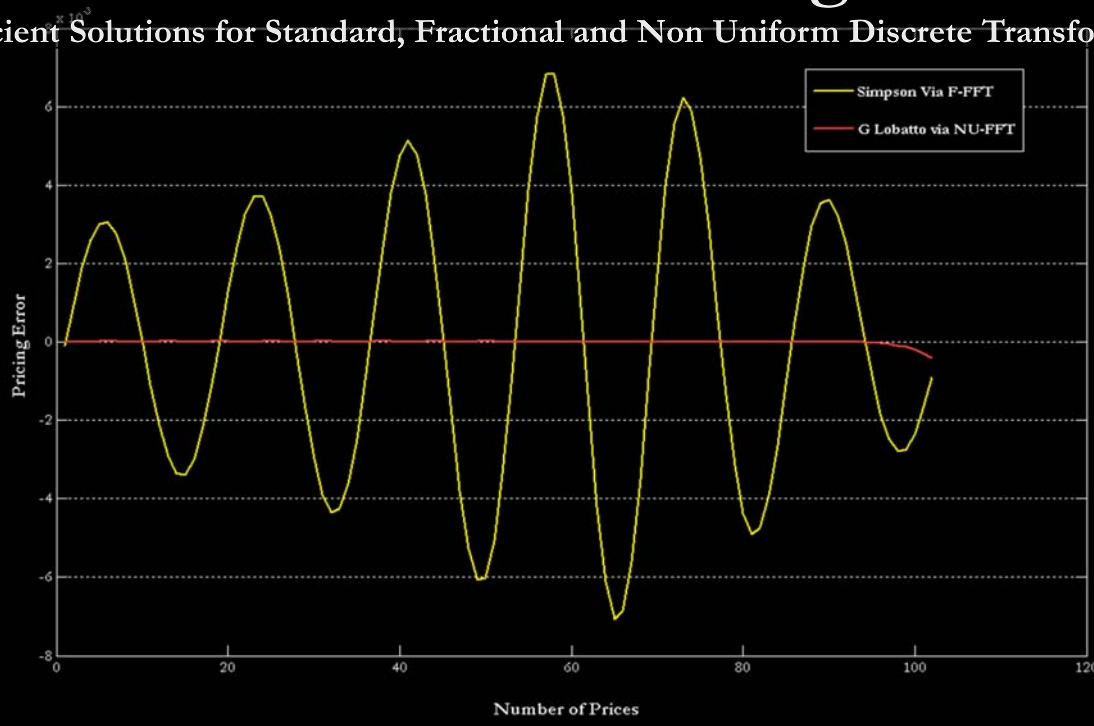
Numerical Methods in Semianalytical Derivatives Pricing

Efficient Solutions for Standard, Fractional and Non Uniform Discrete Transforms



Marcello Minenna – Paolo Verzella
CONSOB

Syllabus of the presentation

- Review of Derivative Pricing via DFT
  - FT Pricing Formulae
  - DFT Convergence to FT
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Derivative Pricing
  - Fractional FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
  - Fractional vs Non Uniform FFT: Empirical Analysis
  - Conclusions

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FT Pricing Formulae

Derivative Price C_t

Spot Price S_t

$$f_2(\ln S_T, \xi \mid \ln S_0) = \int_{-\infty}^{+\infty} e^{i\xi \ln S_T} q_2(\ln S_T \mid \ln S_0)\, d\ln S_T$$

under risk-neutral measure

A linear direct mapping from Fourier Spectral Space

FT

FT Pricing Formulae

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CARR-MADAN REPRESENTATION

FT Pricing Formulae

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A linear direct mapping from Fourier Spectral Space

FT

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i)}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

FT Pricing Formulae

Calibrating α

means choosing a dampened oscillating characteristic function

CARR-MADAN REPRESENTATION

FT Pricing Formulae

Calibrating α

means choosing a dampened oscillating characteristic function

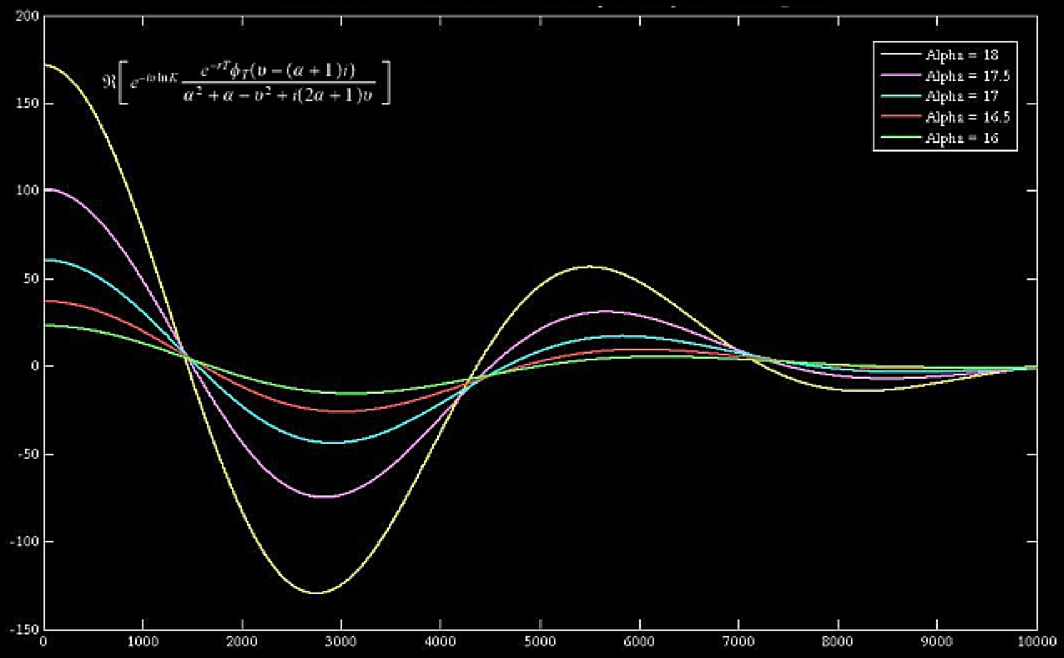
<u>Recent Developments</u>:

