \ldots, n_t$ are the quantities traded by the ith intermediary at time t, and $\tilde{n}_t$ is the number of traders with a value of $\tilde{Q}_t(i)$ other than zero.

The reference model that gives the alert is based on a stochastic differential equation, that is:

$$d\Psi_t = -\omega\Psi_t dt + \sigma dW_t \qquad (12)$$

which, as in the analysis of static concentration, was obtained through the shift to continuous time of the corresponding discrete process (see previous section), that is:

$$\Psi_k - \Psi_{k-1} = -v\Psi_{k-1} + \hat{\sigma}Z_k \qquad (13)$$

This model has the following distributive properties with reference to any constant initial condition identified at time s, with $s < t$, equal to Ψ_s:

$$\Psi_t \sim N\left(\Psi_s e^{-\omega(t-s)}; \sqrt{\frac{\sigma^2}{2\omega}\left(1-e^{-2\omega(t-s)}\right)}\right) \qquad (14)$$

The variable Ψ_t has to be examined with reference to the total quantity traded by each intermediary, that is the difference between purchases and sales (net turnover). Subsequently, three tripwires with the same mathematical formulation were constructed with reference to each of the above aggregates.

The decision to consider net turnover together with purchases and sales for this alert was due to the definition of this aggregate considered in relation to the characteristics of the variable Ψ_t. In fact, since the net turnover at a given time t is the synthesis of the operational behaviour of an intermediary/trader on the market, examining it through the variable Ψ_t – which by construction compares the quantitative trading data of the different intermediaries/traders with the corresponding figures for an earlier period – makes it possible to detect the changes in the operational behaviour of the different intermediaries/traders.

The calibration procedure

The calibration of a MAD procedure consists of:

□ deriving the prediction confidence intervals serving to identify the alerts of the various financial variables on the basis of the properties of the tripwire's reference model;

□ determining the time horizon needed for the specification of the para-

meters used in the prediction confidence intervals;
☐ defining the forecasting horizon of the tripwires;
☐ specifying the algorithm that, by jointly interpreting the various alerts, signals a possible market abuse phenomena to be examined by the enforcement units (so-called warnings); and
☐ defining the temporal validity of a warning generated by the MAD procedure, that is, the number of days the enforcement units must monitor the security following the warning.

The supervisory experience and the analysis of insider trading and market manipulation cases found by Consob permitted, through the *ex post* application of the MAD procedure, the empirical verification for its calibration. In particular, this analysis showed that:
☐ the procedure must operate on a rolling basis with the daily updating of the estimation parameters of the tripwires. In other words, for the securities subject to monitoring, it must identify, on a daily basis, the economic and financial phenomena potentially attributable to a case of market abuse;
☐ the calibration must be carried out using a relatively short series of daily data (equal to or less than one month of trading); the resulting instability of the model makes it possible to capture the changes in the investment opportunities of the various agents operating on the market; and
☐ the predictive capability must be equal to one trading day. The continuous updating of the predictions allows the model to promptly identify the cases that deserve to be analysed in depth by the enforcement units.

The literature on financial markets and the supervisory experience, corroborated by the above-mentioned empirical verification, have provided indications on how to construct the algorithm for the joint interpretation of the alerts produced by the various tripwires. In particular, it was found to be desirable to examine jointly the data on trading volumes, returns and market concentration.

☐ **The calibration of the volume-based tripwire.** Given the diffusion process of (2), a prediction confidence interval can be constructed, on the basis of the data of the trading volumes of the preceding days, that infers the possible values for the following day:

$$P\left(\begin{array}{c} -z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\theta}\left(1-e^{-2\theta}\right)} + Q_{t-1}e^{-\theta} \le \\ \le Q_t \le \\ \le z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\theta}\left(1-e^{-2\theta}\right)} + Q_{t-1}e^{-\theta} \end{array}\right) = \chi \tag{15}$$

The extremes of the interval are the dynamic thresholds that define the alerts for the behaviour of trading volumes. Every time the observed behaviour of trading volumes falls outside the interval, there is an alert.

Having formally defined the fluctuation band, to calibrate the trading volume tripwire it is necessary to estimate the parameters of the stochastic differential equation (2) using the data observed in discrete time.

One effective way to do this, which makes it possible to arrive at an explicit formulation of the parameters and thus to avoid the use of numerical procedures, is to rewrite the discrete-time version of the diffusion process (2), ie, the equation (1), so that it has the same characteristics as (2) in terms of conditional mean and variance (Minenna, 2003a, and Dixit & Pindyck, 1994).

