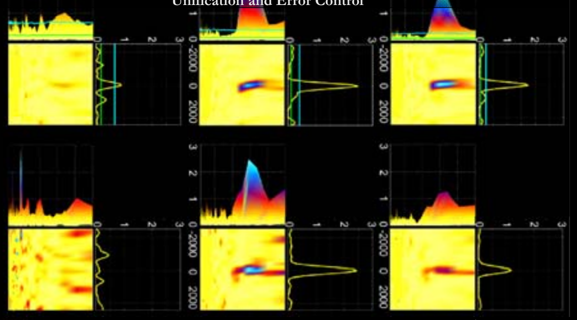
Fast Option Pricing using Non Uniform Discrete Fourier Transform

Unification and Error Control



Marcello Minenna - Paolo Verzella

Syllabus of the presentation

- Review of Option Pricing via DFT
  - FT Pricing formulae
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Option Pricing
  - FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
    - The Computational Framework: Speed, Stability, Accuracy
- Conclusions

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FT Pricing Formulas

Derivative Price C_t

Spot Price S_t

$$f_2(\ln S_T, \xi \mid \ln S_0) = \int_{-\infty}^{+\infty} e^{i\xi \ln S_T} q_2(\ln S_T \mid \ln S_0)\, d\ln S_T$$

under risk-neutral measure

A linear direct mapping from Fourier Spectral Space

FT

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CARR-MADAN REPRESENTATION

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A linear direct mapping from Fourier Spectral Space

FT

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i)}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

FT Pricing Formulas

Calibrating α

means choosing a dampened oscillating characteristic function

CARR-MADAN REPRESENTATION

FT Pricing Formulas

Calibrating α

means choosing a dampened oscillating characteristic function

Recent Developments:

Lee, 2004 - Journal of Computational Finance

Minenna, Verzella - Quant Congress 2006

Lord, Kahl, 2007 - Journal of Computational Finance

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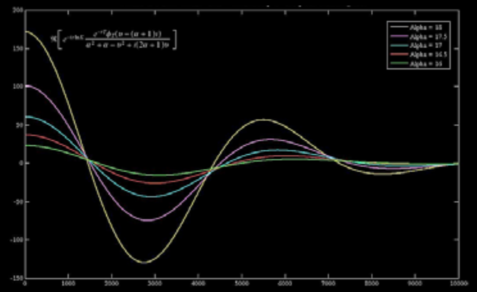
FT Pricing Formulas



CARR-MADAN REPRESENTATION

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under risk-neutral measure

Equivalent Representation in the complex plane

FT

LEWIS REPRESENTATION

FT Pricing Formulas

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$$f_2(\ln S_T, \xi | \ln S_0) = \int_{-\infty}^{+\infty} e^{i\xi \ln S_T} q_2(\ln S_T | \ln S_0)\, d\ln S_T$$

under risk-neutral measure

Equivalent Representation in the complex plane

FT

$$C_t(\ln K) = \Psi(\ln K,\alpha) + \frac{e^{-r(T-t)}}{2\pi}\int_{-\infty-i\alpha}^{\infty-i\alpha} e^{-iz[\ln K + r(T-t)]}\frac{\phi_T(z-i)}{-z(z-i)}dz$$

$$\Psi(\ln K,\alpha) = Se^{-r(T-t)}\cdot 1_{\{\alpha<0\}} - Ke^{-r(T-t)}\cdot 1_{\{\alpha\le -1\}} - \frac{1}{2}\left[Se^{-r(T-t)}\cdot 1_{\{\alpha=0\}} - Ke^{-r(T-t)}\cdot 1_{\{\alpha=-1\}}\right]$$

