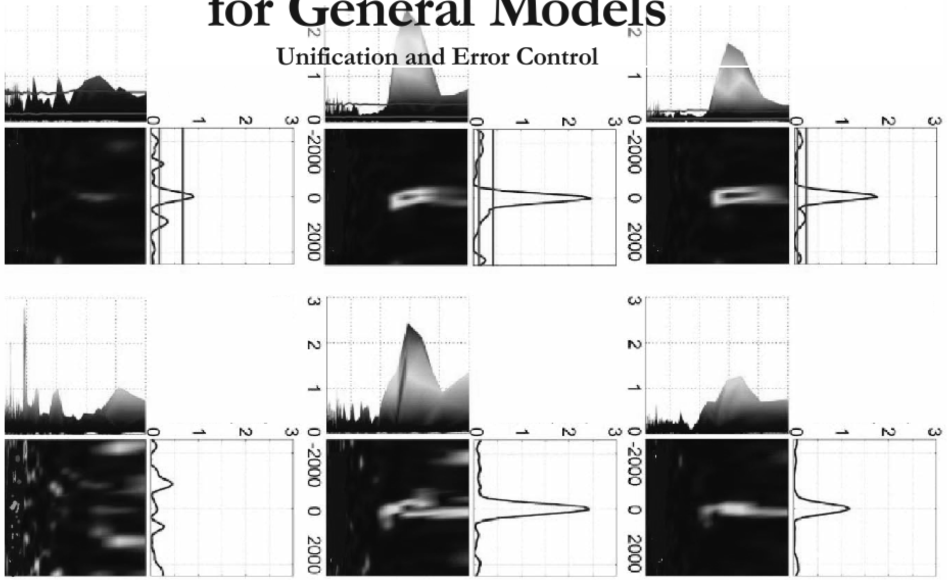
Semi-analytical Fast Option Pricing for General Models

Unification and Error Control



Marcello Minenna - Paolo Verzella

Syllabus of the presentation

- Review of Option Pricing via DFT
  - FT Pricing formulae
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Option Pricing
  - FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
    - The Computational Framework: Speed, Stability, Accuracy
- Conclusions

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FT Pricing Formulas

Derivative Price C_t

Spot Price S_t

$$f_2(\ln S_T, \xi \mid \ln S_0) = \int_{-\infty}^{+\infty} e^{i\xi \ln S_T} q_2(\ln S_T \mid \ln S_0)\, d\ln S_T$$

under risk-neutral measure

A linear direct mapping from Fourier Spectral Space

FT

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FT

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

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CARR-MADAN REPRESENTATION

FT Pricing Formulas

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under risk-neutral measure

Equivalent Representation in the complex plane

FT

$$C_t(\ln K) = \Psi(\ln K, \alpha) + \frac{e^{-r(T-t)}}{2\pi} \int_{-\infty - i\alpha}^{\infty - i\alpha} e^{-iz[\ln K + r(T-t)]} \frac{\phi_T(z-i)}{-z(z-i)} dz$$

$$\Psi(\ln K, \alpha) = Se^{-r(T-t)} \cdot 1_{\{\alpha<0\}} - Ke^{-r(T-t)} \cdot 1_{\{\alpha \leq -1\}} - -\tfrac{1}{2}\left[Se^{-r(T-t)} \cdot 1_{\{\alpha=0\}} - Ke^{-r(T-t)} \cdot 1_{\{\alpha=-1\}}\right]$$