We can therefore rewrite (1) as:

$$Q_k - Q_{k-1} = \left(e^{-\theta} - 1\right)Q_{k-1} + \sqrt{\frac{\sigma^2}{2\theta}\left(1-e^{-2\theta}\right)}Z_k \tag{16}$$

Equation (16) becomes the discrete process that, on the basis of the daily observations of trading volumes, makes it possible to estimate the parameters of (2).

The empirical verification of the tripwire was carried out by examining the various cases of market abuse found by Consob. This made it possible to identify the time horizon for estimating the regression of the data in discrete time, which we found to be 15 trading days. The value of the standardised normal random variable z, which defines the prediction confi-

dence interval, is equal to 2.33, and therefore includes 99% of the possible forecasting scenarios of the financial variable.

☐ **The calibration of the return-based tripwire.** It is possible to construct, in the same way as in the previous section, a prediction confidence interval that, on the basis of the returns of the preceding days' data, infers the possible values for the following day, that is:

$$P\left(\begin{array}{c} -z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2q}\left(1-e^{-2q}\right)} + \left(R_{t-1} - \mu\right)e^{-q} + \mu \le \\ \le R_t \le \\ \le z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2q}\left(1-e^{-2q}\right)} + \left(R_{t-1} - \mu\right)e^{-q} + \mu \end{array}\right) \tag{17}$$

The estimation of the parameters follows similar steps to those described in the previous section.

The empirical verification of the tripwire was carried out by examining the various cases of market abuse found by Consob. It confirms the values of the time horizons and of the random variable z found for the volume tripwire.

☐ **The calibration of the static market-concentration tripwire.** In calibrating this variable on a daily basis with a view to determining the dynamic thresholds that define the alerts, it should be noted that:
☐ by construction, as α increases, the expression (7) becomes more sensitive to the intermediaries that handle a large percentage of the trading volumes;
☐ intermediaries/traders do not necessarily carry out transactions involving a given security every day (so-called discontinuous trading). This implies that the use of daily data on the quantities traded by the ith intermediary/trader – to construct the index and hence the financial variable that defines the tripwire – risks introducing a large noise component that would make it difficult to interpret the values of the variable; and
☐ the reference model is defined by the stochastic differential equation (8), which is the continuous-time version of the discrete-time autoregressive process (9).

Consequently, the same logical and computational approach was used as for the volume-based tripwire (see above, *The calibration of the volume-based tripwire*).

The analysis of the various cases of market abuse found by Consob, filtered through the characteristics of this tripwire, provided an effective empirical verification for the latter's calibration. In particular, it suggested:
☐ analysing the quantities traded by each intermediary over a time horizon of five trading days, which overcomes the problem of the discontinuous trading of some intermediaries/traders;
☐ a value of five for the parameter α, which allows high concentrations to be highlighted;
☐ a reference window of 15 trading days for estimating the regression on the discrete data;
☐ that the value of the standardised normal random variable z, which defines the prediction confidence interval, was equal to 2.33 and therefore included 99% of the possible forecasting scenarios of the financial variable.

The indicator was therefore specified as follows:

$$\Theta_t = \frac{1}{n_t}\sum_{i=1}^{n_t}\left(\frac{\hat{Q}_t(i)}{\mu_t}\right)^5 \tag{18}$$

where $\hat{Q}_t(i) = \Sigma_{i=1}^{5} Q_{t-5}(i)$ is the total quantity traded in the last five days by the ith trader, and:

$$\mu_t = \frac{\sum_{i=1}^{n_t} Q_t(i)}{n_t}$$

As indicated above, in *The analysis of static concentration*, three tripwires are calculated for the static concentration (with reference to, respectively, the gross turnover, purchases and sales of the intermediaries/ traders), since this is considered to represent the evolution of the static concentration, including the directions in which the market moves. The alert of this financial variable is triggered when at least one of the tripwires exceeds the

corresponding threshold given by the prediction confidence interval.

□ **The calibration of the dynamic market-concentration tripwire.** In calibrating this variable on a daily basis with a view to determining the dynamic thresholds that define the alerts, given the characteristics of the construction of the tripwire:

□ the variable $\widetilde{Q}_t(i)$ compares a quantity at time t with the corresponding value at an earlier time, that is, $Q_t(i) - Q_{t-k}(i)$;

□ intermediaries/traders do not necessarily carry out transactions involving a given security every day (so-called discontinuous trading). This implies that the use of daily data on the quantities traded by the ith intermediary/trader to construct the index and hence the financial variable that defines the tripwire risks introduces a large noise component that would make it difficult to interpret the values of the variable; and

□ the reference model is defined by the stochastic differential equation (12), the continuous version of the discrete autoregressive process (13).

