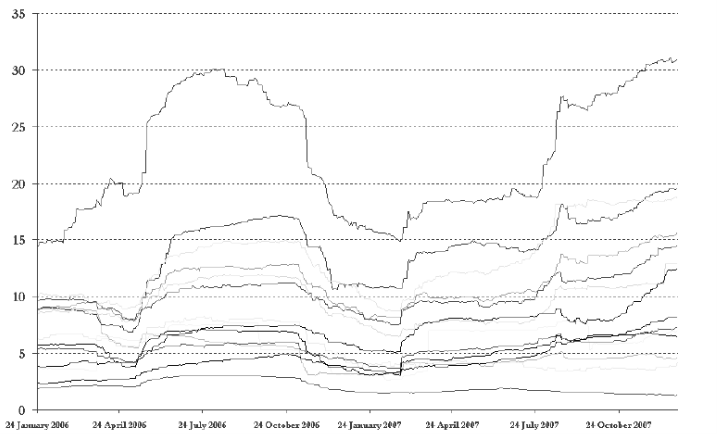
A GARCH Diffusive Approach to Volatility Forecasting: Application for Mutual Funds



MARCELLO MINENNA – GIOVANNA M. BOI

Syllabus

- Volatility
- The GARCH Diffusive Approach
- Application to Flexible Funds Risk Assessment
- Empirical Evidence on the Risk Profile of Flexible Mutual Funds
- Conclusions

Syllabus

- Volatility
  - Importance and Relationship with other Risk Measures
  - Random Variable and Stochastic Process

Volatility: Importance and Relationship with other Risk Measures

Volatility is usefully employed in several problems of mathematical finance, such as:

Derivatives Pricing

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

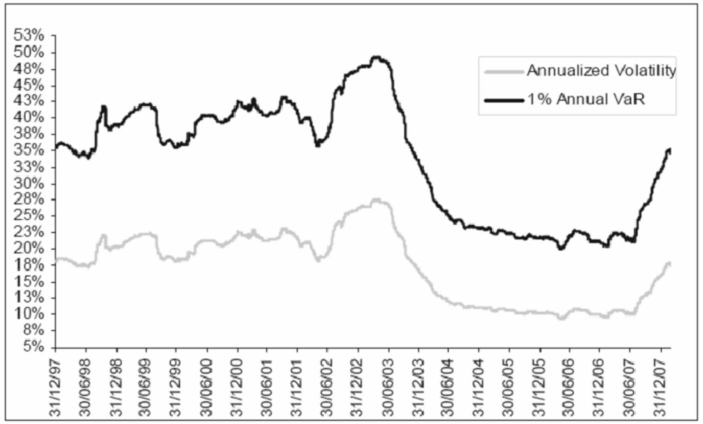
Mutual Funds Risk Assessment

$$d\ln NAV_t = b(t, \ln NAV_t)dt + \sigma\,(t, \ln NAV_t)\,dW_t$$

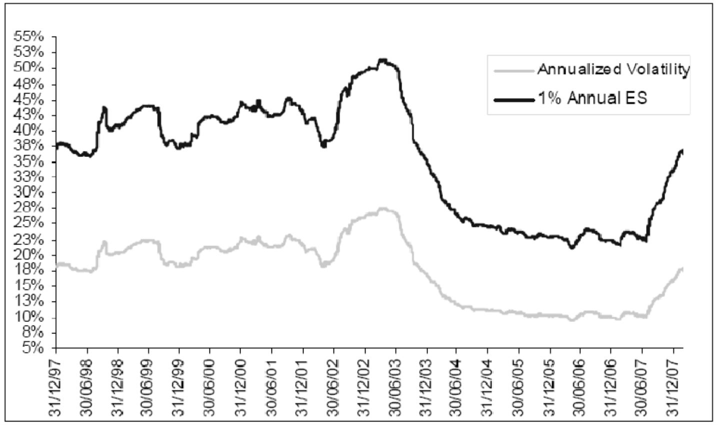
Term Structure Modelling

$$df_t = \alpha(t,T)dt + \sigma(t,T)dW_t$$
$$\alpha(t,T)dt = \sigma(t,T)\int_t^T \sigma(t,s)ds$$

Volatility has a close correspondance with any risk measure, like *Value-at-Risk* (*VaR*) and …



… *Expected Shortfall* (*ES*)



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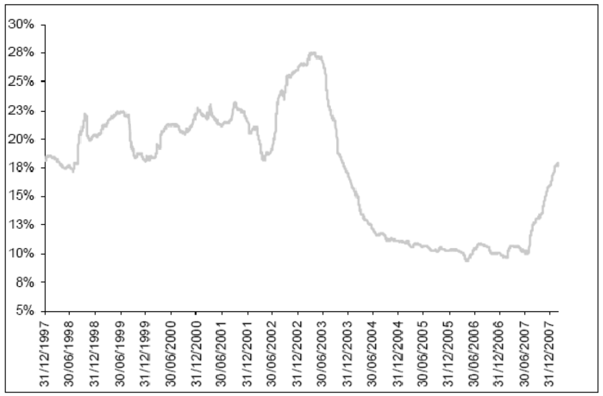
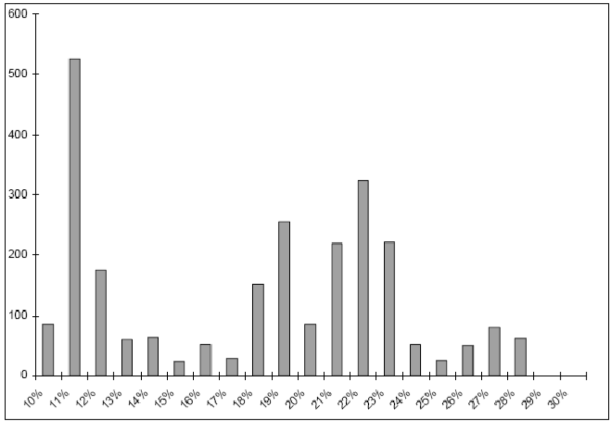
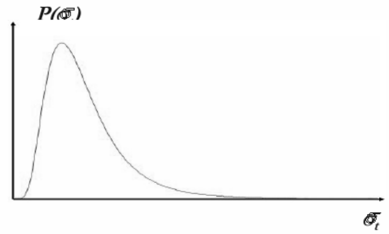
Plot of the Time Series of the Annualized Volatility



Probability Distribution of the Annualized Volatility



Volatility is
a *Random Variable*



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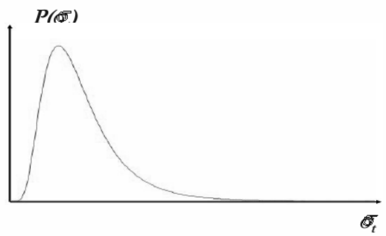
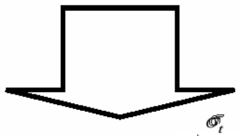
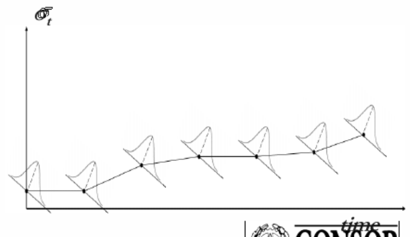




The Time Series of σ_t is
a *Stochastic Process*



Syllabus

- The GARCH Diffusive Approach
  - Intuition
  - The Convergence Theorem on R^2
    - The Statement
    - The Conditions
  - The Diffusion Limit of the M-GARCH(1,1)
    - The Statement
    - The Proof
  - The Predictive Interval for the Volatility
    - The Properties of the Stochastic Differential Equation for the M-GARCH(1,1)
    - The Estimation of the Parameters of the Stochastic Differential Equation
    - Determination of the Interval
  - Other GARCH Models

Need for Volatility Forecasts based on Stochastic Volatility Models

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TIME SERIES ANALYSIS OF VOLATILITY

MODELLING THE TIME SERIES OF VOLATILITY THROUGH THE DIFFUSION LIMIT OF GARCH PROCESSES

MODELLING THE TIME SERIES OF VOLATILITY THROUGH THE DIFFUSION LIMIT OF GARCH PROCESSES

*from: **STOCHASTIC DIFFERENCE EQUATIONS***

*to: **STOCHASTIC DIFFERENTIAL EQUATIONS***

*via: **SHRINKING** of the **TIME INTERVALS***

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The Convergence Theorem on R^2: The Statement

The sequence $\{X_t^h\}$ whose measurable space is $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$, converges weakly for h ↓0 to the process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x,t)dt + \sigma(x,t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4, presented below, are satisfied.

The Convergence Theorem on R^2: The Statement

More explicitly, the sequence $\{X_t^h\}$, composed by the sequences $\{X_{1,t}^h\}$ and $\{X_{2,t}^h\}$, each of them measurable on the space $(\mathbb{R}^1, \mathbb{B}(\mathbb{R}^1))$, converges weakly for h ↓0 to the following system of stochastic differential equations:

$$\begin{aligned} dX_{1,t} &= b(x_1,t)dt + \sigma(x_1,t)dW_t \\ dX_{2,t} &= b(x_2,t)dt + \sigma(x_2,t)dW_t^* \end{aligned}$$

where W_t and W_t^* are two independent uni-dimensional standard Brownian motions, and $X_{1,t}$ and $X_{2,t}$ are two independent processes which take values on $\mathbb{R}^1$, if the conditions 1-4, presented below, are satisfied.