FT Pricing Formulas

Derivative Price C_t

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under risk-neutral measure

Equivalent Representation in the complex plane

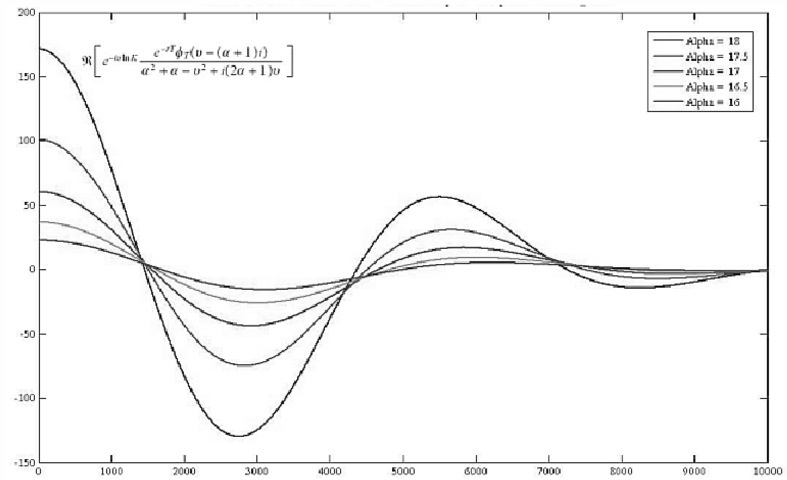
FT

LEWIS REPRESENTATION

Calibrating α

means choosing a dampened oscillating characteristic function

CARR-MADAN REPRESENTATION

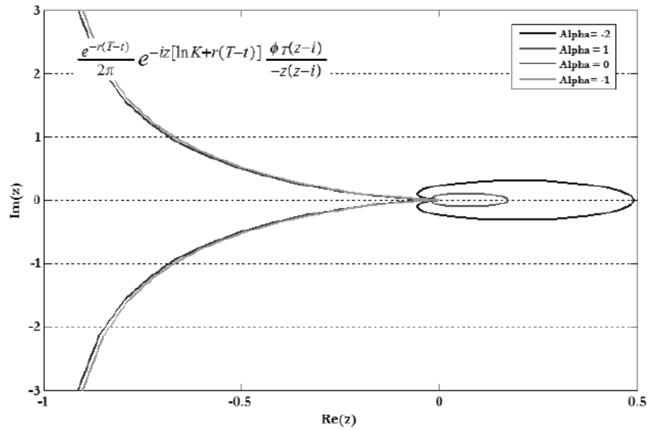


CARR-MADAN REPRESENTATION

Calibrating α

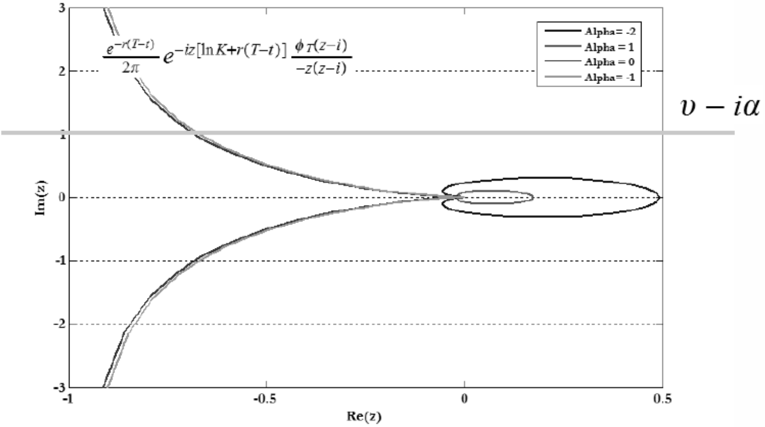
means choosing a fixed horizontal strip of integration in the complex plane

LEWIS REPRESENTATION



LEWIS REPRESENTATION

FT Pricing Formulas

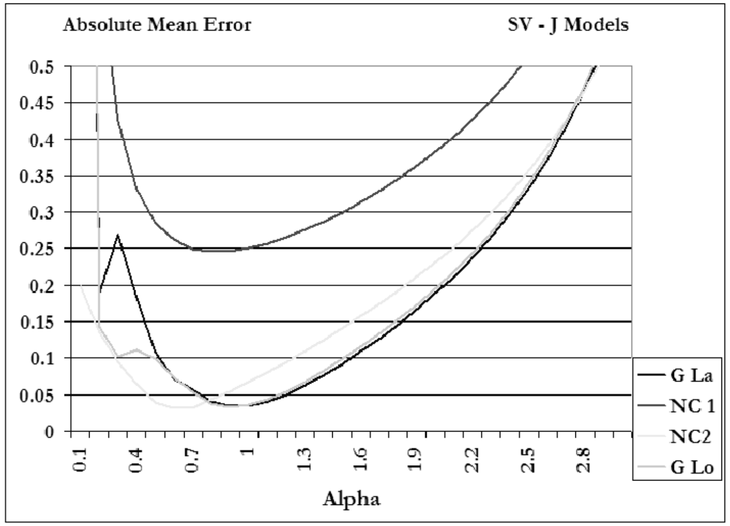


LEWIS REPRESENTATION

FT Pricing Formulas

Accuracy

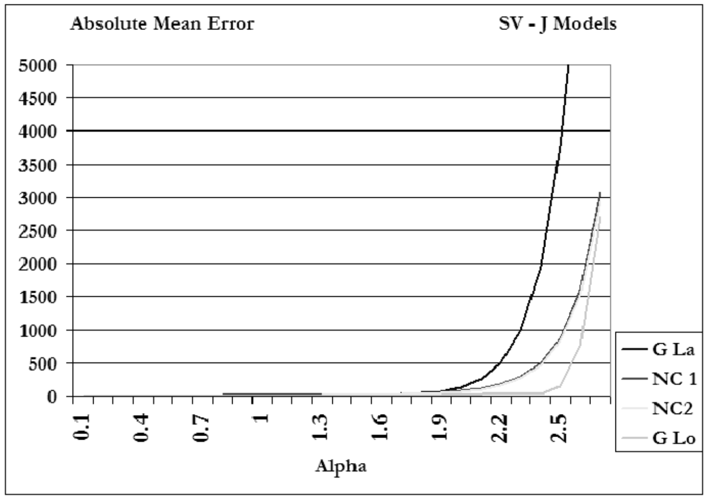
Absolute Mean Error computed w.r.t. α on an (σ, τ) space



FT Pricing Formulas

Stability

Absolute Mean Error computed w.r.t. α on an **Extended** (σ, τ) space



Syllabus of the presentation

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  - **FT Pricing formula**
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1, 2, \ldots M$$

