VOLATILITY METRICS TO ASSESS RELATIVE RISK IN THE QUANTITATIVE PORTFOLIO MANAGEMENT OF MUTUAL FUNDS:

A REGULATORY APPROACH BASED ON DIFFUSIVE GARCH

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Syllabus

- The Risk Profile of Mutual Funds
  - Representation via Qualitative Risk Classes
  - Adequate Underlying Risk Metric
- Diffusive GARCH
  - Intuition
  - The Weak Convergence Theorem on R^2
  - The Continuous Limit of the M-GARCH(1,1)
  - The Prediction Interval for the Volatility
- The Grid of Volatility Intervals
  - Loss Intervals
  - Mapping into Initial Volatility Intervals
  - Intervals Fine-Tuning: the Iterative Procedure
  - Intervals Adequacy
- Empirical Evidence
  - Preliminary Information
  - The Evolution of the Risk Profile over Time
- Conclusions

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The Issue for the Regulator

Disclosure of the risk profile to investors

Quantitative Risk Measurement and Monitoring

The Issue for the Regulator

Disclosure of the risk profile to investors

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Disclosure of the risk profile to investors

UCITS Prospectus requires Qualitative Risk Classes

The Issue for the Regulator

Disclosure of the risk profile to investors

UCITS Prospectus requires Qualitative Risk Classes

Low
Medium-Low
Medium
Medium-High
High
Very High

The Issue for the Regulator

Quantitative Risk Measurement and Monitoring

The Issue for the Regulator

Adequate Requirements in the Definition of the Risk Metric

Quantitative Risk Measurement and Monitoring

risk metric values

must be representative of the corresponding

risk class

risk metric values

must be representative of the corresponding

risk class

Risk Metric	
Min_{low}	Max_{low}
$Min_{medium\text{-}low}$	$Max_{medium\text{-}low}$
Min_{medium}	Max_{medium}
$Min_{medium\text{-}high}$	$Max_{medium\text{-}high}$
Min_{high}	Max_{high}
$Min_{very\ high}$	$Max_{very\ high}$

mapping

Low
Medium-Low
Medium
Medium-High
High
Very High

risk metric values

must be representative of the corresponding

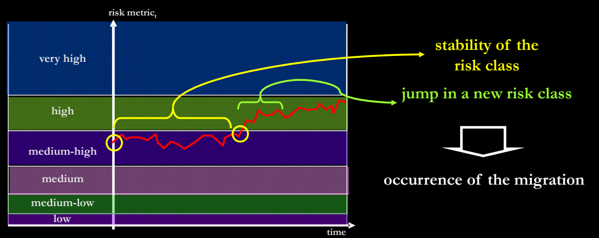
risk class

risk metric values

must timely **capture**

the fund's **migration** to a different **risk class**

definition: fund's migration to a different risk class



migration rule

Prospectus Update: at least yearly

MORE THAN 3 CONSECUTIVE MONTHS

IN A NEW RISK CLASS

risk metric values

must be representative of the corresponding

risk class

risk metric values

must timely **capture**

the fund's **migration** to a different **risk class**

risk metric values

must be **flexible** enough to **accommodate**

"*STANDARD*" FUND'S MANAGER ACTIVITY

in the real evolution of financial markets

risk metric values

must be **flexible** enough to **accommodate**

CALIBRATED INTERVALS OF THE RISK METRIC

MUST BE WIDE ENOUGH …

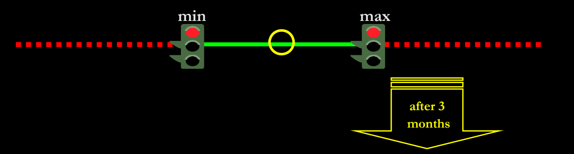
"*STANDARD*" FUND'S MANAGER ACTIVITY

in the real evolution of financial markets

risk metric values

must be **flexible** enough to **accommodate**

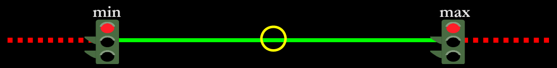
too narrow intervals can imply spurious migration



NO PORTFOLIO REALLOCATION

AND SPURIOUS MIGRATION

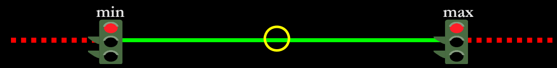
wide enough intervals do imply aware migration



before than 3 months

PORTFOLIO REALLOCATION AND NO MIGRATION

wide enough intervals do imply aware migration



after 3 months

NO PORTFOLIO REALLOCATION AND AWARE MIGRATION

a bunch of risk metrics in finance: which one to choose?

regulator looks for

representativeness — simplicity — common basis to every other risk measure

VOLATILITY

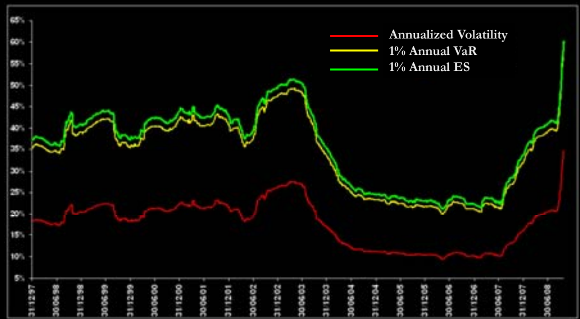
a bunch of volatility measures: which one to choose?