The Convergence Theorem on R^2: The Statement

The process $\{X_t\}$ has a distribution independent on the choice of $\sigma(x,t)$ and it takes finite values over finite time intervals, i.e. $\forall T > 0$:

$$P(\sup_{0 \le t \le T} \|X_t\| < \infty) = 1$$

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CONDITION 1

If there exists a δ>0 such that:

$$\lim_{h\downarrow 0}\begin{pmatrix} c_{h,\delta}(x_1,t) \\ c_{h,\delta}(x_2,t) \end{pmatrix} = 0$$

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Then there exist $a(x,t)$ and $b(x,t)$, continuous measures respectively mapping from $\mathbb{R}^2\times[0,\infty)$ into the space of the *2x2* semi-definite positive matrices, and from $\mathbb{R}^2\times[0,\infty)$ into $\mathbb{R}^2$, such that:

$$\lim_{h\downarrow 0}\begin{pmatrix} b_h(x_1,t) \\ b_h(x_2,t) \end{pmatrix} = \begin{pmatrix} b(x_1,t) \\ b(x_2,t) \end{pmatrix}$$

$$\lim_{h\downarrow 0}\begin{pmatrix} a_h(x_1,t) & a_h((x_1,x_2),t) \\ a_h((x_2,x_1),t) & a_h(x_2,t) \end{pmatrix} = \begin{pmatrix} a(x_1,t) & 0 \\ 0 & a(x_2,t) \end{pmatrix}$$

CONDITION 2

There exists $\sigma(x,t)$, a continuous mapping from $\mathbb{R}^2\times[0,\infty)$ into $\mathbb{R}^2$, such that, $\forall x_1\in\mathbb{R}^1, \forall x_2\in\mathbb{R}^1$, it holds:

$$\begin{pmatrix} \sigma(x_1,t) & 0 \\ 0 & \sigma(x_2,t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1,t)} & 0 \\ 0 & \sqrt{a(x_2,t)} \end{pmatrix}$$

CONDITION 3

For h ↓ 0, X_0^h converges in distribution to a random variable X_0 with probability measure v_0 on $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$

CONDITION 4

v_0, $a(x,t)$ and $b(x,t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution v_0, a conditional second moment $a(x,t)$ and a conditional first moment $b(x,t)$

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Given the equation of the conditional variance* in the M-GARCH(1,1):

$$\ln\sigma_{k+1}^2 - \ln\sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1)\ln\sigma_k^2 + \beta_1^{(k)}\ln Z_k^2$$

or, equivalently:

$$\ln\sigma_{k+1}^2 - \ln\sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1)\ln\sigma_k^2 + 2\beta_1^{(k)}\ln|Z_k|$$

Z_k is i.i.d. *N(0,1)*

*The focus is on the difference equation for the volatility. For the convergence of the first equation see Minenna 2003.

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Z_k is i.i.d. *N(0,1)*

its diffusion limit is:

$$d\ln\sigma_t^2 = \left[\beta_0 + 2\beta_1\mathbf{E}(\ln|Z_t|) + (\beta_1 - 1)\ln\sigma_t^2\right]dt + 2|\beta_1|\sqrt{Var(\ln|Z_t|)}dW_t^*$$

Z_t is *N(0,1)*

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STEP 1:
THE RE-SCALING OF THE PROCESS

The *k* intervals are divided into *1/h* subintervals each one of length *h*

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The *k* intervals are divided into *1/h* subintervals each one of length *h*

$$\ln \sigma^2_{(k+1)h} - \ln \sigma^2_{kh}$$

=

$$\beta_{0h} + (\beta_{1h} - h)\ln \sigma^2_{kh} + 2\beta_{1h}\left\{\sqrt{h}\left[\ln|Z_k| - E\left(\ln|Z_k|\right)\right] + E\left(\ln|Z_k|\right)\right\}$$

STEP 2:
THE CONSTRUCTION OF THE PROCESS $\left\{\ln \sigma_t^{2^h}\right\}$

Definition of the probability measure P_h on the Skorokhod Space D such that:

$$P_h(\ln \sigma_0^{2^h} \in \Gamma) = v_0(\Gamma) \qquad \forall \Gamma \in \mathbb{B}(\mathbb{R}^1)$$

$$P_h(\ln \sigma_t^{2^h} = \ln \sigma^2_{kh}, \quad \forall\ kh \le t < (k+1)h) = 1$$

$$P_h(\ln \sigma^2_{(k+1)h} \in \Gamma | \widehat{\Im}_{kh}) = \Pi_{h,kh}(\ln \sigma^2_{kh}, \Gamma) \text{a.s. under } P_h,\ \forall k \ge 0,\ \forall \Gamma \in \mathbb{B}(\mathbb{R}^1)$$

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$$P_h(\ln \sigma_t^{2^h} = \ln \sigma_{kh}^2, \quad \forall\, kh \le t < (k+1)h) = 1$$

$$P_h(\ln \sigma_{(k+1)h}^2 \in \Gamma | \widehat{\Im}_{kh}) = \Pi_{h,kh}(\ln \sigma_{kh}^2, \Gamma) \text{ a.s. under } P_h,\ \forall k \ge 0,\ \forall \Gamma \in \mathbb{B}(\mathbb{R}^1)$$

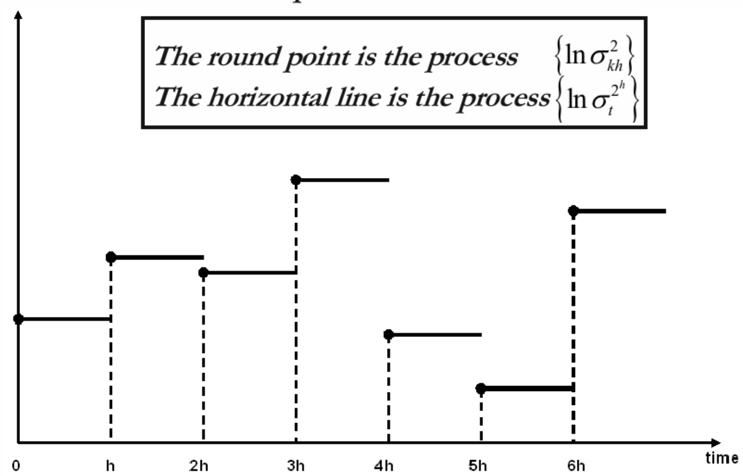
⇓

$$\ln \sigma_{t+1}^{2^h} - \ln \sigma_t^{2^h}$$

$$=$$

$$\beta_{0h} + (\beta_{1h} - h) \ln \sigma_t^{2^h} + 2\beta_{1h} \left\{ \sqrt{h} \left[\ln |Z_t^h| - E\left(\ln |Z_t^h|\right)\right] + E\left(\ln |Z_t^h|\right) \right\}$$

A qualitative idea



STEP 3:

CHECK OF CONDITION 1 OF THE CONVERGENCE THEOREM

Finding the values of β_{0h} and β_{1h} which guarantee

the convergence of the conditional moments

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CHECK OF CONDITION 1 OF THE CONVERGENCE THEOREM

Finding the values of β_{0h} and β_{1h} which guarantee

the convergence of the conditional moments

$$\left.\begin{array}{l} \beta_{0h} := \beta_0 \cdot h \\ \beta_{1h} := \beta_1 \cdot h \end{array}\right\} \Longleftrightarrow \begin{cases} \lim_{h \downarrow 0} c_{h,\delta=1}(\widehat{\ln \sigma^2}, t) = 0 \\ \lim_{h \downarrow 0} b_h(\widehat{\ln \sigma^2}, t) = (\beta_0 + 2\beta_1 \mathbf{E}(\ln|Z_t|) + (\beta_1 - 1)\ln \sigma_t^2) \\ \lim_{h \downarrow 0} a_h(\widehat{\ln \sigma^2}, t) = 4\beta_1^2 Var(\ln|Z_t|) \end{cases}$$

STEP 4:
CHECK OF CONDITIONS 2, 3 and 4 OF THE CONVERGENCE THEOREM

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CHECK OF CONDITIONS 2, 3 and 4 OF THE CONVERGENCE THEOREM

- Condition 2 is verified for every σ>0, i.e.:

$$\sigma(\widehat{\ln \sigma^2}, t) = 2\,|\beta_1|\,\sqrt{Var(\ln|Z_t|)}$$

- Condition 3 is evidently satisfied by construction of the process $\left\{\ln \sigma_t^{2^h}\right\}$
- Consequently, Condition 4 is verified too.

Q.E.D.

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KEY POINT

From the Diffusion Limit of the GARCH Process it is possible to establish a *Predictive Interval for σ_t*

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$$d\ln\sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}\left(\ln|Z_t|\right) + (\beta_1 - 1)\ln\sigma_t^2\right]dt + 2\,|\beta_1|\sqrt{Var(\ln|Z_t|)}dW_t^*$$

$$d\ln\sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}\left(\ln|Z_t|\right) + (\beta_1 - 1)\ln\sigma_t^2\right]dt + 2\,|\beta_1|\sqrt{Var(\ln|Z_t|)}dW_t^*$$

⇓

$$\ln\sigma_t^2 \sim N\left[\left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)};\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}\right]$$

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The relationship between the Stochastic Difference Equation and the Stochastic Differential Equation

$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k|$$

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$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k|$$

$$d \ln \sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}\left(\ln |Z_t|\right) + (\beta_1 - 1) \ln \sigma_t^2\right] dt + 2\left|\beta_1\right| \sqrt{Var(\ln |Z_t|)} dW_t^*$$

Matching of the first two Conditional Moments

Matching of the first two Conditional Moments

$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2$$

$$=$$

$$\left(e^{(\beta_1 - 1)} - 1\right)\left(\frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}\right) + \left(e^{(\beta_1 - 1)} - 1\right) \ln \sigma_k^2 + 2\left(e^{(\beta_1 - 1)} - 1\right) \ln |Z_k|$$

The Maximum Likelihood Method

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$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \widehat{a} + \widehat{b} \ln \sigma_k^2 + e_k$$

The Maximum Likelihood Method

$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \widehat{a} + \widehat{b} \ln \sigma_k^2 + e_k$$

$$\beta_0 = f_1\left(\widehat{a}, \widehat{b}\right)$$

$$\beta_1 = f_2\left(\widehat{a}, \widehat{b}\right)$$

$$2\left|\beta_1\right| \sqrt{Var\left(\ln\left|Z_t\right|\right)} = f_3\left(\widehat{a}, \widehat{b}, e_k\right)$$

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The Predictive Interval for the Volatility: Determination of the Interval

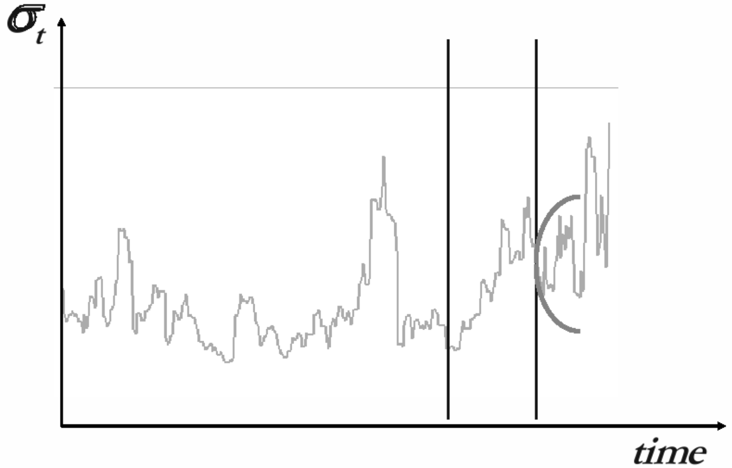
$$P\left(\begin{array}{c} -z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)} \\ \leq \ln\sigma_t^2 \leq \\ z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)} \end{array}\right)=\alpha$$