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1, 2, \ldots M$$

$$M \neq N$$

The Convergence Theorem for General DFT's (C Th)

$$\mathcal{F}[f(x)](t_m) = \lim_{N\to\infty} \sum_{j=1}^{N} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j, X)$$

$$t_m = \frac{2\pi}{X}(m-1)$$

C_0 via FT

Convergence theorems

$\overrightarrow{C_0}$ via DFT

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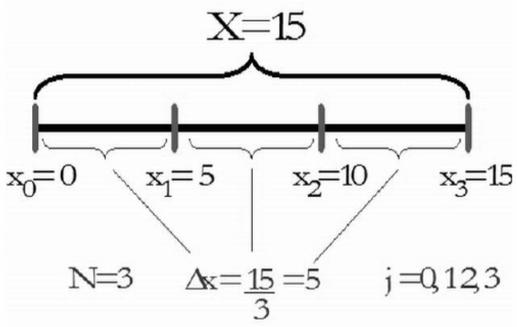
Condition 1

Uniform Discretization Grid



Condition 2

N=M

DFT specialized

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(n-1)} f(x_j) \quad where\ n=1,2,\ldots,N$$

Condition 1 Condition 2

DFT Simplified Formula

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{N}j(n-1)} f(x_j) \quad where\ n = 1,2,\ldots N$$

Nyquist – Shannon Limit (N-S)

$$\mathcal{F}[f(x)](t_n) = \lim_{N\to\infty} \frac{X}{N}\omega(n)$$

$\{t_n\}_{n=1..\frac{N}{2}}$ *for N even*

$\{t_n\}_{n=1..\frac{N+1}{2}}$ *for N odd*

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

Uniform Discretization Grids for *f*

1. $f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]$
2. $f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]\cdot[3+(-1)^{j+1}-\delta_j-\delta_{N-j}]$

1.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]$$

+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{b}\cdot \Re(\omega(u))$$

2.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

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+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{3b}\cdot \Re(\omega(u))$$

Theorems of Equivalence

The Call Price computed via Convergence Theorem is equal to the Call Price computed via Trapezoid/Simpson Quadrature Rule

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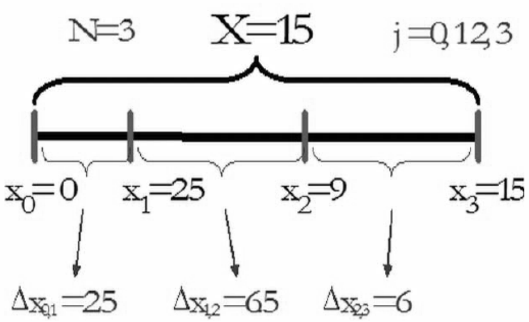
Condition 1

Non Uniform Discretization Grid



Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre

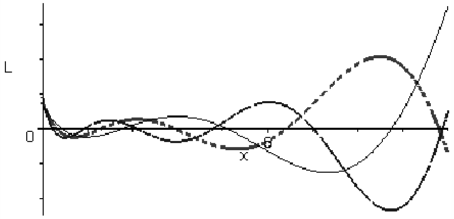
Gander Gautschi

Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre



Zeros of Laguerre Poynomials

Convergence Theorems for Non Uniform Gaussian Grids

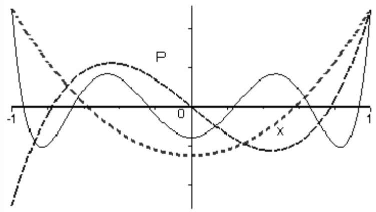
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Lobatto

Zeros of Legendre Poynomials



Convergence Theorems for Non Uniform Gaussian Grids

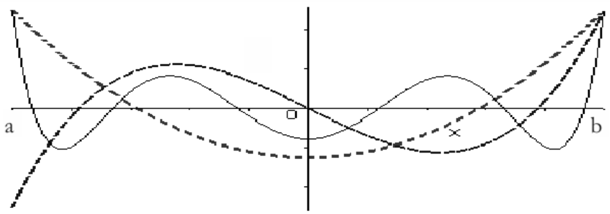
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gander Gautschi

Zeros of rescaled Legendre Poynomials



Convergence Theorems for Non Uniform Gaussian Grids

Condition 2

N≠M

General DFT

$$\omega(m)=\sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m=1,2,\ldots,2M$$

The Convergence Theorem for General DFT's (C Th)

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$$t_m = \frac{2\pi}{X}(m-1)$$

$$C_0 = \frac{e^{-\alpha\ln K}}{\pi}\int_0^{+\infty}\Re\left(e^{i\xi\ln K}\frac{e^{-rT}f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

Gaussian Grids for *f*

1. $f(v_{j-1}) = e^{\left[1+i\left(\frac{M\pi}{a*}-\ln S_t\right)\right]v_{j-1}}\psi_0(v_{j-1}) \quad \frac{1}{L_{N+1}(v_{j-1})L_N'(v_{j-1})}$

2. $f\left(\frac{1}{2}a(1+v_{j-1})\right) = \left[e^{-i\left(\frac{1}{2}a(1+v_{j-1})\right)\left[\ln S_t-\frac{M\pi}{a}\right]}\psi_0\left(\frac{1}{2}a(1+v_{j-1})\right)\right]\frac{1}{[P_{N-1}(v_{j-1})]^2}$

1.

$$C_0 = \frac{e^{-\alpha\ln K}}{\pi}\int_0^{+\infty}\Re\left(e^{i\xi\ln K}\frac{e^{-rT}f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