Lee, 2004 - Journal of Computational Finance

Minenna, Verzella - Quant Congress 2006

Lord, Kahl, 2007 - Journal of Computational Finance

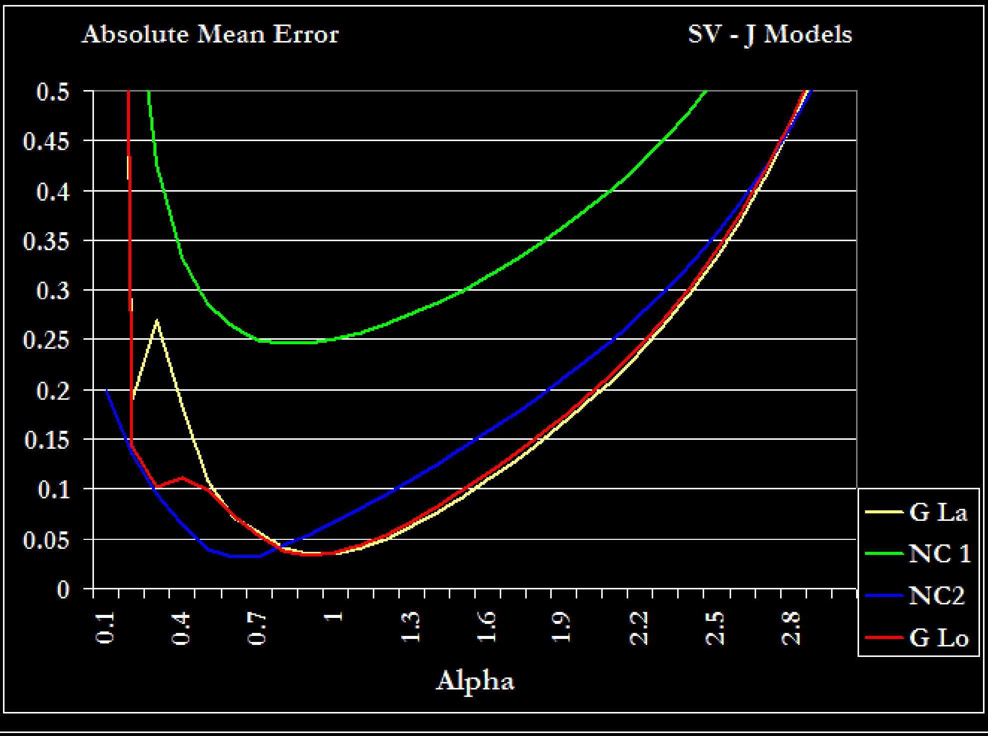
CARR-MADAN REPRESENTATION



CARR-MADAN REPRESENTATION

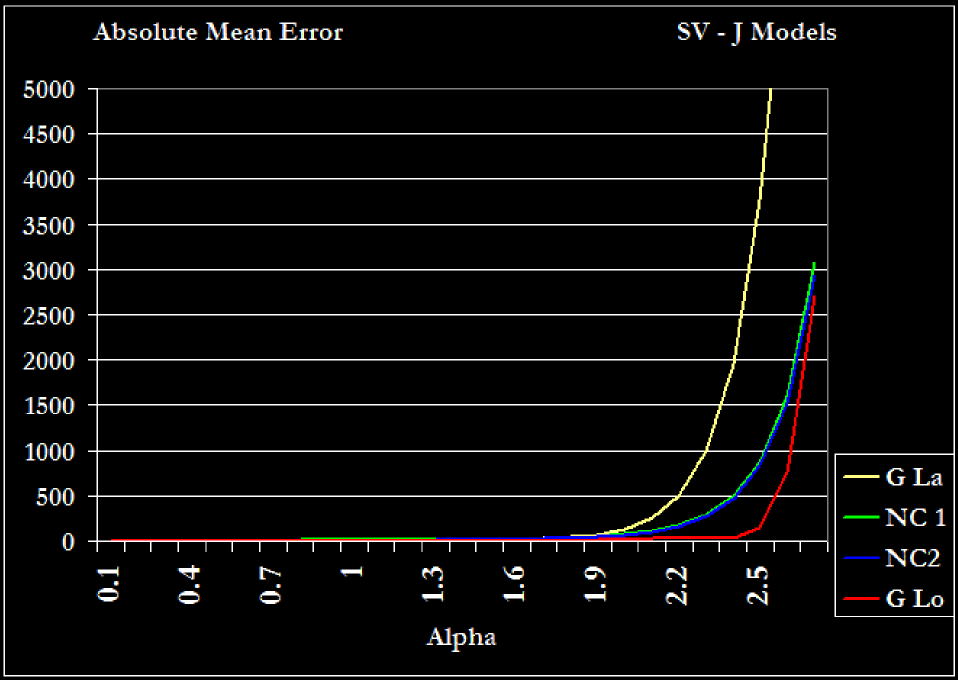
Accuracy

Absolute Mean Error computed w.r.t. α on an (σ, τ) space



Stability

Absolute Mean Error computed w.r.t. α on an Extended (σ, τ) space



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  - Convergence Theorems for Non Uniform Gaussian Grids