The Predictive Interval for the Volatility: Determination of the Interval

$$P\left(\begin{array}{c} -z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)} \\ \leq \ln\sigma_t^2 \leq \\ z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)} \end{array}\right)=\alpha$$

$$\left[\sigma_{t,\min}^G,\sigma_{t,\max}^G\right] = \left[e^{\frac{-z_{\frac{\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}};\; e^{\frac{z_{\frac{\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}}\right]$$

The Predictive Interval for the Volatility: Determination of the Interval



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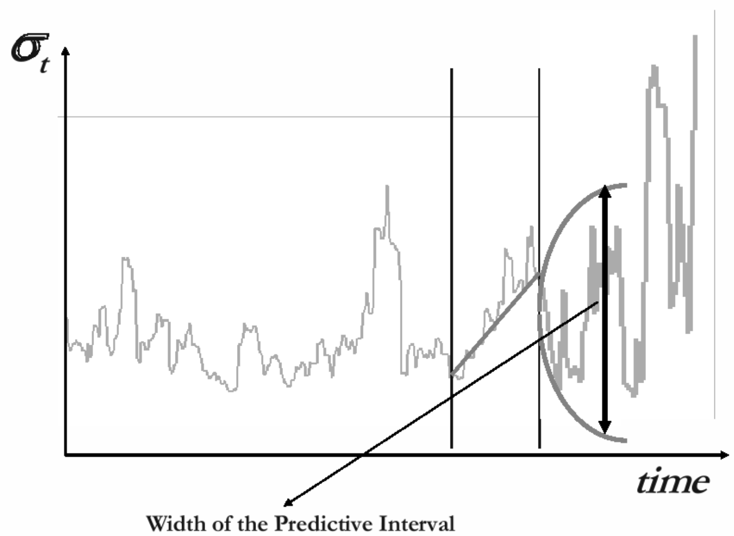

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Analogous Procedure

THE DIFFUSION LIMIT OF THE L-GARCH(1,1)

Given the L-GARCH(1,1) model:

$$\sigma_{k+1}^2 - \sigma_k^2 = \omega + \sigma_k^2(\beta + \varpi Z_k^2 - 1)$$

Z_k is $N(0,1)$

its diffusion limit is:

$$d\sigma_t^2 = [\omega + \vartheta\sigma_t^2]dt + \sqrt{2\varpi}\sigma_t^2 dW_t$$

Analogous Procedure

THE DIFFUSION LIMIT OF THE L-GARCH(1,1)

Given the L-GARCH(1,1) model:

$$\sigma_{k+1}^2 - \sigma_k^2 = \omega + \sigma_k^2(\beta + \varpi Z_k^2 - 1)$$

Z_k is $N(0,1)$

its diffusion limit is:

$$d\sigma_t^2 = [\omega + \vartheta\sigma_t^2]dt + \sqrt{2\varpi}\sigma_t^2 dW_t$$

THE DIFFUSION LIMIT OF THE E-GARCH(1,1)

Given the E-GARCH(1,1) model:

$$\ln\sigma_{k+1}^2 - \ln\sigma_k^2 = \beta_0^{(k)} + \beta_1^{(k)}\ln\sigma_k^2 + \beta_2^{(k)}\left(|Z_k| + \vartheta Z_k\right)$$

Z_k is $N(0,1)$

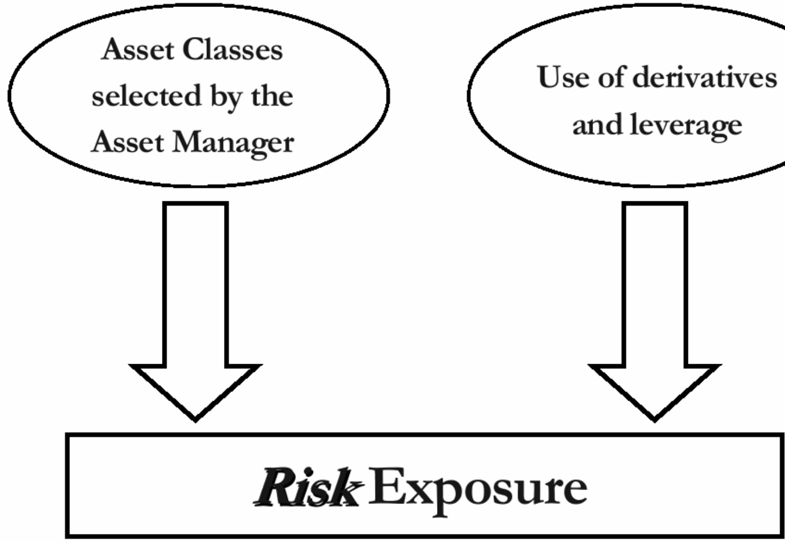
its diffusion limit is:

$$d\ln\sigma_t^2 = \left[\alpha_0 + \frac{2}{\sqrt{2\pi}}\left(\alpha_4 + \frac{\alpha_5}{2}\right) - \alpha_4 - \alpha_5 - \alpha_1 - 1 + (\alpha_1 - 1)\ln\sigma_t^2\right]dt - \frac{\alpha_5}{2}dW_t + \left|\alpha_4 + \frac{\alpha_5}{2}\right|\sqrt{\frac{\pi-2}{\pi}}dW_t^*$$

Syllabus

- **Application to Flexible Funds Risk Assessment**
  - Key Concepts on Flexible Funds
  - Transparency Regulation on the Risk Profile
  - The Perspective of the Asset Manager
  - Quantitative Methodology for Risk Measurement
  - The Solution for the Asset Manager
  - Migration and Prospectus

DEFINITION	Freedom to invest in any market and in any financial instrument and to take leveraged positions
OBJECTIVE	Maximization of the expected return for a given level of risk



Syllabus

- Application to Flexible Funds Risk Assessment
  - Key Concepts on Flexible Funds
  - Transparency Regulation on the Risk Profile
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  - The Solution for the Asset Manager
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QUALITATIVE RISK CLASSES
Low
Medium-Low
Medium
Medium-High
High
Very High

Fundamental Assumption

The quantitative risk assessment

is based on

VOLATILITY MEASURES

Variation of the Volatility over time

Exposure to the ***Migration Risk***

Variation of the Volatility over time

Exposure to the ***Migration Risk***

QUALITATIVE RISK CLASSES

Low
Medium-Low
Medium
Medium-High
High
Very High

Assessment and Delimitation of the *Migration Risk*

Assessment and Delimitation of the *Migration Risk*

Regulatory framework consistent with the markets evolution and the activity of the Asset Manager

Syllabus

- Application to Flexible Funds Risk Assessment
  - Key Concepts on Flexible Funds
  - Transparency Regulation on the Risk Profile
  - The Perspective of the Asset Manager
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  - The Solution for the Asset Manager
  - Migration and Prospectus

Evaluation and Management of the *Migration Risk*

Evaluation and Management of the *Migration Risk*

Position inside the Risk Class declared in the Prospectus

Syllabus

- Application to Flexible Funds Risk Assessment
  - Key Concepts on Flexible Funds
  - Transparency Regulation on the Risk Profile
  - The Perspective of the Asset Manager
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Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Step 1: Definition of the Loss Intervals of the Fund

What is the Loss in a Financial Investment?

Step 1: Definition of the Loss Intervals of the Fund

What is the Loss in a Financial Investment?

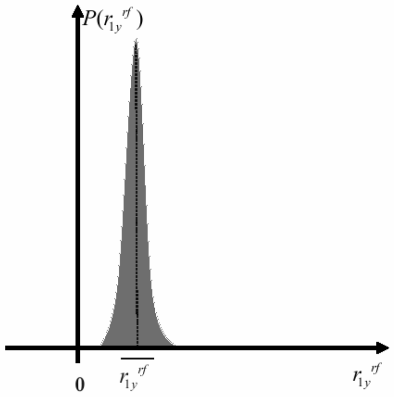
RISK-NEUTRALITY PRINCIPLE

$$\text{LOSS} \in (-100\%, \overline{r^{rf}}\,]$$

where: $\overline{r^{rf}}$ = average of the Probability Distribution of the risk-free rate

Step 1: Definition of the Loss Intervals of the Fund

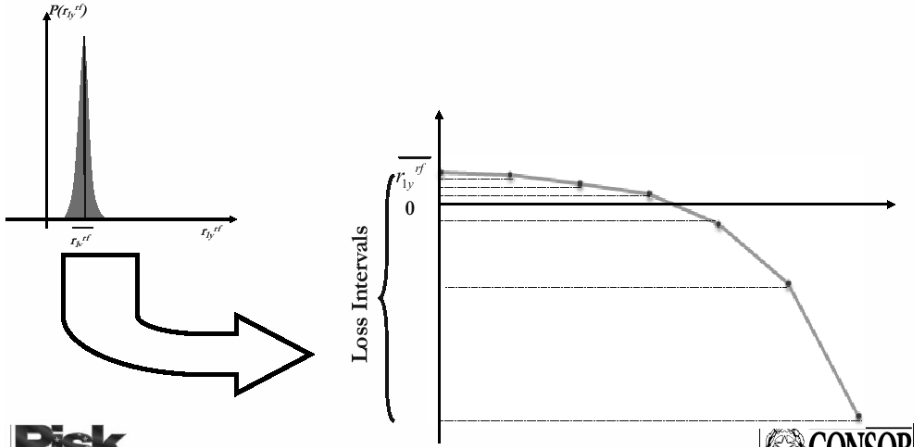
After having selected the Probability Distribution of the 1-year risk-free rate …



$\overline{r_{1y}^{rf}}$ = average of the Probability Distribution of the 1-year risk-free rate

Step 1: Definition of the Loss Intervals of the Fund

… to each Qualitative Risk Class is associated the corresponding annual Loss Interval (multiple of r_{1y}^{rf} according to an exponential function) …



Step 1: Definition of the Loss Intervals of the Fund

... obtaining six initial loss intervals:

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

Step 2: Mapping of the Loss Intervals of the Fund to the corresponding Volatility Intervals of the Fund

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
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Step 2: Mapping of the Loss Intervals of the Fund to the corresponding Volatility Intervals of the Fund

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medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

$$dR_t = q(\mu - R_t)dt + \sigma dW_t$$

$$dR_t = \mu R_t dt + \sigma R_t dW_t$$

....

Step 2: Mapping of the Loss Intervals of the Fund to the corresponding Volatility Intervals of the Fund

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

$$dR_t = q(\mu - R_t)dt + \sigma dW_t$$

$$dR_t = \mu R_t dt + \sigma R_t dW_t$$

....