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\+

C-Th

$$C_0([\ln K]_u^*) \approx -\Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a*}+\frac{2\pi}{a*}(u-1)\right)}}{\pi}\frac{1}{N+1}\cdot\omega^*(u)\right]$$

2.

$$C_0 = \frac{e^{-\alpha\ln K}}{\pi}\int_0^{+\infty}\Re\left(e^{i\xi\ln K}\frac{e^{-rT}f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f\left(\frac{1}{2}a(1+v_{j-1})\right) = \left[e^{-i\left(\frac{1}{2}a(1+v_{j-1})\right)\left[\ln S_t-\frac{M\pi}{a}\right]}\psi_0\left(\frac{1}{2}a(1+v_{j-1})\right)\right]\frac{1}{[P_{N-1}(v_{j-1})]^2}$$

\+

C-Th

$$C_0([\ln K]_u^*) \approx \Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a*}+\frac{2\pi}{a*}(u-1)\right)}}{\pi}\frac{1}{N(N-1)}\cdot\omega^*\left(\tfrac{1}{2}a(1+v_{j-1})\right)\right]$$

Convergence Theorems for Non Uniform Gaussian Grids

Theorems of Equivalence

The Call Price computed via Convergence Theorem is equal to the Call Price computed via Gauss Laguerre/Gander Gautschi Quadrature Rule

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Fast Option Pricing

$\overrightarrow{C_t}$ via DFT

allows

Fast Fourier Trasform Algorithms

Fast Option Pricing

$\overrightarrow{C_t}$ via DFT

Newton-Cotes

Uniform FFT

Fast Option Pricing

$\overrightarrow{C_t}$ via DFT

Gauss

NonUniform FFT

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Uniform FFT

Cooley-Tukey DFT Characterization

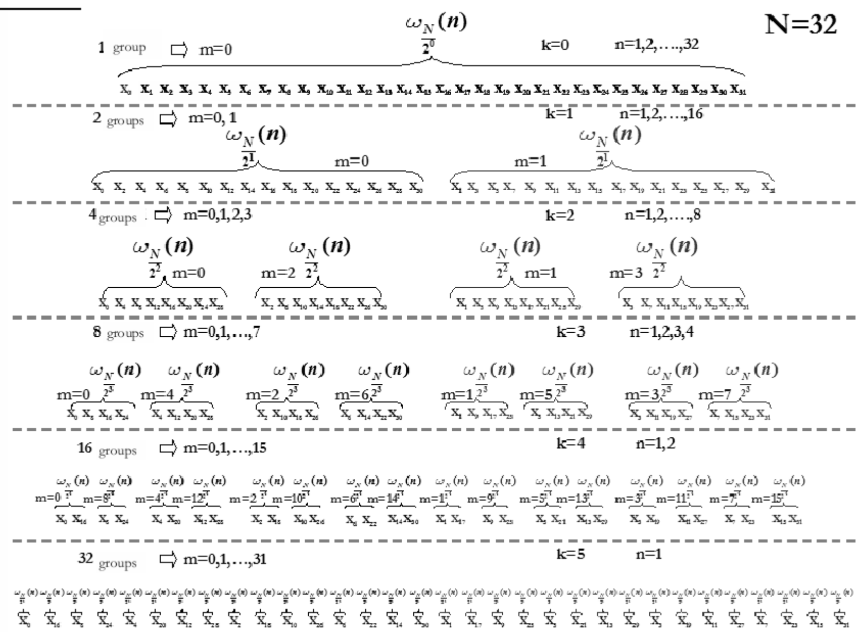
$$\omega_2^m(n) = f(x_m) + W_2^{(n-1)} f(x_{m+N/2}) \quad for\ n = 1,2$$

Iterated Bottom – Up for N stages

It gives the FFT Cooley – Tukey Algorithm

Uniform FFT

N=32



FFT Cooley – Tukey Algorithm

↓

The DFT computational cost drops

↓ from

$O\left(N^2\right)$

↓ to

$O\left(N \log_2 N\right)$

Since the Nyquist – Shannon Limit, the pricing formulas

↓ via FFT

Give accurate prices

ONLY

Around the Nyquist Frequency

Since the Nyquist – Shannon Limit, the pricing formulas

↓ via FFT

Give accurate prices

ONLY

Around the Nyquist Frequency

↓

Approx. 25% of prices can be accepted

The Nyquist relation

$$\lambda = \frac{2\pi}{N}$$

↓ via FFT

must hold between the spectral and log-strike domain

The Nyquist relation

$$\lambda = \frac{2\pi}{N}$$

via FFT

must hold between
the spectral and log-strike domain

The grids cannot be independently choosen

The problem of
Nyquist relation
can be overcome

via FFT

The problem of
Nyquist relation
can be overcome

via FFT

ONLY using the
Fractional FFT - Chourdakis (2005)