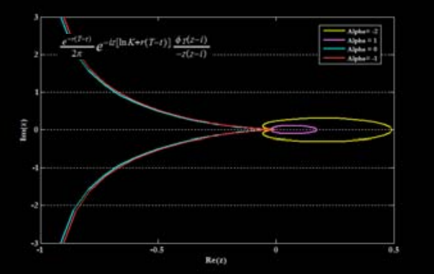
FT Pricing Formulas

Calibrating α

means choosing a fixed horizontal strip of integration in the complex plane

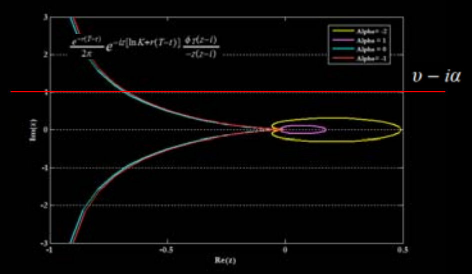
LEWIS REPRESENTATION

FT Pricing Formulas



LEWIS REPRESENTATION

FT Pricing Formulas

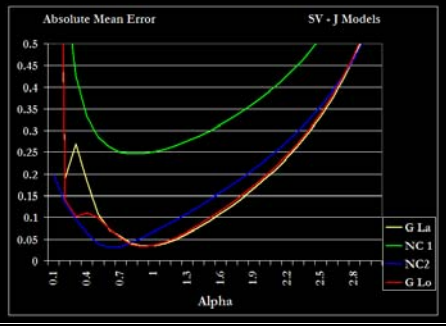


LEWIS REPRESENTATION

FT Pricing Formulas

Accuracy

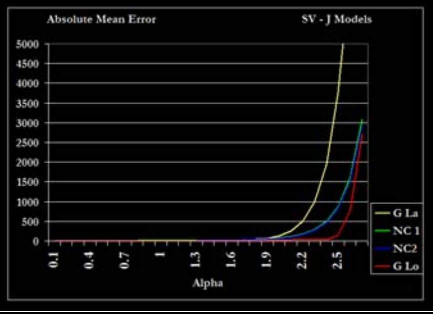
Absolute Mean Error computed w.r.t. α on an (σ, τ) space



FT Pricing Formulas

Stability

Absolute Mean Error computed w.r.t. α on an Extended (σ, τ) space



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DFT Convergence to FT

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{\lambda}x_j(m-1)} f(x_j) \quad \text{where } m = 1, 2, \ldots M$$

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad \text{where } m = 1,2,\ldots M$$

$$M \neq N$$

The Convergence Theorem for General DFT's (C Th)

$$\mathcal{F}[f(x)](t_m) = \lim_{N\to\infty} \sum_{j=1}^{N} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j, X)$$

$$t_m = \frac{2\pi}{X}(m-1)$$

C_0 via FT

Convergence theorems

$\overrightarrow{C_0}$ via DFT

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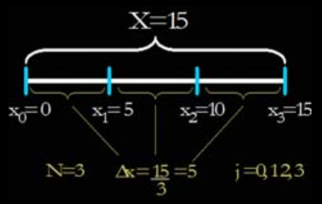
Condition 1

Uniform Discretization Grid



Condition 2

N=M

DFT specialized

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(n-1)} f(x_j) \quad \text{where } n=1,2,\ldots,N$$

Condition 1 Condition 2

DFT Simplified Formula

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{N}j(n-1)} f(x_j) \quad \text{where } n = 1,2,\ldots N$$

Nyquist – Shannon Limit (N-S)

$$\mathcal{F}[f(x)](t_n) = \lim_{N\to\infty} \frac{X}{N}\omega(n)$$

$\{t_n\}_{n=1..\frac{N}{2}}$ *for N even*

$\{t_n\}_{n=1..\frac{N+1}{2}}$ *for N odd*

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

Uniform Discretization Grids for *f*

1. $f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]$
2. $f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]\cdot[3+(-1)^{j+1} - \delta_j - \delta_{N-j}]$

1.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi} \right) d\xi$$

$$f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]$$

\+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{b} \cdot \Re(\omega(u))$$

