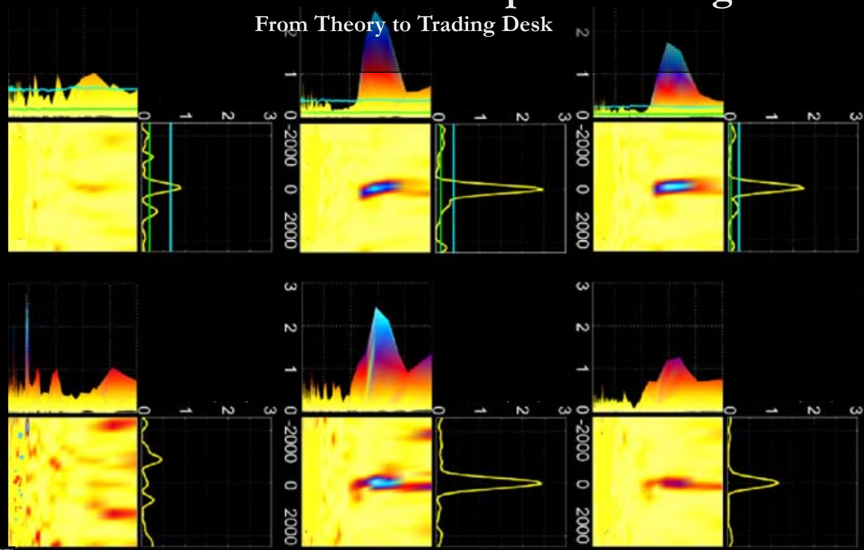
DFT methods for Fast Option Pricing

From Theory to Trading Desk



Marcello Minenna - Paolo Verzella

International Summer School on Risk Measurement and Control,
University La Sapienza, Rome 16th June 2007

Syllabus of the presentation

- Option Pricing via DFT
  - FT Pricing formula
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Option Pricing
  - FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
    - The Computational Framework: Speed, Stability, Accuracy
- Conclusions

Syllabus of the presentation

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FT Pricing Formulas

European Call Price C_t

Spot Price S_t

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln K)$$

under different martingale measures

FT Pricing Formulas

European Call Price C_t

Spot Price S_t

$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln K)$

under different martingale measures

$$C_t$$

FT Pricing Formulas

European Call Price C_t

Spot Price S_t

$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln K)$

under different martingale measures

$$C_t$$

PDE

$$C_t = h\left[P_1(\Theta), P_2(\Theta)\right]$$

FT Pricing Formulas

European Call Price C_t

Spot Price S_t

$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln K)$

under different martingale measures

$$C_t$$

PDE

$$C_t = h\left[P_1(\Theta), P_2(\Theta)\right]$$

FT

$$C_t = u\left\{g^{-1}\left[P_2(\Theta, \alpha)\right]\right\}$$

FT Pricing Formulas

European Call Price C_t

Spot Price S_t

$f_j(\ln S_T, \xi \,|\, \ln S_t) = \int_{-\infty}^{+\infty} e^{i\xi \ln S_T} q_j(\ln S_T \,|\, \ln S_t) d\ln S_T$

under different martingale measures

$$C_t$$

PDE

$$C_t = h\left[P_1(f_1), P_2(f_2)\right]$$

FT

$$C_t = u\left(f_2\right)$$

European Call Price C_t

Spot Price S_t

$$f_2(\ln S_T,\xi \mid \ln S_t) = \int_{-\infty}^{+\infty} e^{i\xi \ln S_T} q(\ln S_T \mid \ln S_t)\, d\ln S_T$$

Risk-neutral measure

$$C_t$$

PDE | **FT**

$$C_t = h\left[P_1(\Lambda_{1,2} f_2), P_2(f_2)\right] \qquad C_t = u(f_2)$$

$$\Lambda_{1,2} = \frac{S_T}{e^{r(T-t)} S_t}$$

$$C_t = S_t\left[\frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} \Re\left(\frac{e^{i\xi \ln S_T}}{i\xi} f_1\right) d\xi\right] - Ke^{-r(T-t)}\left[\frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} \Re\left(\frac{e^{i\xi \ln S_T}}{i\xi} f_2\right) d\xi\right]$$

PDE

$$C_t$$

$$C_t$$

FT

$$C_t = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right) d\xi$$

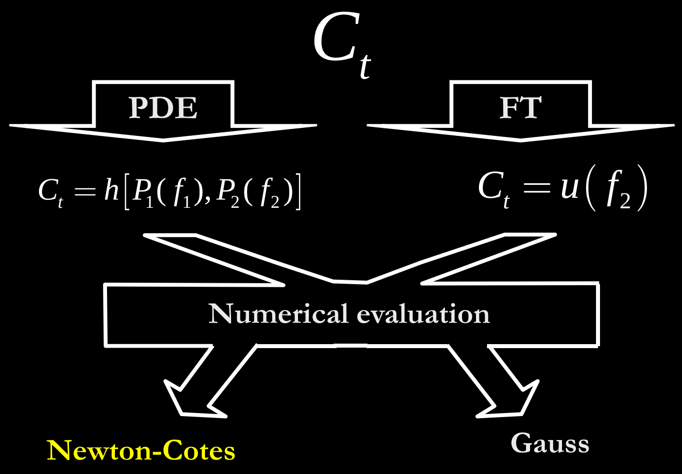
$$C_t$$

PDE | **FT**

$$C_t = h\left[P_1(f_1), P_2(f_2)\right] \qquad C_t = u(f_2)$$

Numerical evaluation

Newton-Cotes | **Gauss**



Newton-Cotes schemes compute C_t
Trapezoid rule

$$C_t \approx \left(\frac{1}{2}+\frac{1}{\pi}\right)\left[S_t \sum_{j=0}^{N} \frac{1}{2} a_j^N \cdot \Re\left(\frac{e^{-ij\frac{a}{N}\ln K}}{i(j+\varepsilon\delta_j)} f_1\left(j\frac{a}{N}\right)\right) - Ke^{-r(T-t)} \sum_{j=0}^{N} \frac{1}{2} a_j^N \cdot \Re\left(\frac{e^{-ij\frac{a}{N}\ln K}}{i(j+\varepsilon\delta_j)} f_2\left(j\frac{a}{N}\right)\right)\right]$$

PDE

Newton-Cotes schemes compute C_t
Trapezoid rule

Newton-Cotes schemes compute C_t
Trapezoid rule

FT

$$C_t \approx \frac{e^{-\alpha \ln K}}{2\pi} \frac{a}{N} \Re\left[\sum_{j=0}^{N} e^{-ijh\ln K} f_2\left(j\frac{a}{N}\right)\right]$$