The problem of
Nyquist relation
can be overcome

via FFT

ONLY using the
Fractional FFT - Chourdakis (2005)

at the cost of increasing
complexity

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Gaussian Gridding

Gaussian Gridding

Step 1

Gaussian Gridding

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

Gaussian Gridding

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j-x-2k\pi)^2}{4\tau}}$$

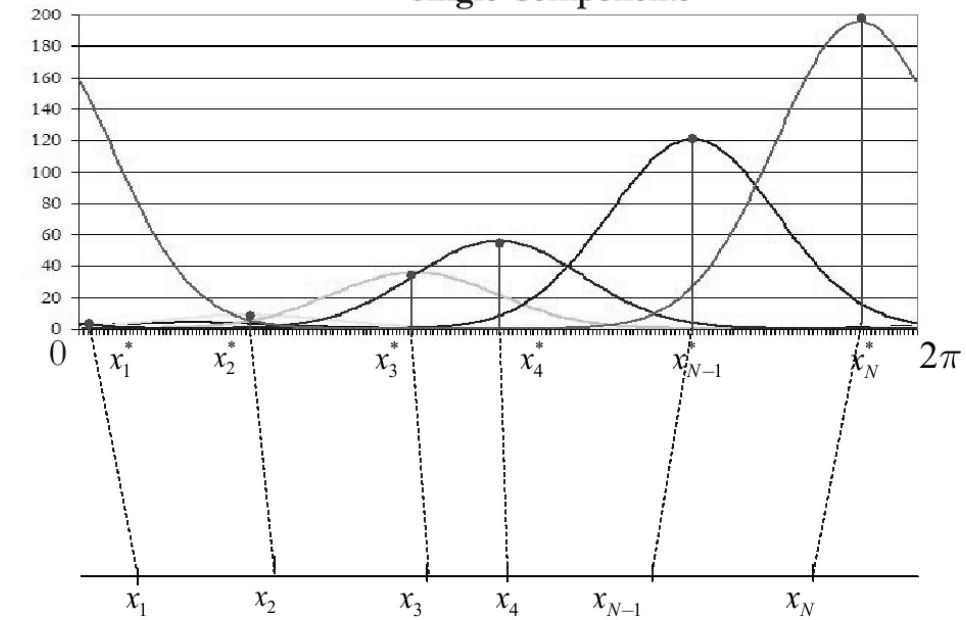
Gaussian Gridding

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \boxed{\sum_{k=-\infty}^{\infty} e^{-\frac{(x_j-x-2k\pi)^2}{4\tau}}}$$

Single Components

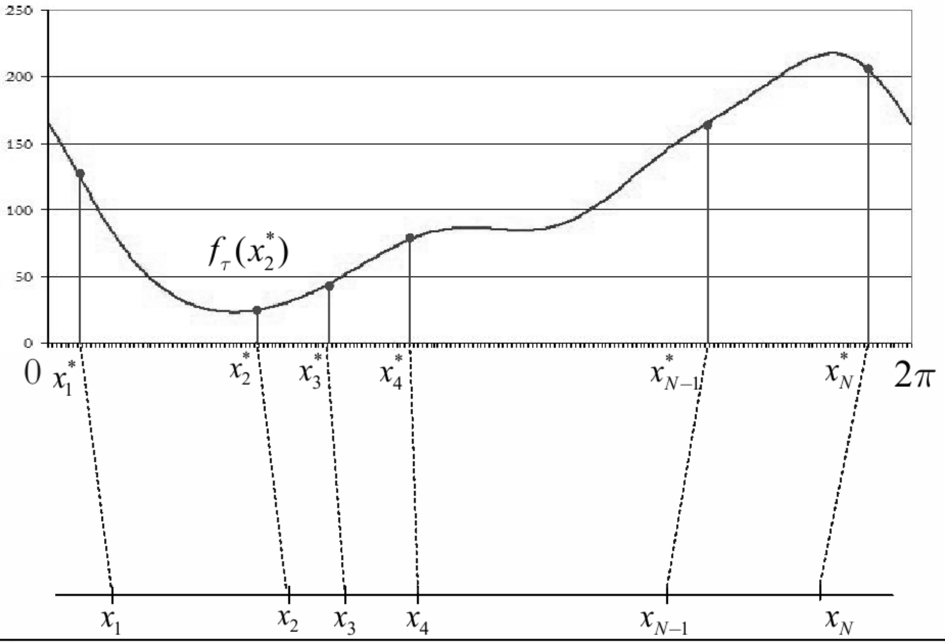


Gaussian Gridding

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

$$\boxed{f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j-x-2k\pi)^2}{4\tau}}}$$



