4 Pricing via Quadrature: Empirical Performances

4.1 Stability Assessment

A wide performance assessment has been done on three very popular models: the Heston (1993) stochastic volatility model, the Merton (1976) jump diffusion model and the Bates (1996) stochastic volatility–jump diffusion model. The Black–Scholes model via Fourier transform is used as a benchmark to assess the performance measures and the schemes' parameters.

4.1.1. Oscillations of the Characteristic Functions

The characteristic function embedded in Formulae (1.2) and (1.14) in Section 1 may be very sensitive to the movements of the model's parameters. By simply moving the level of volatility or the time to maturity it can become either strongly peaked or highly oscillatory. In the following charts, some examples are calculated for Black–Scholes (via Fourier transform), Heston, Merton and Bates model.

4.1.1.1. Black-Scholes Model

The analytic form of the risk-neutral characteristic function is:

$$\tilde{f}_2(v) = \phi_T(v) = e^{riv\tau + \frac{1}{2}\sigma^2 iv(iv-1)\tau + iv\ln S_t}$$

The following initial values for the model's parameters are used: $r = 0.02$, $\sigma = 0.3$, $\tau = 0.5$. The characteristic function is spanned on the truncated interval $[0, 10]$. $\alpha = 10$, $K = 1$. See Figure 4.1.

4.1.1.2. Heston Model

The analytic form of the risk-neutral characteristic function is:

$$\tilde{f}_2(v) = \phi_t(v) = \exp\left[riv\tau - \frac{2\kappa\theta}{\sigma^2}\left(\alpha_2\tau + \ln\frac{\frac{\alpha_2}{\alpha_1}e^{d\tau} - 1}{\frac{\alpha_2}{\alpha_1} - 1}\right)\right.$$
$$\left. - \left(\frac{2\alpha_2}{\sigma^2}\frac{1 - e^{d\tau}}{1 - \frac{\alpha_2}{\alpha_1}e^{d\tau}}\right)v_t + iv[\ln S_t + r\tau]\right]$$

where:

$$d = \sqrt{(\rho\sigma vi - b_j)^2 + \sigma^2(vi - v^2)}$$

$$\alpha_1 = \frac{\rho\sigma vi - \kappa + d}{2}, \qquad \alpha_2 = \frac{\rho\sigma vi - \kappa - d}{2}$$

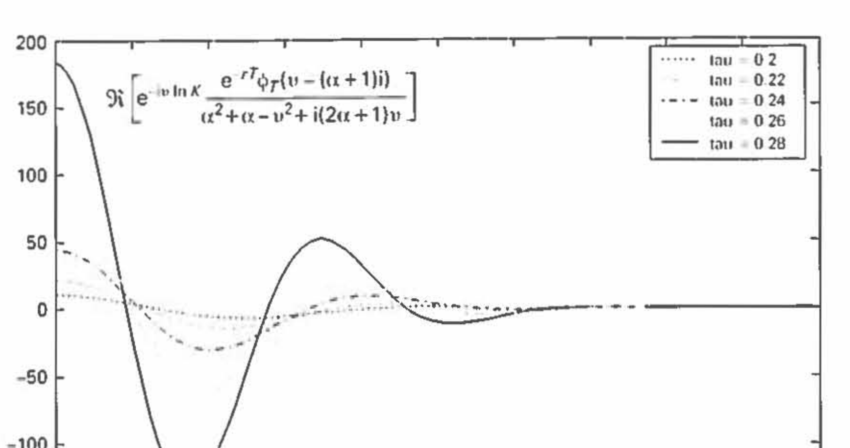


Figure 4.1. Black–Scholes model – stability analysis on tau

- ρ is the correlation coefficient between the spot price and variance;
- v is the current level of variance;
- θ is the long-run mean of the variance process;
- σ is the diffusion coefficient of the variance process;
- κ is the speed of mean reversion of the variance process.

The following initial values for the model's parameters are used: $r = 0.02$, $v = 0.3$, $\rho = -1$, $\kappa = 1$, $\sigma = 0.1$, $\theta = 0.5$, $\tau = 0.5$. The characteristic function is spanned on the truncated interval $[0, 10]$. $\alpha = 10$, $K = 1$. See Figures 4.2 and 4.3.

