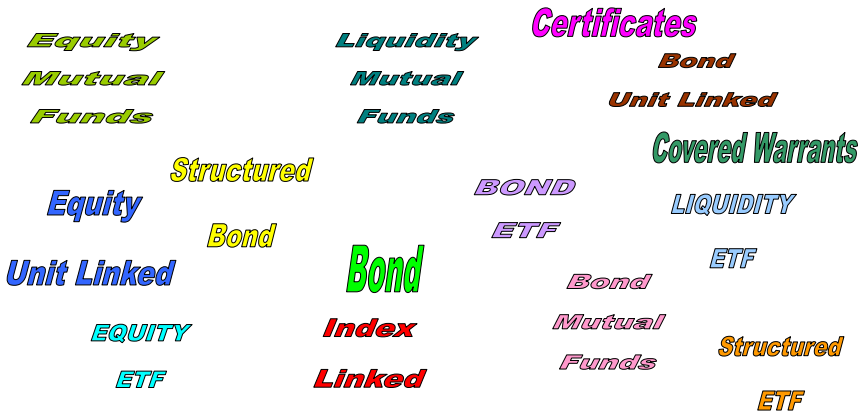
CONSOB
COMMISSIONE NAZIONALE
PER LE SOCIETÀ E LA BORSA

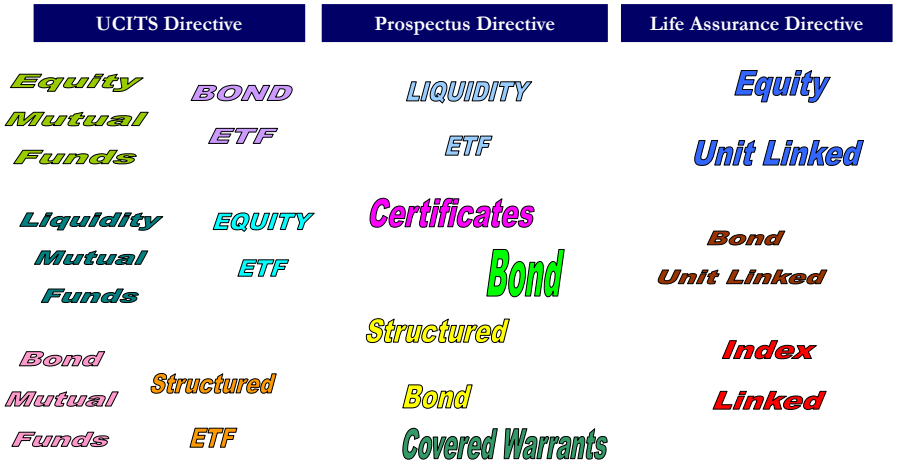
Quantitative measures for a comprehensive approach to risks disclosure in structured products

Risk Europe Frankfurt, 04-05 June 2009

The implementation of the disclosure regulation on the risk-profile of non-equity investment products should allow the investor, even assisted by a financial advisor, to choose the financial product more suitable to his investment objectives.



THREE DIFFERENT DIRECTIVES FOR THE SAME FINANCIAL ENGINEERING



The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

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Time goal: liquidity/investment horizon

INVESTMENT HORIZON

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Time goal: liquidity/investment horizon

INVESTMENT HORIZON

Risk profile: risk limit in terms of downside

RISKS

The information to be provided to the investor, in a simple, clear and fair way, must allow an assessment of his needs in terms of:

Time goal: liquidity/investment horizon

INVESTMENT HORIZON

Risk profile: risk limit in terms of downside

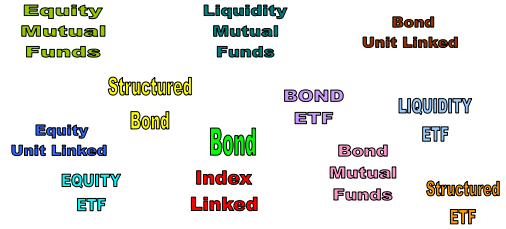
RISKS

Return goal: desired returns

RETURNS

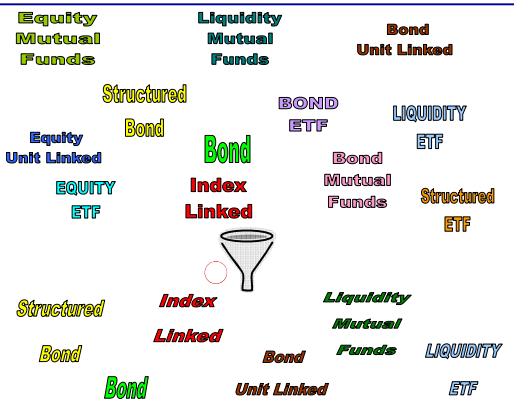
INVESTMENT HORIZON

(less than 3 years)



INVESTMENT HORIZON
(less than 3 years)

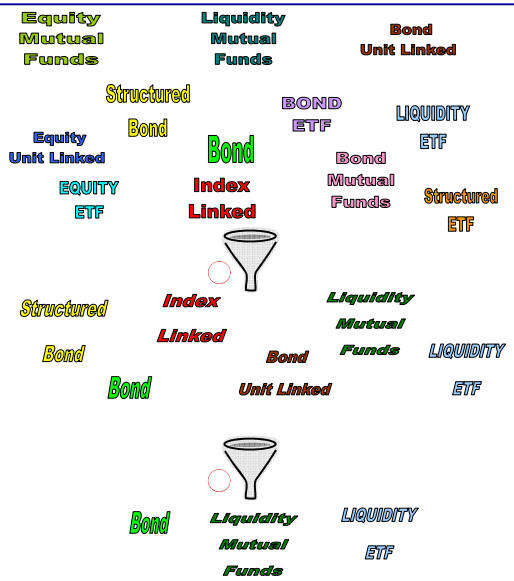
RISKS
(medium-low)



INVESTMENT HORIZON
(less than 3 years)

RISKS
(medium-low)

RETURNS
(maximum return)



RETURNS **RISKS** **INVESTMENT HORIZON**

… allow the investor to match his needs with the features of the financial products and to make an informed investment decision

RETURNS **RISKS** **INVESTMENT HORIZON**

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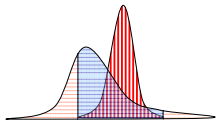
PREVENT MISBUYING

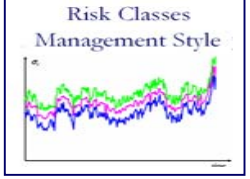
RETURNS | RISKS | INVESTMENT HORIZON

The key qualitative information is made objective by using a three-pillars approach based on quantitative measures.

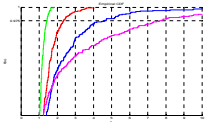
1st — Table of probabilistic performance scenarios



2nd — Synthetic Risk Indicator



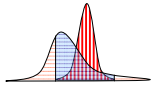
3rd — The recommended minimum investment horizon



Identification and representation of risk-reward by a three-pillars approach

RETURNS | RISKS | INVESTMENT HORIZON

1st — Table of probabilistic performance scenarios



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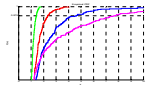
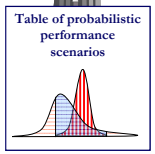


The three-pillars approach must be implemented via the proprietary models of risk management used by the industry, according to the general principles specified in the transparency regulation.

Identification and representation of risk-reward by a three-pillars approach

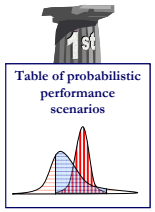
1st — Table of probabilistic performance scenarios



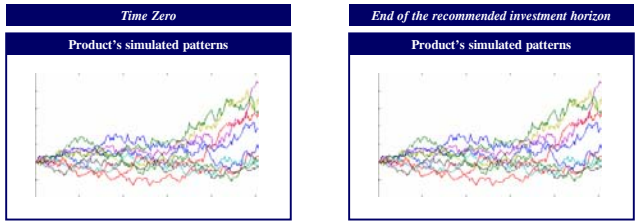
Performance Risk
w.r.t. the risk-free asset
under
the risk-neutral probability measure

… illustrates the unbundling of the price of the financial products at the time of subscription and provides clear and concise information about the possible outcomes and costs.