Risk Classes	Volatility Intervals σ_{min}	σ_{max}
low	${}_0\sigma_{1,min}$	${}_0\sigma_{1,max}$
medium-low	${}_0\sigma_{2,min}$	${}_0\sigma_{2,max}$
medium	${}_0\sigma_{3,min}$	${}_0\sigma_{3,max}$
medium -high	${}_0\sigma_{4,min}$	${}_0\sigma_{4,max}$
high	${}_0\sigma_{5,min}$	${}_0\sigma_{5,max}$
very high	${}_0\sigma_{6,min}$	${}_0\sigma_{6,max}$

*The subscript *0* preceding the volatility indicates that this is the initial interval, i.e. before the calibration

Step 3: Calibration of the Intervals

REQUIREMENTS

- *Ability to express in a robust and significant way the risk level "typical" of the corresponding Qualitative Class*
- *Stability over time also when the yield curve changes significantly*

Step 3: Calibration of the Intervals

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TOOLS

- *GARCH Diffusive Models*
- *Stochastic Non-Linear Programming*

Step 3: Calibration of the Intervals

REQUIREMENTS

- *Ability to express in a robust and significant way the risk level "typical" of the corresponding Qualitative Class*
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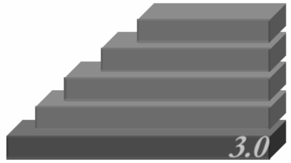
TOOLS

- *GARCH Diffusive Models*
- *Stochastic Non-Linear Programming*

Fine-tuning Intervention on the Volatility Intervals

Step 3: Calibration of the Intervals

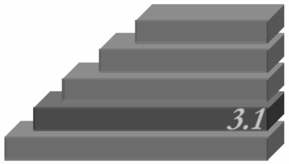


Selection of an initial Volatility Interval

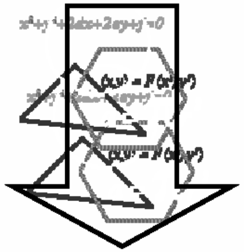
Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	$_0\sigma_{1,min}$	$_0\sigma_{1,max}$
medium-low	$_0\sigma_{2,min}$	$_0\sigma_{2,max}$
medium	$_0\sigma_{3,min}$	$_0\sigma_{3,max}$
medium-high	$_0\sigma_{4,min}$	$_0\sigma_{4,max}$
high	$_0\sigma_{5,min}$	$_0\sigma_{5,max}$
very high	$_0\sigma_{6,min}$	$_0\sigma_{6,max}$

$[_0\sigma_{4,min} \quad _0\sigma_{4,max}]$

Step 3: Calibration of the Intervals

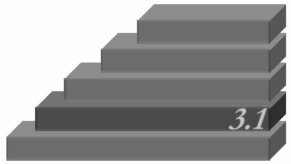


Simulation of the Fund pattern



NAV **Stochastic Differential Equation**

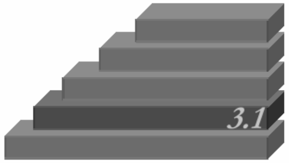
Step 3: Calibration of the Intervals



Simulation of the Fund pattern

NAV **S.D.E.** ⇒ **What Parameters?**

Step 3: Calibration of the Intervals



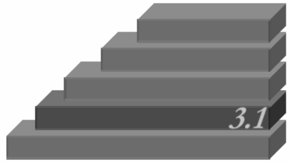
Simulation of the Fund pattern

NAV **S.D.E.** ⇒ **What Parameters?**

The Drift

Application to Flexible Funds Risk Assessment: Quantitative Methodology for Risk Measurement

Step 3: Calibration of the Intervals



Simulation of the Fund pattern

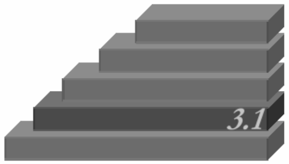
Risk-Neutrality Principle

⇓

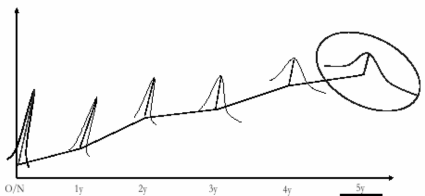
$$\text{Drift} = r^{rf}$$

Application to Flexible Funds Risk Assessment: Quantitative Methodology for Risk Measurement

Step 3: Calibration of the Intervals



Simulation of the Fund pattern



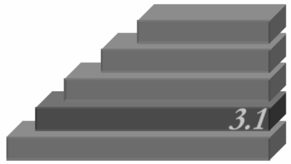
Robustness of the Volatility Intervals

⇓

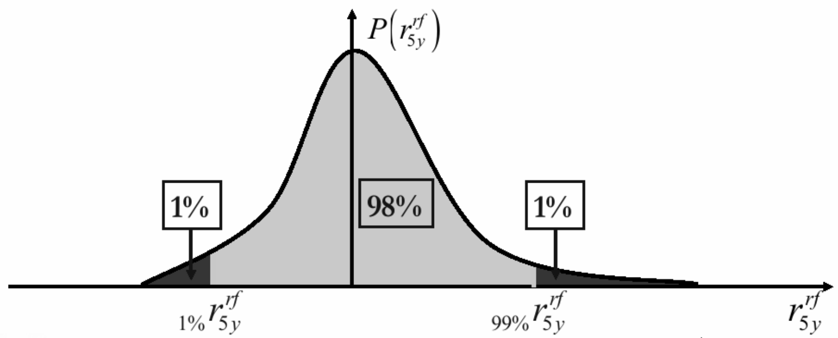
$$\text{Drift} = r_{5y}^{rf}$$

Application to Flexible Funds Risk Assessment: Quantitative Methodology for Risk Measurement

Step 3: Calibration of the Intervals

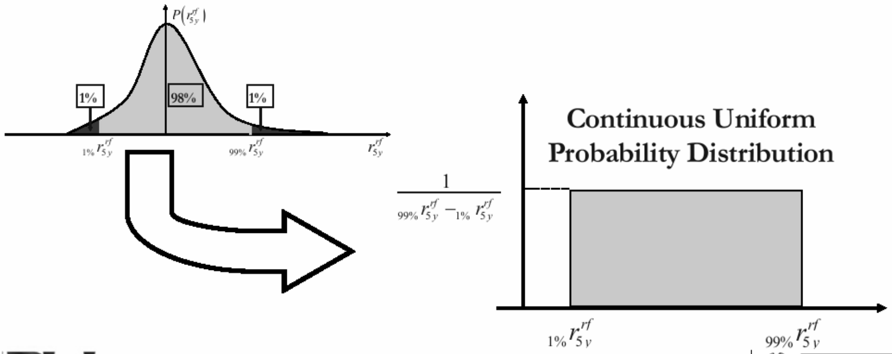


Simulation of the Fund pattern



Step 3: Calibration of the Intervals

3.1 *Simulation of the Fund pattern*



Step 3: Calibration of the Intervals

3.1 *Simulation of the Fund pattern*

***NAV* S.D.E.** ⇒ **What Parameters?**

The Diffusion

Step 3: Calibration of the Intervals

3.1 *Simulation of the Fund pattern*

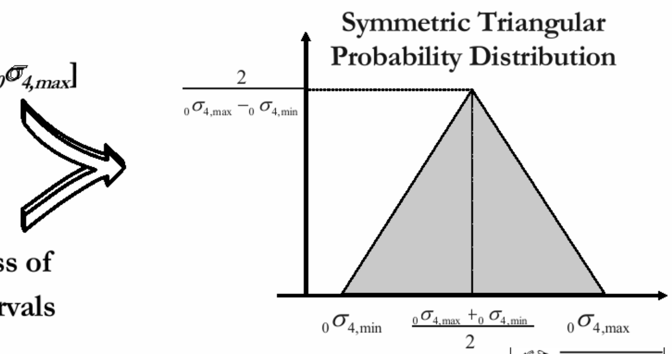
Initial Volatility Interval: $[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Representativeness of the Volatility Intervals

Step 3: Calibration of the Intervals

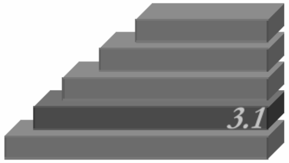
3.1 *Simulation of the Fund pattern*

Initial Volatility Interval: $[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

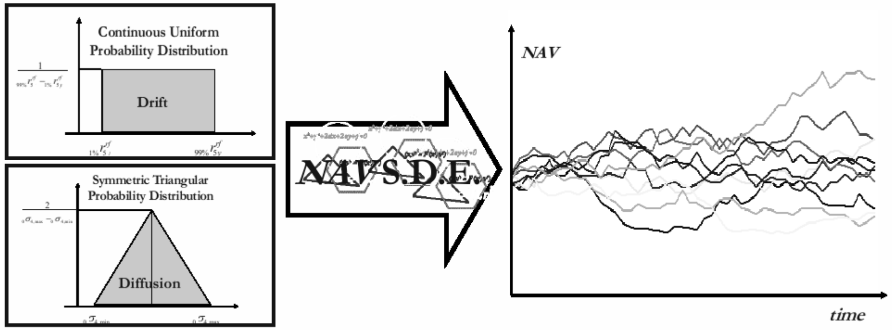


Representativeness of the Volatility Intervals

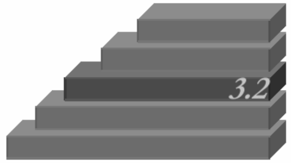
Step 3: Calibration of the Intervals



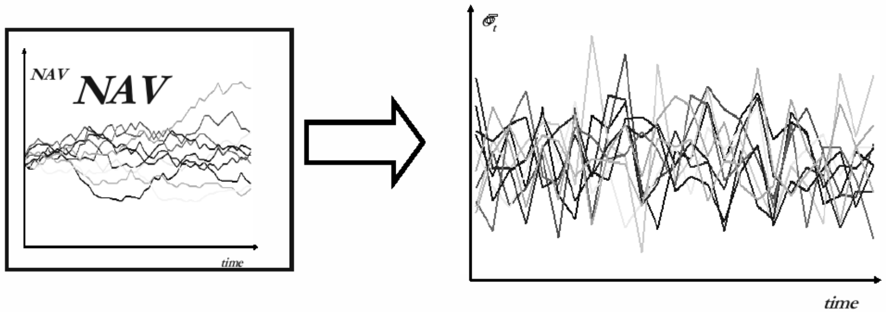
Simulation of the Fund pattern



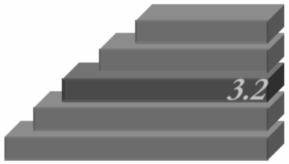
Step 3: Calibration of the Intervals



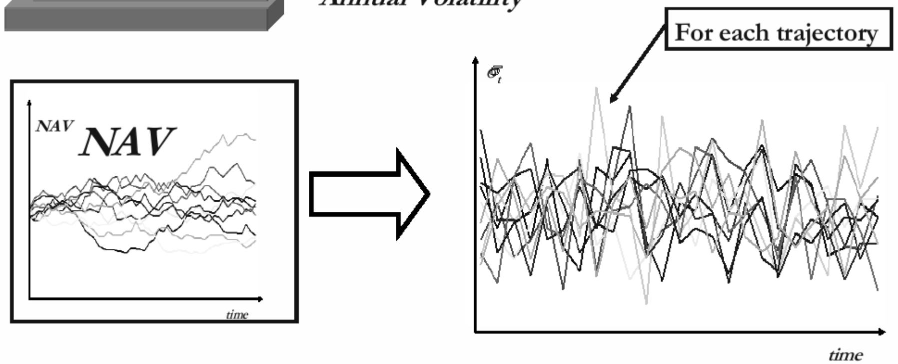
Determination of the Time Series of the Annual Volatility