Gaussian Gridding

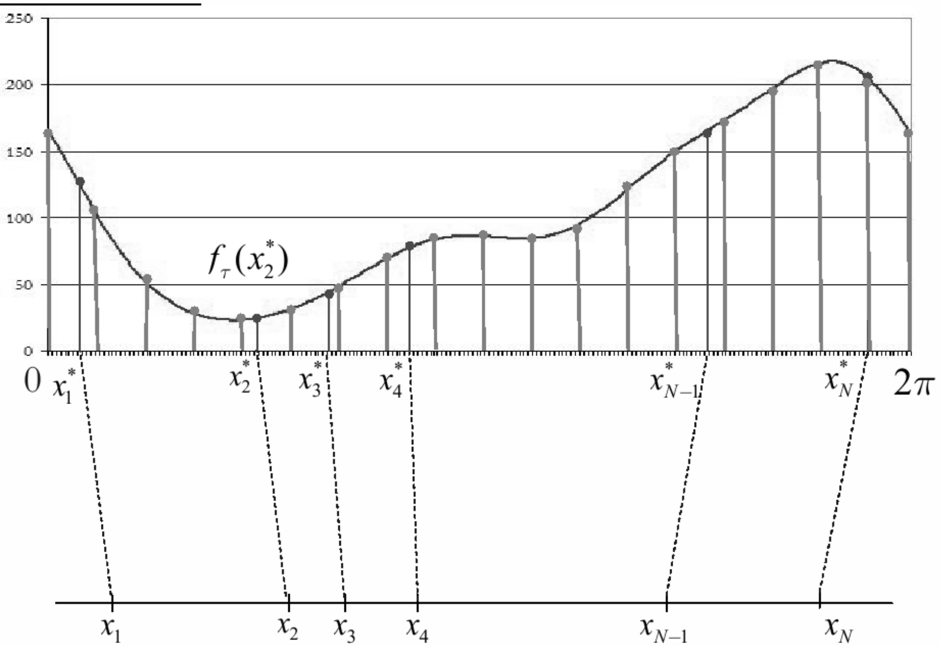
Step 2

Discretization on an uniform oversampled grid of $f_\tau(x)$

$$\widetilde{f}_\tau(y_m) = \sum_{j=0}^{N-1} f(x_j)\, \widetilde{K}(y_m, x_j, \sqrt{2\tau}; 2\pi)$$

where

$$\widetilde{K}(y_m, x_j, \sqrt{2\tau}; 2\pi) = \begin{cases} \sum_{k=-\infty}^{\infty} e^{-\frac{\left(2\pi \frac{x_j}{X} - 2\pi \frac{m}{M_\tau} - 2\pi k\right)^2}{4\tau}} \\ oppure \\ \sum_{k=-\infty}^{\infty} e^{-\frac{\left(x_j^* - 2\pi \frac{m}{M_\tau} - 2\pi k\right)^2}{4\tau}} \end{cases}$$



Gaussian Gridding

Step 3

Computation of the Fourier Coefficient of $f_\tau(x)$ **discretised**

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \sum_{m=0}^{M_\tau - 1} \widetilde{f}_\tau\left(m \frac{2\pi}{M_\tau}\right) e^{-im\frac{2\pi}{M_\tau}(n-1)}$$

Gaussian Gridding

Step 4

NU-DFT representation of the Fourier Coefficient $F_\tau(n)$

$$\widetilde{\omega}(n) = \sqrt{\frac{\pi}{\tau}} e^{n^2 \tau} F_\tau(n)$$

Gaussian Gridding

Step 5

DFT representation of the Fourier Coefficient $F_\tau(n)$

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \omega(n)$$

for $n = 1, 2, \ldots, \frac{M_\tau}{2}$

Gaussian Gridding

Step 6

NU-DFT derivation as a function of DFT

Gaussian Gridding

Step 6

NU-DFT derivation as a function of DFT

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \omega(n)$$

$$\widetilde{\omega}(n) = \sqrt{\frac{\pi}{\tau}} e^{n^2 \tau} F_\tau(n)$$

$$\widetilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2 \tau^*} \frac{1}{M_\tau} \omega(n)$$

for $n = 1, 2, \ldots, \frac{M_\tau}{2}$

Gaussian Gridding

Step 7
NU-FFT computation

Gaussian Gridding

Step 7
NU-FFT computation

$$\tilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}}\, e^{n^2 \tau^*} \frac{1}{M_\tau}\, \omega(n)$$

FFT

Gaussian Gridding

Step 7
NU-FFT computation

$$\tilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}}\, e^{n^2 \tau^*} \frac{1}{M_\tau}\, \omega(n)$$

NU-FFT — FFT

Computational Cost

The major computational cost of the Procedure is the FFT on the oversampled grid

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Choosing the oversampling ratio

$$M_\tau = 2M$$

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The major computational cost of the Procedure is the FFT on the oversampled grid

Choosing the oversampling ratio

$$M_\tau = 2M$$

The total cost of the procedure is $\simeq 2M \log 2M$

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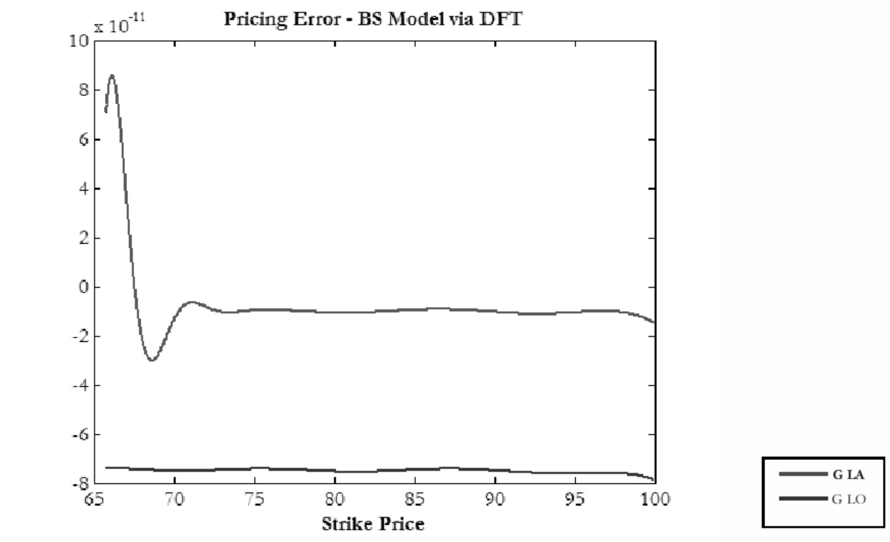
ACCURACY