Newton-Cotes schemes compute C_t

Simpson rule

$$C_t \approx \left(\frac{1}{2}+\frac{1}{\pi}\right)\left[S_t \sum_{j=0}^{N}\left[3+(-1)^{j+1}-\delta_j\right]\cdot \Re\left(\frac{e^{-ij\frac{a}{N}\ln K}}{i\left(j+\varepsilon\delta_j\right)} f_1\left(j\frac{a}{N}\right)\right) - K e^{-r(T-t)}\sum_{j=0}^{N}\left[3+(-1)^{j+1}-\delta_j\right]\cdot \Re\left(\frac{e^{-ij\frac{a}{N}\ln K}}{i\left(j+\varepsilon\delta_j\right)} f_2\left(j\frac{a}{N}\right)\right)\right]$$

PDE

Newton-Cotes schemes compute C_t

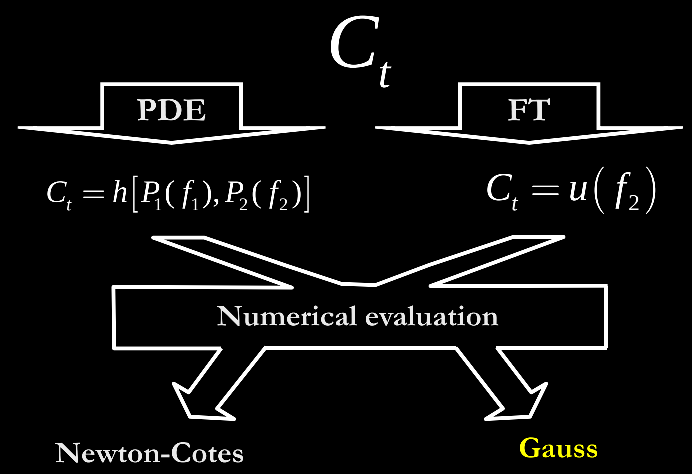
Simpson rule

Newton-Cotes schemes compute C_t

Simpson rule

FT

$$C_t \approx \frac{e^{-\alpha \ln K}}{3\pi}\frac{a}{N}\Re\left[\sum_{j=0}^{N}\left[3+(-1)^{j+1}-\delta_j\right]\cdot e^{-ijh\ln K} f_2\left(j\frac{a}{N}\right)\right]$$



Gauss schemes compute C_t

Gauss-Lobatto rule

$$C_t \approx S_t\left(\frac{1}{2}+\frac{1}{\pi}\right)a\left[\frac{1}{N(N-1)}\Re\left(\frac{e^{-i\varepsilon\ln K}}{i\varepsilon}f_1(\varepsilon)\right)+\Re\left(\frac{e^{-ia\ln K}}{ia}f_1(a)\right)+\right.$$
$$\left.+\sum_{j=2}^{N-1}\frac{1}{N(N-1)\left[P_{N-1}(\xi_j)\right]^2}\cdot\Re\left(\frac{e^{-i\left(\frac{1}{2}a(1+\xi_j)\right)\ln K}}{i\left(\frac{1}{2}a(1+\xi_j)\right)}f_1\left(\frac{1}{2}a(1+\xi_j)\right)\right)\right]-$$
$$-Ke^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\right)a\left[\frac{1}{N(N-1)}\Re\left(\frac{e^{-i\varepsilon\ln K}}{i\varepsilon}f_2(\varepsilon)\right)+\Re\left(\frac{e^{-ia\ln K}}{ia}f_2(a)\right)+\right.$$
$$\left.+\sum_{j=2}^{N-1}\frac{1}{N(N-1)\left[P_{N-1}(\xi_j)\right]^2}\cdot\Re\left(\frac{e^{-i\left(\frac{1}{2}a(1+\xi_j)\right)\ln K}}{i\left(\frac{1}{2}a(1+\xi_j)\right)}f_2\left(\frac{1}{2}a(1+\xi_j)\right)\right)\right]$$

PDE (↑)

Gauss schemes compute C_t

Gauss-Lobatto rule

Gauss schemes compute C_t

Gauss-Lobatto rule

FT (↓)

$$C_t \approx \frac{e^{-\alpha\ln K}}{\pi}a\left[\frac{1}{N(N-1)}\Re\left(f_2(0)\right)+\Re\left(e^{-ia\ln K}f_2(a)\right)+\right.$$
$$\left.+\sum_{j=2}^{N-1}\frac{1}{N(N-1)\left[P_{N-1}(\xi_j)\right]^2}\cdot\Re\left(e^{-i\left(\frac{1}{2}a(1+\xi_j)\right)\ln K}f_2\left(\frac{1}{2}a(1+\xi_j)\right)\right)\right]$$

$$C_t \approx -S_t\left(\frac{1}{2N}+\frac{1}{N\pi}\right)\sum_{j=1}^{N}\frac{1}{L_{N-1}(\xi_j)L'_N(\xi_j)}\Re\left(\frac{e^{-\xi_j(i\ln K-1)}}{i\xi_j}f_1(\xi_j)\right)+$$
$$+Ke^{-r(T-t)}\left(\frac{1}{2N}+\frac{1}{N\pi}\right)\sum_{j=1}^{N}\frac{1}{L_{N-1}(\xi_j)L'_N(\xi_j)}\Re\left(\frac{e^{-\xi_j(i\ln K-1)}}{i\xi_j}f_2(\xi_j)\right)$$

PDE (↑)

Gauss schemes compute C_t

Gauss-Laguerre rule

Gauss schemes compute C_t

Gauss-Laguerre rule

FT

$$C_t \approx -\frac{e^{-\alpha \ln K}}{N\pi} \sum_{j=1}^{N} \frac{1}{L_{N-1}(\xi_j) L'_{N-1}(\xi_j)} \cdot \Re\left(e^{-\xi_j (i \ln K - 1)} f_2(\xi_j)\right)$$

C_t via FT

$$C_t = u\left\{g^{-1}\left[P_2(\Theta, \alpha)\right]\right\}$$

calibration of α

by minimizing

C_t via PDE

$$C_t = h\left[P_1(\Theta), P_2(\Theta)\right]$$

C_t via FT

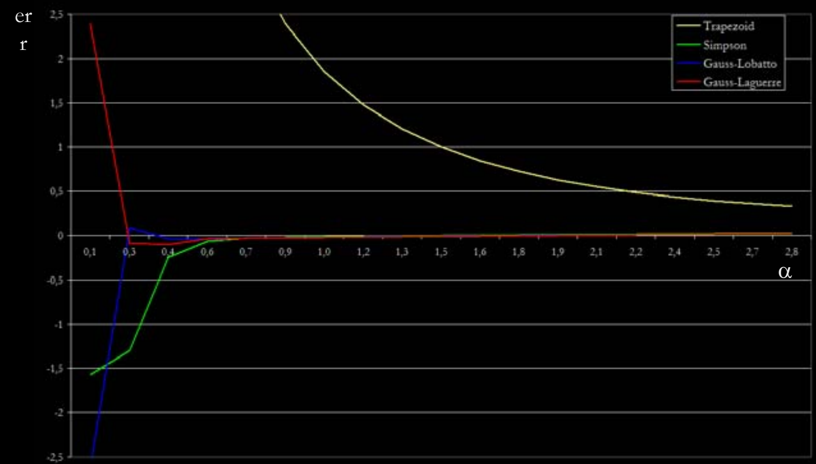
$$C_t = u\left\{g^{-1}\left[P_2(\Theta, \alpha)\right]\right\}$$