Step 3: Calibration of the Intervals

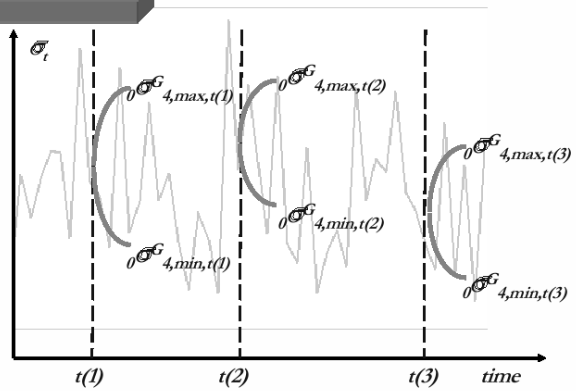


Determination of the Time Series of the Annual Volatility



Step 3: Calibration of the Intervals

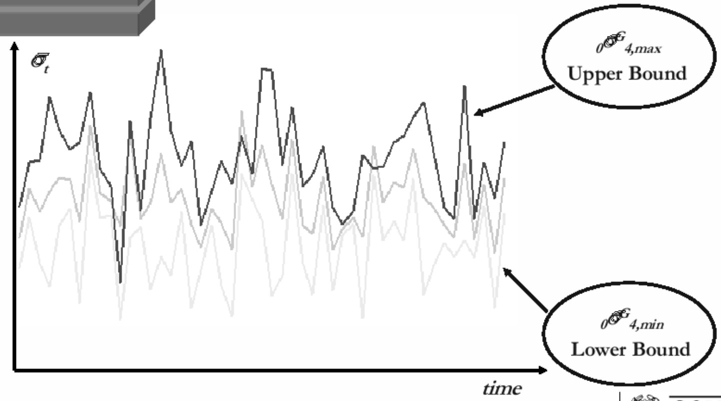
Volatility Forecast Band through GARCH Diffusive Models



Step 3: Calibration of the Intervals

3.3

Volatility Forecast Band through GARCH Diffusive Models



Step 3: Calibration of the Intervals

3.4

Validation of the initial Volatility Interval

$[_0\sigma_{4,min} \quad _0\sigma_{4,max}]$

VS

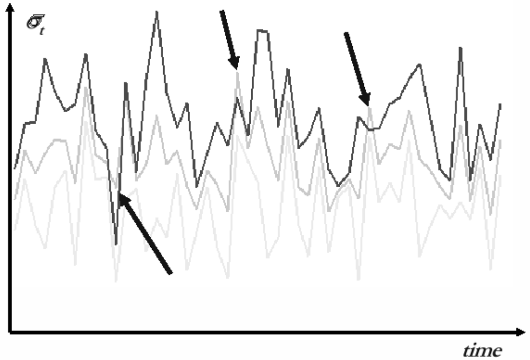
$[_0\sigma^G_{4,min} \quad _0\sigma^G_{4,max}]$

Step 3: Calibration of the Intervals

3.4

Validation of the initial Volatility Interval

Calculus of the number of observations outside the Band



Step 3: Calibration of the Intervals

3.4

Validation of the initial Volatility Interval

Calculus of the number of observations outside the Band

Trajectory	n. obs. $\in$ $[_0\sigma_{4,min} \quad _0\sigma_{4,max}]$	n. obs. < $_0\sigma_{4,min}$	n. obs. > $_0\sigma_{4,max}$
1			
2			
...			
n			
	Tot $\in$ $[_0\sigma_{4,min} \quad _0\sigma_{4,max}]$	Tot. < $_0\sigma_{4,min}$	Tot. > $_0\sigma_{4,max}$

Hp.: n. of observations of σ_t = 250 for each trajectory

Step 3: Calibration of the Intervals

3.4

Validation of the initial Volatility Interval

Number of observations outside the Band in 1 year

$$\Delta = \frac{[Tot. > {}_0\sigma_{4,max}] + [Tot. < {}_0\sigma_{4,min}]}{n*250}$$

$$\Delta_{up} = \frac{[Tot. > {}_0\sigma_{4,max}]}{n*250} \qquad \Delta_{down} = \frac{[Tot. < {}_0\sigma_{4,min}]}{n*250}$$

Step 3: Calibration of the Intervals

3.4.1

Update of the initial Volatility Interval
Iteration of the Procedure

$\Delta > 5\%$ and $\Delta_{up} > \Delta_{down}$ ⇒ $[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$ ⇒ 3.1

Step 3: Calibration of the Intervals

3.4.2

Update of the initial Volatility Interval
Iteration of the Procedure

$\Delta > 5\%$ and $\Delta_{up} < \Delta_{down}$ ⇒ $[{}_1\sigma_{4,min} \quad {}_0\sigma_{4,max}]$ ⇒ 3.1

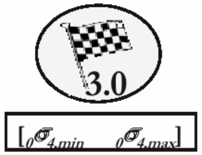
Step 3: Calibration of the Intervals

3.4.3

End of the Procedure

$\Delta \leq 5\%$ ⇒ $[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}] = [\sigma_{4,min} \quad \sigma_{4,max}]$

Step 3: Calibration of the Intervals



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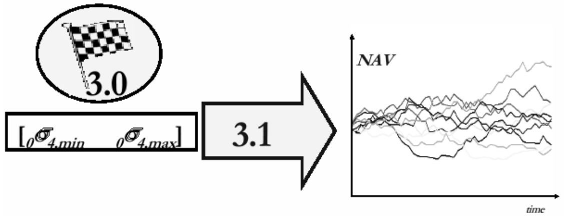

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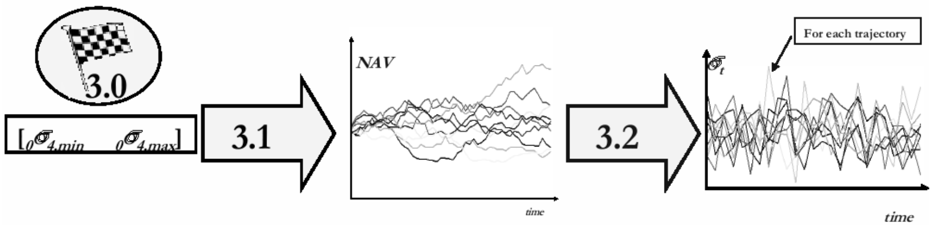

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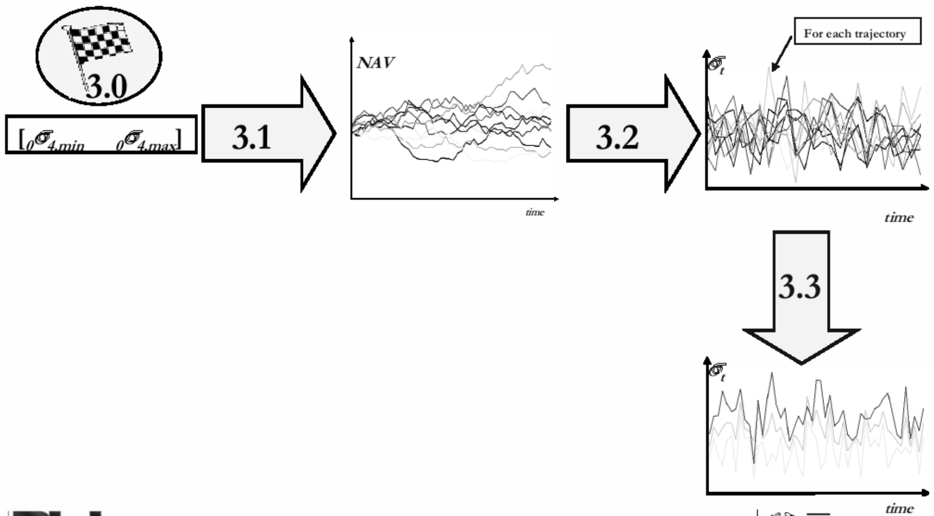

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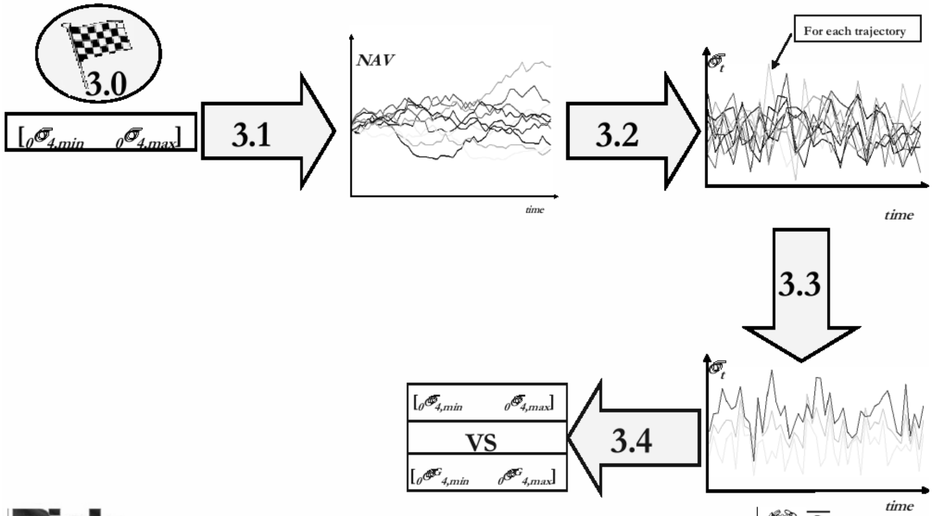
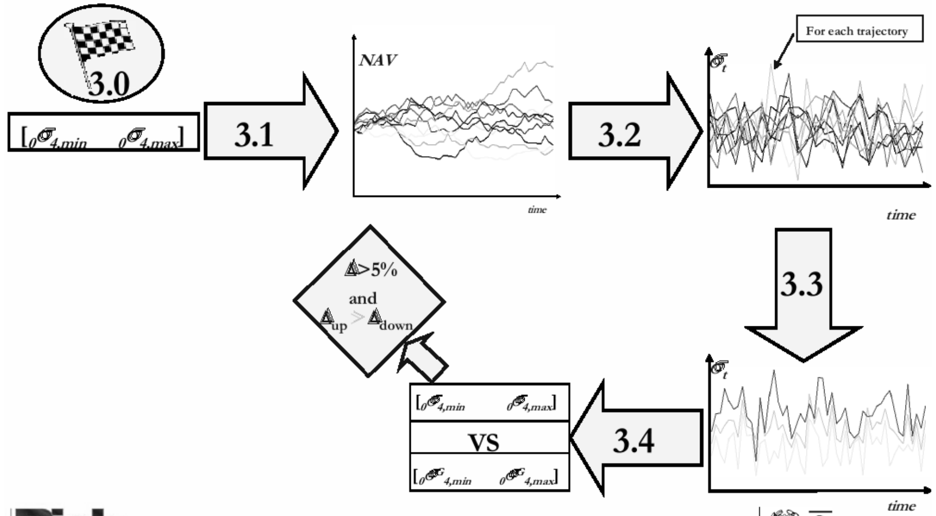