G LA
G LO

The Computational Framework



2000 Prices computed

The Computational Framework

Absolute Mean Error	Black - Scholes	
Alpha 1.89	G Lobatto	G Laguerre
Tau 0.5	0.000584925943635	0.000584925944845
Tau 1	0.000587732250634	0.000587732252305
Tau 1.5	0.000591973600440	0.000591973603290
Tau 2	0.000597240904278	0.000597240928148
Tau 2.5	0.000603064341251	0.000603065896247
Tau 3	0.000609132584764	0.000609148908355
Tau 3.5	0.000615260627437	0.000615453604830
Tau 4	0.000621355197636	0.000624276301093
Tau 4.5	0.000627781650046	0.000662038720675
Tau 5	0.000634392387803	0.000757217959670

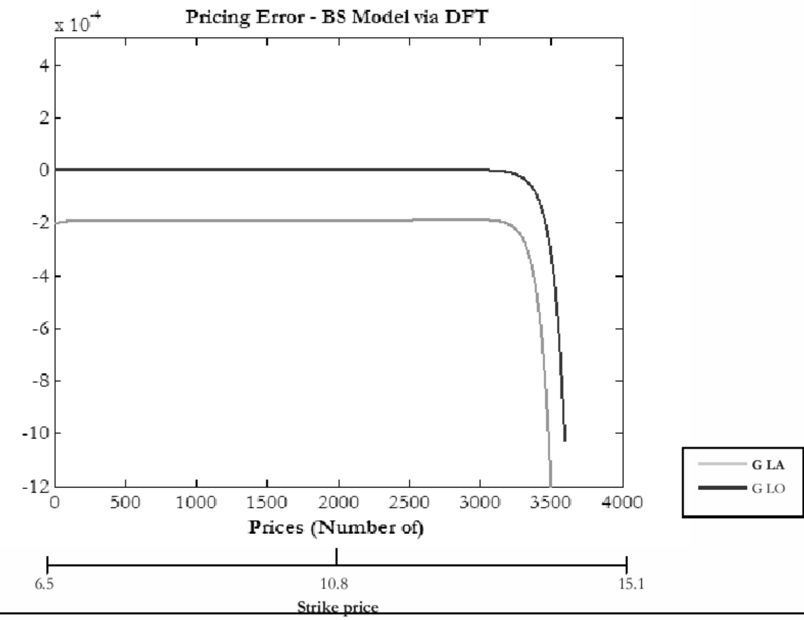
Absolute Mean Error	Black - Scholes	
Alpha 1.89	G Lobatto	G Laguerre
V 0.1	0.000602754964299	0.000584898424795
V 0.25	0.000584897741615	0.000584897745704
V 0.4	0.000584697746586	0.000584897747804
V 0.55	0.000584699535138	0.000584899535695
V 0.7	0.000584972304642	0.000584972304568
V 0.85	0.000585458279806	0.000585458281222
V 1	0.000586861399914	0.000586861402748
V 1.15	0.000589509729395	0.000589509732674
V 1.3	0.000593470665503	0.000593470774300
V 1.45	0.000598636201797	0.000598635557999