Consequently, the same logical and computational approach was used as for the volume-based tripwire (see above, *The calibration of the volume-based tripwire*).

The analysis of the various cases of market abuse found by Consob, filtered through the characteristics of this tripwire, provided an effective empirical verification for the latter's calibration. In particular, it suggested:

□ constructing the variable $\widetilde{Q}_t(i)$ by comparing the quantities traded by each intermediary at time t with the corresponding value observed at time $t-5$ (in other words, the lag is equal to the trading week); and

□ a reference window of 15 trading days for estimating the regression on the discrete data.

The indicator was therefore specified as follows:

$$\Psi_t = \sqrt{\frac{1}{\widetilde{n}_t}\sum_{i=1}^{\widetilde{n}_t}\left[Q_t(i) - Q_{t-5}(i)\right]^2} \qquad (19)$$

As explained above, in *The analysis of dynamic concentration*, three tripwires are calculated for the dynamic concentration (with reference to, respectively, the net turnover, purchases and sales of the intermediaries-traders), since this is considered to represent the evolution of the dynamic concentration, including the directions in which the market moves. This financial variable's alert is triggered when at least one of the tripwires exceeds the corresponding threshold given by the prediction confidence interval.

The algorithm for reading the alerts

The tripwire identifies the dynamic thresholds and thus the abnormalities in the behaviour of the financial variable analysed, on a daily basis, with reference to a rolling set of observations. In particular, the dynamic thresholds that define the prediction confidence interval for the financial variable at time t are calculated with reference to the interval $[t-k, t]$ where $k = 15$. The alerts are thus of the rolling type with a reference window of 15 trading days. The crossing of the dynamic thresholds corresponding to the extremes of the prediction confidence interval defined by the generic tripwire signals an anomaly in the behaviour of the financial variable in question. The extremes of the prediction confidence intervals that define the alerts for each financial variable are listed below:

1. $Q_t \notin \left(Q_{\text{inf}}; Q_{\text{sup}}\right)$

$$Q_{\text{inf}} = -z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta}\right)} + Q_{t-1}e^{-\theta}$$

$$Q_{\text{sup}} = +z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta}\right)} + Q_{t-1}e^{-\theta}$$

2. $R_t \notin \left(R_{\text{inf}}; R_{\text{sup}}\right)$

$$R_{\text{inf}} = \mu - z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2q}\left(1 - e^{-2q}\right)} + (R_{t-1} - \mu)e^{-q}$$

$$R_{\text{sup}} = \mu + z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2q}\left(1 - e^{-2q}\right)} + (R_{t-1} - \mu)e^{-q}$$

3. $\Theta_t \notin \left(\Theta_{\text{inf}}; \Theta_{\text{sup}}\right)$

$$\Theta_{\text{inf}} = -z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\zeta}\left(1 - e^{-2\zeta}\right)} + \Theta_{t-1}e^{-\zeta}$$

$$\Theta_{\text{sup}} = +z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\zeta}\left(1 - e^{-2\zeta}\right)} + \Theta_{t-1}e^{-\zeta}$$

4. $\Psi_t \notin \left(\Psi_{\text{inf}}; \Psi_{\text{sup}}\right)$

$$\Psi_{\text{inf}} = -z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\omega}\left(1 - e^{-2\omega}\right)} + \Psi_{t-1}e^{-\omega}$$

$$\Psi_{\text{sup}} = +z_{\frac{\chi}{2}}\sqrt{\frac{\sigma^2}{2\omega}\left(1 - e^{-2\omega}\right)} + \Psi_{t-1}e^{-\omega}$$

The analysis of the literature and the practical experience showed that the data on trading volumes, the behaviour of returns and the evolution of market concentration needed to be analysed jointly. In this sense the examination of the various cases of market abuse found by Consob, filtered by means of the *ex post* application of the procedure, has shown that an effective algorithm for the joint reading of the various alerts produced by the tripwires aimed at identifying a warning for a security is to consider a trading day to be anomalous if at least three of the four tripwires have signalled an alert.

The critical period (that is, the period of validity of the warning) was taken to be the five subsequent trading days. In the event of more than three alerts on successive days, the critical period starts from the last day on which there was a warning.