Identification and representation of risk-reward by a three-pillars approach



Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon



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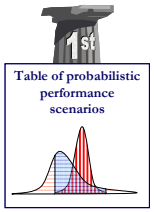

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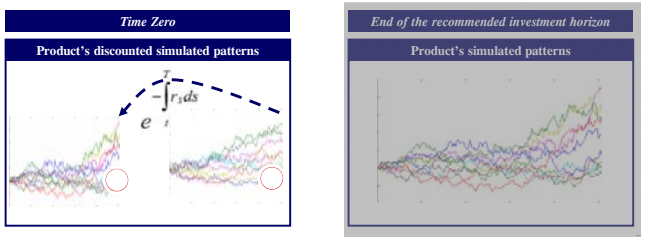

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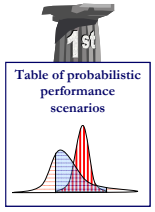

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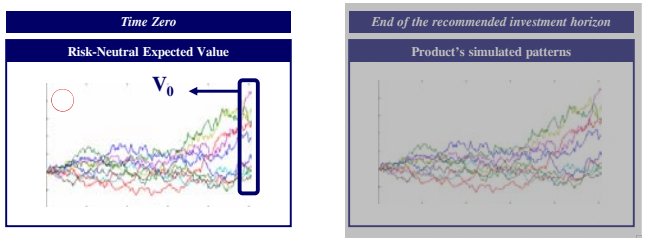

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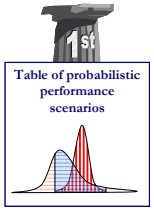

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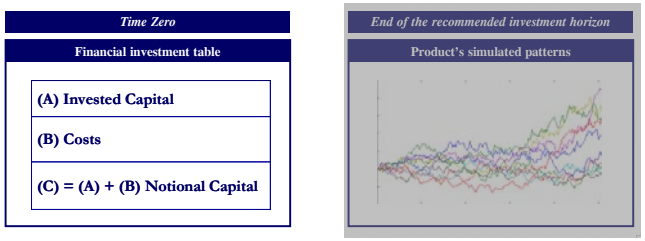

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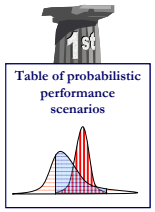
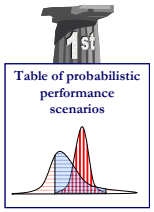


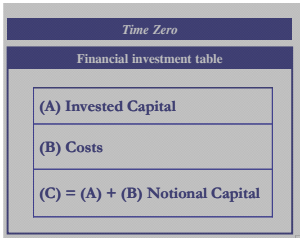
Unbundling of the financial investment at time zero

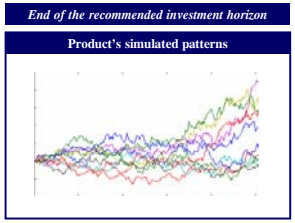
Financial investment table
(A) Invested Capital
(B) Costs
(C) = (A) + (B) Notional Capital

Identification and representation of risk-reward by a three-pillars approach

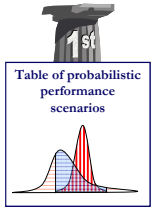


Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon





Identification and representation of risk-reward by a three-pillars approach



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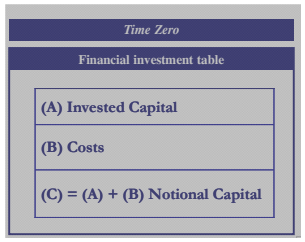

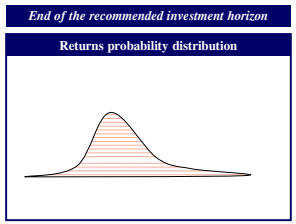

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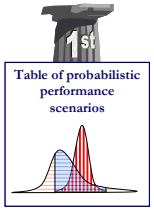

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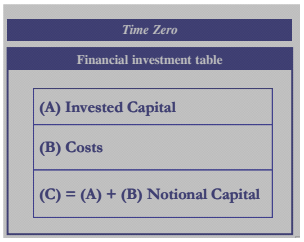

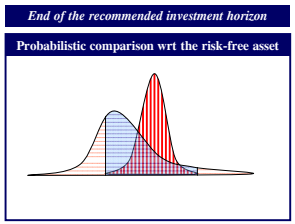

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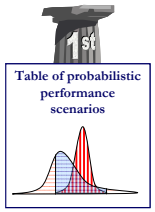


Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon

Time Zero

Financial investment table

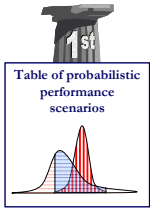
(A) Invested Capital
(B) Costs
(C) = (A) + (B) Notional Capital

End of the recommended investment horizon

Table of probabilistic performance scenarios

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	%	€
The performance is positive but lower than risk-free asset	%	€
The performance is positive and in line with risk-free asset	%	€
The performance is positive and higher than risk-free asset	%	€

Identification and representation of risk-reward by a three-pillars approach



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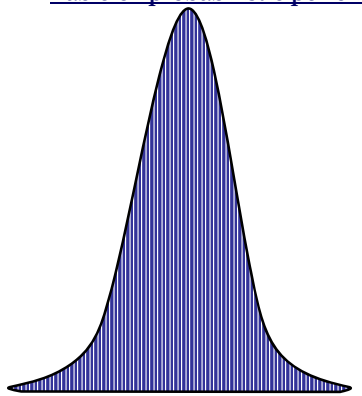
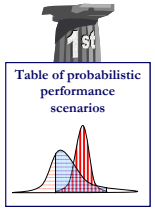


Probability Distribution of the final value of the Notional Capital invested in the risk-free asset

Identification and representation of risk-reward by a three-pillars approach



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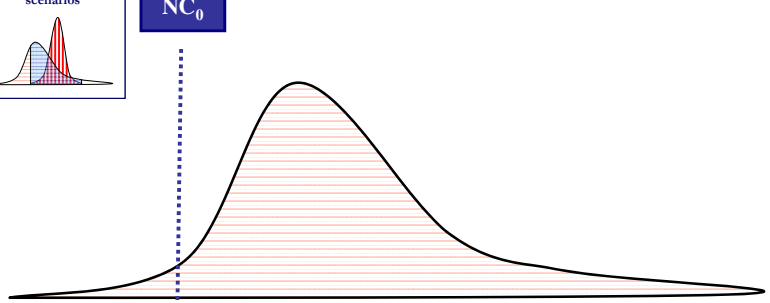
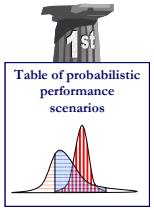


Probability Distribution of the final value of the Invested Capital

Identification and representation of risk-reward by a three-pillars approach



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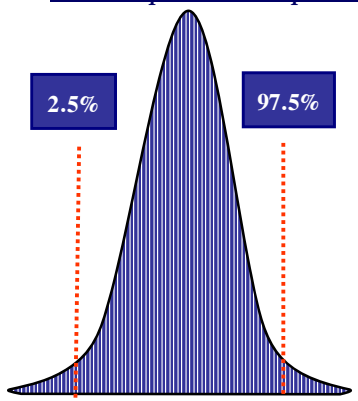

Identification and representation of risk-reward by a three-pillars approach

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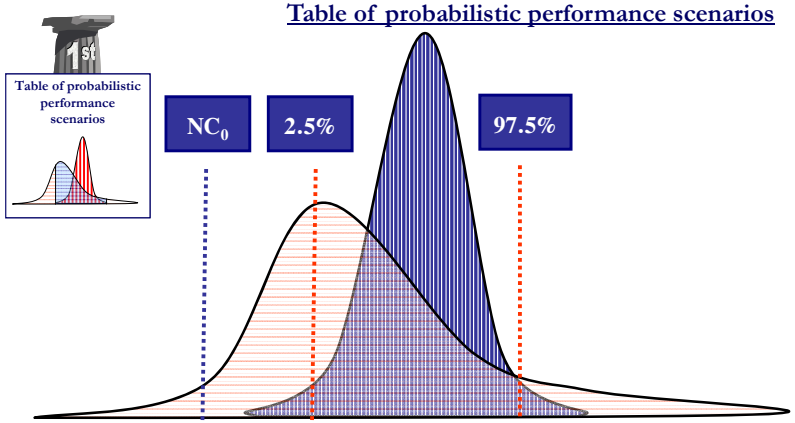