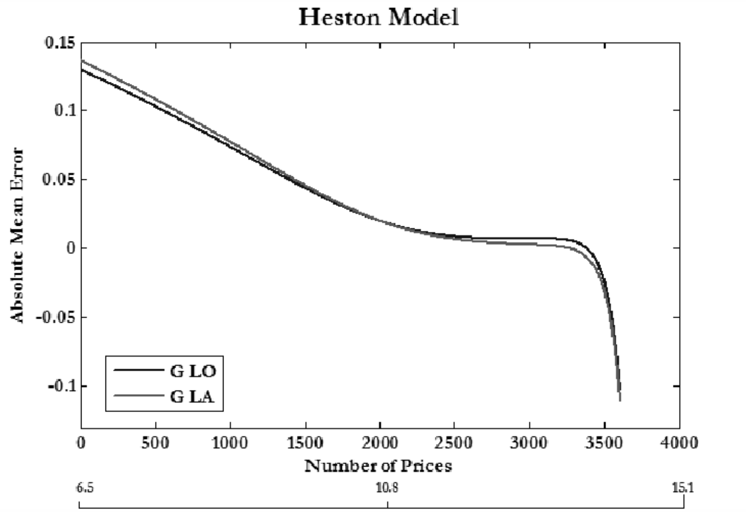
Weighted Absolute Mean Error

The Computational Framework

STABILITY

The Computational Framework

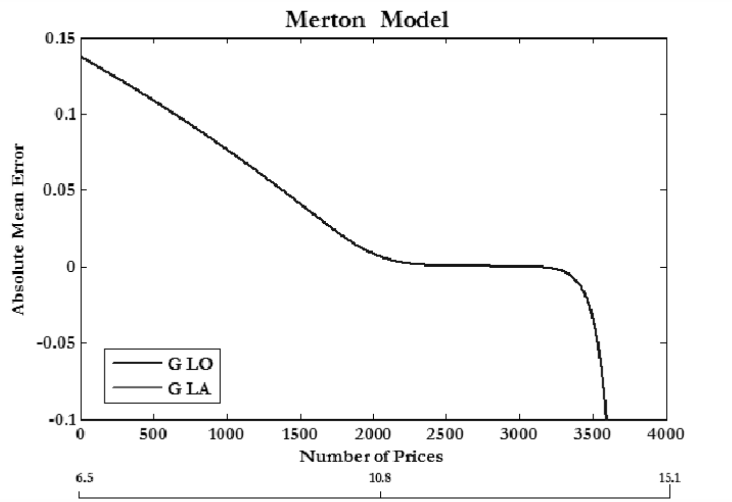




Absolute Mean Error	Heston $\rho = 0.5, \sigma = 0.1, \theta = 0.2, \kappa = 1.5$	
Alpha 1.01	G Lobatto	G Laguerre
Tau 0.5	0.203261248814126	0.206643694502080
Tau 1	0.467697723243552	0.485753402024083
Tau 1.5	0.696314515533637	0.747261110439642
Tau 2	0.833488166510773	0.942225866624220
Tau 2.5	0.866433188291042	1.028827463647480
Tau 3	1.019847958296090	0.967109842505122
Tau 3.5	1.346164593625500	0.718496902407599
Tau 4	1.883548224773260	0.580938209657741
Tau 4.5	2.688023770252460	0.853649681327116
Tau 5	3.827669167590610	1.631212045578270

Absolute Mean Error	Heston $\rho = 0.5, \sigma = 0.1, \theta = 0.2, \kappa = 1.5$	
Alpha 1.01	G Lobatto	G Laguerre
V 0.1	0.147730398465110	0.142723772804352
V 0.25	0.155895724840067	0.151205138597161
V 0.4	0.171938403925182	0.167802215395030
V 0.55	0.192410056590754	0.188536412450069
V 0.7	0.213529795635996	0.210174014069956
V 0.85	0.231570136049150	0.228214515934763
V 1	0.243104933587585	0.239749289632671
V 1.15	0.245011201669546	0.241655420417753
V 1.3	0.233972501913545	0.230617079312524
V 1.45	0.206211823504124	0.202855410504408

Weighted Absolute Mean Error



Absolute Mean Error	Merton $\lambda = 0.1, \mu = 0.1, \sigma = 0.1$	
Alpha 1.03	G Lobatto	G Laguerre
Tau 0.5	0.197726949941278	0.197726981416182
Tau 1	0.523196336061597	0.523196435234984
Tau 1.5	0.932063187975280	0.932062986627531
Tau 2	1.404395549255810	1.404398379226510
Tau 2.5	1.928130639602080	1.928129244375190
Tau 3	2.494696907909670	2.494670694411140
Tau 3.5	3.097427062903490	3.097431353211300
Tau 4	3.731015827279840	3.731142256371510
Tau 4.5	4.391299242353300	4.391521929678760
Tau 5	5.074887056777140	5.074989789421280

Absolute Mean Error	Merton $\lambda = 0.1, \mu = 0.1, \sigma = 0.1$	
Alpha 1.03	G Lobatto	G Laguerre
V 0.1	0.109839205124085	0.109854035372817
V 0.25	0.121771105302441	0.121771107322680
V 0.4	0.145036319844125	0.145036326108331
V 0.55	0.175289110861558	0.175289128224079
V 0.7	0.209392928147416	0.209392968959966
V 0.85	0.245582556867865	0.245582630825178
V 1	0.282531302622102	0.282531560078724
V 1.15	0.319429963153446	0.319429660594360
V 1.3	0.355954455247725	0.355958929678353
V 1.45	0.391915427517519	0.391903553950703

Weighted Absolute Mean Error

STABILITY

The error of 90% of prices computed lies in the

STABILITY

The error of 90% of prices computed lies in the

10^{-2}

RANGE OF PRECISION

SPEED

SPEED

the NU – FFT is around 2 time slower than FFT

SPEED

At very low time scales, the differences disappear

SPEED

At very low time scales, the differences disappear

	NC2	G – LA	G - LO
FFT	0.01 sec.	N/A	N/A
NU – FFT	0.02 sec.	0.0261 sec.	0.0301 sec.

Computation of 4000 prices on a Centrino 1600Mhz – 2gb RAM
Mean Value over 1000 runs

Syllabus of the presentation

- Review of Option Pricing via DFT
  - FT Pricing formulae
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Option Pricing
  - Uniform FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
    - The Computational Framework: Speed, Stability, Accuracy
- Conclusions

Conclusions

- **NU – FFT allows the use of Gaussian Grids**

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- NU – FFT is more stable than FFT

- **NU – FFT allows the use of Gaussian Grids**
- **NU – FFT is indifferent to Nyquist _Shannon Limit**
- **NU – FFT does not need the Nyquist relation**
- **NU – FFT is at least as accurate as FFT**
- **NU – FFT is more stable than FFT**
- **NU – FFT speed performances are indistinguishable from FFT's ones**

NU – FFT
is a natural candidate for operational use on trading desks