2.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi} \right) d\xi$$

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\+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{3b} \cdot \Re(\omega(u))$$

Theorems of Equivalence

The Call Price computed via Convergence Theorem is equal to the Call Price computed via Trapezoid/Simpson Quadrature Rule

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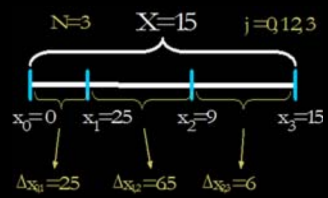
Condition 1

Non Uniform Discretization Grid



Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre

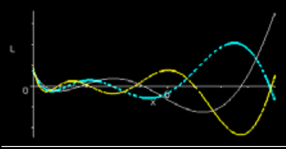
Gander Gautschi

Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre



Zeros of Laguerre Poynomials

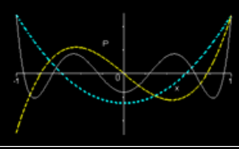
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Lobatto

Zeros of Legendre Poynomials



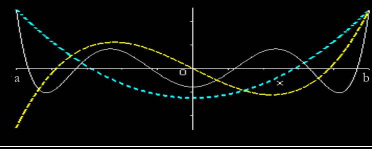
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gander Gautschi

Zeros of rescaled Legendre Poynomials



Condition 2

N≠M

General DFT

$$\omega(m)=\sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad \text{where } m=1,2,\ldots,2M$$

The Convergence Theorem for General DFT's (C Th)

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$$C_0=\frac{e^{-\alpha\ln K}}{\pi}\int_0^{+\infty}\Re\left(e^{i\xi\ln K}\frac{e^{-rT}f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

Gaussian Grids for *f*

1. $f(\upsilon_{j-1})=e^{\left[1+i\left(\frac{M\pi}{a^*}-\ln S_t\right)\right]\upsilon_{j-1}}\psi_0(\upsilon_{j-1})\ \frac{1}{L_{N+1}(\upsilon_{j-1})L'_N(\upsilon_{j-1})}$

2. $f\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)=\left[e^{-i\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)\left[\ln S_t-\frac{M\pi}{a}\right]}\psi_0\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)\right]\frac{1}{[P_{N-1}(\upsilon_{j-1})]^2}$

1.

$$C_0=\frac{e^{-\alpha\ln K}}{\pi}\int_0^{+\infty}\Re\left(e^{i\xi\ln K}\frac{e^{-rT}f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f(\upsilon_{j-1})=e^{\left[1+i\left(\frac{M\pi}{a^*}-\ln S_t\right)\right]\upsilon_{j-1}}\psi_0(\upsilon_{j-1})\ \frac{1}{L_{N+1}(\upsilon_{j-1})L'_N(\upsilon_{j-1})}$$

\+

C-Th

$$C_0([\ln K]^*_u)\ \approx\ -\Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a^*}+\frac{2\pi}{a^*}(u-1)\right)}}{\pi}\frac{1}{N+1}\cdot\omega^*(u)\right]$$

2.

$$C_0=\frac{e^{-\alpha\ln K}}{\pi}\int_0^{+\infty}\Re\left(e^{i\xi\ln K}\frac{e^{-rT}f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)=\left[e^{-i\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)\left[\ln S_t-\frac{M\pi}{a}\right]}\psi_0\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)\right]\frac{1}{[P_{N-1}(\upsilon_{j-1})]^2}$$

\+

C-Th

$$C_0([\ln K]^*_u)\ \approx\ \Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a^*}+\frac{2\pi}{a^*}(u-1)\right)}}{\pi}\frac{1}{N(N-1)}\cdot\omega^*\left(\frac{1}{2}a(1+\upsilon_{j-1})\right)\right]$$

Theorems of Equivalence

The Call Price computed via Convergence Theorem is equal to the Call Price computed via Gauss Laguerre/Gander Gautschi Quadrature Rule