regulator looks for

a measure consistent with the 3 months migration rule

Annualized Volatility of Daily NAV Returns

1:1 correspondance with other risk measures



Syllabus

need for volatility forecasts

need for volatility forecasts

through the diffusion limit of GARCH

need for volatility forecasts

through the diffusion limit of GARCH

Weak Convergence Theorem

from: STOCHASTIC DIFFERENCE EQUATIONS

to: STOCHASTIC DIFFERENTIAL EQUATIONS

via: SHRINKING of the TIME INTERVALS

statement

The sequence $\{X_t^h\}$, whose measurable space is $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$, converges weakly for h ↓0 to the process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x,t)dt + \sigma(x,t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4, presented below, are satisfied.

statement

The process $\{X_t\}$ has a distribution independent on the choice of $\sigma(x,t)$ and it takes finite values over finite time intervals, i.e. $\forall T > 0$:

$$P(\sup_{0 \le t \le T} \|X_t\| < \infty) = 1$$

conditions:

n. 1

If ∃ a δ>0 s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1,t) \\ c_{h,\delta}(x_2,t) \end{pmatrix} = 0$$

conditions:

n. 1

If ∃ a δ>0 s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1,t) \\ c_{h,\delta}(x_2,t) \end{pmatrix} = 0$$

then ∃

$a(x,t)$

$$\lim_{h \downarrow 0} \begin{pmatrix} a_h(x_1,t) & a_h((x_1,x_2),t) \\ a_h((x_2,x_1),t) & a_h(x_2,t) \end{pmatrix} = \begin{pmatrix} a(x_1,t) & 0 \\ 0 & a(x_2,t) \end{pmatrix}$$

s.t.

$b(x,t)$

$$\lim_{h \downarrow 0} \begin{pmatrix} b_h(x_1,t) \\ b_h(x_2,t) \end{pmatrix} = \begin{pmatrix} b(x_1,t) \\ b(x_2,t) \end{pmatrix}$$

conditions:

n. 2

$\exists\ \sigma(x,t)$ s.t.: $\forall x_1 \in \mathbb{R}^1, \forall x_2 \in \mathbb{R}^1$,

it holds

$$\begin{pmatrix} \sigma(x_1,t) & 0 \\ 0 & \sigma(x_2,t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1,t)} & 0 \\ 0 & \sqrt{a(x_2,t)} \end{pmatrix}$$

conditions:

n. 3

For h↓0, X_0^h converges in distribution to a random variable X_0 with probability measure v_0 on $(\mathbb{R}^2, \mathbb{B}(\mathbb{R}^2))$

n. 4

v_0, $a(x,t)$ and $b(x,t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution v_0, a conditional second moment $a(x,t)$ and a conditional first moment $b(x,t)$

Syllabus

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \tilde{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

$\tilde{Z}_k$ and Z_k are i.i.d. *N(0,1)*

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \tilde{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

$\tilde{Z}_k$ and Z_k are i.i.d. *N(0,1)*

Weak Convergence Theorem

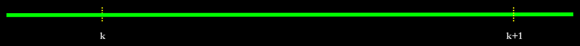
$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = \left(\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right) dt + 2|\beta_1| \sqrt{Var(\ln |Z_t|)} dW_t^*$$

Z_t is *N(0,1)*

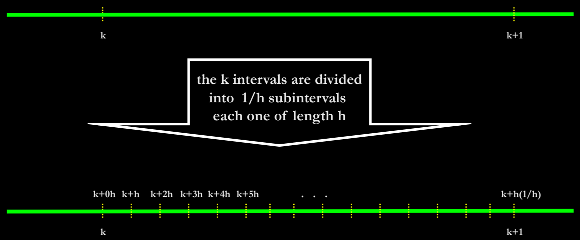
proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process



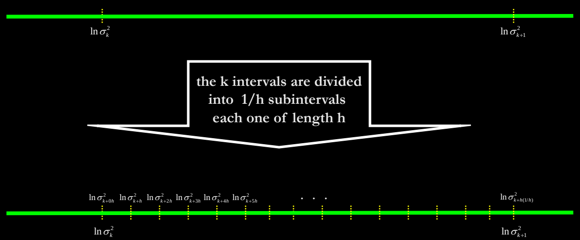
proof

step 1: rescaling of the discrete process

* The focus is on the difference equation for the volatility.
For the convergence of the first equation see Minenna 2003.

proof

step 1: rescaling of the discrete process



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For the convergence of the first equation see Minenna 2003.

proof

step 1: rescaling of the discrete process

relationship from rescaling

$$\ln \sigma_k^2 - \ln \sigma_{k-1}^2 = \sum_{j=1}^{\frac{1}{h}} \left(\ln \sigma_{(k-1)+jh}^2 - \ln \sigma_{(k-1)+jh-h}^2 \right)$$

proof

step 1: rescaling of the discrete process

rescaling tools

$$(\ln|Z|)_{kh} = \sqrt{h} \ln|Z_{k-1}| + \left(h - \sqrt{h}\right) E\left(\ln|Z_{k-1}|\right)$$

where: $f_{Z_{kh}}(z_{kh}) = (1/\sqrt{h}) f_{Z_k}(z_{kh})$

proof

step 1: rescaling of the discrete process

rescaling tools

$$\beta_{0h} = \beta_0^{(k)} \cdot h$$

proof

step 1: rescaling of the discrete process

rescaling tools

β_{1h} s.t.:

$$0 = \left(\beta_0 + \beta_1 \ln \sigma_{k-1}^2 + 2\beta_1 \ln|Z_{k-1}|\right) - \beta_{1h}^{\frac{1}{h}} \ln \sigma_{k-1}^2 -$$
$$- \left(\beta_0 h + 2\beta_{1h} \left(\sqrt{h} \ln|Z_{k-1}| + \left(h - \sqrt{h}\right) E\left(\ln|Z_{k-1}|\right)\right)\right) \sum_{j=1}^{\frac{1}{h}-1} \beta_{1h}^j$$