calibration of α

C_t via FT - spanning Θ, α

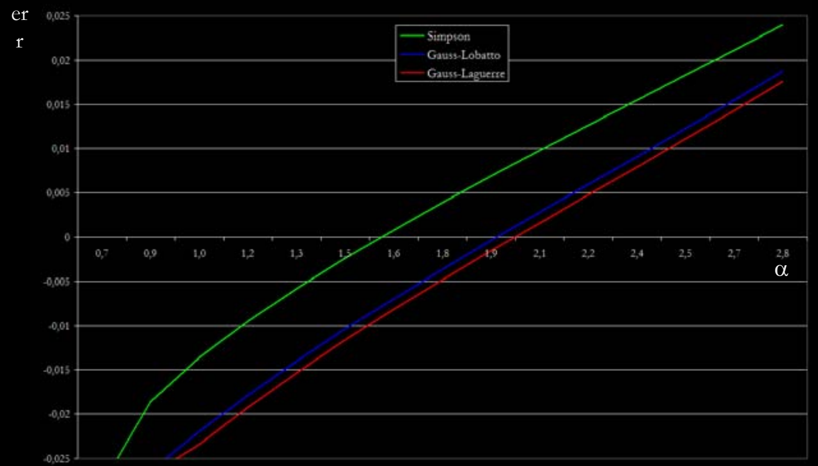
Stochastic volatility models

Low volatility | Short maturity



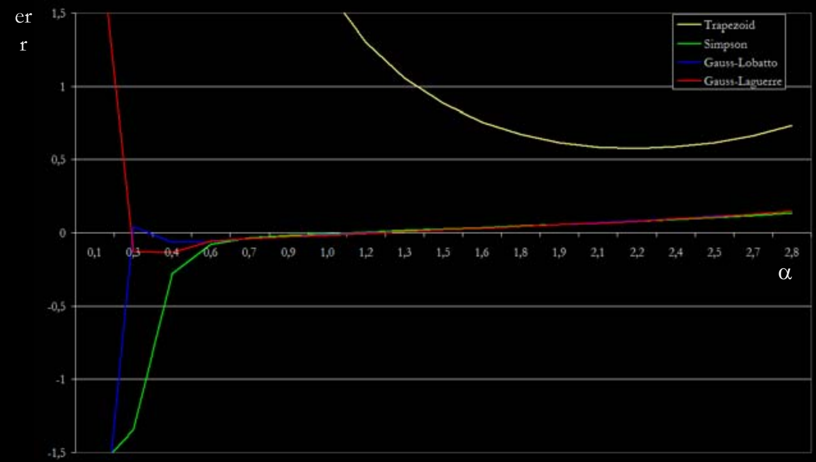
C_t via FT - spanning Θ, α
Stochastic volatility models

Low volatility **zoom** Short maturity



C_t via FT - spanning Θ, α
Stochastic volatility models

High volatility Short maturity



C_t via FT - spanning Θ, α
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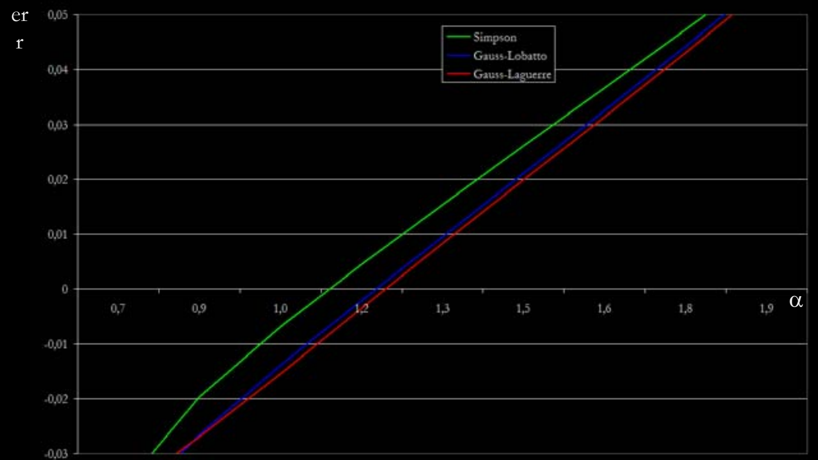
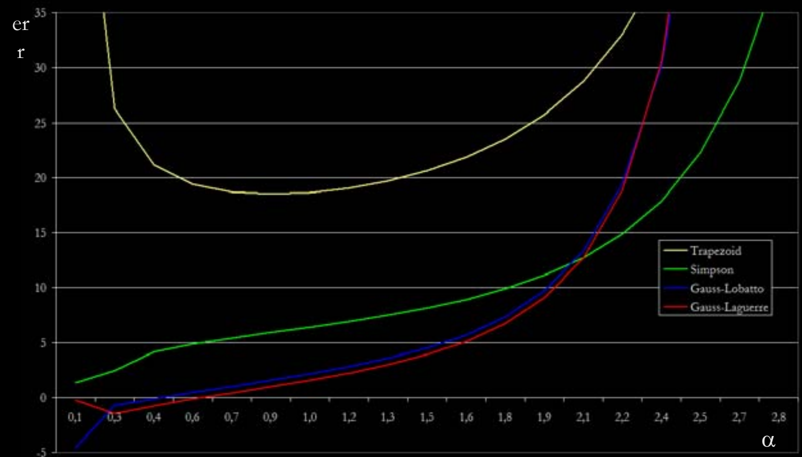
High volatility **zoom** Short maturity



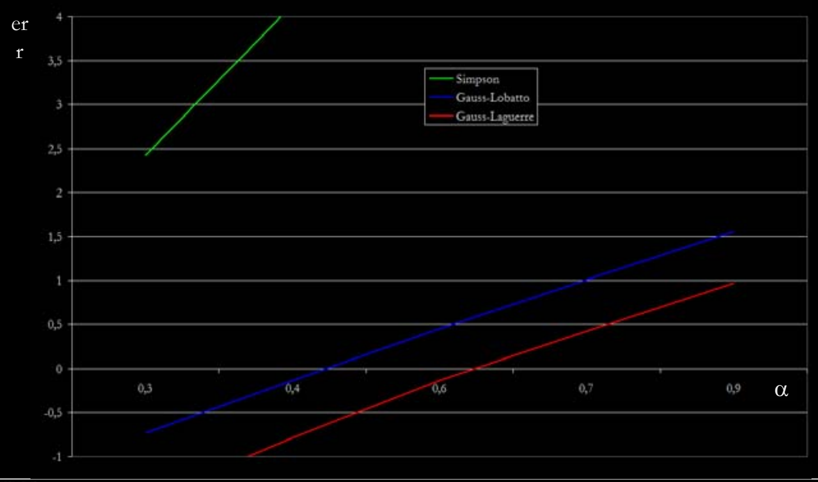
C_t via FT - spanning Θ, α
Stochastic volatility models