4.1.1.3. Merton Model

The analytic form of the "risk-neutral" characteristic function is:

$$\bar{f}_2(v) = \phi_T(v) = e^{\frac{1}{2}\sigma^2 iv\tau(iv-1) + riv\tau - \lambda\mu iv\tau + \lambda\tau e^{\frac{\sigma^2}{2}iv(iv-1)}(1+\mu)^{iv} - \lambda\tau + ivx},$$

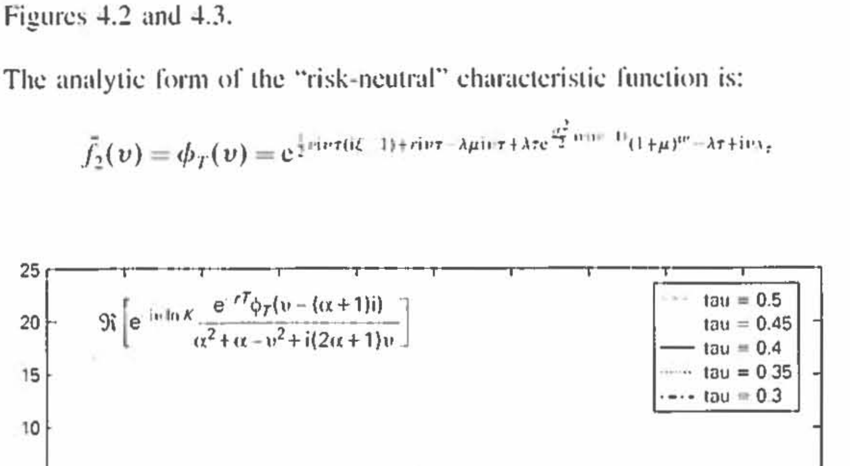
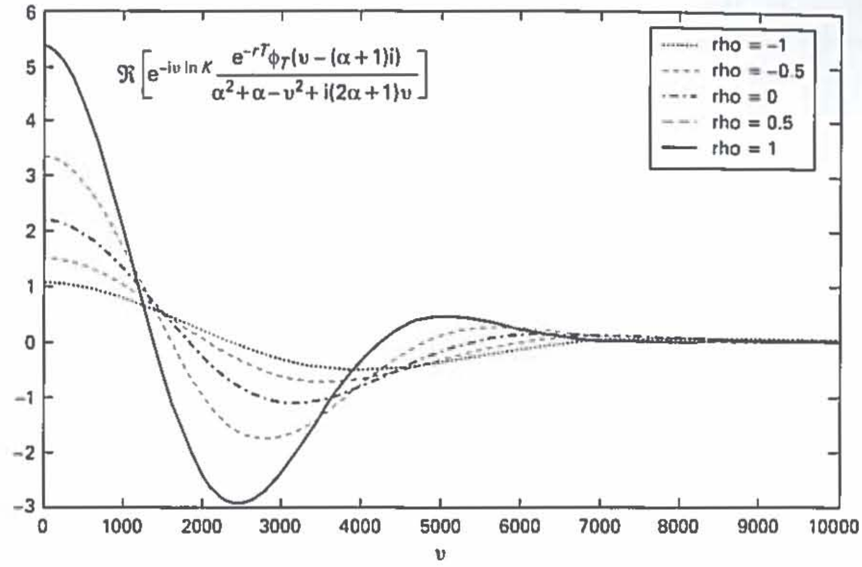


Figure 4.2. Heston model – stability analysis on tau

Figure 4.3. Heston model – stability analysis on rho



where:

- λ is the frequency of jumps in the unit of time;
- v is the constant level of variance;
- μ is the mean of jumps that occur in the unit of time;
- σ is the volatility of jumps that occur in the unit of time.

The following initial values for the model's parameters are used: $r=0.02$, $v=0.3$, $\lambda=0.1$, $\mu=0.1$, $\sigma=0.1$, $\tau=0.5$. The characteristic function is spanned on the truncated interval [0, 10]. $\alpha=10$, $K=1$. See Figures 4.4 and 4.5.

4.1.1.4. Bates Model

The analytic form of the "risk-neutral" characteristic function is:

$$\tilde{f}_2(v)=\phi_T(v)=e^{[C_\tau+D_\tau v_t+i\xi\ln S_t]}$$

Figure 4.4. Merton model – stability analysis on tau

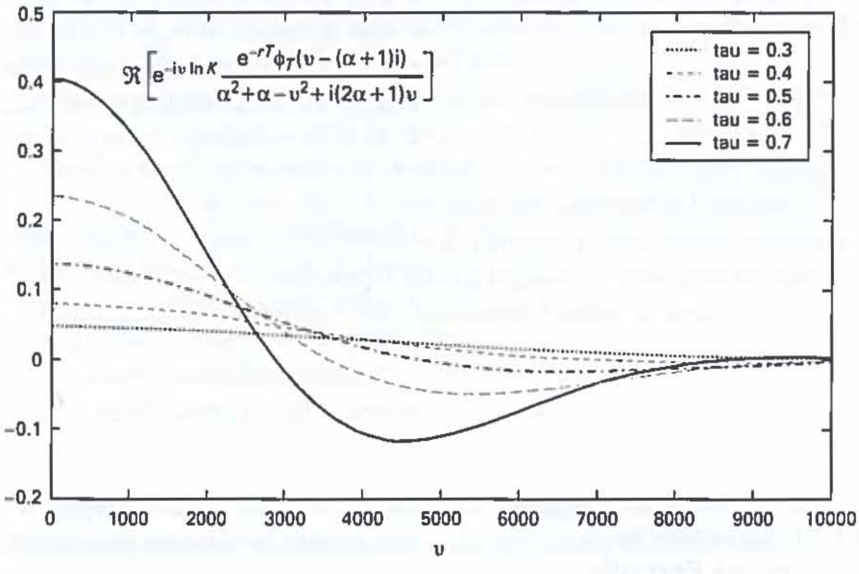
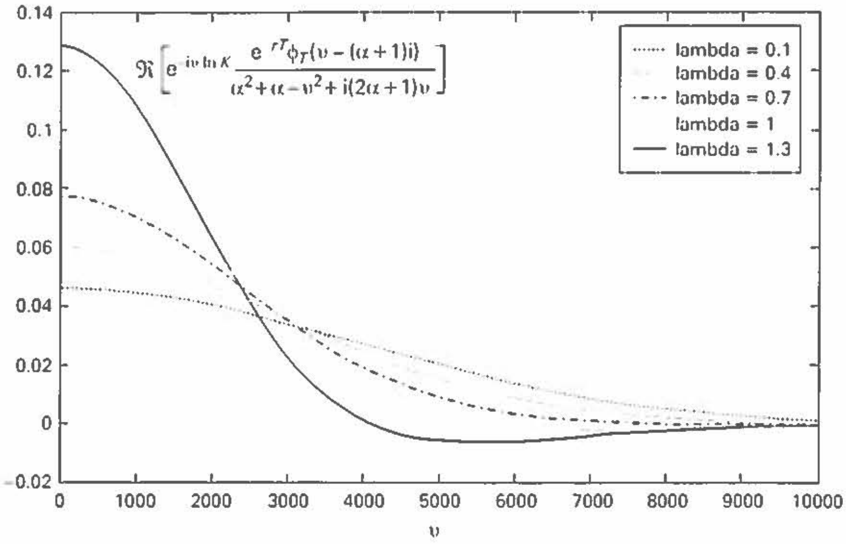


Figure 4.5. Merton model – stability analysis on lambda



where:

$$C_\tau = riv\tau - \lambda\mu_j iv\tau - \frac{\kappa\theta}{\sigma^2}[[\rho\sigma iv - \kappa] + d_2]\tau$$
$$-\frac{2\kappa\theta}{\sigma^2}\ln\left[1 - \frac{[\rho\sigma iv - \kappa - \widetilde{\chi} + d_2](1 - e^{-d_2\tau})}{2d_2}\right]$$
$$+\lambda\tau\left[e^{\frac{\sigma_j^2}{2}iv(iv-1)}(1+\mu_j)^{iv} - 1\right]$$
$$D_\tau = \frac{iv(iv-1)(1 - e^{-d_2\tau})}{2d_2 - (d_2 + \rho\sigma iv - \kappa)(1 - e^{-d_2\tau})}$$
$$d_2 = \sqrt{[\rho\sigma iv - \kappa]^2 - \sigma^2 iv(iv-1)}$$

- ρ is the correlation coefficient between the spot price and variance;
- v is the current level of variance;
- θ is the long-run mean of the variance process;
- σ is the diffusion coefficient of the variance process;
- κ is the speed of mean reversion of the variance process;
- λ is the frequency of jumps in the unit of time;
- μ_j is the mean of jumps that occur in the unit of time;
- σ_j is the volatility of jumps that occur in the unit of time.

The following initial values for the model's parameters are used: $r = 0.02$, $v = 0.3$, $\lambda = 0.1$, $\rho = -1$, $\kappa = 1$, $\sigma = 0.1$, $\theta = 0.5$, $\mu_j = 0.1$, $\sigma_j = 0.1$, $\tau = 0.5$. The characteristic function is spanned on the truncated interval $[0, 10]$. $\alpha = 10$, $K = 1$. See Figures 4.6 and 4.7.

Different valuation criteria are defined to assess the algorithms performance with respect to the pricing formulae.

4.1.2. The Stability Impact of Alpha in Single Integration Formula

Let us notice also that simply by changing α, the integrand in Equation (1.14) can become either strongly peaked when getting close to the