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$\overrightarrow{C_t}$ via DFT

allows

Fast Fourier Trasform Algorithms

$\overrightarrow{C_t}$ via DFT

Newton-Cotes

Uniform FFT

$\overrightarrow{C_t}$ via DFT

Gauss

NonUniform FFT

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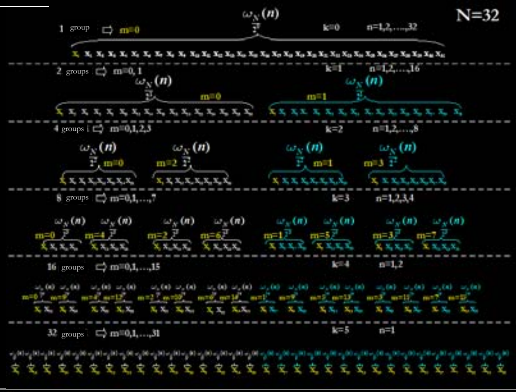
Cooley-Tukey DFT Characterization

$$\omega_2^m(n) = f(x_m) + W_2^{(n-1)} f(x_{m+N/2}) \quad for\ n = 1,2$$

Iterated Bottom – Up for N stages

It gives the FFT Cooley – Tukey Algorithm



FFT Cooley – Tukey Algorithm

The DFT computational cost drops

from

$O(N^2)$

to

$O(N \log_2 N)$

Since the Nyquist – Shannon Limit,
the pricing formulas

via FFT

Give accurate prices
ONLY
Around the Nyquist Frequency

Since the Nyquist – Shannon Limit,
the pricing formulas

via FFT

Give accurate prices
ONLY
Around the Nyquist Frequency

Approx. 25% of prices can be accepted

The Nyquist relation

$$\lambda = \frac{2\pi}{N}$$

via FFT

must hold between
the spectral and log-strike domain

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$$\lambda = \frac{2\pi}{N}$$

via FFT

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The grids cannot be independently choosen

The problem of
Nyquist relation
can be overcome

via FFT

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via FFT

ONLY using the
Fractional FFT - Chourdakis (2005)

The problem of
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at the cost of increasing complexity

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Gaussian Gridding

Gaussian Gridding

Step 1

Gaussian Gridding

Step 1
Gaussian Convolution of the non uniformly sampled characteristic function

Gaussian Gridding

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Gaussian Convolution of the non uniformly sampled characteristic function

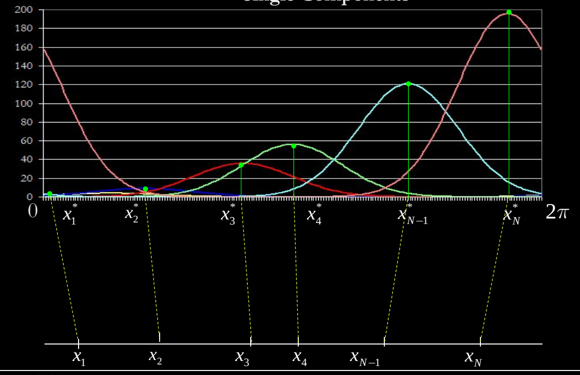
$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j - x - 2k\pi)^2}{4\tau}}$$

Gaussian Gridding

Step 1
Gaussian Convolution of the non uniformly sampled characteristic function

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j - x - 2k\pi)^2}{4\tau}}$$

Single Components



Gaussian Gridding

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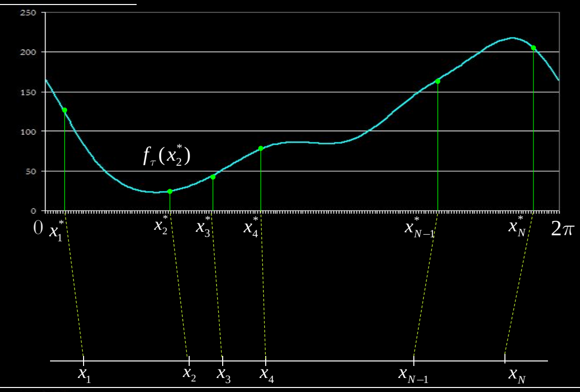