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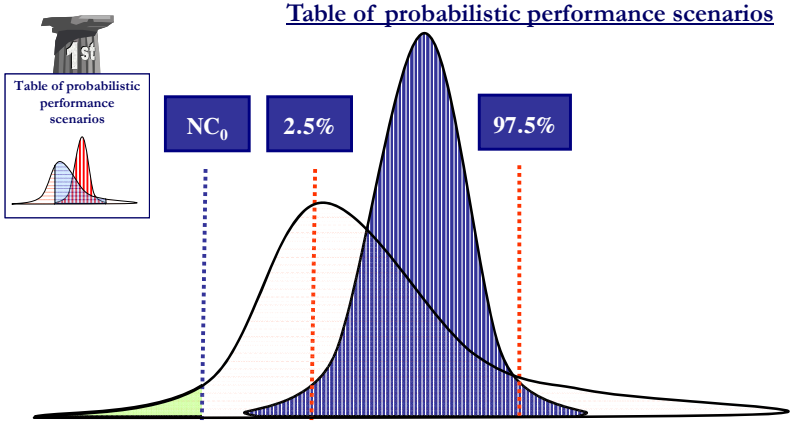
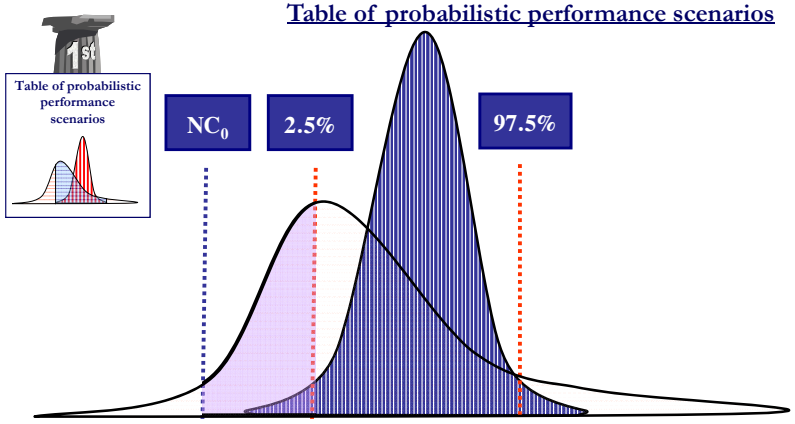


The performance
is negative

Identification and representation of risk-reward by a three-pillars approach

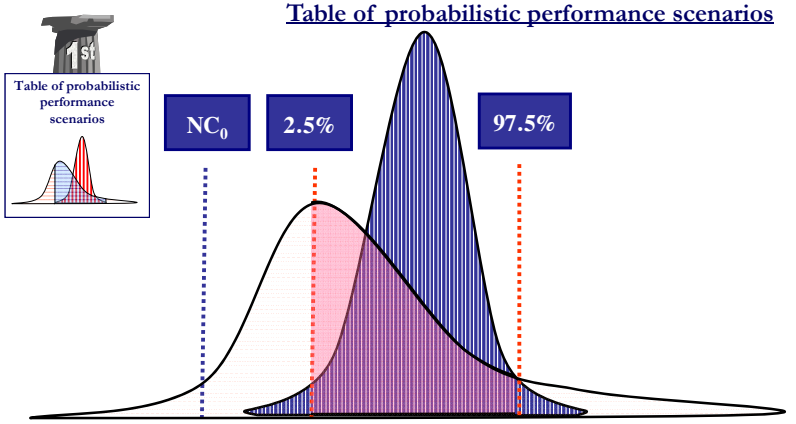
Table of probabilistic performance scenarios



The performance
is positive
but lower than
the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

Table of probabilistic performance scenarios



The performance
is positive
and in line with
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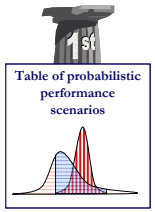
Identification and representation of risk-reward by a three-pillars approach

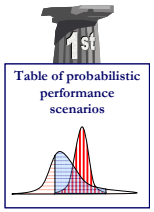
Table of probabilistic performance scenarios



The performance is positive higher than the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

Table of probabilistic performance scenarios



The performance is negative

The performance is positive but lower than the risk-free asset

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The performance is positive higher than the risk-free asset

Identification and representation of risk-reward by a three-pillars approach

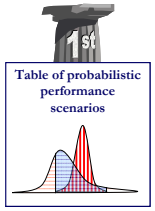

Table of probabilistic performance scenarios

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	%	€
The performance is positive but lower than the risk-free asset	%	€
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Identification and representation of risk-reward by a three-pillars approach

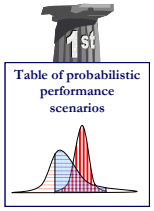

Table of probabilistic performance scenarios
Connection between the risk-neutral price at time zero and at the end of recommended minimum investment horizon

Time Zero

Financial investment table

(A) Invested Capital
(B) Costs
(C) = (A) + (B) Notional Capital

End of the recommended investment horizon

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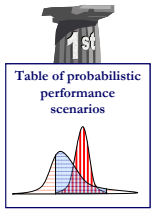
1:1 Relationship

Identification and representation of risk-reward by a three-pillars approach

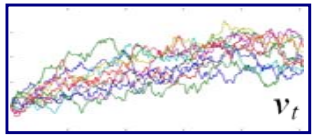


Model Risk Assessment

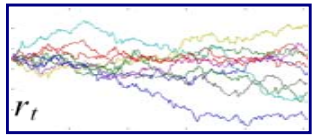
The Recommended Time Horizon has a significant influence on the choice of the model

For Time Horizons greater than 1 year…..

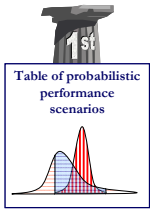
I



II



Identification and representation of risk-reward by a three-pillars approach



Model Risk Assessment

The Recommended Time Horizon has a significant influence on the choice of the model

Two possible choices…

I

HESTON

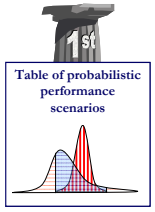
Stochastic Volatility Model

II

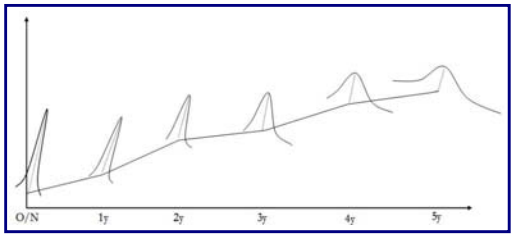
BAKSHI CAO CHEN

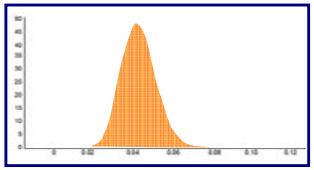
Stochastic Volatility - Stochastic Interest Rate - Jump Model

Identification and representation of risk-reward by a three-pillars approach

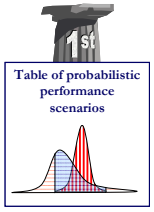


Step 1: Calculation of the Probability Distribution of the Notional Capital at the end of recommended time horizon