proof

step 1: rescaling of the discrete process

rescaled equation

$$\ln \sigma_{kh}^2 = \beta_{0h} + \beta_{1h} \ln \sigma_{(k-1)h}^2 +$$
$$+ 2\beta_{1h} \left(\sqrt{h} \ln|Z_{k-1}| + \left(h - \sqrt{h}\right) E\left(\ln|Z_{k-1}|\right)\right)$$

proof

step 2: construction of the process $\left\{\ln\sigma_t^{2^h}\right\}$

definition of the prob. measure P_h on the Skorokhod space D s.t.:

$$P_h(\ln\sigma_0^{2^h}\in\Gamma)=v_0(\Gamma)\qquad\forall\Gamma\in\mathbb{B}(\mathbb{R}^1)$$

$$P_h(\ln\sigma_t^{2^h}=\ln\sigma_{kh}^2,\quad\forall\, kh\le t<(k+1)h)=1$$

$$P_h(\ln\sigma_{(k+1)h}^2\in\Gamma|\widehat{\Im}_{kh})=\Pi_{h,kh}(\ln\sigma_{kh}^2,\Gamma)\text{ a.s. under } P_h,\ \forall k\ge 0,\ \forall\Gamma\in\mathbb{B}(\mathbb{R}^1)$$

proof

step 2: construction of the process $\left\{\ln\sigma_t^{2^h}\right\}$

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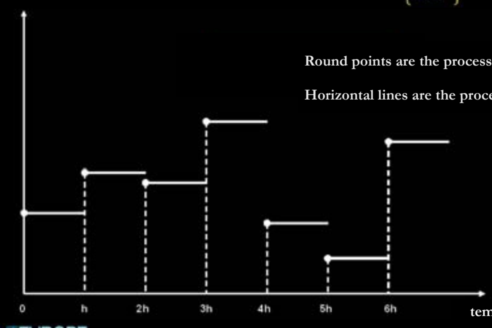
$$\ln\sigma_{t+1}^{2^h}-\ln\sigma_t^{2^h}=\beta_{0h}+(\beta_{1h}-h)\ln\sigma_t^{2^h}+ +2\beta_{1h}\left\{\sqrt{h}\ln|Z_t^h|+\left(h-\sqrt{h}\right)E\left(\ln|Z_t^h|\right)\right\}$$

proof

step 2: construction of the process $\left\{\ln\sigma_t^{2^h}\right\}$



proof

step 3: check of condition n. 1 of the weak convergence th.

$$\beta_{0h}:=\beta_0\cdot h$$
$$\beta_{1h}:=\beta_1\cdot h$$

$$\lim_{h\downarrow 0}c_{h,\delta=1}(\widehat{\ln\sigma^2},t)=0$$

$$\lim_{h\downarrow 0}b_h(\widehat{\ln\sigma^2},t)=(\beta_0+2\beta_1\mathbf{E}(\ln|Z_t|)+(\beta_1-1)\ln\sigma_t^2)$$

$$\lim_{h\downarrow 0}a_h(\widehat{\ln\sigma^2},t)=4\beta_1^2Var(\ln|Z_t|)$$

proof

step 3: check of condition n. 1 of the weak convergence th.

Condition n. 2 is verified for every $\sigma>0$, i.e.:

$$\sigma\left(\ln\widehat{\sigma^2},t\right)=2|\beta_1|\sqrt{Var\left(\ln|Z_t|\right)}$$

Condition n. 3 is satisfied by construction of the process $\left\{\ln\sigma_t^{2^h}\right\}$

Consequently, Condition 4 is verified too

statement

from the M-GARCH(1,1)

$$\ln\sigma_{k+1}^2-\ln\sigma_k^2=\beta_0^{(k)}+(\beta_1^{(k)}-1)\ln\sigma_k^2+\beta_1^{(k)}\ln Z_k^2$$

Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem

$$d\ln\sigma_t^2=\left(\beta_0+2\beta_1E(\ln|Z_t|)+(\beta_1-1)\ln\sigma_t^2\right)dt+2|\beta_1|\sqrt{Var(\ln|Z_t|)}dW_t^*$$

Z_t is $N(0,1)$

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key point

From the Diffusion Limit of the M-GARCH(1,1) Process it is possible to establish a ***Predictive Interval for*** σ_t

distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d\ln\sigma_t^2=\left[\beta_0+2\beta_1\mathbf{E}(\ln|Z_t|)+(\beta_1-1)\ln\sigma_t^2\right]dt+2|\beta_1|\sqrt{Var(\ln|Z_t|)}dW_t^*$$

O-U process

$$\ln\sigma_t^2\sim N\left[\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)};\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}\right]$$

matching of the first two conditional moments

discrete process

$$E\left(\ln\sigma_k^2\right) = \beta_0^{(k)} + \beta_1^{(k)}\ln\sigma_{k-1}^2 + 2\beta_1^{(k)}E\left(\ln|Z_{k-1}|\right)$$

$$Var\left(\ln\sigma_k^2\right) = 4\left(\beta_1^{(k)}\right)^2 Var\left(\ln|Z_{k-1}|\right)$$

matching of the first two conditional moments

$$E\left(\ln\sigma_t^2\right) = \left(\ln\sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E\left(\ln|Z_t|\right)}{(\beta_1 - 1)}\right)e^{(\beta_1-1)} - \frac{\beta_0 + 2\beta_1 E\left(\ln|Z_t|\right)}{(\beta_1 - 1)}$$