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1,2,\ldots M$$

DFT Convergence to FT

Given the General DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1,2,\ldots M$$

$$M \neq N$$

The Convergence Theorem for General DFT's (C Th)

$$\mathcal{F}[f(x)](t_m) = \lim_{N\to\infty} \sum_{j=1}^{N} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j, X)$$

$$t_m = \frac{2\pi}{X}(m-1)$$

C_0 **via FT**

Convergence theorems

$\overrightarrow{C_0}$ **via DFT**

- **Review of Derivative Pricing via DFT**
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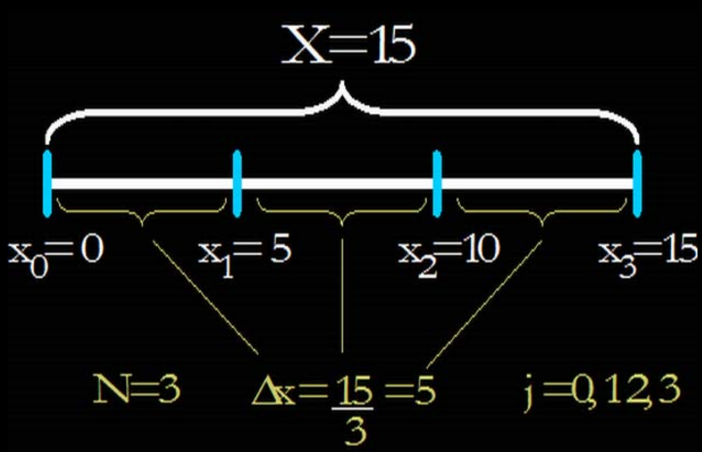
Condition 1

Uniform Discretization Grid



Condition 2

N=M

DFT specialized

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(n-1)} f(x_j) \quad \text{where } n=1,2,\ldots,N$$

Condition 1 Condition 2

DFT Simplified Formula

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i2\pi kj\gamma} f(x_j) \quad \text{where} \quad n = 1\ldots N$$

Nyquist – Shannon Limit (N-S)

$$\mathcal{F}[f(x)](t_n) = \lim_{N\to\infty} \frac{X}{N}\,\omega(n)$$

$$\{t_n\}_{n=1..\frac{N}{2}} \qquad \textit{for N even}$$

$$\{t_n\}_{n=1..\frac{N+1}{2}} \qquad \textit{for N odd}$$

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i)}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

Uniform Discretization Grids for *f*

1. $\phi_T(\xi_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]$

2. $\phi_T(\xi_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]}\Psi_0[(j-1)\eta]\cdot[3+(-1)^{j+1} - \delta_j - \delta_{N-j}]$

Convergence Theorems for Uniform Grids

1.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i)}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

$$\phi_T(\xi_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]} \Psi_0[(j-1)\eta]$$

+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{b} \cdot \Re(\omega(u))$$

Convergence Theorems for Uniform Grids

2.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi - (\alpha+1)i)}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi} \right) d\xi$$

$$\phi_T(\xi_{j-1}) = e^{-i(j-1)\eta[\ln S_t - b]} \Psi_0[(j-1)\eta] \cdot [3 + (-1)^{j+1} - \delta_j - \delta_{N-j}]$$

+

N-S

$$C_0[\ln K]_u^- \approx \frac{e^{-\alpha[\ln S_t - b + \lambda(u-1)]}}{3b} \cdot \Re(\omega(u))$$

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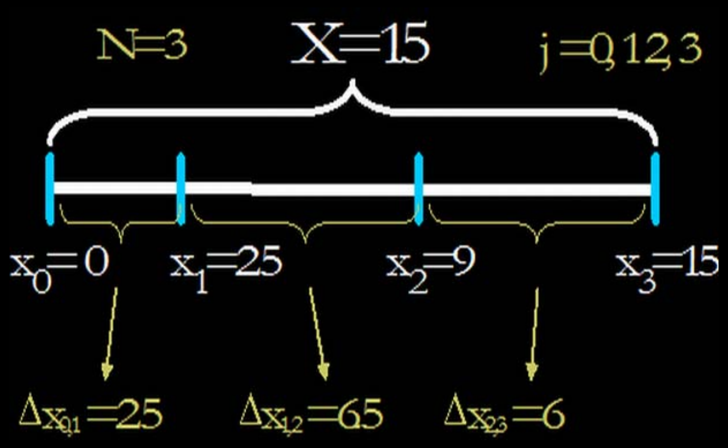
Convergence Theorems for Non Uniform Gaussian Grids