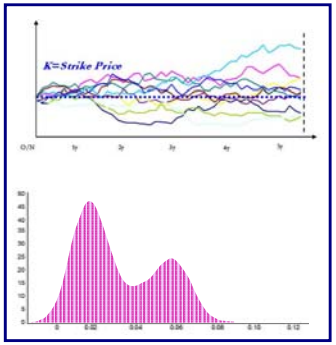


Identification and representation of risk-reward by a three-pillars approach



Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

Heston



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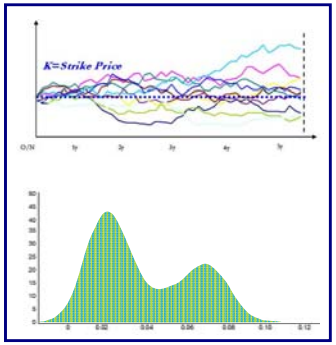
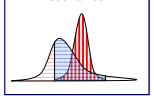


Identification and representation of risk-reward by a three-pillars approach

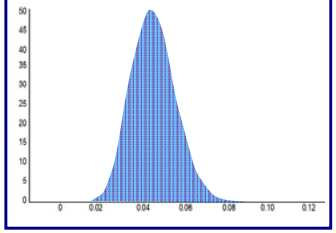
1st

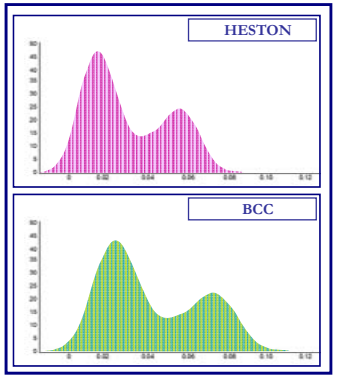
Table of probabilistic performance scenarios



Step 2: Calculation of the Probability Distribution of the Invested Capital at the end of recommended time horizon

Probability Distribution of the Risk-Free Asset

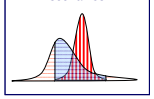




Identification and representation of risk-reward by a three-pillars approach

1st

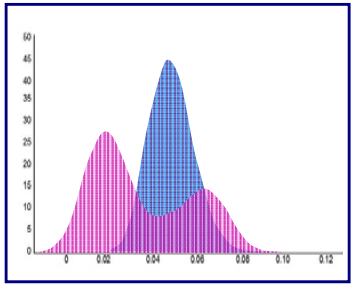
Table of probabilistic performance scenarios



Step 3: Probabilistic comparison with the Risk-Free Asset

Analysing the two probability distribution…

Heston



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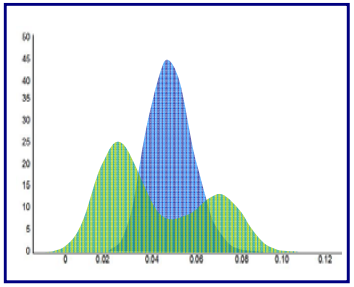

Identification and representation of risk-reward by a three-pillars approach

1st

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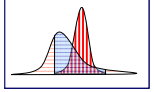


Step 3: Probabilistic comparison with the Risk-Free Asset

The following output is obtained:

Heston

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	4.80 %	97.44 €
The performance is positive but lower than risk-free asset	52.1 %	100.01 €
The performance is positive and in line with risk-free asset	16.42 %	115.63 €
The performance is positive and higher than risk-free asset	24.68 %	135.07 €

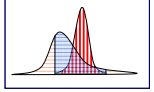
BCC

EVENTS	PROBABILITY	MEDIAN RETURN
The performance is negative	7.20 %	96.92 €
The performance is positive but lower than risk-free asset	45.51 %	100.11 €
The performance is positive and in line with risk-free asset	16.2 %	117.22 €
The performance is positive and higher than risk-free asset	27.79 %	137.56 €

Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



Assessing the model risk

Heston

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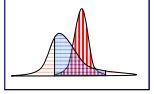
$|\Delta| = 2{,}40\%$

Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



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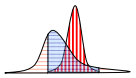
$|\Delta| = 3{,}29\%$

Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



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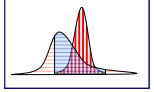
$|\Delta| = 2{,}22\%$

Identification and representation of risk-reward by a three-pillars approach

1st

Table of probabilistic performance scenarios



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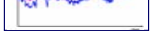
$|\Delta| = 3{,}11\%$

Identification and representation of risk-reward by a three-pillars approach

2nd

Synthetic Risk Indicator



Synthetic Risk Indicator

… provides a description, on a qualitative scale, of the risk level of the financial products based on volatility measures.

… represents in an explicit way the riskiness of the product embedded in the probabilistic performance scenarios of the first pillar.

Identification and representation of risk-reward by a three-pillars approach

Synthetic Risk Indicator

The model has to take in account the following steps …

Time evolution of the Volatility

↓

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Identification and representation of risk-reward by a three-pillars approach

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Phase 1: Calibration of Volatility Intervals

↓

Step 1: Definition of Loss Intervals

Step 2: Mapping of Loss Intervals to the corresponding Volatility Intervals

Step 3: *Fine-tuning* of Volatility Intervals

Identification and representation of risk-reward by a three-pillars approach

Step 1: **Definition of Loss Intervals**

What is a loss in a financial investment?

RISK NEUTRALITY PRINCIPLE

↓

$$LOSS \in (-100\%, \overline{r^{rf}}]$$

$\overline{r^{rf}}$ = average of the probability distribution of the risk-free rate

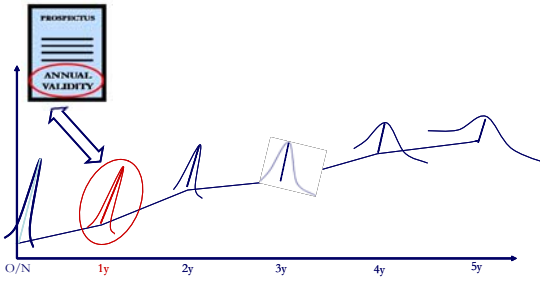
Identification and representation of risk-reward by a three-pillars approach

Step 1: **Definition of Loss Intervals**

given the risk-free yield curve and the associated volatility surface…

↓

Risk-free yield curve and *volatility surfaces*

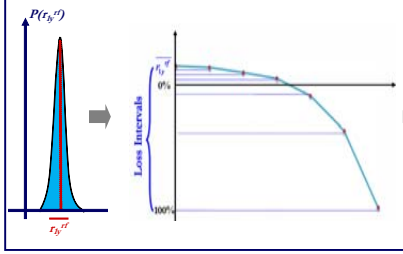


Identification and representation of risk-reward by a three-pillars approach

Step 1: Definition of Loss Intervals

the corresponding annual loss interval (multiple of r_{1y}^{rf} according to an exponential function) is associated to each risk class

From *risk-free* yield curve to one year loss intervals



Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

Identification and representation of risk-reward by a three-pillars approach

Step 2: Mapping into Initial Volatility Intervals

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

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medium	${}_0\sigma_{3,min}$	${}_0\sigma_{3,max}$
medium -high	${}_0\sigma_{4,min}$	${}_0\sigma_{4,max}$
high	${}_0\sigma_{5,min}$	${}_0\sigma_{5,max}$
very high	${}_0\sigma_{6,min}$	${}_0\sigma_{6,max}$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

TOOLS

√ GARCH Diffusive Models

√ Non linear Stochastic Programming

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: GARCH Diffusive Models

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

The Weak Convergence Theorem on R^2

The sequence $\{X_t^h\}$, whose measurable space is $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$, converges weakly for h ↓ 0 to the process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x,t)dt + \sigma(x,t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4, presented below, are satisfied.