Low volatility Long maturity



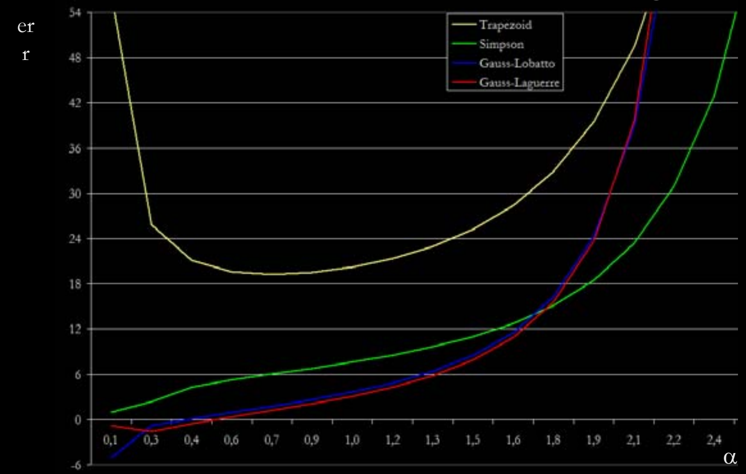
C_t via FT - spanning Θ, α
Stochastic volatility models

Low volatility — zoom — Long maturity



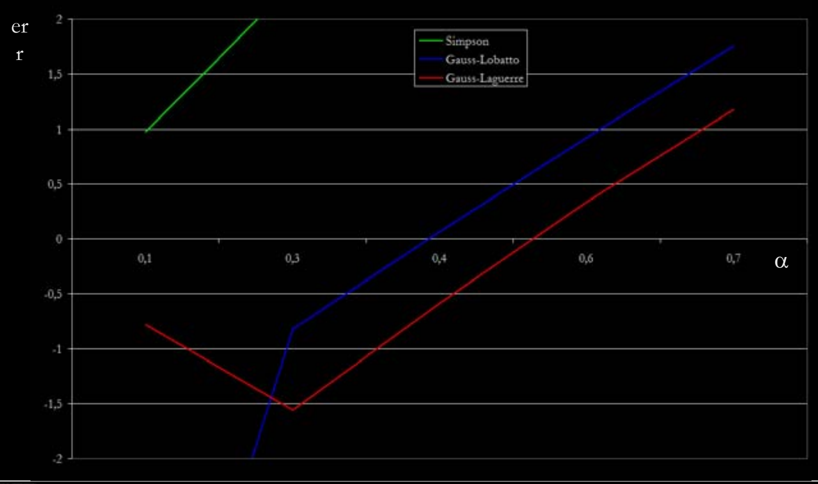
C_t via FT - spanning Θ, α
Stochastic volatility models

High volatility — Long maturity



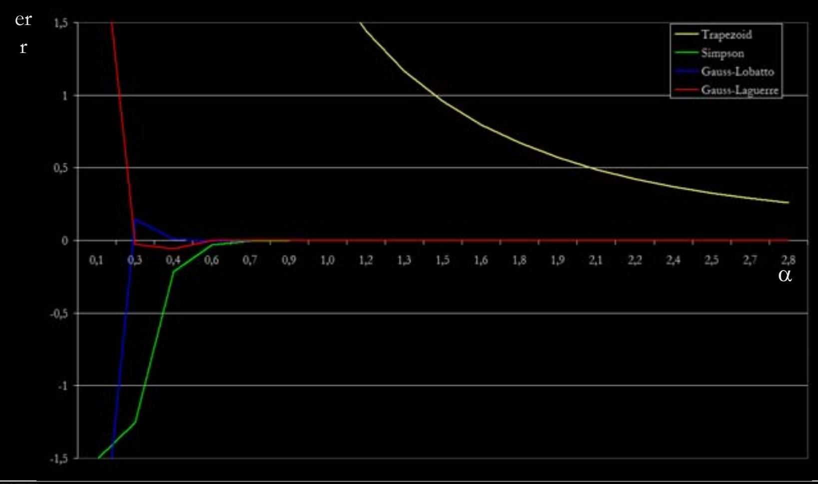
C_t via FT - spanning Θ, α
Stochastic volatility models

High volatility — zoom — Long maturity



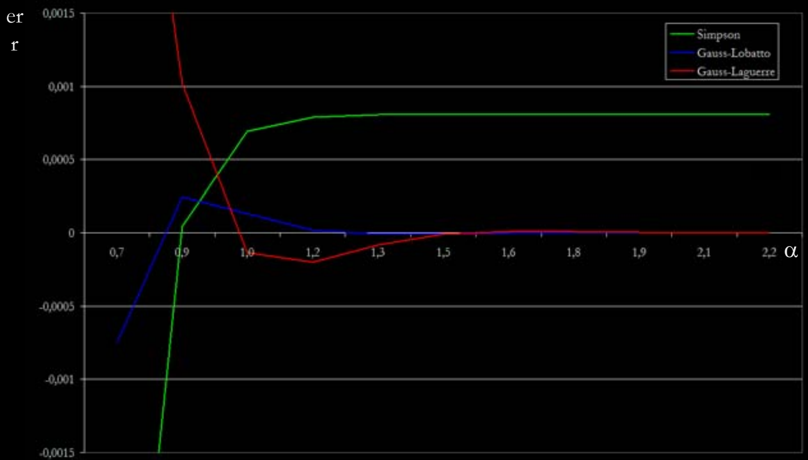
C_t via FT - spanning Θ, α
Jump Diffusion models