Condition 1

Non Uniform Discretization Grid



Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre

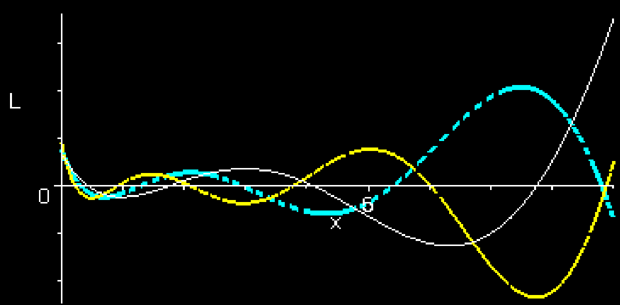
Gander Gautschi

Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Laguerre



Zeros of
Laguerre Poynomials

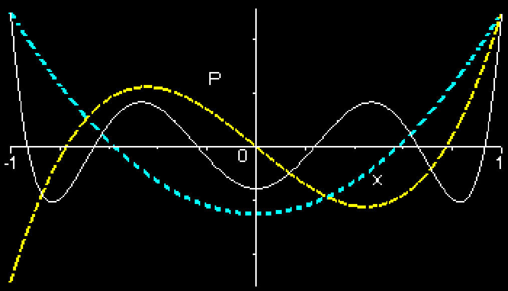
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gauss Lobatto

Zeros of
Legendre Poynomials



Convergence Theorems for Non Uniform Gaussian Grids

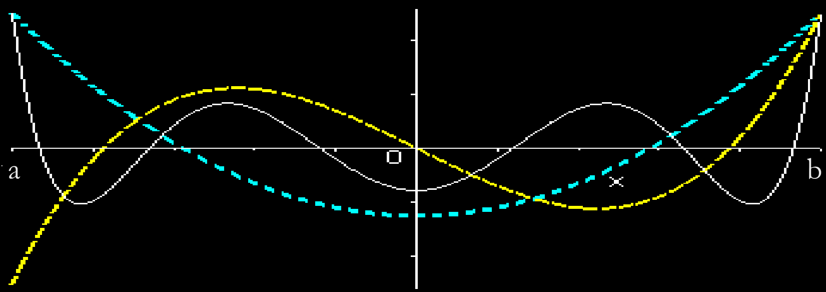
Condition 1

Gaussian Grids

Optimal choice of discretization points

Gander Gautschi

Zeros of rescaled
Legendre Poynomials



Condition 2

N≠M

General DFT

$$\omega(m)=\sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m=1,2,\ldots,2M$$

The Convergence Theorem for General DFT's (C Th)

$$\mathcal{F}[f(x)](t_m) = \lim_{N\to\infty}\sum_{j=1}^{N} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j,X)$$

$$t_m = \frac{2\pi}{X}(m-1)$$

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi} \right) d\xi$$

Gaussian Grids for *f*

1. $$\phi_T(\xi_{j-1}) = e^{\left[1+i\left(\frac{M\pi}{a^*}-\ln S_t\right)\right]\xi_{j-1}} \Psi_0[\xi_{j-1}] \cdot \frac{1}{L_{N+1}(\xi_{j-1}) L'_N(\xi_{j-1})}$$

2. $$\phi_T\left(\frac{1}{2}a(1+\xi_{j-1})\right) = e^{\left[-i\left(\frac{1}{2}a(1+\xi_{j-1})\right)\left(\ln S_t - \frac{M\pi}{a^*}\right)\right]} \Psi_0\left[\frac{1}{2}a(1+\xi_{j-1})\right] \cdot \frac{1}{\left[P_{N-1}(\xi_{j-1})\right]^2}$$

1.

$$C_0 = \frac{e^{-\alpha \ln K}}{\pi} \int_0^{+\infty} \Re\left(e^{i\xi \ln K} \frac{e^{-rT} f_2(\xi-(\alpha+1)i}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi} \right) d\xi$$

$$\phi_T(\xi_{j-1}) = e^{\left[1+i\left(\frac{M\pi}{a^*}-\ln S_t\right)\right]\xi_{j-1}} \Psi_0[\xi_{j-1}] \cdot \frac{1}{L_{N+1}(\xi_{j-1}) L'_N(\xi_{j-1})}$$

+

C-Th

$$C_0([\ln K]_u^*) \approx -\Re\left[\frac{e^{-\alpha\left(\ln S_t - \frac{M\pi}{a^*} + \frac{2\pi}{a^*}(u-1)\right)}}{\pi} \frac{1}{N+1} \cdot \omega^*(u) \right]$$

Convergence Theorems for Non Uniform Gaussian Grids

2.

$$\boxed{C_0 = \frac{e^{-\alpha \ln K}}{\pi}\int_0^{+\infty} \Re\left(e^{i\xi \ln K}\frac{e^{-rT} f_2(\xi-(\alpha+1)i)}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right)d\xi}$$

$$\phi_T\left(\frac{1}{2}a\left(1+\xi_{j-1}\right)\right) = e^{\left[-i\left(\frac{1}{2}a\left(1+\xi_{j-1}\right)\right)\left(\ln S_t-\frac{M\pi}{a^*}\right)\right]}\Psi_0\left[\frac{1}{2}a\left(1+\xi_{j-1}\right)\right]\cdot\frac{1}{\left[P_{N-1}\left(\xi_{j-1}\right)\right]^2}$$

\+

C-Th

$$C_0([\ln K]^*_u) \approx \Re\left[\frac{e^{-\alpha\left(\ln S_t-\frac{M\pi}{a*}+\frac{2\pi}{a*}(u-1)\right)}}{\pi}\frac{1}{N(N-1)}\cdot\omega^*\left(\tfrac{1}{2}a(1+\upsilon_{j-1})\right)\right]$$

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- **Fast Derivative Pricing**
  - Fractional FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
  - Fractional vs Non Uniform FFT: Empirical Analysis
  - Conclusions