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

Condition

n. 1

If $\exists$ a $\delta>0$ s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1,t) \\ c_{h,\delta}(x_2,t) \end{pmatrix} = 0$$

then $\exists$

$$a(x,t) \qquad \lim_{h \downarrow 0} \begin{pmatrix} a_h(x_1,t) & a_h((x_1,x_2),t) \\ a_h((x_2,x_1),t) & a_h(x_2,t) \end{pmatrix} = \begin{pmatrix} a(x_1,t) & 0 \\ 0 & a(x_2,t) \end{pmatrix}$$

s.t.

$$b(x,t) \qquad \lim_{h \downarrow 0} \begin{pmatrix} b_h(x_1,t) \\ b_h(x_2,t) \end{pmatrix} = \begin{pmatrix} b(x_1,t) \\ b(x_2,t) \end{pmatrix}$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

Condition

n. 2

$$\exists\ \sigma(x,t) \quad \text{s.t.: } \forall x_1 \in \mathbb{R}^1, \forall x_2 \in \mathbb{R}^1,$$

it holds

$$\begin{pmatrix} \sigma(x_1,t) & 0 \\ 0 & \sigma(x_2,t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1,t)} & 0 \\ 0 & \sqrt{a(x_2,t)} \end{pmatrix}$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals:
GARCH Diffusive Models

Conditions

n. 3

For h ↓ 0, X_0^h converges in distribution to a random variable X_0 with probability measure v_0 on $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$

n. 4

v_0, $a(x,t)$ and $b(x,t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution v_0, a conditional second moment $a(x,t)$ and a conditional first moment $b(x,t)$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

The Continuous Limit of the M-GARCH(1,1)

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \tilde{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

$\tilde{Z}_k$ and Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem

$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = (\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2)\, dt + 2|\beta_1| \sqrt{Var(\ln |Z_t|)}\, dW_t^*$$

Z_t is $N(0,1)$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

The Prediction Interval for the Volatility

key point

From the Diffusion Limit of the M-GARCH(1,1) Process it is possible to establish a ***Predictive Interval for*** σ_t

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

The Prediction Interval for the Volatility

distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d \ln \sigma_t^2 = \left[\beta_0 + 2\beta_1 \mathbf{E}(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right] dt + 2|\beta_1| \sqrt{Var(\ln |Z_t|)}\, dW_t^*$$

O-U process

$$\ln \sigma_t^2 \sim N\left[\left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}\right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}; \sqrt{\frac{\left(2|\beta_1| \sqrt{Var(\ln |Z_t|)}\right)^2}{2(\beta_1 - 1)} \left(e^{2(\beta_1 - 1)} - 1\right)}\right]$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

matching of the first two conditional moments

$$\begin{aligned} \ln \sigma_k^2 - \ln \sigma_{k-1}^2 = & \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)]\left(e^{(\beta_1 - 1)} - 1\right)}{\beta_1 - 1} - \\ & - 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} E(\ln |Z_{k-1}|) + \\ & + \left(e^{(\beta_1 - 1)} - 1\right) \ln \sigma_{k-1}^2 + \\ & + 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} \ln |Z_{k-1}| \end{aligned}$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

maximum likelihood estimation

setting:

$$Y_k = \ln \sigma_k^2 - \ln \sigma_{k-1}^2$$

$$a = \frac{[\beta_0 + 2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1} - E(\ln|Z_{k-1}|)\,|\beta_1|\sqrt{\frac{2\left(e^{2(\beta_1-1)}-1\right)}{(\beta_1-1)}}$$

$$b = \left(e^{(\beta_1-1)}-1\right)$$

$$c = |\beta_1|\sqrt{\frac{2\left(e^{2(\beta_1-1)}-1\right)}{(\beta_1-1)}}$$

$$X_{k-1} = \ln \sigma_{k-1}^2$$

$$Z = \ln|Z_{k-1}|$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

maximum likelihood estimation

leads to:

$$Y_k = a + bX_{k-1} + cZ$$

log-likelihood function

$$\ln L(Y;\beta_0,\beta_1) = \; n\ln\left(\frac{2}{c\sqrt{2\pi}}\right) + \sum_{k=1}^{n}\left(\frac{Y_k - a - bX_{k-1}}{c} - \frac{1}{2}e^{2\left(\frac{Y_k-a-bX_{k-1}}{c}\right)}\right)$$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**

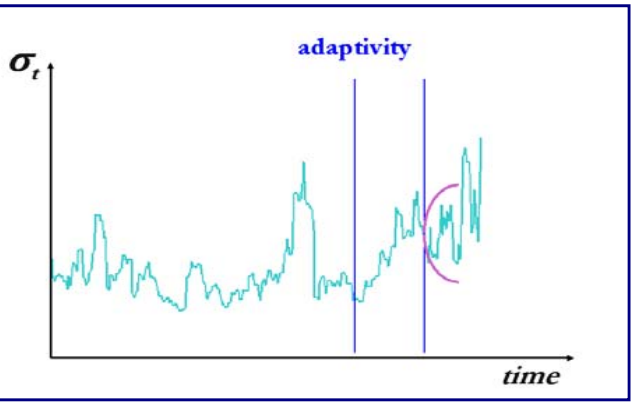
$$P\left(-z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}} \le \ln\sigma_t^2 \le z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}\right) = \alpha$$

hence:

$$\left[\sigma^G_{t,\min};\sigma^G_{t,\max}\right] = \left[e^{\frac{-z_{\frac{\alpha}{2}}\sqrt{\frac{(2.3214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)+\left(\ln\sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}}{2}};\; e^{\frac{z_{\frac{\alpha}{2}}\sqrt{\frac{(2.3214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)+\left(\ln\sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}}{2}}\right]$$

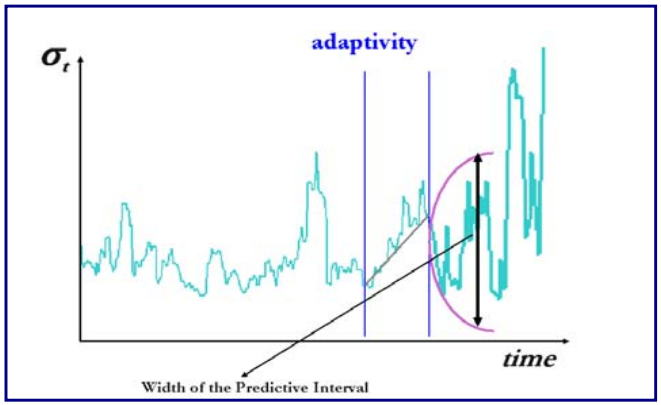
Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals: **GARCH Diffusive Models**



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

√ **Non linear Stochastic Programming**

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

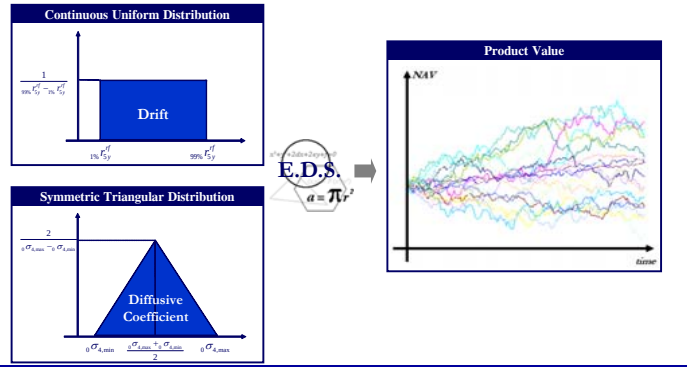
1. **The Product Pattern is simulated for each Initial Volatility Interval**

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	${}_0\sigma_{1,min}$	${}_0\sigma_{1,max}$
medium-low	${}_0\sigma_{2,min}$	${}_0\sigma_{2,max}$
medium	${}_0\sigma_{3,min}$	${}_0\sigma_{3,max}$
medium -high	${}_0\sigma_{4,min}$	${}_0\sigma_{4,max}$
high	${}_0\sigma_{5,min}$	${}_0\sigma_{5,max}$
very high	${}_0\sigma_{6,min}$	${}_0\sigma_{6,max}$

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

1. **The Product Pattern is simulated for each Initial Volatility Interval**



Identification and representation of risk-reward by a three-pillars approach

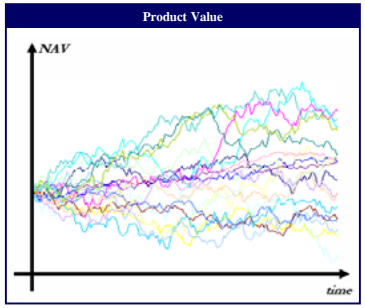
Step 3: *Fine-tuning* of Volatility Intervals