Gaussian Gridding

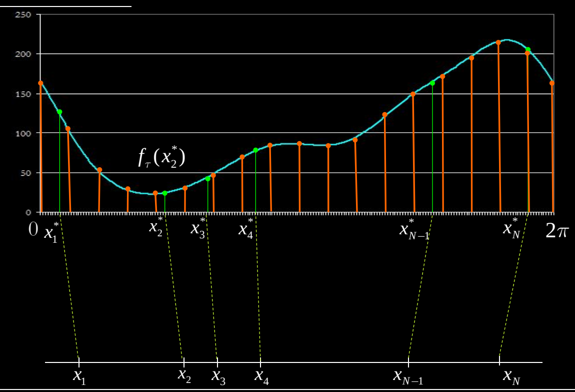
Step 2

Discretization on an uniform oversampled grid of $f_\tau(x)$

$$\widetilde{f}_\tau(y_m) = \sum_{j=0}^{N-1} f(x_j)\, \widetilde{K}(y_m, x_j, \sqrt{2\tau}; 2\pi)$$

where

$$\widetilde{K}(y_m, x_j, \sqrt{2\tau}; 2\pi) = \begin{cases} \sum_{k=-\infty}^{\infty} e^{-\frac{(\ldots - 2\pi k)^2}{4\tau}} \\ \text{oppure} \\ \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j^* - 2\pi\frac{m}{M_\tau} - 2\pi k)^2}{4\tau}} \end{cases}$$



Gaussian Gridding

Step 3

Computation of the Fourier Coefficient of $f_\tau(x)$ discretised

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \sum_{m=0}^{M_\tau - 1} \widetilde{f}_\tau\left(m\frac{2\pi}{M_\tau}\right) e^{-im\frac{2\pi}{M_\tau}(n-1)}$$

Gaussian Gridding

Step 4

NU-DFT representation of the Fourier Coefficient $F_\tau(n)$

$$\widetilde{\omega}(n) = \sqrt{\frac{\pi}{\tau}} e^{n^2\tau} F_\tau(n)$$

Gaussian Gridding

Step 5

DFT representation of the Fourier Coefficient $F_\tau(n)$

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \omega(n)$$

for $n = 1, 2, \ldots, \frac{M_\tau}{2}$

Gaussian Gridding

Step 6

NU-DFT derivation as a function of DFT

Gaussian Gridding

Step 6
NU-DFT derivation as a function of DFT

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau}\omega(n)$$

$$\tilde{\omega}(n) = \sqrt{\frac{\pi}{\tau}} e^{n^2\tau} F_\tau(n)$$

$$\tilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2\tau^*} \frac{1}{M_\tau}\omega(n)$$

for $n = 1, 2, \ldots, \frac{M_\tau}{2}$

Gaussian Gridding

Step 7
NU-FFT computation

Gaussian Gridding

Step 7
NU-FFT computation

$$\tilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2\tau^*} \frac{1}{M_\tau}\omega(n)$$

FFT

Gaussian Gridding

Step 7
NU-FFT computation

$$\tilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2\tau^*} \frac{1}{M_\tau}\omega(n)$$

NU-FFT ---------- FFT

Computational Cost

The major computational cost of the
Procedure is the FFT on the oversampled grid

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Choosing the oversampling ratio
$M_\tau = 2M$

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The major computational cost of the
Procedure is the FFT on the oversampled grid