Fast Option Pricing

$\overrightarrow{C_t}$ via DFT

allows

Fast Fourier Trasform Algorithms

Fast Option Pricing

$\overrightarrow{C_t}$ via DFT

Newton-Cotes

Fractional FFT

$\overrightarrow{C}_t$ via DFT

Gauss

NonUniform FFT

- Review of Derivative Pricing via DFT
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The Fractional DFT

The Fractional DFT

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i2\pi kj\gamma} f(x_j) \quad where \quad n = 1 \ldots N$$

The Fractional DFT

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i2\pi kj\gamma} f(x_j) \quad where \quad n = 1\ldots N$$

with γ that can be any complex number

If $\gamma = \frac{1}{N}$

$$\omega(n) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{N}j(n-1)} f(x_j) \quad where\ n = 1,2,\ldots N$$

The standard DFT definition

Choosing two indipendent uniform grids

Fractional FFT

Choosing two indipendent uniform grids

$x_j = jg\left(\frac{a}{N}\right) \quad for \quad j = 1 \ldots N$	**Spectral Grid**
$[\ln K]_u^* = \ln S_t - b + \lambda_u \quad for \quad u = 1, \ldots, N$	**Log-Strike Grid**

Choosing two indipendent uniform grids

Choosing two indipendent uniform grids

Implies choosing a specific value of γ

Choosing two indipendent uniform grids

Implies choosing a specific value of γ

$$\boxed{\gamma = \frac{\lambda g\left(\frac{a}{N}\right)}{2\pi}}$$

Fast Fractional Reconstruction

Step 1

Bailey-Swarztrauber F-DFT Characterization

$$\hat{\omega}(n) = e^{-i\pi(n-1)^2\gamma} \sum_{j=0}^{N-1} y_j z_{n-j-1} \text{ where } n = 1, 2, \ldots N$$

$$\boxed{y_j = f(x_j) e^{-i\pi j^2 \gamma}} \qquad \boxed{z_j = e^{i\pi j^2 \gamma}}$$

Fast Fractional Reconstruction

Step 2

2p-extension of DFT's coefficients

$$y = \left\{ \left(f\left((j-1)g\left(\tfrac{a}{N}\right)\right) e^{-i\pi j^2 \gamma} \right)_{j=0}^{N-1}, (0)_{j=0}^{N-1} \right\}$$

$$z = \left\{ \left(e^{i\pi j^2 \gamma} \right)_{j=0}^{N-1}, \left(e^{i\pi (N-j)^2 \gamma} \right)_{j=0}^{N-1} \right\}$$

Fast Fractional Reconstruction

Step 3

Bailey's Lemma

$$\hat{\omega}(n) = e^{-i\pi (n-1)^2 \gamma} \cdot \overline{\Lambda}_n$$

$$\overline{\Lambda}_n = \sum_{j=0}^{2p-1} \overline{y}_j \left[\overline{z}_{n-j-1} \right]_{2p}$$

$$0 \le n \le N-1$$

Fast Fractional Reconstruction

Step 4

2p points DFT's computation

$$\overline{Z}(m) = \sum_{j=0}^{2p-1} e^{-i\frac{2\pi}{2p}j(m-1)}\,\overline{z}(x_j) \quad \text{where } m = 0,1,\ldots 2p-1$$

$$\overline{Y}(m) = \sum_{j=0}^{2p-1} e^{-i\frac{2\pi}{2p}j(m-1)}\,\overline{y}(x_j) \quad \text{where } m = 0,1,\ldots 2p-1$$

Fast Fractional Reconstruction

Step 5

Circular Convolution Theorem

$$\boxed{\overline{\Delta}(m) = \overline{Y}(m) \cdot \overline{Z}(m)}$$

FFT

Fast Fractional Reconstruction

Step 6
F-DFT derivation as a function of DFT

$$\hat{\omega}(n) = e^{-i\pi(n-1)^2\gamma} \cdot \overline{\Lambda}_n$$

$$\overline{\Delta}(m) = \overline{Y}(m) \cdot \overline{Z}(m)$$

Fractional FFT

Fast Fractional Reconstruction

Step 6
F-DFT derivation as a function of DFT

$$\hat{\omega}(n) = e^{-i\pi(n-1)^2\gamma} \cdot \overline{\Lambda}_n$$

$$\overline{\Delta}(m) = \overline{Y}(m) \cdot \overline{Z}(m)$$

$$\hat{\omega}(n) = \frac{e^{-i\pi(n-1)^2\gamma}}{2p} \cdot \sum_{m=0}^{2p-1} e^{i\frac{\pi}{p}m(n-1)}\, \overline{\Delta}(m+1)$$

for $n = 1, 2, \ldots, N$

Fast Fractional Reconstruction

Step 7
F-FFT computation

Fast Fractional Reconstruction

Step 7
F-FFT computation

$$\boxed{\hat{\omega}(n)} = \frac{e^{-i\pi(n-1)^2\gamma}}{2p} \cdot \boxed{\sum_{m=0}^{2p-1} e^{i\frac{\pi}{p}m(n-1)} \bar{\Delta}(m+1)}$$

FFT

Fast Fractional Reconstruction

Step 7

F-FFT computation

$$\boxed{\hat{\omega}(n)} = \frac{e^{-i\pi(n-1)^2\gamma}}{2p} \cdot \sum_{m=0}^{2p-1} e^{i\frac{\pi}{p}m(n-1)}\,\overline{\Delta}(m+1)$$