1. The Product Pattern is simulated for each Initial Volatility Interval

Initial Volatily Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

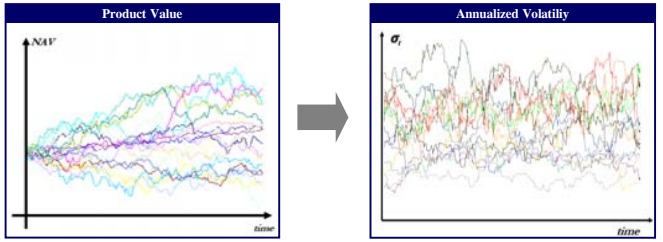
E-GARCH



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Determination of the Time Series of the Annualized Volatility of Product Daily Returns

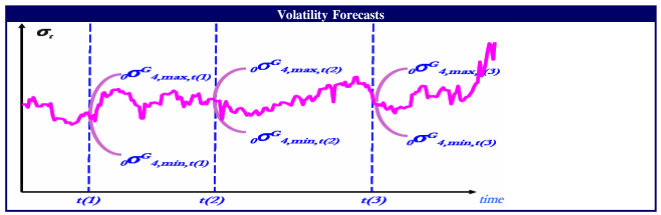


Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

3. For each trajectory the Volatility forecast band is calculated using GARCH Diffusive Models

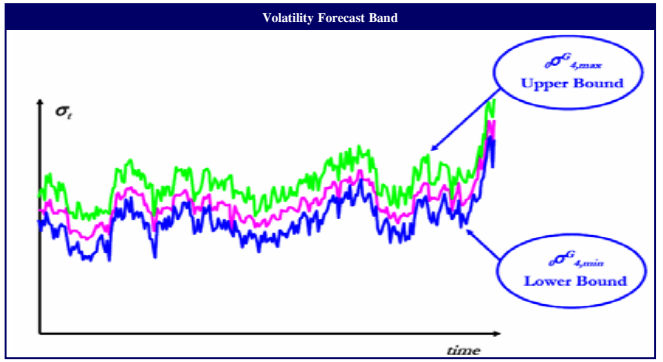
$$\left[\sigma^{G}_{t,\min};\sigma^{G}_{t,\max}\right] = \left[e^{\frac{-\alpha\frac{\beta}{2}\sqrt{\frac{(1.2214\beta_1)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)+\left(\ln\sigma^2_{t-1}+\frac{\beta_0-1.27045}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.27045}{(\beta_1-1)}}}{2}}; e^{\frac{+\alpha\frac{\beta}{2}\sqrt{\frac{(1.2214\beta_1)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)+\left(\ln\sigma^2_{t-1}+\frac{\beta_0-1.27045}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.27045}{(\beta_1-1)}}}{2}}\right]$$



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

3. For each trajectory the Volatility forecast band is calculated using GARCH Diffusive Models

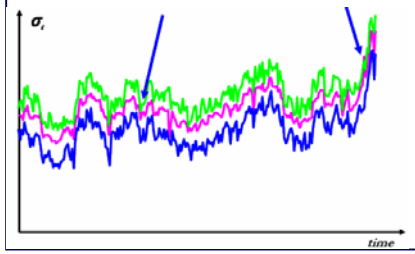


Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

4. Validation of Initial Volatility Interval through an iterative procedure that minimizes the number of observations outside the band

Volatility Forecast Band



$$\Delta = \frac{[\text{Tot.} > {}_0\sigma^G_{4,max}] + [\text{Tot.} < {}_0\sigma^G_{4,min}]}{n*250}$$

$$\Delta_{up} = \frac{[\text{Tot.} > {}_0\sigma^G_{4,max}]}{n*250}$$

$$\Delta_{down} = \frac{[\text{Tot.} < {}_0\sigma^G_{4,min}]}{n*250}$$

$\Delta > 5\%$

The Interval is updated

Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

BEGIN PROCEDURE

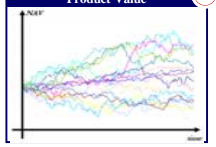
Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



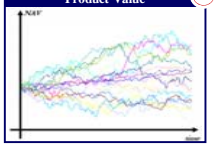
Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

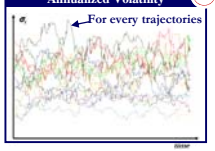
Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Annualized Volatility



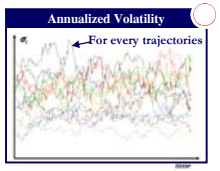
Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

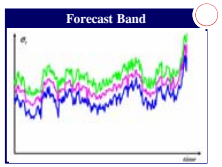
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value



Annualized Volatility

Forecast Band



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value

Annualized Volatility

Initial Interval

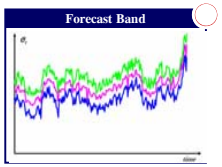
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval

$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$

Forecast Band



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Product Value

Annualized Volatility

$\Delta > 5\%$ e $\Delta_{up} > \Delta_{down}$

Initial Interval

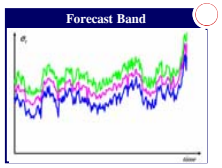
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval

$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$

Forecast Band



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

Update

$[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$

Product Value

Annualized Volatility

$\Delta > 5\%$ e $\Delta_{up} > \Delta_{down}$

Initial Interval

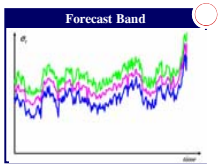
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval

$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$

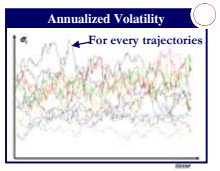
Forecast Band



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

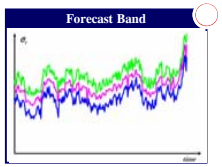


$\Delta > 5\%$ e $\Delta_{up} < \Delta_{down}$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

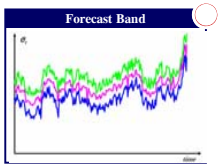
Update
$[{}_1\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

$\Delta > 5\%$ e $\Delta_{up} < \Delta_{down}$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$



Identification and representation of risk-reward by a three-pillars approach

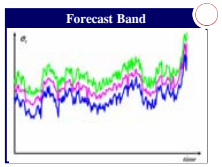
Step 3: *Fine-tuning* of Volatility Intervals

$\Delta \leq 5\%$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

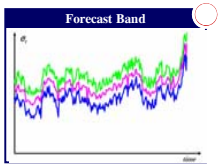
END PROCEDURE
$[{}_k\sigma_{4,min} \quad {}_k\sigma_{4,max}]$
$=$
$[\sigma_{4,min} \quad \sigma_{4,max}]$

$\Delta \leq 5\%$

Initial Interval
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

Garch Interval
$[{}_0\sigma^{G}_{4,min} \quad {}_0\sigma^{G}_{4,max}]$



Identification and representation of risk-reward by a three-pillars approach

Step 3: *Fine-tuning* of Volatility Intervals

OUTPUT

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
Low	0,01%	0,49%
Medium-Low	0,50%	1,59%
Medium	1,60%	3,99%
Medium-High	4,00%	9,99%
High	10,00%	24,99%
Very High	25,00%	>25,00%

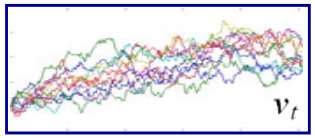
Identification and representation of risk-reward by a three-pillars approach

Synthetic Risk Indicator

Time evolution of the Volatility

Mapping of the Qualitative Risk Classes into corresponding Volatility Intervals

Structured Products



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High	10,00%	24,99%
Very High	25,00%	>25,00%

Identification and representation of risk-reward by a three-pillars approach

Synthetic Risk Indicator

The model has to take in account the following steps …

Benchmark Products

Time evolution of the "intensity" of the Management Style

Mapping of each Volatility Interval into corresponding Intervals of a suitable Volatility Measure for every Management Style

Identification and representation of risk-reward by a three-pillars approach

Synthetic Risk Indicator

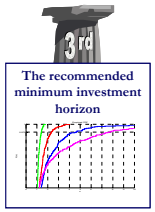
The model has to take in account the following steps …