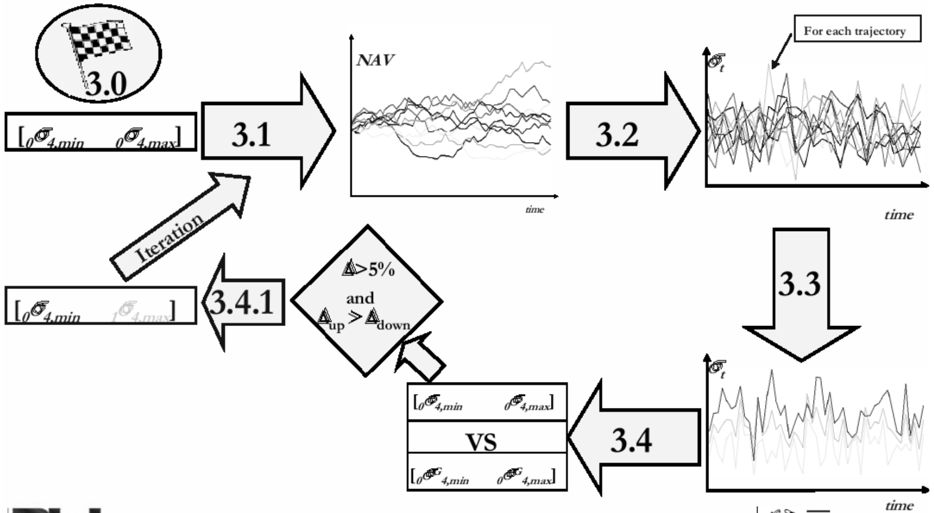
Application to Flexible Funds Risk Assessment: Quantitative Methodology for Risk Measurement

Step 3: Calibration of the Intervals



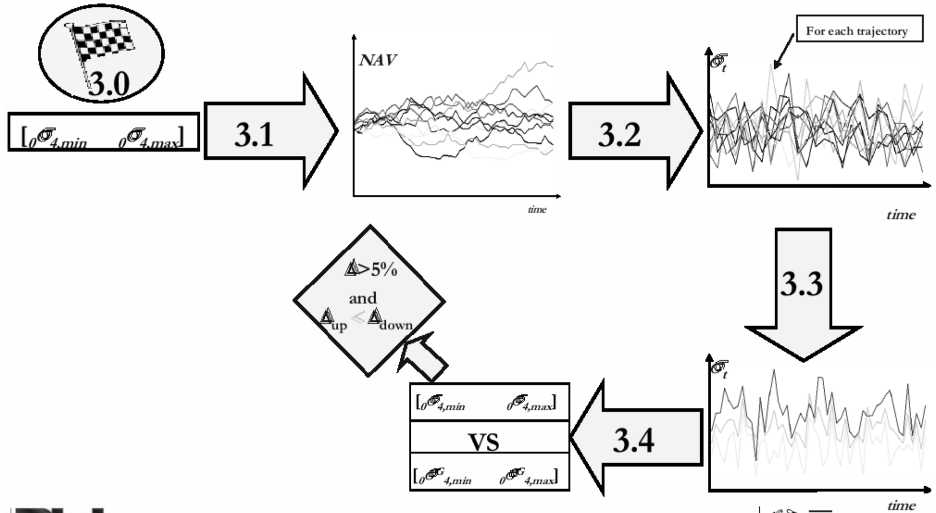
Application to Flexible Funds Risk Assessment: Quantitative Methodology for Risk Measurement

Step 3: Calibration of the Intervals

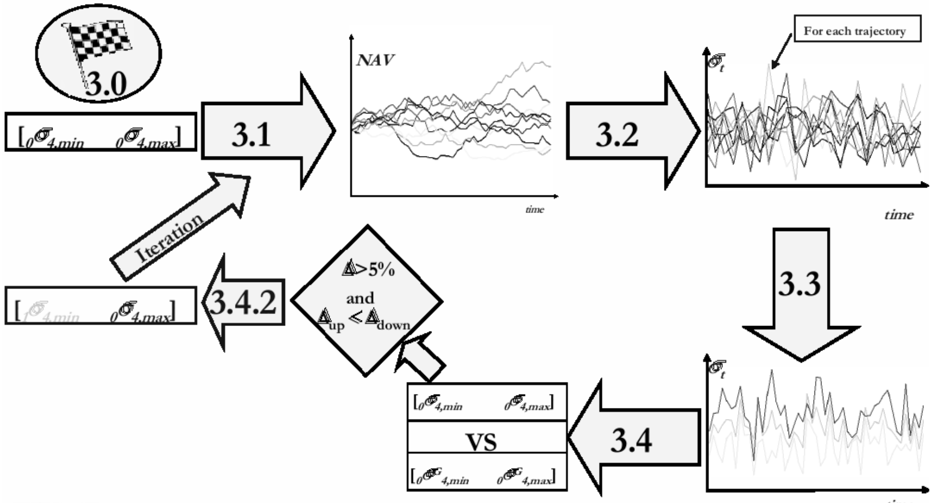


Application to Flexible Funds Risk Assessment: Quantitative Methodology for Risk Measurement

Step 3: Calibration of the Intervals



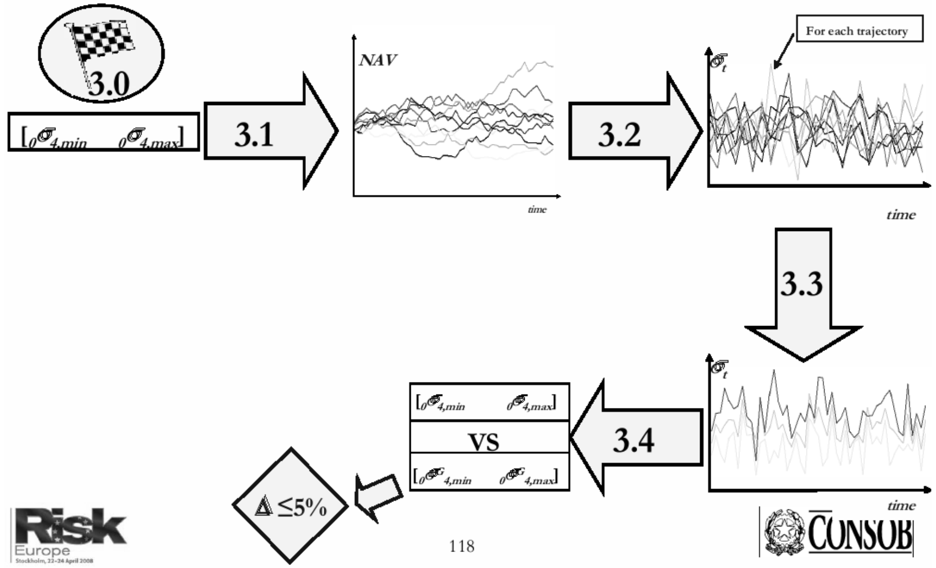
Step 3: Calibration of the Intervals



Risk Europe

CONSOB

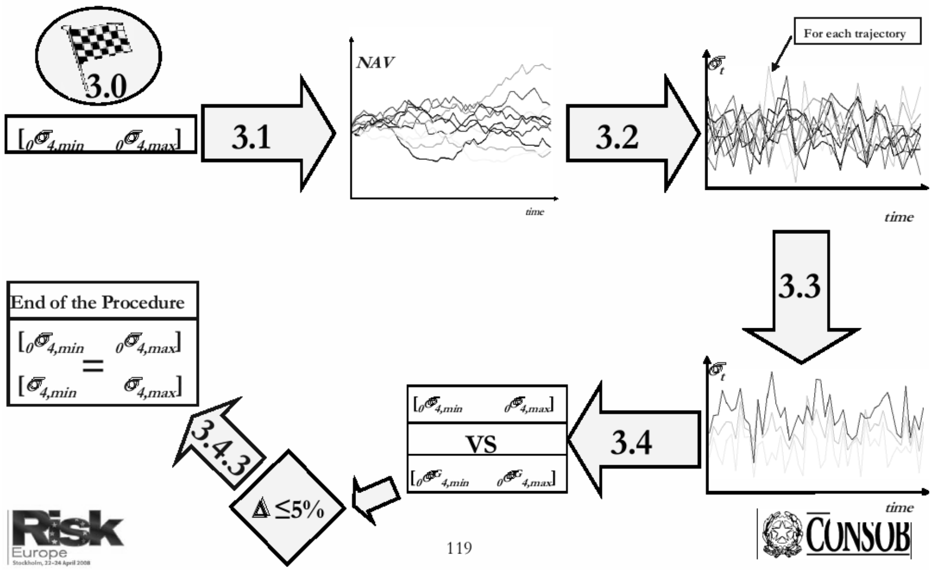
Step 3: Calibration of the Intervals



Risk Europe

CONSOB

Step 3: Calibration of the Intervals



Risk Europe

CONSOB

Step 3: Calibration of the Intervals

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	*0.01%*	*0.49%*
medium-low	*0.50%*	*1.59%*
medium	*1.60%*	*3.99%*
medium-high	*4.00%*	*9.99%*
high	*10.00%*	*24.99%*
very high	*25.00%*	*above 25.00%*

Risk Europe

CONSOB

Syllabus

- Application to Flexible Funds Risk Assessment
  - Key Concepts on Flexible Funds
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  - The Solution for the Asset Manager
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Mapping of the QualitativeRisk Classes to corresponding Volatility Intervals