F-FFT

FFT

Fast Fractional Reconstruction

The total computational cost drops

from

$$O(N^2)$$

to

$$O(6N\log_2 2N)$$

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Non Uniform FFT

The Non Uniform DFT

$$\omega(m) = \sum_{j=0}^{N-1} e^{-i\frac{2\pi}{X}x_j(m-1)} f(x_j) \quad where\ m = 1,2,\ldots M$$

$$M \neq N$$

Choosing two perfectly indipendent grids

$x_j \quad arbitrary \quad for \ j = 1,\ldots,N$	**Spectral Grid**
$[\ln K]_u = \ln S_t - b + \lambda_u \quad for\ j = 1,\ldots,M$	**Log-Strike Grid**

Non Uniform FFT

Choosing two perfectly indipendent grids

It's a natural property of the Non Uniform Approach

Gaussian Gridding Reconstruction

Gaussian Gridding Reconstruction

Step 1

Gaussian Gridding Reconstruction

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

Non Uniform FFT

Gaussian Gridding Reconstruction

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j - x - 2k\pi)^2}{4\tau}}$$

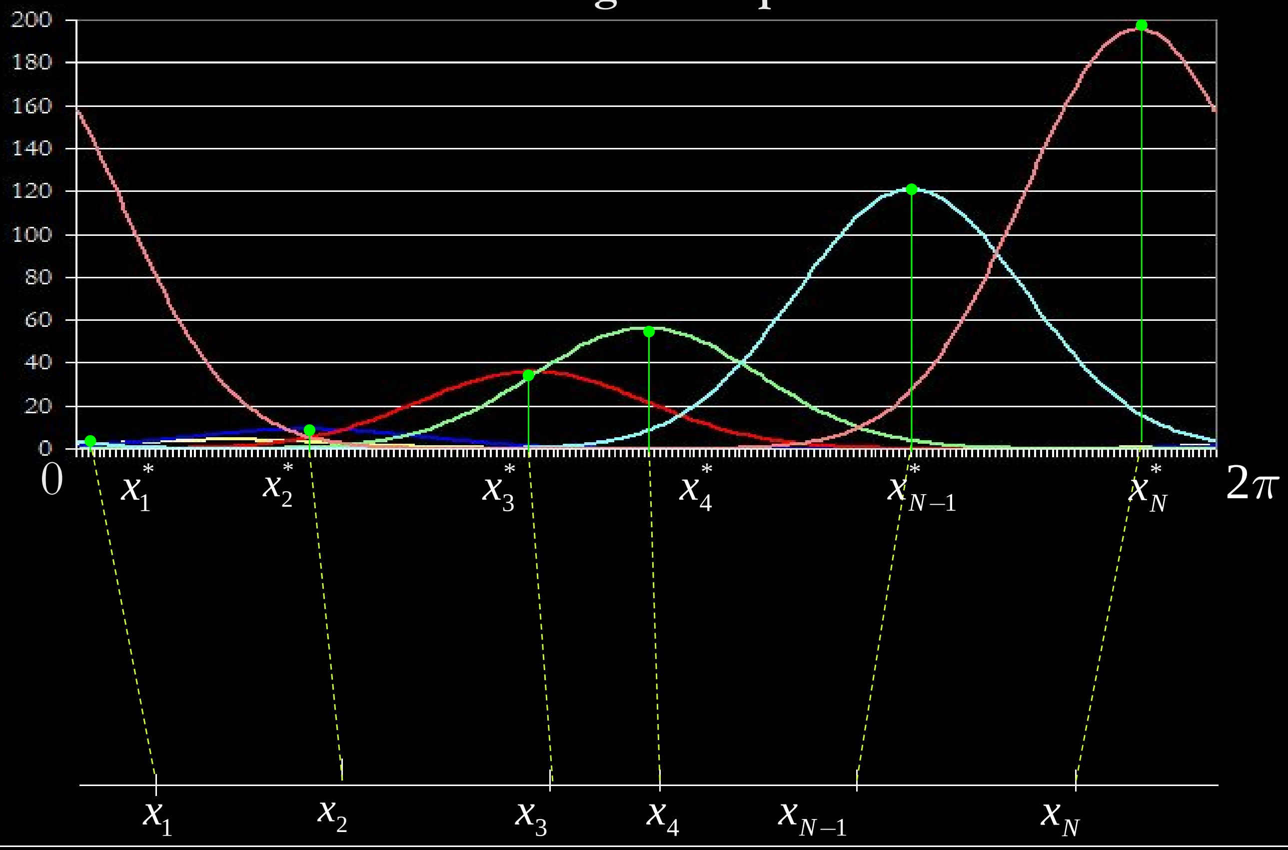
Gaussian Gridding Reconstruction

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \boxed{\sum_{k=-\infty}^{\infty} e^{-\frac{\left(x_j - x - 2k\pi\right)^2}{4\tau}}}$$