$$Var\left(\ln\sigma_t^2\right) = \frac{4\beta_1^2 Var\left(\ln|Z_t|\right)}{2(\beta_1 - 1)}\left(e^{2(\beta_1-1)} - 1\right)$$

continuous process

matching of the first two conditional moments

$$\left|\beta_1^{(k)}\right| = |\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}}{2(\beta_1-1)}}$$

$$\beta_0^{(k)} = -2|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}}{2(\beta_1-1)}}E\left(\ln|Z_{k-1}|\right) - |\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}}{2(\beta_1-1)}}\ln\sigma_{k-1}^2 + \\ + e^{(\beta_1-1)}\ln\sigma_{k-1}^2 + \frac{[\beta_0+2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1-1}$$

matching of the first two conditional moments

$$\ln\sigma_k^2 - \ln\sigma_{k-1}^2 = \frac{[\beta_0+2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1-1} - \\ -2|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}}{2(\beta_1-1)}}E\left(\ln|Z_{k-1}|\right) + \\ + \left(e^{(\beta_1-1)} - 1\right)\ln\sigma_{k-1}^2 + \\ + 2|\beta_1|\sqrt{\frac{e^{2(\beta_1-1)}}{2(\beta_1-1)}}\ln|Z_{k-1}|$$

maximum likelihood estimation

setting:

$$Y_k = \ln\sigma_k^2 - \ln\sigma_{k-1}^2$$

$$a = \frac{[\beta_0 + 2\beta_1 E\left(\ln|Z_{k-1}|\right)]\left(e^{(\beta_1-1)} - 1\right)}{\beta_1 - 1} - E\left(\ln|Z_{k-1}|\right)|\beta_1|\sqrt{\frac{2\left(e^{2(\beta_1-1)} - 1\right)}{(\beta_1 - 1)}}$$

$$b = \left(e^{(\beta_1-1)} - 1\right)$$

$$c = |\beta_1|\sqrt{\frac{2\left(e^{2(\beta_1-1)} - 1\right)}{(\beta_1 - 1)}}$$

$$X_{k-1} = \ln\sigma_{k-1}^2$$

$$Z = \ln|Z_{k-1}|$$

maximum likelihood estimation

leads to:

$$Y_k = a + bX_{k-1} + cZ$$

maximum likelihood estimation

leads to:

$$Y_k = a + bX_{k-1} + cZ$$

log-likelihood function

$$\ln L(Y;\beta_0,\beta_1) = n\ln\left(\frac{2}{c\sqrt{2\pi}}\right) + \sum_{k=1}^{n}\left(\frac{Y_k - a - bX_{k-1}}{c} - \frac{1}{2}e^{2\left(\frac{Y_k - a - bX_{k-1}}{c}\right)}\right)$$

maximum likelihood estimation

$\widehat{\beta_0}$

is given, solving numerically the F.O.C.

$$\frac{\partial}{\partial\beta_0}L(Y;\beta_0,\beta_1) = 0$$

$\widehat{\beta_1}$

is given, solving numerically the F.O.C.

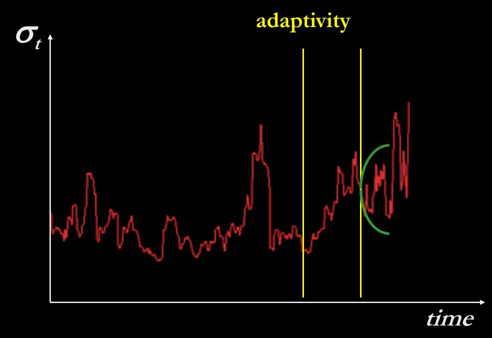
$$\frac{\partial}{\partial\beta_1}L(Y;\beta_0,\beta_1) = 0$$

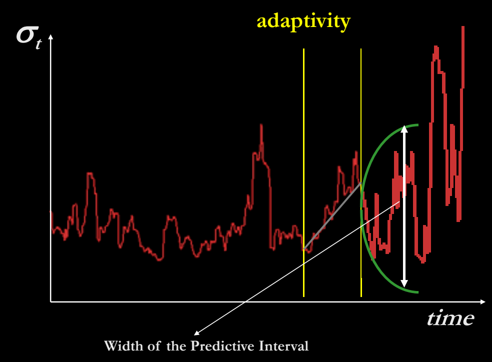
$$P\left(\begin{array}{c} -z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)} \\ \leq \ln\sigma_t^2 \leq \\ z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)} + \left(\ln\sigma_{t-1}^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)} \end{array}\right) = \alpha$$

$$P\left(\begin{array}{c} -z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)} \\ \le \ln\sigma_t^2 \le \\ z_{\frac{\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1E(\ln|Z_t|)}{(\beta_1-1)} \end{array}\right)=\alpha$$

hence:

$$\left[\sigma_{t,\min}^G,\sigma_{t,\max}^G\right]=\left[e^{\frac{-z_{\frac{\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}};\ e^{\frac{z_{\frac{\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)}\left(e^{2(\beta_1-1)}-1\right)}+\left(\ln\sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}}\right]$$





Syllabus

- The Risk Profile of Mutual Funds
  - Representation via Qualitative Risk Classes
  - Adequate Underlying Risk Metric
- Diffusive GARCH
  - Intuition
  - The Weak Convergence Theorem on R^2
  - The Continuous Limit of the M-GARCH(1,1)
  - The Prediction Interval for the Volatility
- The Grid of Volatility Intervals
  - Loss Intervals
  - Mapping into Initial Volatility Intervals
  - Intervals Fine-Tuning: the Iterative Procedure
  - Intervals Adequacy
- Empirical Evidence
  - Preliminary Informations
  - The Evolution of the Risk Profile over Time
- Conclusions

what is the loss in a financial investment?