Low volatility — Short maturity



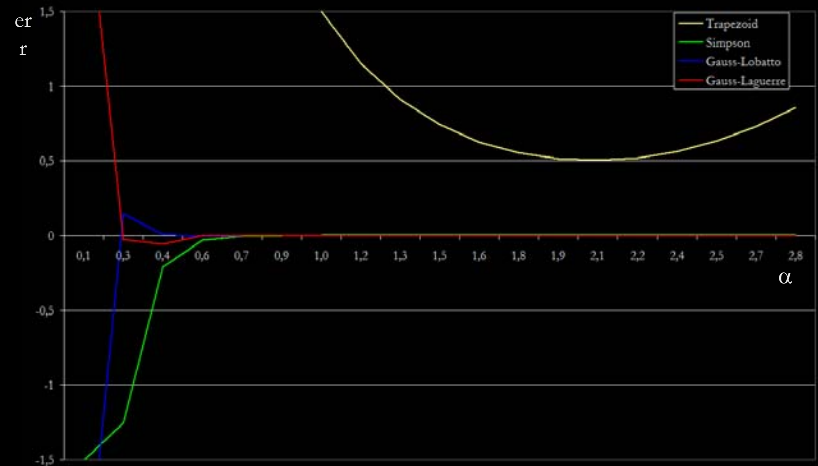
C_t via FT - spanning Θ, α Jump Diffusion models

Low volatility zoom Short maturity



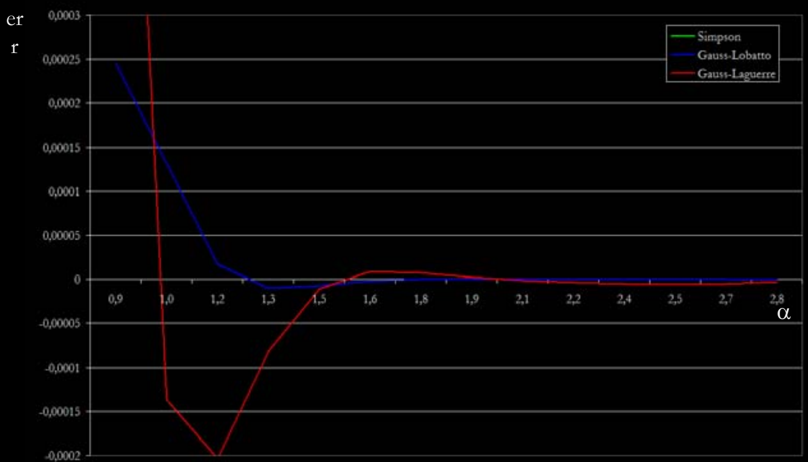
C_t via FT - spanning Θ, α Jump Diffusion models

High volatility Short maturity



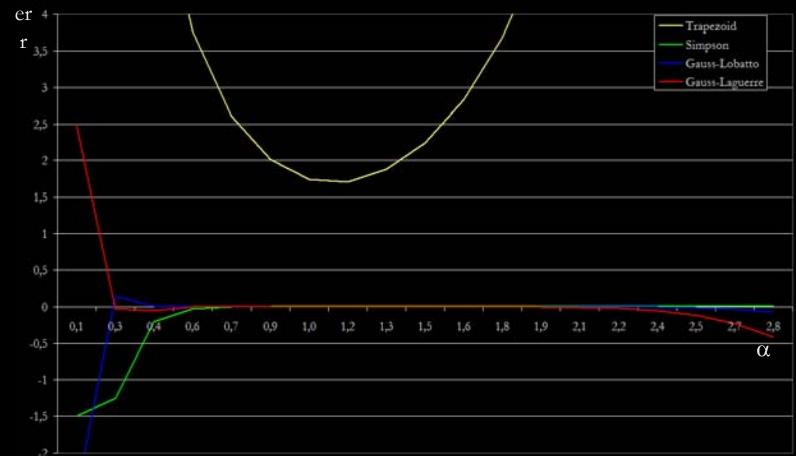
C_t via FT - spanning Θ, α Jump Diffusion models

High volatility zoom Short maturity



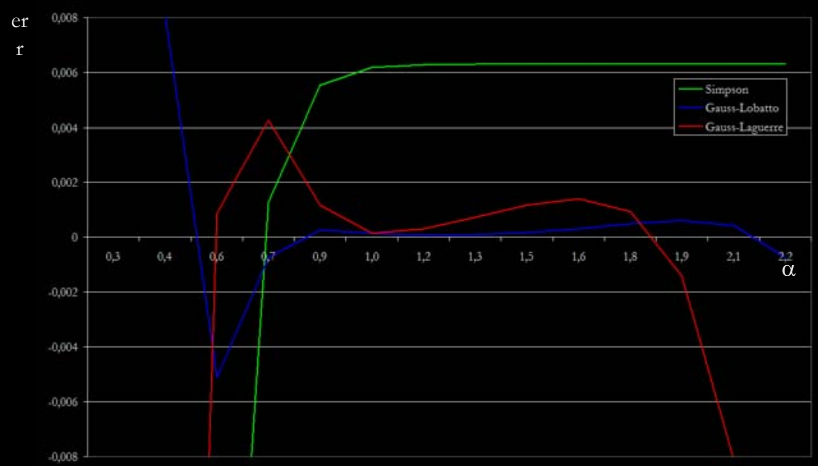
C_t via FT - spanning Θ, α Jump Diffusion models

Low volatility Long maturity



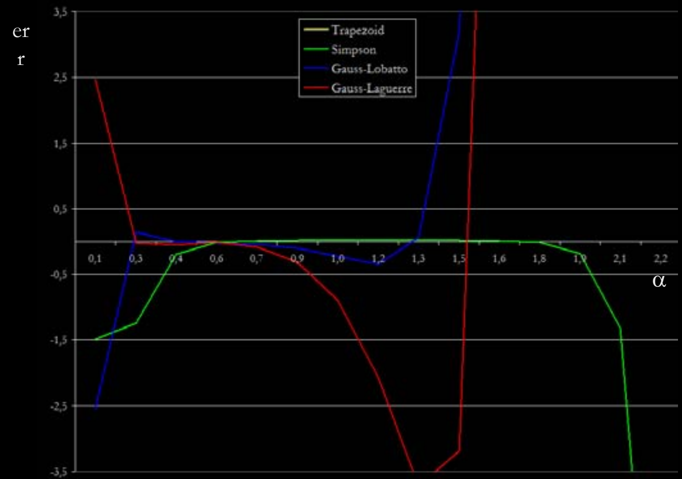
C_t via FT - spanning Θ, α
Jump Diffusion models

Low volatility **zoom** Long maturity



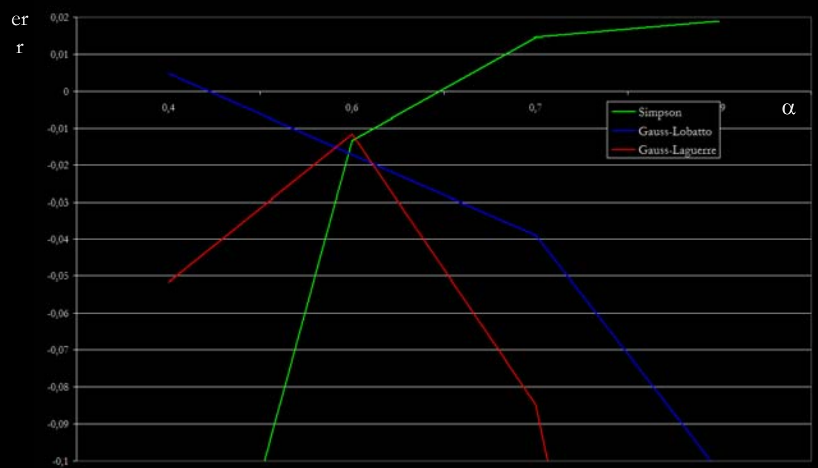
C_t via FT - spanning Θ, α
Jump Diffusion models