Single Components

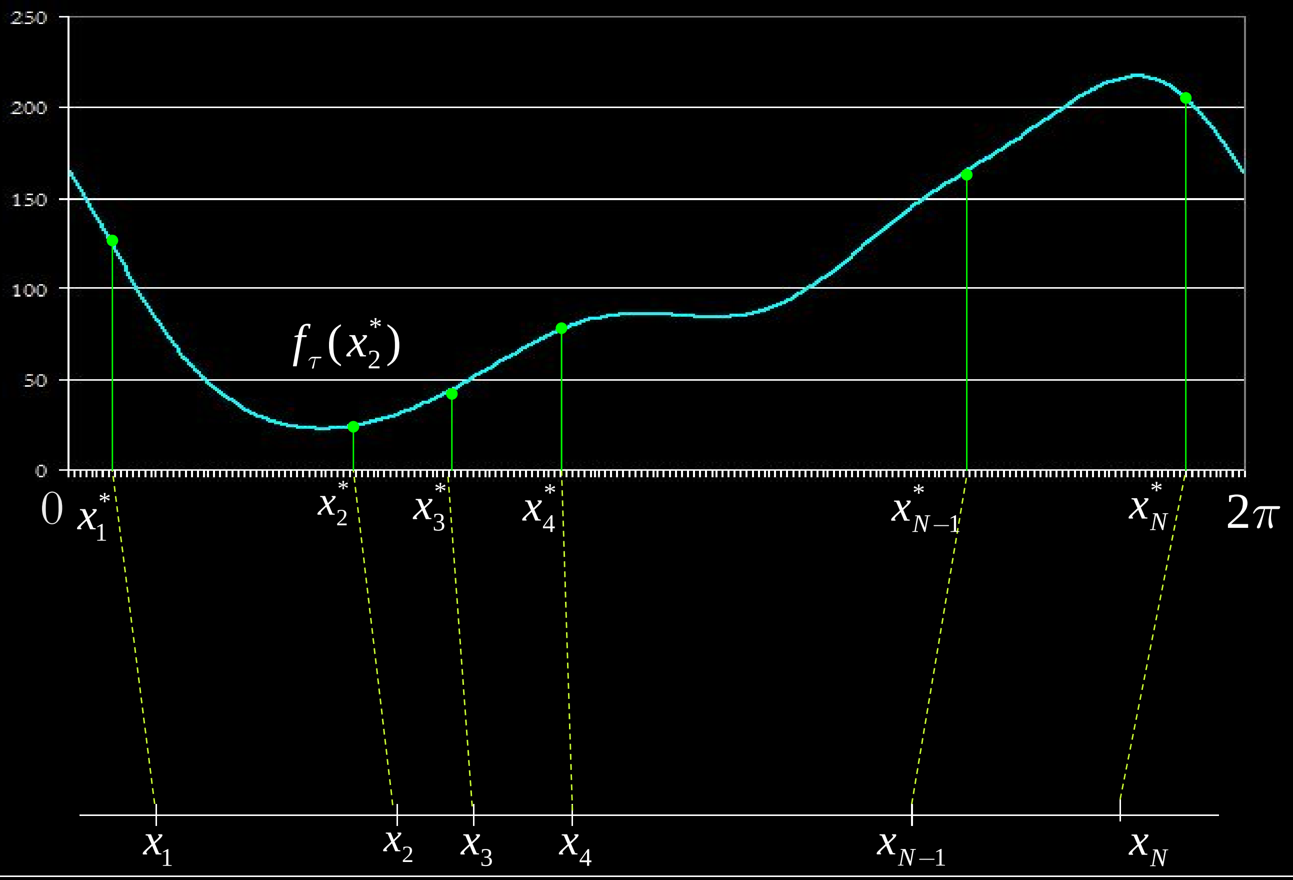


Gaussian Gridding Reconstruction

Step 1

Gaussian Convolution of the non uniformly sampled characteristic function

$$f_\tau(x) = \sum_{j=0}^{N-1} f(x_j) \sum_{k=-\infty}^{\infty} e^{-\frac{(x_j - x - 2k\pi)^2}{4\tau}}$$