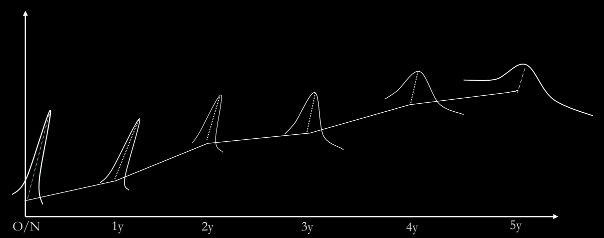
what is the loss in a financial investment?

RISK NEUTRALITY PRINCIPLE

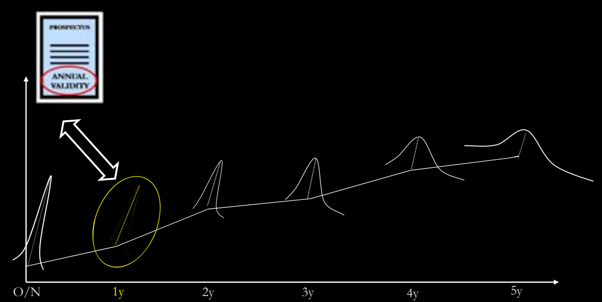
$$\text{LOSS} \in (-100\%, \overline{r^{rf}}\,]$$

$\overline{r^{rf}}$= average of the probability distribution of the risk-free rate

given the risk-free yield curve and the associated volatility surface…



… the probability distribution of the 1-yr risk-free rate is selected…



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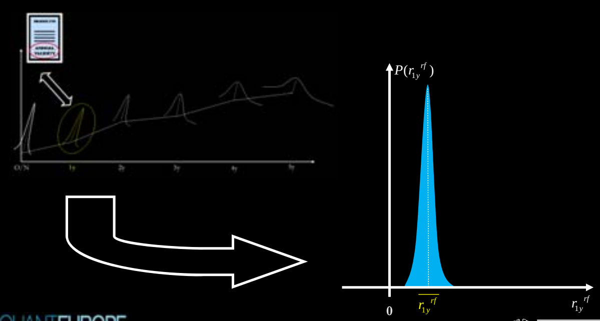
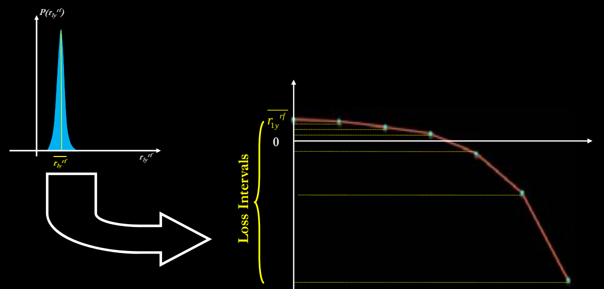


The Grid of Volatility Intervals: Loss Intervals

…to each risk class is associated the corresponding annual loss interval

(multiple of r_{1y}^{rf} according to and exponential function)



The Grid of Volatility Intervals: Loss Intervals

identification of six initial loss intervals

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

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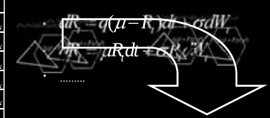
The Grid of Volatility Intervals: Mapping into Initial Volatility Intervals

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$

The Grid of Volatility Intervals: Mapping into Initial Volatility Intervals

Risk Classes	Loss Intervals L_{min}	L_{max}
low	${}_0L_{1,min}$	${}_0L_{1,max}$
medium-low	${}_0L_{2,min}$	${}_0L_{2,max}$
medium	${}_0L_{3,min}$	${}_0L_{3,max}$
medium-high	${}_0L_{4,min}$	${}_0L_{4,max}$
high	${}_0L_{5,min}$	${}_0L_{5,max}$
very high	${}_0L_{6,min}$	${}_0L_{6,max}$



Risk Classes	Volatility Intervals σ_{min}	σ_{max}
low	${}_0\sigma_{1,min}$	${}_0\sigma_{1,max}$
medium-low	${}_0\sigma_{2,min}$	${}_0\sigma_{2,max}$
medium	${}_0\sigma_{3,min}$	${}_0\sigma_{3,max}$
medium -high	${}_0\sigma_{4,min}$	${}_0\sigma_{4,max}$
high	${}_0\sigma_{5,min}$	${}_0\sigma_{5,max}$
very high	${}_0\sigma_{6,min}$	${}_0\sigma_{6,max}$

*The subscript *0* preceding the volatility indicates that this is the initial interval, i.e. before the calibration

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The Grid of Volatility Intervals: Intervals Fine-Tuning: the Iterative Procedure

tools

Diffusive GARCH Models

Stochastic Non-Linear Programming

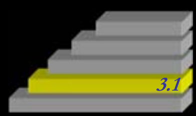
The Grid of Volatility Intervals: Intervals Fine-Tuning: the Iterative Procedure

Selection of an initial Volatility Interval

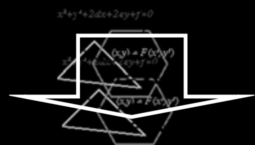
Risk Classes	Volatility Intervals σ_{min}	σ_{max}
low	${}_0\sigma_{1,min}$	${}_0\sigma_{1,max}$
medium-low	${}_0\sigma_{2,min}$	${}_0\sigma_{2,max}$
medium	${}_0\sigma_{3,min}$	${}_0\sigma_{3,max}$
medium-high	${}_0\sigma_{4,min}$	${}_0\sigma_{4,max}$
high	${}_0\sigma_{5,\mu in}$	${}_0\sigma_{5,max}$
very high	${}_0\sigma_{6,min}$	${}_0\sigma_{6,max}$

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

The Grid of Volatility Intervals: Intervals Fine-Tuning: the Iterative Procedure



Simulation of the Fund pattern



***NAV* Stochastic Differential Equation**

3.1 *Simulation of the Fund pattern*

NAV S.D.E. → What Parameters?