High volatility Long maturity



C_t via FT - spanning Θ, α
Jump Diffusion models

High volatility **zoom** Long maturity



FT Pricing Formulas

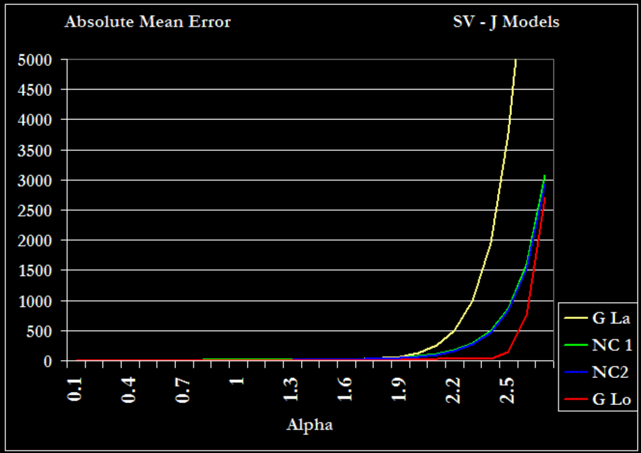
Accuracy

Absolute Mean Error computed w.r.t. α on an (σ, τ) space

FT Pricing Formulas

Stability

Absolute Mean Error computed w.r.t. α on an Extended (σ,τ) space



CONSOB

C_t via FT - spanning Θ,α
Stochastic volatility models

MATURITY \ VOLATILITY	LOW	HIGH
SHORT	Simpson [α≈(1,5;1,6)] Gauss-Lobatto [α≈(1,9;2)] Gauss-Laguerre [α≈(1,9;2)]	Simpson [α≈(1,05;1,1)] Gauss-Lobatto [α≈(1,15;1,2)] Gauss-Laguerre [α≈(1,15;1,2)]
LONG	Gauss-Lobatto [α≈(0,35;0,45)] Gauss-Laguerre [α≈(0,6;0,65)]	Gauss-Lobatto [α≈(0,4;0,45)] Gauss-Laguerre [α≈(0,5;0,55)]

CONSOB

C_t via FT - spanning Θ,α
Jump diffusion models

MATURITY \ VOLATILITY	LOW	HIGH
SHORT	Gauss-Lobatto [α≈(1,4;2)] Gauss-Laguerre [α≈(1,55;2)]	Gauss-Lobatto [α≈(1,8;2,2)] Gauss-Laguerre [α≈(1,8;2,2)]
LONG	Gauss-Lobatto [α≈(0,9;1,2)] Gauss-Laguerre [α≈(0,95;1,05)]	Simpson [α≈(0,6;0,7)] Gauss-Lobatto [α≈(0,35;0,45)] Gauss-Laguerre [α≈(0,6)]

CONSOB

Syllabus of the presentation

- **Option Pricing via DFT**
  - FT Pricing formula
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids

CONSOB

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1,2,\ldots M$$

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1,2,\ldots M$$

$$M \neq N$$

The Convergence Theorem (C Th)

$$\mathcal{F}[f(x)](t_m) = \lim_{N\to\infty} \frac{X}{N}\omega(m)$$

$$t_m = \frac{2\pi}{X}(m-1)$$

C_0 via FT

Convergence theorems

$\overrightarrow{C_0}$ via DFT

- **Option Pricing via DFT**
  - FT Pricing formula
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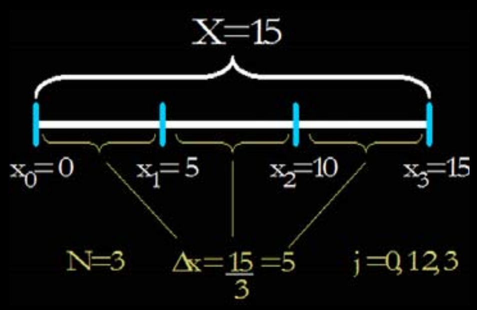
Condition 1

Uniform Discretization Grid



Condition 2

N=M

DFT specialized

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(n-1)} f(x_j) \qquad \textit{where } n=1,2,\ldots,N$$

Condition 1 Condition 2

DFT Simplified Formula

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{N}j(n-1)} f(x_j) \quad \textit{where } n = 1,2,\ldots N$$

Nyquist – Shannon Limit (N-S)

$$\mathcal{F}[f(x)](t_n) = \lim_{N\to\infty} \frac{X}{N}\omega(n)$$

$$\{t_n\}_{n=1..\frac{N}{2}} \qquad \text{for } N \text{ even}$$

$$\{t_n\}_{n=1..\frac{N+1}{2}} \qquad \text{for } N \text{ odd}$$

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

Uniform Discretization Grids for *f*

1. $f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\psi_0((j-1)\eta)$

2. $f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\psi_0((j-1)\eta) \cdot \left(3+(-1)^j - \delta_{j-1}\right)$

1.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\psi_0((j-1)\eta)$$

+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{b} \cdot \Re(\omega(u))$$

2.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f(v_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\psi_0((j-1)\eta) \cdot \left(3+(-1)^j - \delta_{j-1}\right)$$

+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{3b} \cdot \Re(\omega(u))$$

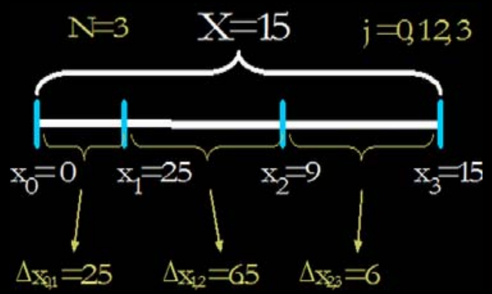
Theorems of Equivalence

The Call Price computed via Convergence Theorem is equal to the Call Price computed via Trapezoid/Simpson Quadrature Rule

- Option Pricing via DFT
  - FT Pricing formula
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids

Condition 1

Non Uniform Discretization Grid



Condition 1

Gaussian Grids

Optimal choice of discretization points

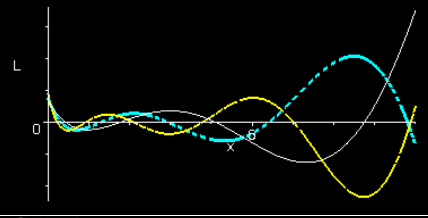
Gauss Laguerre — Gander Gautschi

Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre



Zeros of
Laguerre Poynomials

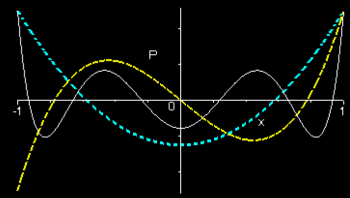
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Lobatto

Zeros of
Legendre Poynomials



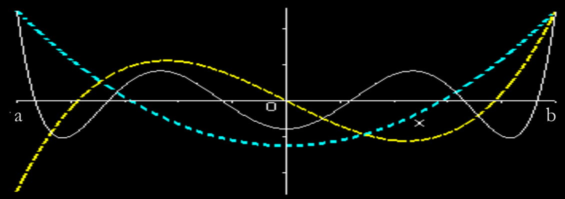
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gander Gautschi

Zeros of rescaled
Legendre Poynomials



Condition 2

N≠M

General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \qquad \omega\eta\epsilon\rho\epsilon\ \mu=1,2,\ldots,M$$

The Convergence Theorem (C Th)

$$\mathcal{F}[f(x)](t_m) = \lim_{N\to\infty} \frac{X}{N}\omega(m)$$

$$t_m = \frac{2\pi}{X}(m-1)$$

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

Gaussian Grids for *f*

1. $f(\upsilon_{j-1}) = e^{\left[1+i\left(\frac{M\pi}{a*}-\ln S_t\right)\right]\upsilon_{j-1}}\psi_0(\upsilon_{j-1}) \quad \frac{1}{L_{N+1}(\upsilon_{j-1})L'_N(\upsilon_{j-1})}$

2. $f(\frac{1}{2}a(1+\upsilon_{j-1})) = e^{[1+i(\frac{M\pi}{a*}-\ln S_t)]\frac{1}{2}a(1+\upsilon_{j-1})}\psi_0(\frac{1}{2}a(1+\upsilon_{j-1})) \quad \frac{1}{[P_{N-1}(\upsilon_{j-1})]^2}$

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\+

C-Th

$$C_0([\ln K]_u^*) \approx -\Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a*}+\frac{2\pi}{a*}(u-1)\right)}}{\pi}\frac{1}{N+1}\cdot\omega^*(u)\right]$$

2.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi$$

$$f(\tfrac{1}{2}a(1+\upsilon_{j-1})) = e^{[1+i(\frac{M\pi}{a*}-\ln S_t)]\frac{1}{2}a(1+\upsilon_{j-1})}\psi_0(\tfrac{1}{2}a(1+\upsilon_{j-1})) \quad \frac{1}{[P_{N-1}(\upsilon_{j-1})]^2}$$

\+

C-Th

$$C_0([\ln K]_u^*) \approx \Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a*}+\frac{2\pi}{a*}(u-1)\right)}}{\pi}\frac{1}{N(N-1)}\cdot\omega^*(\tfrac{1}{2}a(1+\upsilon_{j-1}))\right]$$

Theorems of Equivalence

The Call Price computed via Convergence Theorem is equal to the Call Price computed via Gauss Laguerre/Gander Gautschi Quadrature Rule

- Option Pricing via DFT
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- Conclusions

$\overrightarrow{C_t}$ via DFT

allows

Fast Fourier Trasform Algorithms

$\overrightarrow{C_t}$ via DFT

Newton-Cotes

Uniform FFT

$\overrightarrow{C_t}$ via DFT

Gauss

NonUniform FFT

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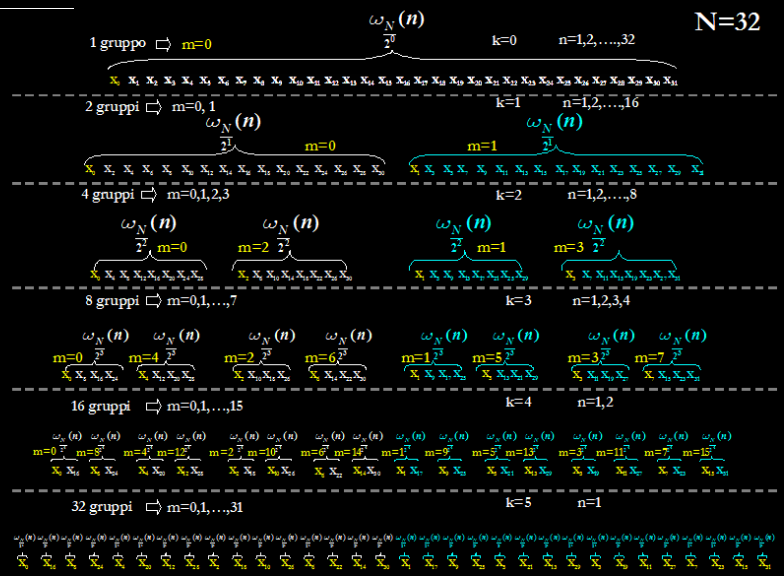
Cooley-Tukey DFT Characterization

$$\omega_2^m(n) = f(x_m) + W_2^{(n-1)} f(x_{m+N/2}) \quad for\ n = 1,2$$

Iterated Bottom – Up for N stages

It gives the FFT Cooley – Tukey Algorithm



FFT Cooley – Tukey Algorithm

The DFT computational cost drops

from

$O\left(N^2\right)$

to

$O\left(N \log_2 N\right)$

Since the Nyquist – Shannon Limit,
the pricing formulas

via FFT

Give accurate prices
ONLY
Around the Nyquist Frequency

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via FFT

Give accurate prices
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Approx. 25% of prices can be accepted

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Gaussian Gridding

Gaussian Gridding

Step 1

Gaussian Gridding

Step 1

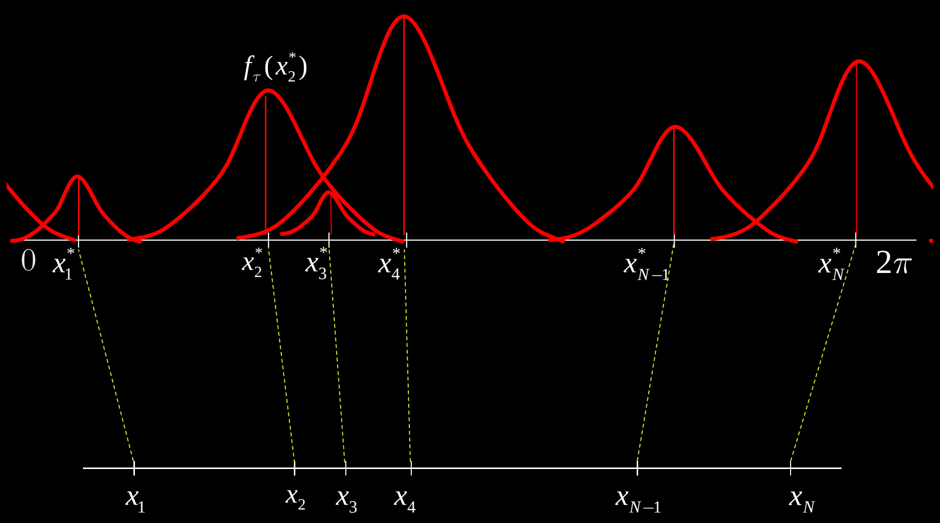
Gaussian Projection of the non uniformly sampled characteristic function on a oversampled uniform grid