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	*0.01%*	*0.49%*
medium-low	*0.50%*	*1.59%*
medium	*1.60%*	*3.99%*
medium-high	*4.00%*	*9.99%*
high	*10.00%*	*24.99%*
very high	*25.00%*	*above 25.00%*

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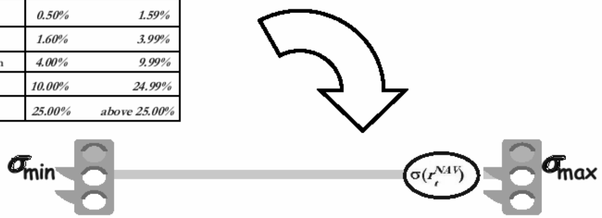

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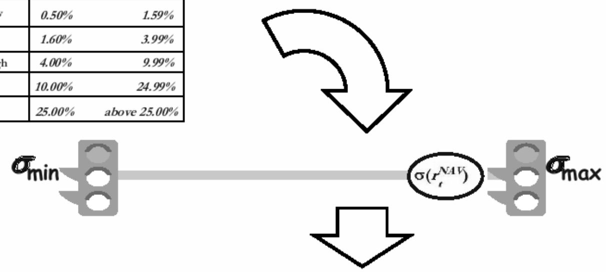

SAFE ASSETS

Mapping of the QualitativeRisk Classes to corresponding Volatility Intervals

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	*0.01%*	*0.49%*
medium-low	*0.50%*	*1.59%*
medium	*1.60%*	*3.99%*
medium-high	*4.00%*	*9.99%*
high	*10.00%*	*24.99%*
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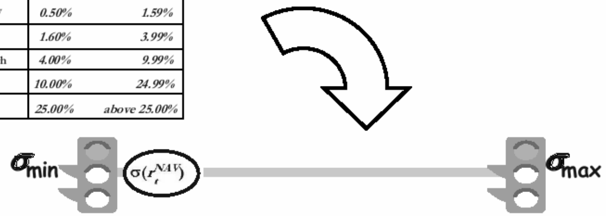

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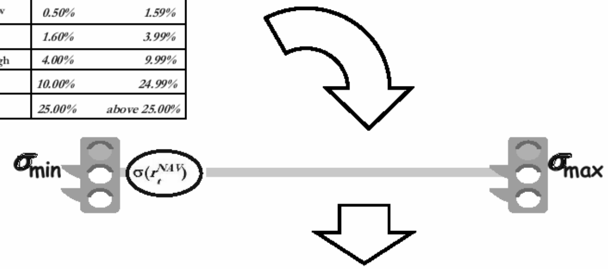


RISKY ASSETS

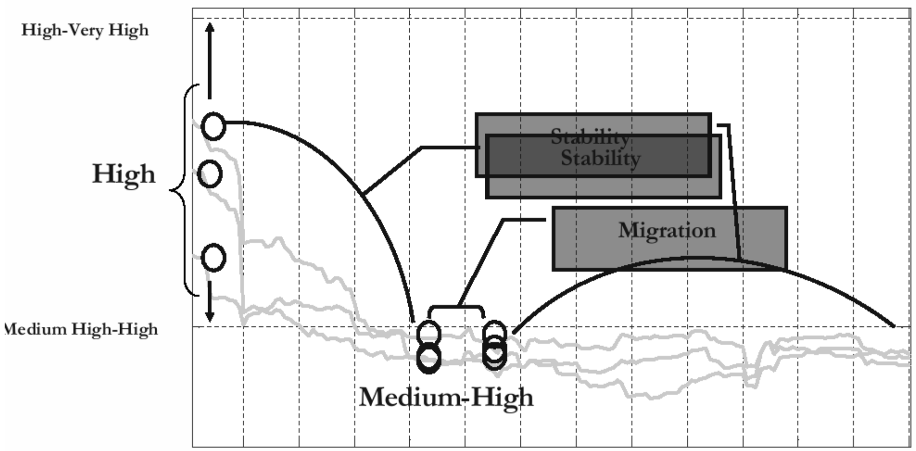
Syllabus

- Application to Flexible Funds Risk Assessment
  - Key Concepts on Flexible Funds
  - Transparency Regulation on the Risk Profile
  - The Perspective of the Asset Manager
  - Quantitative Methodology for Risk Measurement
  - The Solution for the Asset Manager
  - Migration and Prospectus

When does the Migration occur?

The Migration occurs when the fund remains for a significant period outside the qualitative class declared in the Prospectus …

Application to Flexible Funds Risk Assessment: Migration and Prospectus



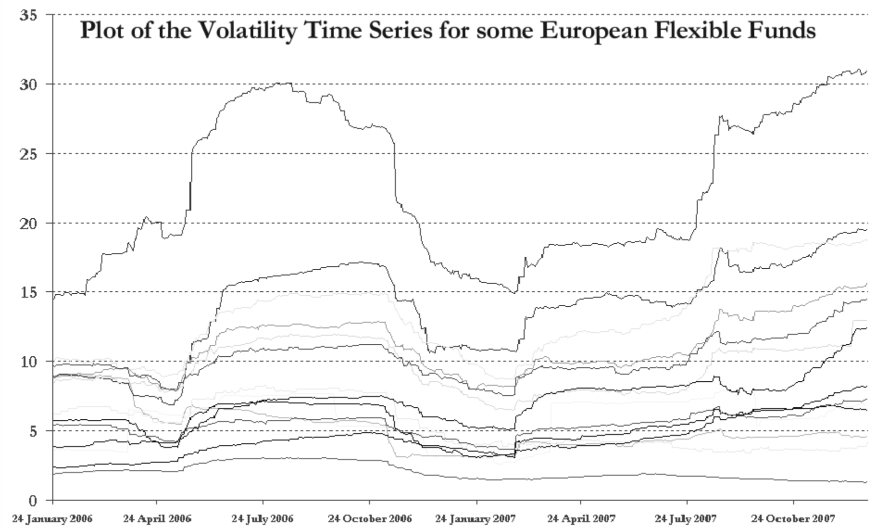
Syllabus

- Empirical Evidence on the European Industry
  - Preliminary Informations
  - The Evolution of the Risk-Profile over time

Empirical Evidence on the European Industry: Preliminary Informations

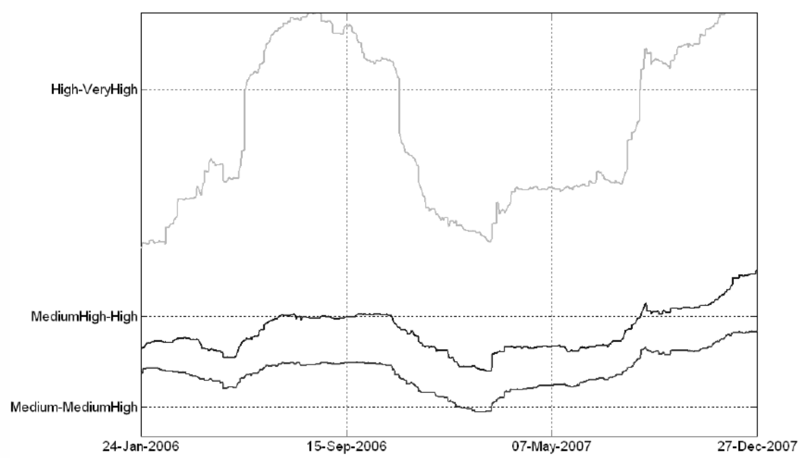
UNIVERSE	Total (A)	Selected (B)	Representativity (B/A)
Austria	17	13	76.5%
France	92	53	57.6%
Germany	63	45	71.4%
Ireland	2	1	50.0%
Italy	58	52	89.7%
Luxembourg	252	153	60.7%
Spain	224	130	58.0%
UK	8	7	87.5%
Total	716	454	63.4%