Choosing the oversampling ratio
$M_\tau = 2M$

The total cost of the procedure is $\simeq 2M \log 2M$

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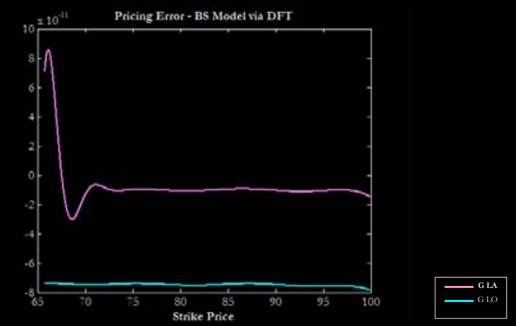
ACCURACY



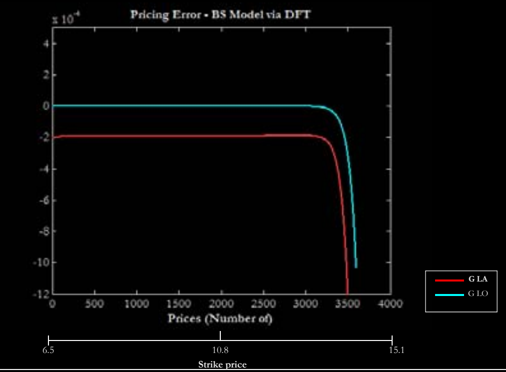
2000 Prices computed

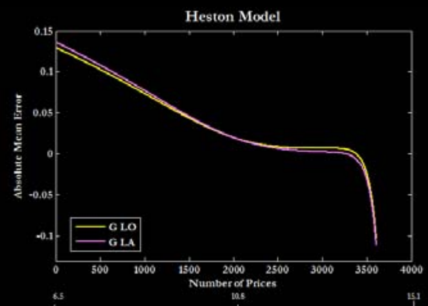
Absolute Mean Error	Black - Scholes	
Alpha 1.89	G Lobatto	G Laguerre
Tau 0.5	0.000384922943633	0.000384923944543
Tau 1	0.000387732230634	0.000387732232305
Tau 1.5	0.000591973600440	0.000591973602290
Tau 2	0.000597240904278	0.000597240928348
Tau 2.5	0.000603064341231	0.000603063896247
Tau 3	0.000609132394764	0.000609148906323
Tau 3.5	0.000615260627437	0.000615453604520
Tau 4	0.000621388197636	0.000624276301093
Tau 4.5	0.000627781650046	0.000662039720675
Tau 5	0.000634392287803	0.000757217939670

Absolute Mean Error	Black - Scholes	
Alpha 1.89	G Lobatto	G Laguerre
V 0.1	0.000602754964299	0.000384999424795
V 0.25	0.000584897741613	0.000384897740704
V 0.4	0.000384897746586	0.000384897747504
V 0.55	0.000584899533138	0.000384999532695
V 0.7	0.000384972304642	0.000384972204568
V 0.85	0.000385438279806	0.000385438281222
V 1	0.000586561399914	0.000386862402748
V 1.15	0.000389809729393	0.000389309732674
V 1.3	0.000393470665503	0.000393470774300
V 1.45	0.000599636201797	0.000399638827999

Weighted Absolute Mean Error

STABILITY

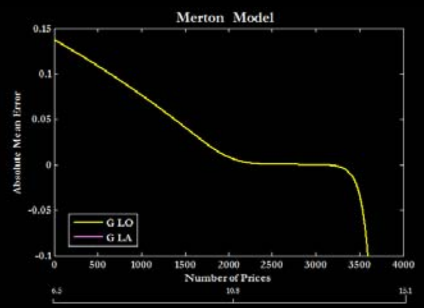




Absolute Mean Error	Heston $\rho = 0.5, \sigma = 0.1, \theta = 0.2, \kappa = 1.5$	
Alpha 1.01	G Lobatto	G Laguerre
Tau 0.5	0.203261249834126	0.206643694502080
Tau 1	0.467697723243352	0.485753402024063
Tau 1.5	0.696343815533637	0.747261110430642
Tau 2	0.833488166010773	0.942228866324220
Tau 2.5	0.966433188291042	1.029827463647480
Tau 3	1.019947918296090	0.967109842303122
Tau 3.5	1.344816409262500	0.718496902407399
Tau 4	1.853548224773260	0.580938209607741
Tau 4.5	2.698023770252460	0.553649681327516
Tau 5	3.527669167590610	1.631212045578270