3.1 *Simulation of the Fund pattern*

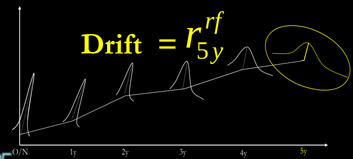
NAV S.D.E. → What Parameters?

The Drift

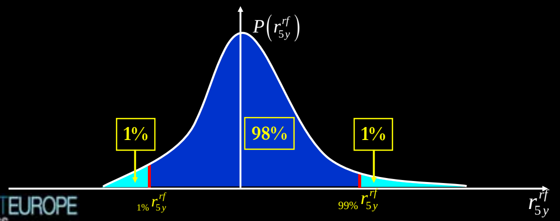
3.1 *Simulation of the Fund pattern*

risk-neutrality principle

robustness of volatility intervals



3.1 *Simulation of the Fund pattern*



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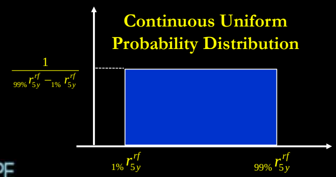


3.1 *Simulation of the Fund pattern*

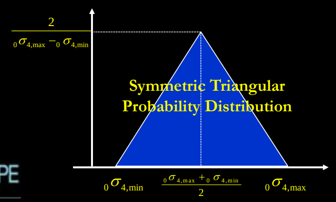
NAV S.D.E. → What Parameters?

The Diffusion

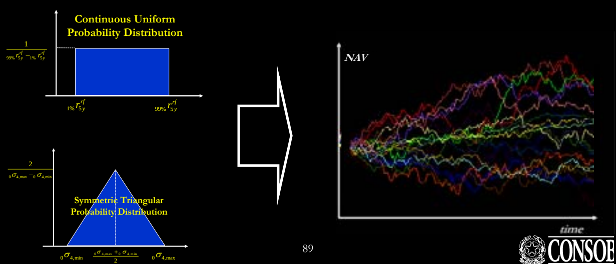
3.1 *Simulation of the Fund pattern*

$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

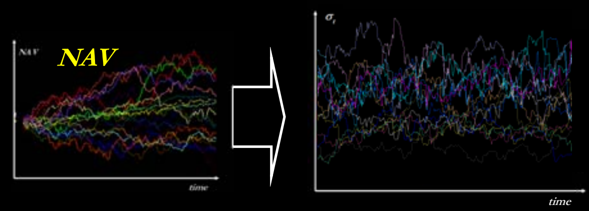
representativeness of volatility intervals



3.1 *Simulation of the Fund pattern*



3.2 *Determination of the Time Series of the Annual Volatility*



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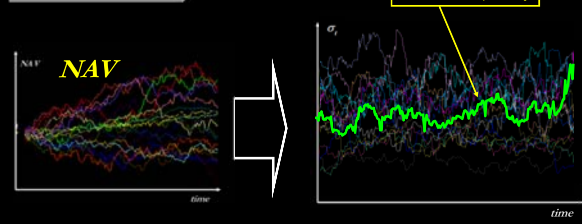
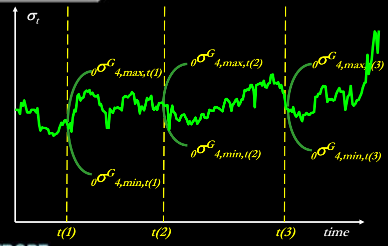


3.3 *Volatility Forecast Band through GARCH Diffusive Models*



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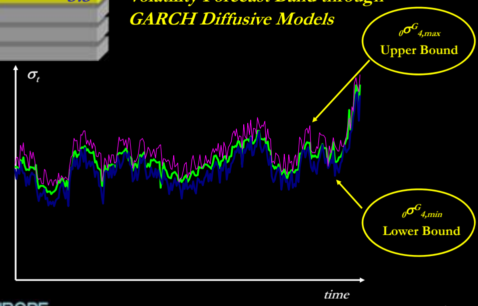


3.4 *Validation of the initial Volatility Interval*

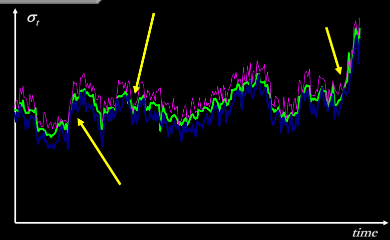
$[{}_0\sigma_{4,min} \quad {}_0\sigma_{4,max}]$

VS

$[{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$

3.4 *Validation of the initial Volatility Interval*

Calculus of the number of observations outside the Band



3.4 *Validation of the initial Volatility Interval*

Number of observations outside the Band in 1 year

Trajectory	n° obs $\in [{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$	n° obs. $< {}_0\sigma^G_{4,min}$	n° obs. $> {}_0\sigma^G_{4,max}$
1			
2			
...			
n			
	Tot. $\in [{}_0\sigma^G_{4,min} \quad {}_0\sigma^G_{4,max}]$	Tot. $< {}_0\sigma^G_{4,min}$	Tot. $> {}_0\sigma^G_{4,max}$