Benchmark Products

Time evolution of the "intensity" of the Management Style

Risk Classes	Delta-Vol Intervals					
	Limited		Intermediate		Considerable	
	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$
Low	-0.118%	0.118%	-0.176%	0.176%	-0.235%	0.235%
Medium-Low	-0.239%	0.239%	-0.358%	0.358%	-0.477%	0.477%
Medium	-0.600%	0.600%	-0.900%	0.900%	-1.200%	1.200%
Medium-High	-1.250%	1.250%	-1.875%	1.875%	-2.500%	2.500%
High	-3.125%	3.125%	-4.668%	4.668%	-6.249%	6.249%
Very High	-6.250%	6.250%	-9.375%	9.375%	-12.500%	12.500%

Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

Investment period which can be deemed appropriate having regard to the risk-reward profile and to the costs of the product

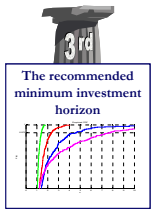
Identification and representation of risk-reward by a three-pillars approach

The recommended minimum investment horizon

The recommended minimum investment horizon crucially depends on types of financial products…

Risk Target Products	Performance Target Products	Benchmark Products

Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

… for performance target products and for guaranteed products the recommended minimum investment horizon is inherent to their financial engineering, as:

Identification and representation of risk-reward by a three-pillars approach

The recommended minimum investment horizon

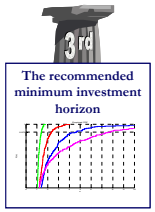
… for performance target products and for guaranteed products the recommended minimum investment horizon is inherent to their financial engineering, as:

the recommended minimum investment horizon is

=

the period of validity (or the time to maturity) of their target/guarantee mechanisms

Identification and representation of risk-reward by a three-pillars approach



The recommended minimum investment horizon

… for risk target products or benchmark products is calculated as the break-even time of the financial investment, i.e. the time needed to recover the initial charges and to offset the ongoing costs at least once, from a probabilistic perspective.

Identification and representation of risk-reward by a three-pillars approach

The recommended minimum investment horizon

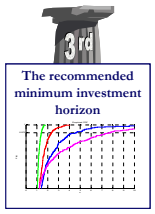
Formally speaking, the probability of the event

The investiment recovers the initial charges and offsest the ongoing costs at least once

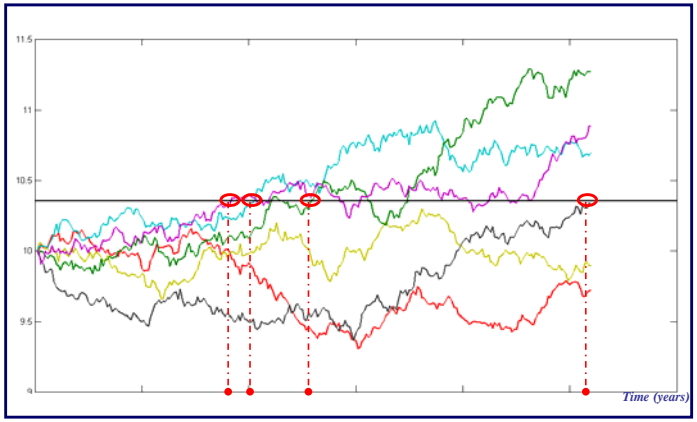
can be calculated using the mathematical concept of

First Hitting Time

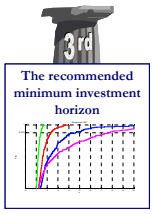
Identification and representation of risk-reward by a three-pillars approach



First Hitting Time of a Structured Product:
first time (expressed in years) at which the value of the product recovers the initial cost and offsets the ongoing costs



Identification and representation of risk-reward by a three-pillars approach



The probability of the event

The investiment recovers the initial charges and offsest the ongoing costs at least once

is perfectly represented using the cumulative distribution of first hitting times, i.e:

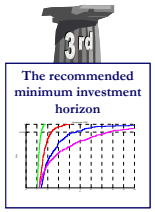
$$\mathrm{P}\left[t^{*} \leq T\right] = X\%$$

where

$$t^{*} = \inf\left[t \in \mathbb{R} : CI_t > CN\right]$$

is the first hitting time

Identification and representation of risk-reward by a three-pillars approach



The probability of the event

The investiment recovers the initial charges and offseet the ongoing costs at least once

given a level of confidence α, identifies univocally a time T on the cumulative distribution of first hitting times, i.e.:

$$T^* = \left\{ t \in \mathbb{R} : \mathrm{P}\left[t^* \le T \right] = \alpha \right\}$$

where

$$t^* = \inf \left[t \in \mathbb{R} : CI_t > CN \right]$$

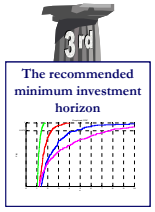
is the first hitting time

Identification and representation of risk-reward by a three-pillars approach

$$T^* = \left\{ t \in \mathbb{R} : \mathrm{P}\left[t^* \le T \right] = \alpha \right\}$$

is defined as the recommended minimum investiment horizon

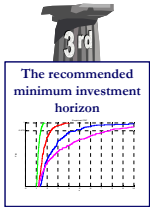
Identification and representation of risk-reward by a three-pillars approach



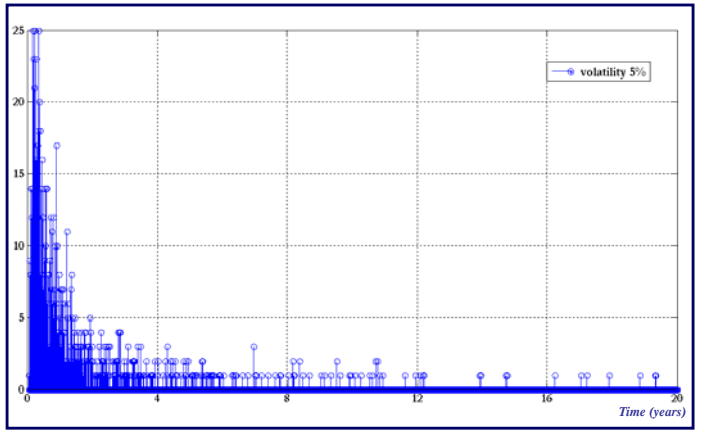
$$T^* = \left\{ t \in \mathbb{R} : \mathrm{P}\left[t^* \le T \right] = \alpha \right\}$$

Computational Steps

Identification and representation of risk-reward by a three-pillars approach



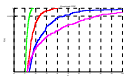
1. The First Hitting Time Distribution of the Structured Product is calculated:



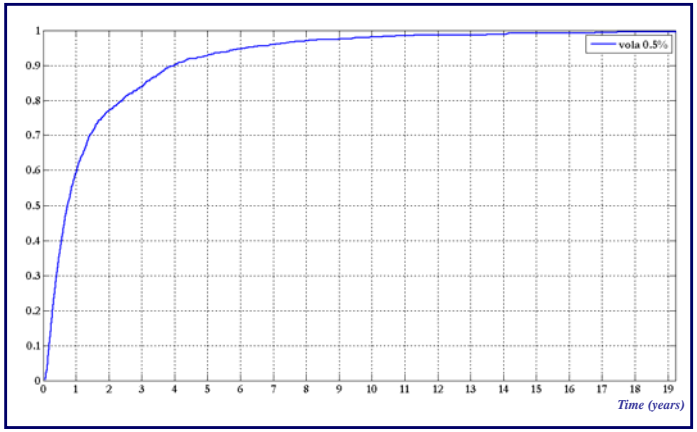
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



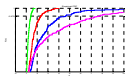
2. The First Hitting Time Cumulative Distribution of the Structured Product is then represented:



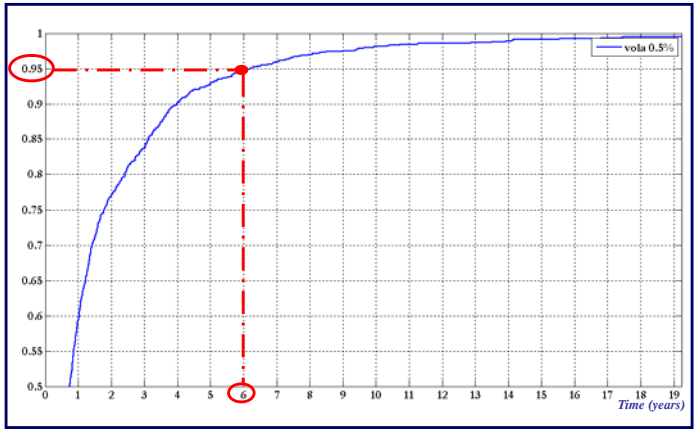
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



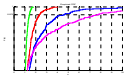
3. The level of confidence α identifies univocally T on the cumulative distribution of first hitting times:



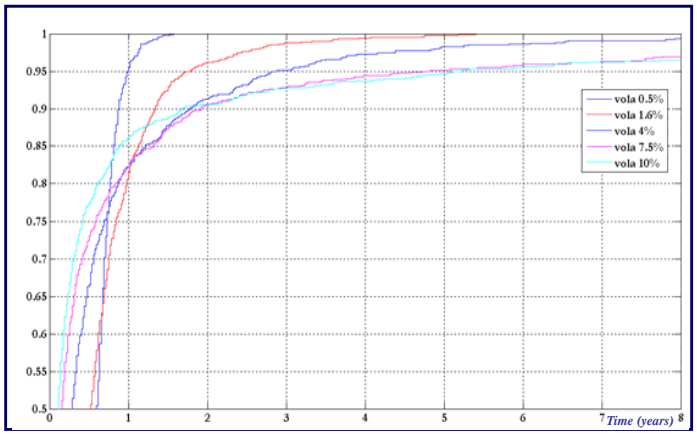
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



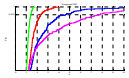
The recommended minimum investment horizon is heavily depedent from the measured level of volatility:



Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



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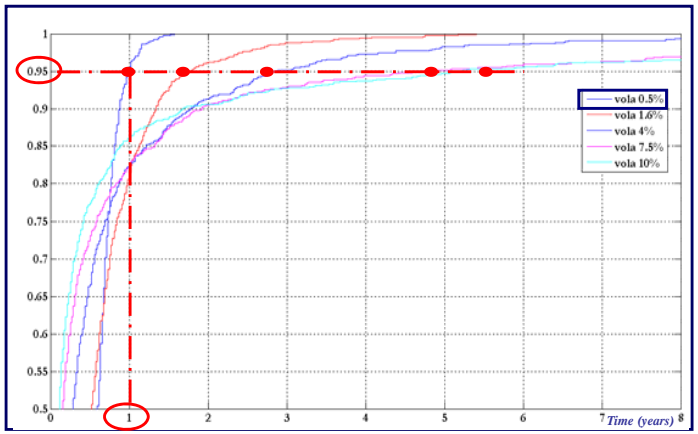

Identification and representation of risk-reward by a three-pillars approach

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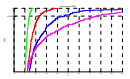
The recommended minimum investment horizon



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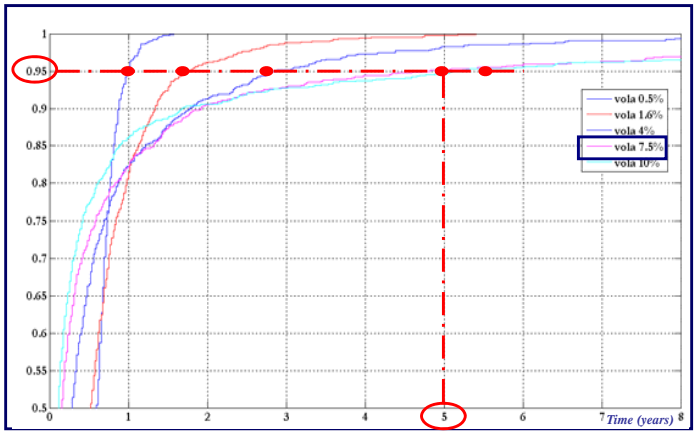
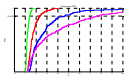


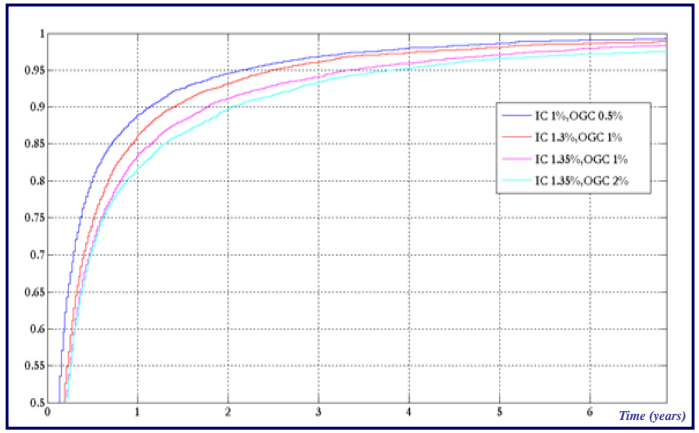
Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



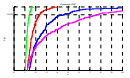
The recommended minimum investiment horizon is heavily depedent from the level of initial charges and ongoing costs:



Identification and representation of risk-reward by a three-pillars approach

3rd

The recommended minimum investment horizon



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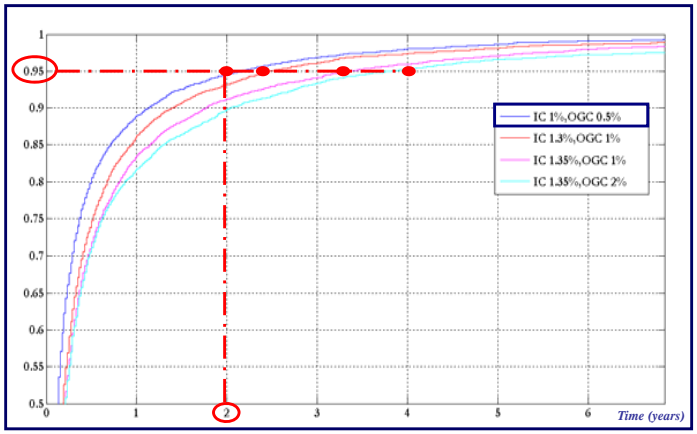

Identification and representation of risk-reward by a three-pillars approach

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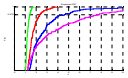

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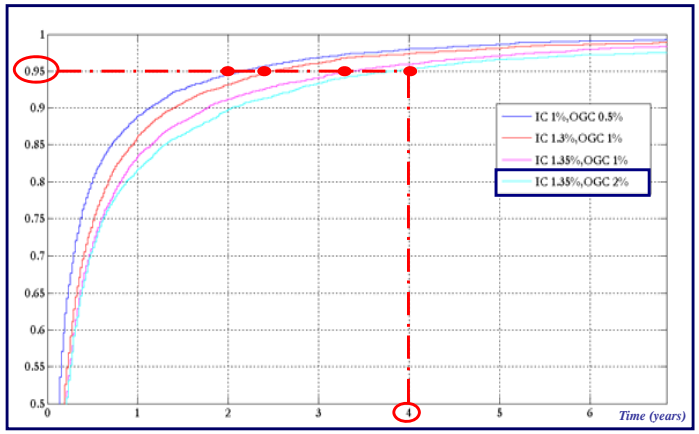
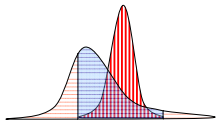


RETURNS | **RISKS** | **INVESTMENT HORIZON**

The key qualitative information is made objective by using a three-pillars approach based on quantitative measures.

1st

Table of probabilistic performance scenarios

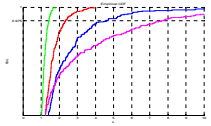


2nd

Synthetic Risk Indicator

3rd

The recommended minimum investment horizon



CONSOB
COMMISSIONE NAZIONALE PER LE SOCIETÀ E LA BORSA

Quantitative measures for a comprehensive approach to risks disclosure in structured products

Risk Europe Frankfurt, 04-05 June 2009