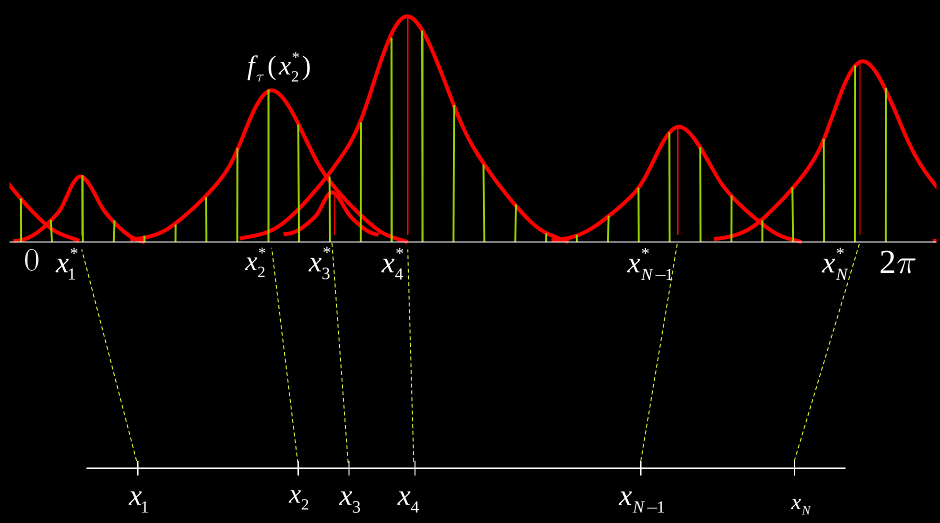
Gaussian Gridding

Step 1

Gaussian Projection of the non uniformly sampled characteristic function on a oversampled uniform grid

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j - x - 2k\pi)^2}{4\tau}}$$





Gaussian Gridding

⇓

Step 2

Gaussian Gridding

⇓

Step 2

FFT computation on the oversampled grid of the Fourier Coefficient of the reprojected characteristic function

Gaussian Gridding

Step 2

FFT computation on the oversampled grid of the Fourier Coefficient of the reprojected characteristic function

$$F_\tau(n) = \frac{1}{2\pi}\int_0^{2\pi} f_\tau(x)e^{-ix(n-1)}dx$$

Gaussian Gridding

Step 3

Elimination of frequencies greater than Nyquist – Shannon Limit

Gaussian Gridding

Step 4

homothetic rescaling from Gaussian scale

Gaussian Gridding

Step 4

homothetic rescaling from Gaussian scale

$$\omega(n) = \sqrt{\frac{\pi}{\tau}}\, e^{n^2\tau} F_\tau(n)$$

Computational Cost

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The major computational cost of the
Procedure is the FFT on the oversampled grid

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Choosing the oversampling ratio

$M_\tau = 2M$

Computational Cost

The major computational cost of the
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Choosing the oversampling ratio

$M_\tau = 2M$

The total cost of the procedure is $\simeq 2M \log_2 2M$

Syllabus of the presentation

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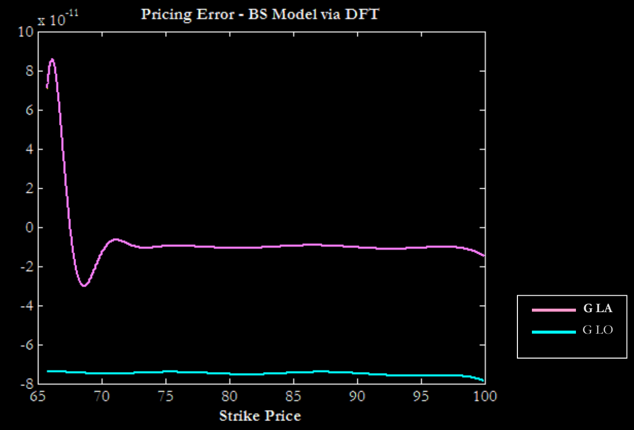
The Computational Framework

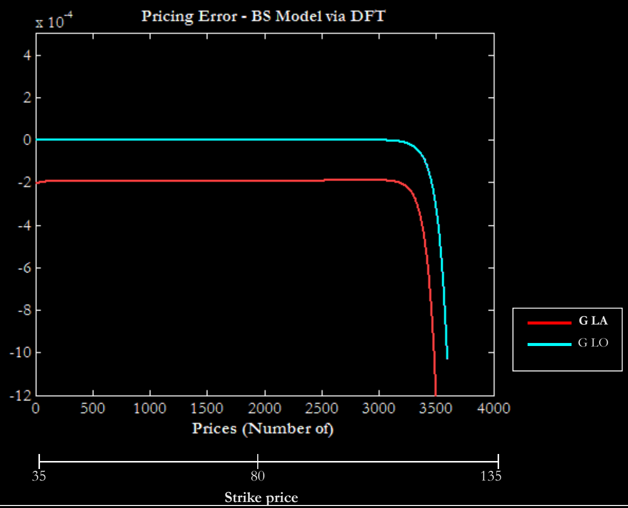
ACCURACY



2000 Prices computed

The Computational Framework

STABILITY



The Computational Framework

STABILITY

The error of 90% of prices computed lies in the

STABILITY

The error of 90% of prices computed lies in the

10^{-3}

RANGE OF PRECISION

SPEED

the NU – FFT is around 2 time slower than FFT

SPEED

At very low time scales, the differences disappear

SPEED

At very low time scales, the differences disappear

	NC2	G – LA	G - LO
FFT	0.01 sec.	N/A	N/A
	NC2	G - LA	G - LO
NU – FFT	0.02 sec.	0.0261 sec.	0.0301 sec.

Computation of 4000 prices on a Centrino 1600Mhz – 2gb RAM
Mean Value over 1000 runs

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Conclusions

- NU – FFT allows the use of Gaussian Grids

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- NU – FFT is indifferent to Nyquist _Shannon Limit

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- NU – FFT is more stable than FFT
- NU – FFT speed performances are indistinguishable from FFT's ones

NU – FFT
is a natural candidate for
operational use on trading desks