The supervisory experience and the market abuse detection cases

The model proposed is the first approach in this field that uses stochastic limit theory to detect irregularities in the market that could be a sign of market abuse, instead of traditional econometric methods that rely on a more standard statistical analysis.

Hence, the probability density functions adopted are not an assumption but a result of the quantitative procedure adopted. Therefore, the probability density functions identified by the procedure should not be evaluated in light of a standard statistical estimation procedure but in light of the way in which the model performs and estimates the parameters. In particular, the model used for the financial variables in the discrete time is an autoregressive model. By construction of the stochastic limit theory its continuous time version is an Ornstein-Uhleneck stochastic differential equation, or alternatively a specific form of the Kolmogorov backward equation that entails a normal distribution and a particular relationship between the parameters of the discrete time process and the ones of the continuous time process. In other words, the distribution of the financial variables is derived from applying the stochastic limit theory, specifically after a particular parameters estimation procedure. This procedure works if:

□ there is some corresponding treatable stochastic differential equation that ensures the weak convergence of the discrete time process in continuous time; and

□ it is possible to identify some parameters' estimation to assess a consistent bridge between the discrete and the continuous time process parameters.

The model represents an improvement in the detection of the market abuse phenomena, as it is able to overcome the structural weaknesses of the standard statistical approaches in discrete time.

On this point, for the return tripwire (a financial variable largely analysed in the literature with these approaches) it is sufficient to mention some of these structural weaknesses – overcome by construction with the MAD procedure – such as (Hamilton, 1994, and Minenna, 2003b):

□ the request of a time-series data set that may not be available if the stock has been recently quoted on the stock exchange;

□ the employment of a particularly long time horizon (generally more then 150 trading days) in order to overcome the statistical issues of the econometric procedures – such as the presence of serial correlation phenomena, parameters stability, etc – could include too many relevant events for

A. Detail of alerts

Company	Abnormal transactions		Report to judicial authority	Case description
	Date of beginning	Date of ending		
Alleanza Assicurazioni	08/11/01	12/11/01	21/05/02	Insider trading involving some transactions on Alleanza Assicurazioni saving shares in the period preceding the conversion of these shares into the ordinary ones
Allianz	06/09/00		12/10/01	Insider trading involving some transactions on Allianz Subalpina shares in the period preceding the announcement of the decision taken by Ras SpA to launch a takeover bid on these shares
Banca Legnano	18/12/00	19/12/00	30/11/01	Insider trading involving some transactions on Banca di Legnano shares in the period preceding the announcement of the disposal to Banca Popolare di Milano of the control stock held by Banca Intesa to Banca Popolare di Milano
Banca Profilo	11/01/00		10/01/01	Insider trading in view of the abnormal pattern of Banca Profilo shares in the period preceding the announcement of the activation of a plan for the realisation of an innovative service on network data transmission
Burgo	19/04/00		27/12/00	Insider trading in view of the abnormal performance of the shares of Cartiere Burgo SpA in the period preceding the announcement of a takeover bid launched by Dieci srl
Cir	13/09/00		09/07/01	Insider trading on Cir saving shares in the period preceding the date in which the issuing company disclosed the approval – by the board of directors – of a proposal of converting the saving shares and the preferred shares into the ordinary ones
Cremonini	12/01/01	12/01/01	02/10/01	Insider trading involving some transactions on Cremonini SpA shares immediately before the announcement of the discovery of the first Italian case of bovine spongiform encephalopaty
Dmail	05/11/01	06/11/01	25/03/02	Action-based manipulation in view of the abnormal pattern recorded by the price of Dmail shares in the period immediately before the disclosure of the information concerning an interest in purchasing from the main shareholders at least the 81% of the of the equity capital of Dmail.it SpA
Fiat	06/04/01	06/04/01	06/12/02	Action-based manipulation on Fiat ordinary shares related to the fact that these shares were the underlying of a put option embedded in the financial engineering of reverse convertible bonds
Generali	04/09/00	04/09/00	26/09/01	Action-based manipulation in view of some transactions executed on Assicurazioni Generali shares during the pre-opening phase and related to the fact that these shares were the underlying of put options embedded in the financial engineering of reverse convertible bonds
HDP	01/02/00	14/02/00	10/04/00	Action-based manipulation involving some transactions on the shares of HDP
Inv. Imm. Lombardi	07/03/02	20/03/02	08/01/03	Insider trading involving some transactions on Investimenti Immobiliari Lombardi Spa shares (now Bipielle Investimenti Spa) in the period before the announcement of the takeover bid launched by Banca Popolare di Lodi on these shares
Italiana Ass.	before 20/04/01		21/12/01	Insider trading involving some transactions on Italiana Assicurazioni shares in the period preceding the announcement of the takeover bid launched on these shares by Reale Mutua Assicurazioni
Magneti Marelli	13/03/00	05/05/00	25/05/01	Insider trading involving some transactions on Toro Assicurazioni and Magneti Marelli ordinary shares in the period preceding the announcement of the takeover bid launched by Fiat Spa on these shares
Pirelli & Co	25/09/00	26/09/00	01/10/01	Insider trading involving some transactions on Pirelli SpA and on Pirelli & Co ordinary shares in the period before the announcement of the disposal of the indirect holding of Pirelli SpA in Optical Technologies USA Corp to Corning Inc
Recodati	01/09/00	19/09/00	03/08/01	Insider trading involving some transactions on Recordati saving non-convertible shares in the period preceding the announcement of the conversion of the saving non-convertible shares into the ordinary ones
Rotondi Evolution	01/06/99	19/07/99	08/05/00	Action-based manipulation on Rotondi Evolution shares in view of the abnormal trading conditions recorded by these shares
Seat PG	before 10/02/00		30/07/02	Insider trading involving some transactions on Seat ordinary shares, Seat saving non-convertible shares and on financial derivatives tools connected to Seat shares in the period before the announcement of the merger between Tin.it and Seat
Seat PG	before 10/02/00		30/07/02	Action-based manipulation finalised to alter the price of Seat ordinary shares
Snai	04/01/01	04/01/01	05/12/01	Action-based manipulation finalised to alter the price of Snai shares
Toro	13/03/00	05/05/00	25/05/01	Insider trading involving some transactions on Toro Assicurazioni and Magneti Marelli ordinary shares in the period preceding the announcement of the takeover bid launched by Fiat Spa on these shares
Unim	before 29/09/99		24/01/01	Insider trading involving the abnormal pattern of the Unim SpA shares in the period preceding the announcement of a takeover bid launched by Milano Centrale on these shares