Empirical Evidence on the European Industry: Preliminary Informations



Empirical Evidence on the European Industry: Preliminary Informations

LUXEMBOURG

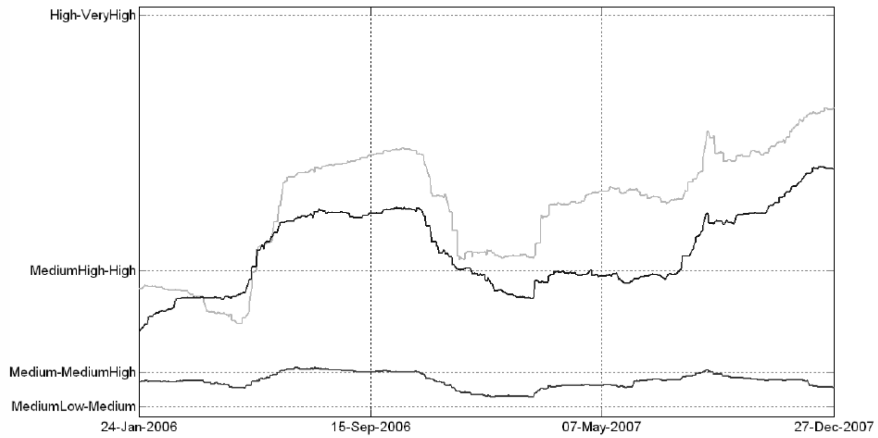


Risk Europe

CONSOB

Empirical Evidence on the European Industry: Preliminary Informations

GERMANY

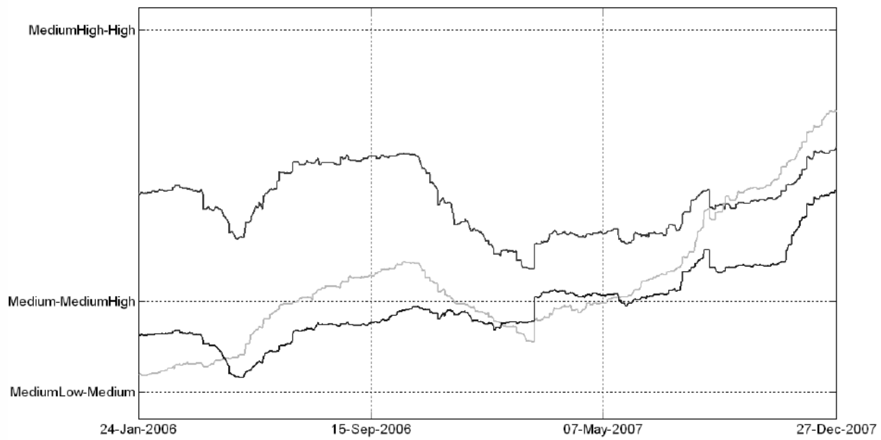


Risk Europe

CONSOB

Empirical Evidence on the European Industry: Preliminary Informations

ITALY

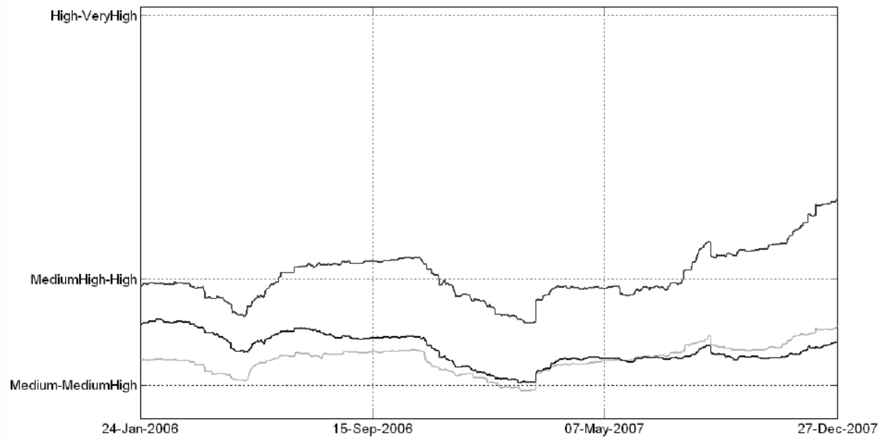


Risk Europe

CONSOB

Empirical Evidence on the European Industry: Preliminary Informations

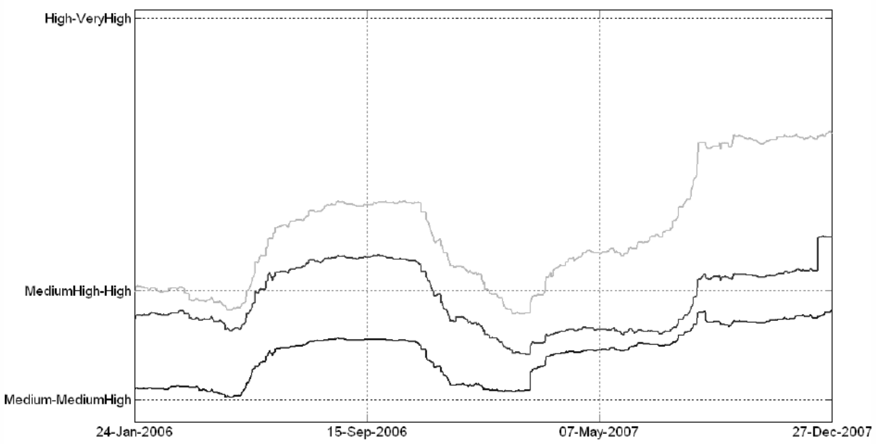
SPAIN



Risk Europe

CONSOB

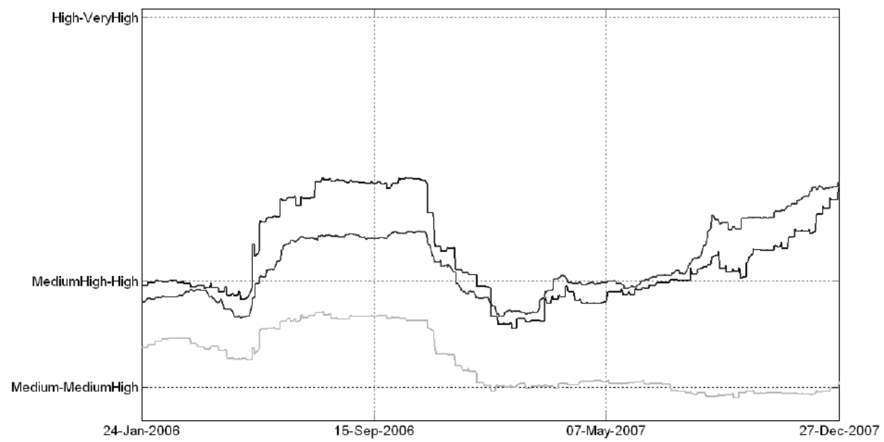
FRANCE



Risk Europe CONSOB

Empirical Evidence on the European Industry: Preliminary Informations

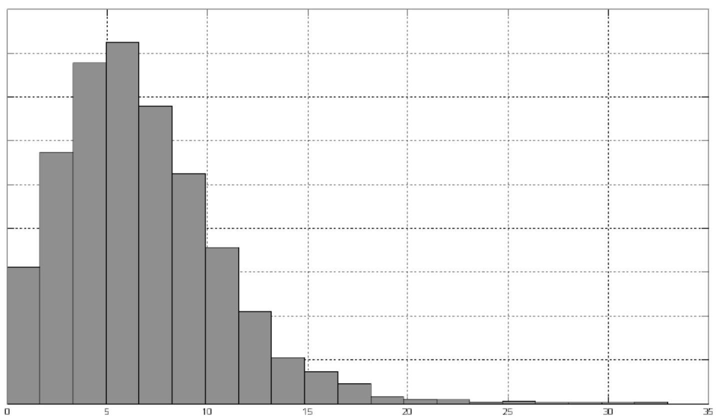
UNITED KINGDOM



Risk Europe CONSOB

Empirical Evidence on the European Industry: Preliminary Informations

Histogram of the Volatility Time Series of the Flexible Funds selected



Risk Europe CONSOB

Empirical Evidence on the European Industry: Preliminary Informations

Initial Distribution of the 454 Funds between the 6 risk classes (abs. values)

	Initial Risk Class as from 1st January 2006						
Country	1	2	3	4	5	6	TOTAL
Austria	0	0	4	8	1	0	13
France	0	2	9	37	5	0	53
Germany	0	2	10	26	7	0	45
Ireland	0	1	0	0	0	0	1
Italy	1	11	11	28	1	0	52
Luxembourg	1	6	30	100	16	0	153
Spain	0	23	33	62	12	0	130
UK	0	0	0	5	2	0	7
TOTAL	2	45	97	266	44	0	454

Risk Europe CONSOB

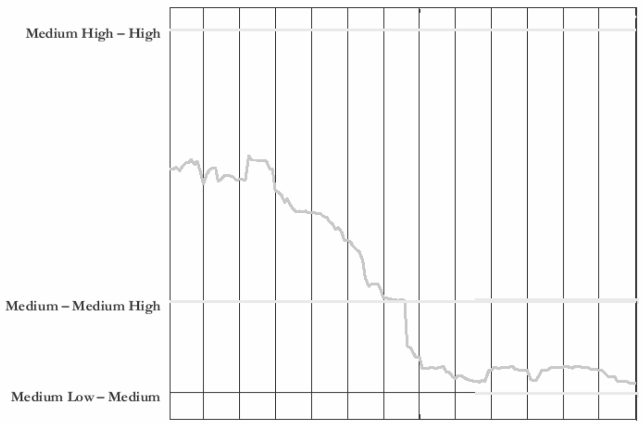
Initial Distribution of the 454 Funds between the 6 risk classes (perc. values)

	Initial Risk Class as from 1st January 2006						
Country	1	2	3	4	5	6	TOTAL
Austria	0.0%	0.0%	30.8%	61.5%	7.7%	0%	100%
France	0.0%	3.8%	17.0%	69.8%	9.4%	0%	100%
Germany	0.0%	4.4%	22.2%	57.8%	15.6%	0%	100%
Ireland	0.0%	100.0%	0.0%	0.0%	0.0%	0%	100%
Italy	1.9%	21.2%	21.2%	53.8%	1.9%	0%	100%
Luxembourg	0.7%	3.9%	19.6%	65.4%	10.5%	0%	100%
Spain	0.0%	17.7%	25.4%	47.7%	9.2%	0%	100%
UK	0.0%	0.0%	0.0%	71.4%	28.6%	0%	100%
TOTAL	0.4%	9.9%	21.4%	58.6%	9.7%	0%	100%

Syllabus

- Empirical Evidence on the Italian industry
  - Preliminary Informations
  - The Evolution of the Risk-Profile over time

MIGRATION



Risk Class as from the Prospectus
Risk effectively taken

Number of Migrations occurred between different risk classes over the period 01/01/2006 – 12/31/2007 (abs. values)

	Number of Migrations over the period January 2006 - December 2007						
Country	0	1	2	3	4	5	Total
Austria	2	6	2	3	0	0	13
France	20	13	8	11	1	0	53
Germany	18	6	13	8	0	0	45
Ireland	0	1	0	0	0	0	1
Italy	15	12	17	8	0	0	52
Luxembourg	63	28	34	23	4	1	153
Spain	44	30	31	21	4	0	130
UK	1	3	1	2	0	0	7
Total	163	99	106	76	9	1	454

Number of Migrations occurred between different risk classes over the period 01/01/2006 – 12/31/2007 (perc. values)

Number of Migrations over the period January 2006 - December 2007						
0	1	2	3	4	5	Total
35.9%	21.8%	23.3%	16.7%	2.0%	0.2%	100%

Conclusions

- ✓ GARCH Diffusive Approach to make robust and reliable Volatility Forecast *(adaptiveness, no echoes effects)*
- ✓ Financial Application to the Transparency regulation of Flexible Mutual Funds
  - mapping of qualitative risk classes to calibrated, increasing and non overlapping intervals of the annualized volatility of NAV returns
  - usefulness of this quantitative methodology to monitor the exposure to the migration risk and to promptly capture the occurrence of the migrations which requires a timely update of the Prospectus.

- ✓ Empirical Evidence:
  the phenomenon of the migration interests more than the 60% of the universe examined
- ✓ Closing Recommendations:
  exploring other fields of application of the described methodology, especially to move faster towards a really levelled playing field

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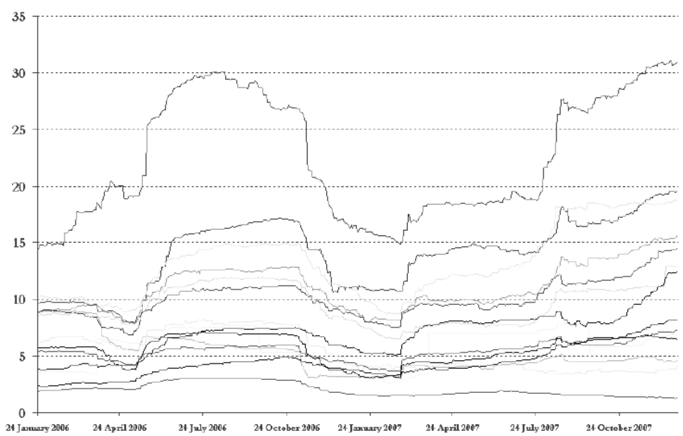

MARCELLO MINENNA – GIOVANNA M. BOI