Hp.: n. of obs. of σ_t = 250 for each trajectory

3.4 *Validation of the initial Volatility Interval*

Number of observations outside the Band in 1 year

$$\Delta = \frac{\sum_{i=1}^{n}\sum_{j=0}^{250} 1\left(\sigma_{i,j} > {}_0\sigma^G_{i,j,\max}\right) + 1\left(\sigma_{i,j} < {}_0\sigma^G_{i,j,\min}\right)}{n \cdot 250}$$

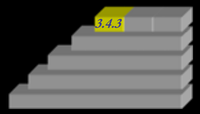
$$\Delta_{up} = \frac{\sum_{i=1}^{n}\sum_{j=0}^{250} 1\left(\sigma_{i,j} > {}_0\sigma^G_{i,j,\max}\right)}{n \cdot 250} \qquad \Delta_{down} = \frac{\sum_{i=1}^{n}\sum_{j=0}^{250} 1\left(\sigma_{i,j} < {}_0\sigma^G_{i,j,\min}\right)}{n \cdot 250}$$

3.4.1 *Update of the initial Volatility Interval*
Iteration of the Procedure

$\Delta > 5\%$ and $\Delta_{up} > \Delta_{down}$ ⇒ $[{}_0\sigma_{4,min} \quad {}_1\sigma_{4,max}]$ ⇒ 3.1

3.4.2 *Update of the initial Volatility Interval*
Iteration of the Procedure

$\Delta > 5\%$ and $\Delta_{up} < \Delta_{down}$ ⇒ $[{}_1\sigma_{4,min} \quad {}_0\sigma_{4,max}]$ ⇒ 3.1



End of the Procedure

$\Delta \leq 5\%$

$$[{}_k\sigma_{4,min} \quad {}_k\sigma_{4,max}] = [\sigma_{4,min} \quad \sigma_{4,max}]$$

k = n. di iterations done

output volatility intervals

Risk Classes	Volatility Intervals (annualized values)	
	σ_{min}	σ_{max}
low	*0.01%*	*0.49%*
medium-low	*0.50%*	*1.59%*
medium	*1.60%*	*3.99%*
medium-high	*4.00%*	*9.99%*
high	*10.00%*	*24.99%*
very high	*25.00%*	*above 25.00%*

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the iterative procedure guarantees that a fund belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 yr

the iterative procedure guarantees that a fund belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 yr

no more than 15 days over 250

but

migration risk is measured against fixed volatility intervals

but

migration risk is measured against fixed volatility intervals

hence

output intervals are inherently prudential w.r.t. the 3 months migration rule

but

migration risk is measured against fixed volatility intervals

hence

output intervals are inherently prudential w.r.t. the 3 months migration rule

i.e.:

output intervals are wide enough to avoid spurious migrations

as confirmed by back-testing simulation

as confirmed by back-testing simulation

risk-neutral drift

hp.: risk class = medium-high

as confirmed by back-testing simulation

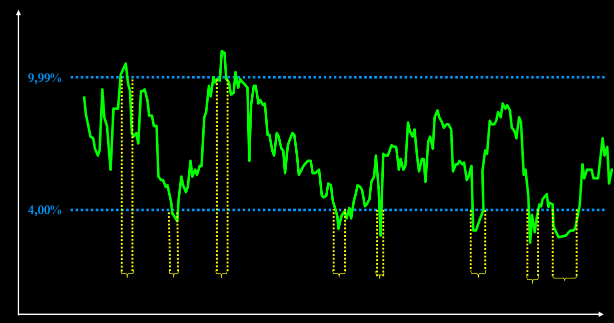
risk-neutral drift

hp.: risk class = medium-high

drawing diffusion from a symm. triang. distrib. bounded at $[\sigma_{min}\ \sigma_{max}]=[4\%\ \ 9.99\%]$
to simulate an hypothetic daily NAV and to calculate the corresponding realized volatility

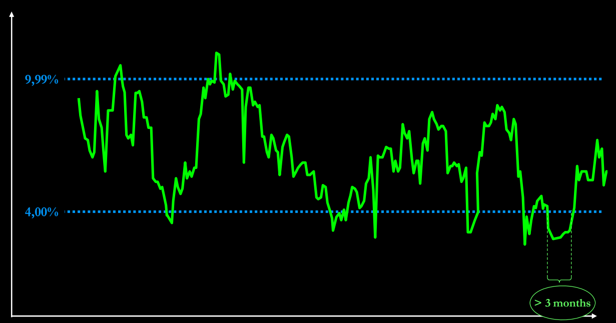
for each trajectory

as confirmed by back-testing simulation



ALL THE BREACHES MARKED IN YELLOW LAST LESS THAN 3 MONTHS

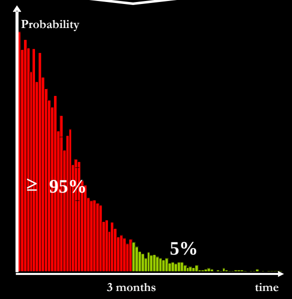
as confirmed by back-testing simulation



THE OUTLIER MARKED IN GREEN LASTS MORE THAN 3 MONTHS

as confirmed by back-testing simulation

considering all trajectories



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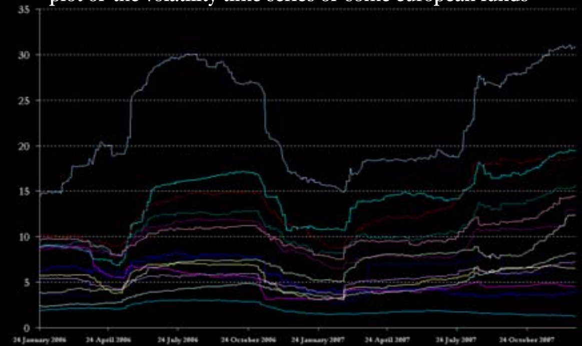
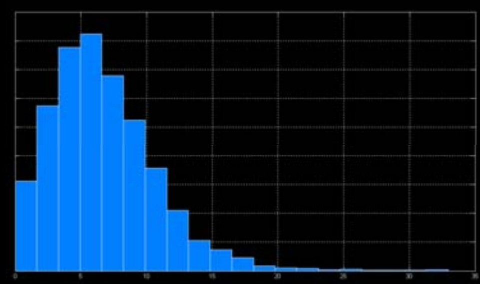
universe VS sample