Absolute Mean Error	Heston $\rho = 0.5, \sigma = 0.1, \theta = 0.2, \kappa = 1.5$	
Alpha 1.01	G Lobatto	G Laguerre
V 0.1	0.147730399468110	0.142722377204352
V 0.25	0.155857249400067	0.151260138597561
V 0.4	0.171936403925182	0.167902218395030
V 0.55	0.192410056990734	0.188536412450069
V 0.7	0.213529795635996	0.210174014069956
V 0.85	0.231570126049130	0.228214813934763
V 1	0.243104933587585	0.239749289632671
V 1.15	0.245011201669548	0.241655420417753
V 1.3	0.233972203913048	0.230617079312324
V 1.45	0.206211823004124	0.202855410204405

Weighted Absolute Mean Error



Absolute Mean Error	Merton $\lambda = 0.1, \mu = 0.1, \sigma = 0.1$	
Alpha 1.03	G Lobatto	G Laguerre
Tau 0.5	0.197726949941278	0.197726981416182
Tau 1	0.522196336061597	0.522196432234964
Tau 1.5	0.932062187975280	0.932062996627531
Tau 2	1.404395149215810	1.404398537922810
Tau 2.5	1.928130539602080	1.928129244375190
Tau 3	2.494696907909670	2.494670694481140
Tau 3.5	3.097427062903490	3.097431353211300
Tau 4	3.731015827279840	3.731142296371010
Tau 4.5	4.391299242353300	4.391521929678760
Tau 5	5.074887056777140	5.074989789421280

Absolute Mean Error	Merton $\lambda = 0.1, \mu = 0.1, \sigma = 0.1$	
Alpha 1.03	G Lobatto	G Laguerre
V 0.1	0.109539205124058	0.109554035372817
V 0.25	0.121771100302448	0.121771107322680
V 0.4	0.145036319944125	0.145036326108331
V 0.55	0.175289110981558	0.175289128224079
V 0.7	0.209392928147416	0.209392969939966
V 0.85	0.245382156967865	0.245382630825178
V 1	0.282531302622102	0.282531560078724
V 1.15	0.319429963153448	0.319429660594360
V 1.3	0.355954458247725	0.355938929678383
V 1.45	0.391915427517519	0.391902853930703

Weighted Absolute Mean Error

STABILITY

The error of **90%** of prices computed lies in the

STABILITY

The error of 90% of prices computed lies in the

10^{-2}

RANGE OF PRECISION

SPEED

SPEED

the NU – FFT is around 2 time slower than FFT

SPEED

At very low time scales, the differences disappear

SPEED

At very low time scales, the differences disappear

FFT	NC2 0.01 sec.	G - LA N/A	G - LO N/A
NU – FFT	NC2 0.02 sec.	G - LA 0.0261 sec.	G - LO 0.0301 sec.

Computation of 4000 prices on a Centrino 1600Mhz – 2gb RAM
Mean Value over 1000 runs

- Review of Option Pricing via DFT
  - FT Pricing formulae
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Option Pricing
  - Uniform FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
    - The Computational Framework: Speed, Stability, Accuracy
- Conclusions

- NU – FFT allows the use of Gaussian Grids

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- NU – FFT is more stable than FFT

Conclusions

- NU – FFT allows the use of Gaussian Grids
- NU – FFT is indifferent to Nyquist _Shannon Limit
- NU – FFT does not need the Nyquist relation
- NU – FFT is at least as accurate as FFT
- NU – FFT is more stable than FFT
- NU – FFT speed performances are indistinguishable from FFT's ones

Conclusions

NU – FFT
is a natural candidate for
operational use on trading desks