the company. This circumstance renders the results of the security analyses biased, difficult to interpret and to support statistically; and

☐ the need of a reference index that would be statistically meaningful as a regressor for an analysis on a specific security. Often the presence of a high number of thin stocks hampers the regression analysis. This market feature implies that there would be some stocks that account for the bulk of the market reference index. Hence, the results of the regression analysis would appear to be statistically meaningful since these reflect a self-explanatory regression where the relationship between the two variables is endogenous.
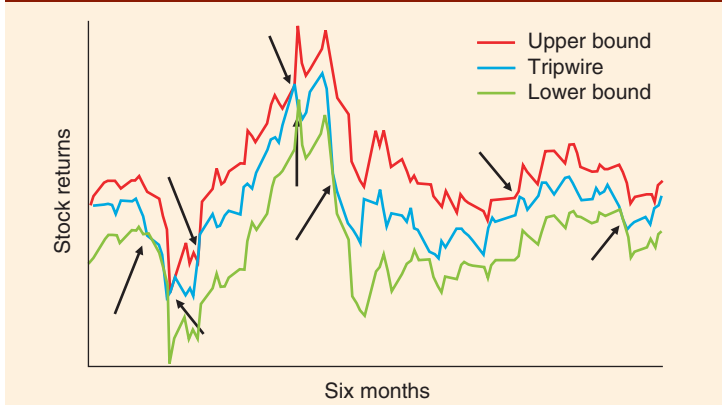
A unique feature of the MAD procedure is that it is able to discard wrong echo signals in the days after the first stock failure thanks to its adaptive nature. In fact, if the procedure identifies a stock failure on a given day, it will incorporate this market occurrence. By doing so, it will widen its prediction bands and consequently adapt its forecasts to the new market behaviour without giving wrong echo signals in successive days.

A further consequence of this behaviour is that the tripwire tends to selectively capture the beginning and the ending of the market micro-structural issue.

An analysis of the return tripwire behaviour for one stock over a period of six months should render graphically what has been stated above (see figure 1).

1. Prediction confidence interval for the return tripwire

Stock returns

- Upper bound
- Tripwire
- Lower bound

Six months

2. Frequency distribution of the warnings generated by the MAD procedure

n° security

One year

- $z = 2.33$
- $z = 1.96$

n° warning

Moreover, since the MAD procedure uses exclusively closed formulas, the implementation is easy and immediate as long as the securities transactions data is available and structured in a relational database. This feature also entails that there would not be a need for numerical simulations or statistical tests, allowing us to drop all the implications in terms of the fine-tuning of the estimate/forecast models.