Country	Total (A)	Selected (B)	Representativity (B/A)
Austria	17	13	76.5%
France	92	53	57.6%
Germany	63	45	71.4%
Ireland	2	1	50.0%
Italy	58	52	89.7%
Luxembourg	252	153	60.7%
Spain	224	130	58.0%
UK	8	7	87.5%
Total	716	454	63.4%

plot of the volatility time series of some european funds



volatility histogram for the sampled funds



initial distribution of the 454 funds between the 6 risk classes
(abs. values)

	Initial Risk Class as from 1st January 2006					
Country	1	2	3	4	5	Total
Austria	0	0	4	8	1	13
France	0	2	9	37	5	53
Germany	0	2	10	26	7	45
Ireland	0	1	0	0	0	1
Italy	1	11	11	28	1	52
Luxembourg	1	6	30	100	16	153
Spain	0	23	33	62	12	130
UK	0	0	0	5	2	7
Total	2	45	97	266	44	454

initial distribution of the 454 funds between the 6 risk classes
(perc. values)

	Initial Risk Class as from 1st January 2006					
Country	1	2	3	4	5	Total
Austria	0.0%	0.0%	30.8%	61.5%	7.7%	100%
France	0.0%	3.8%	17.0%	69.8%	9.4%	100%
Germany	0.0%	4.4%	22.2%	57.8%	15.6%	100%
Ireland	0.0%	100.0%	0.0%	0.0%	0.0%	100%
Italy	1.9%	21.2%	21.2%	53.8%	1.9%	100%
Luxembourg	0.7%	3.9%	19.6%	65.4%	10.5%	100%
Spain	0.0%	17.7%	25.4%	47.7%	9.2%	100%
UK	0.0%	0.0%	0.0%	71.4%	28.6%	100%
Total	0.4%	9.9%	21.4%	58.6%	9.7%	100%

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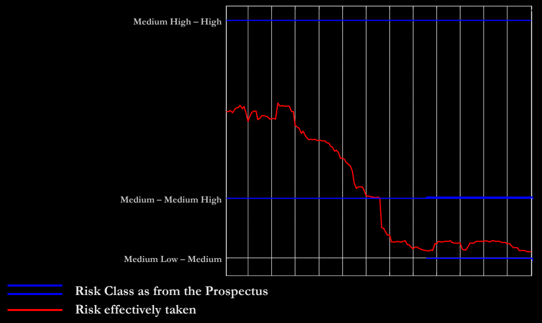
MIGRATION



n. of migrations between different risk classes: 01.01.2006 – 12.31.2007
(abs. values)

	Number of Migrations over the period January 2006 - December 2007						
Country	0	1	2	3	4	5	Total
Austria	2	6	2	3	0	0	13
France	20	13	8	11	1	0	53
Germany	18	6	13	8	0	0	45
Ireland	0	1	0	0	0	0	1
Italy	15	12	17	8	0	0	52
Luxembourg	63	28	34	23	4	1	153
Spain	44	30	31	21	4	0	130
UK	1	3	1	2	0	0	7
Total	163	99	106	76	9	1	454

migrations *per* Country: 01.01.2006 – 12.31.2007
(perc. values)

Country	0	1	2	3	4	5	Total
Austria	15.4%	46.2%	15.4%	23.1%	0.0%	0.0%	100%
France	37.7%	24.5%	15.1%	20.8%	1.9%	0.0%	100%
Germany	40.0%	13.3%	28.9%	17.8%	0.0%	0.0%	100%
Ireland	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	100%
Italy	28.8%	23.1%	32.7%	15.4%	0.0%	0.0%	100%
Luxembourg	41.2%	18.3%	22.2%	15.0%	2.6%	0.7%	100%
Spain	33.8%	23.1%	23.8%	16.2%	3.1%	0.0%	100%
UK	14.3%	42.9%	14.3%	28.6%	0.0%	0.0%	100%

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✓ REGULATORY ISSUE:
measurement and representation of mutual funds risk profile

✓ ADEQUACY REQUIREMENT OF THE APPROACH:
ability to pursue full risk disclosure consistently with the "standard" activity of fund managers

✓ DIFFUSIVE GARCH:
definition of adaptive volatility prediction bands

✓ EMPIRICAL EVIDENCE:
the phenomenon of the migration interests more the funds belonging to the riskiest classes

✓ CLOSING RECOMMENDATION:
exploring other fields of application, especially to move faster towards a really levelled playing field

QUANTEUROPE CONGRESS CONSOB

VOLATILITY METRICS TO ASSESS RELATIVE RISK IN THE QUANTITATIVE PORTFOLIO MANAGEMENT OF MUTUAL FUNDS:

A REGULATORY APPROACH BASED ON DIFFUSIVE GARCH

London, November 6th 2008

Marcello Minenna
Giovanna Maria Boi