An analysis of the warnings generated on average over one year can give an idea of the dimension of the signals generated by the procedure. The MAD procedure has identified about 1,800 warnings for all the listed securities on the Italian stock exchange (about 270) – determined according to the parameter specifications detailed above. It is important to highlight that by construction of the procedure, generally, several warnings are reconductable to the same market phenomenon. In other words, there is not a one-to-one relationship between the warnings and the case that will be investigated.

The histogram in figure 2 classifies all the Italian-listed securities in relation to the warnings generated by the procedure. Moreover, the same histogram shows the frequency distribution of the warnings at a lower value of z – that is, 1.96 – determining thus narrower prediction bands for the four tripwires and hence a lower selectivity in the generation of the warnings.[1]

This aspect highlights another of the procedure's strengths – the extreme effectiveness and efficiency of the calibration of the warnings' selectivity. In fact, the simple fine-tuning of the random variable z controls the entire calibration of the MAD procedure.

Table A presents a sample of 22 cases of market abuse that the procedure has been able to detect.[2] In particular, it shows the period of the abnormal transactions, the signal date of the judicial authority (that is, the ending date of the investigations carried out by Consob's enforcement units) and the description of the market abuse recorded by the model, in terms of the four alerts.
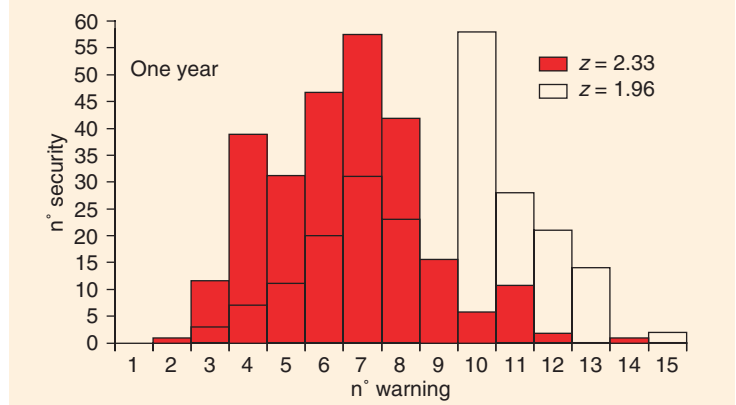
A further consideration arises from observing the results of a standard econometric procedure to identify the abnormal returns for the cases reported in the table[3] (Hamilton, 1994, and Minenna, 2003b). The daily estimate of the abnormal returns, conducted through the discrete model (4) adopting an hypothesis testing with a type I error of 1%, has not generated any alerts for all the cases highlighted in the table.[4]

Conclusions

A proper market abuse detection procedure is essential for regulators, as an effective instrument for preventing and repressing criminal behaviour in the forms of market manipulation and insider trading.

The MAD procedure is a first step in a quantity-based approach to the supervision of financial markets. This procedure does not claim to be exhaustive, but it offers a different perspective in analysing the financial data when compared with the traditional econometric procedures.

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The procedure developed can undoubtedly be improved by:
☐ introducing new tripwires on other financial variables, such as volatility;
☐ by examining data on intra-day trading, such as the inter-arrival time; and
☐ developing some adaptations in order to be applicable to the derivatives markets. In fact, the convexity and the other issues typical of the micro-structure of the derivatives markets render the procedure not applicable in its current version.

These are tasks for future research. ■

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[1] *In particular, the reduction of the value of the random variable z determines the correspondent reduction to the value of 97.5% for the probability scenarios considered and an increase in the warnings generated up to about 2.400*

[2] *The choice of these 22 cases was led on one side by the need to offer a significant representation of the different market abuse conducts detected by the model, and by the fact that the case description would be no longer covered by the duty of secrecy*

[3] *The choice of discussing the results of the econometric analysis with reference to the 'return' financial variable arises from the following considerations: this variable is largely analysed in the literature, so the procedures are quite standardised; and the historical series data of the returns are public and available in the database of the main data providers. These circumstances render the reading of these results immediately verifiable*

[4] *This analysis has been conducted by: using the same time period for the financial time series defined for the MAD procedure in section 3.2; and valuing as irrelevant the failure in the verification of the model hypotheses, fundamentally due to the low number of observations considered*