Gaussian Gridding Reconstruction

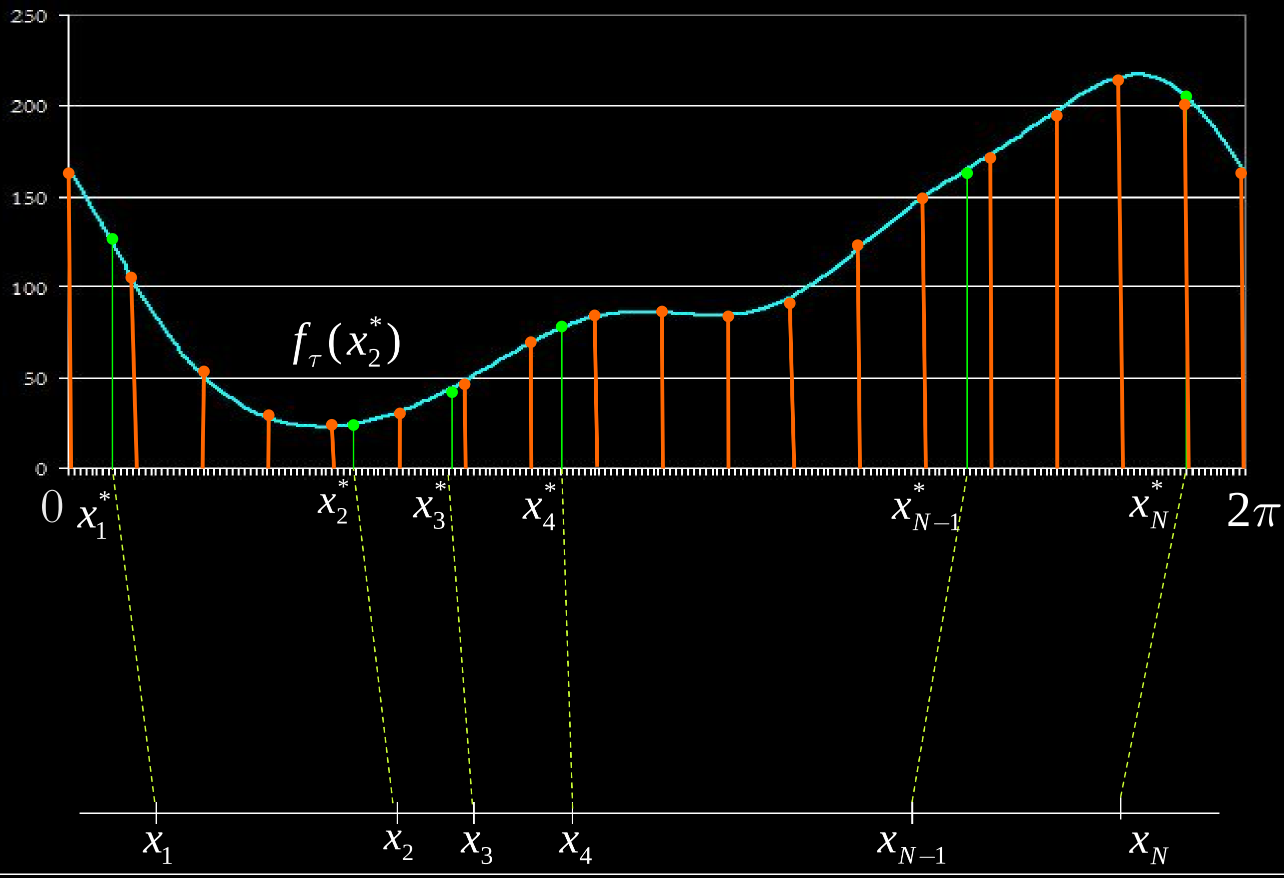
Step 2

Discretization on an uniform oversampled grid of $f_\tau(x)$

$$\widetilde{f}_\tau(y_m) = \sum_{j=0}^{N-1} f(x_j)\,\widetilde{K}(y_m, x_j, \sqrt{2\tau}; 2\pi)$$

where

$$\widetilde{K}(y_m, x_j, \sqrt{2\tau}; 2\pi) = \begin{cases} \displaystyle\sum_{k=-\infty}^{\infty} e^{-\frac{\left(2\pi\frac{x_j}{X} - 2\pi\frac{m}{M_\tau} - 2\pi k\right)^2}{4\tau}} \\ oppure \\ \displaystyle\sum_{k=-\infty}^{\infty} e^{-\frac{\left(x_j^* - 2\pi\frac{m}{M_\tau} - 2\pi k\right)^2}{4\tau}} \end{cases}$$



Gaussian Gridding Reconstruction

Step 3

Computation of the Fourier Coefficient of $f_\tau(x)$ discretised

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \sum_{m=0}^{M_\tau - 1} \widetilde{f}_\tau\left(m \frac{2\pi}{M_\tau}\right) e^{-im\frac{2\pi}{M_\tau}(n-1)}$$

Non Uniform FFT

Gaussian Gridding Reconstruction

Step 4

NU-DFT representation of the Fourier Coefficient $F_\tau(n)$

$$\widetilde{\omega}(n) = \sqrt{\frac{\pi}{\tau}} e^{n^2 \tau} F_\tau(n)$$

Gaussian Gridding Reconstruction

Step 5

DFT representation of the Fourier Coefficient $F_\tau(n)$

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau}\,\omega(n)$$

for $n = 1, 2, \ldots, \frac{M_\tau}{2}$

Gaussian Gridding Reconstruction

Step 6

NU-DFT derivation as a function of DFT

Gaussian Gridding Reconstruction

Step 6

NU-DFT derivation as a function of DFT

$$F_\tau(n) = \lim_{M_\tau \to \infty} \frac{1}{M_\tau} \omega(n)$$

$$\widetilde{\omega}(n) = \sqrt{\frac{\pi}{\tau}} e^{n^2 \tau} F_\tau(n)$$

$$\widetilde{\omega}(n) = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2 \tau^*} \frac{1}{M_\tau} \omega(n)$$

for $n = 1, 2, \ldots, \frac{M_\tau}{2}$

Non Uniform FFT

Gaussian Gridding Reconstruction

Step 7

NU-FFT computation

Gaussian Gridding Reconstruction

Step 7
NU-FFT computation

$$\boxed{\tilde{\omega}(n)} = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2 \tau^*} \frac{1}{M_\tau} \boxed{\omega(n)}$$

FFT

Non Uniform FFT

Gaussian Gridding Reconstruction

Step 7
NU-FFT computation

$$\boxed{\tilde{\omega}(n)} = \lim_{M_\tau \to \infty} \sqrt{\frac{\pi}{\tau^*}} e^{n^2 \tau^*} \frac{1}{M_\tau} \boxed{\omega(n)}$$

NU-FFT

FFT

Computational Cost

The major computational cost of the
Procedure is the FFT on the oversampled grid

Non Uniform FFT

Computational Cost

The major computational cost of the
Procedure is the FFT on the oversampled grid

Choosing the oversampling ratio

$$M_{\tau} = 2M$$

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The total cost of the procedure is $\simeq 2M \log_2 2M$

- Review of Derivative Pricing via DFT
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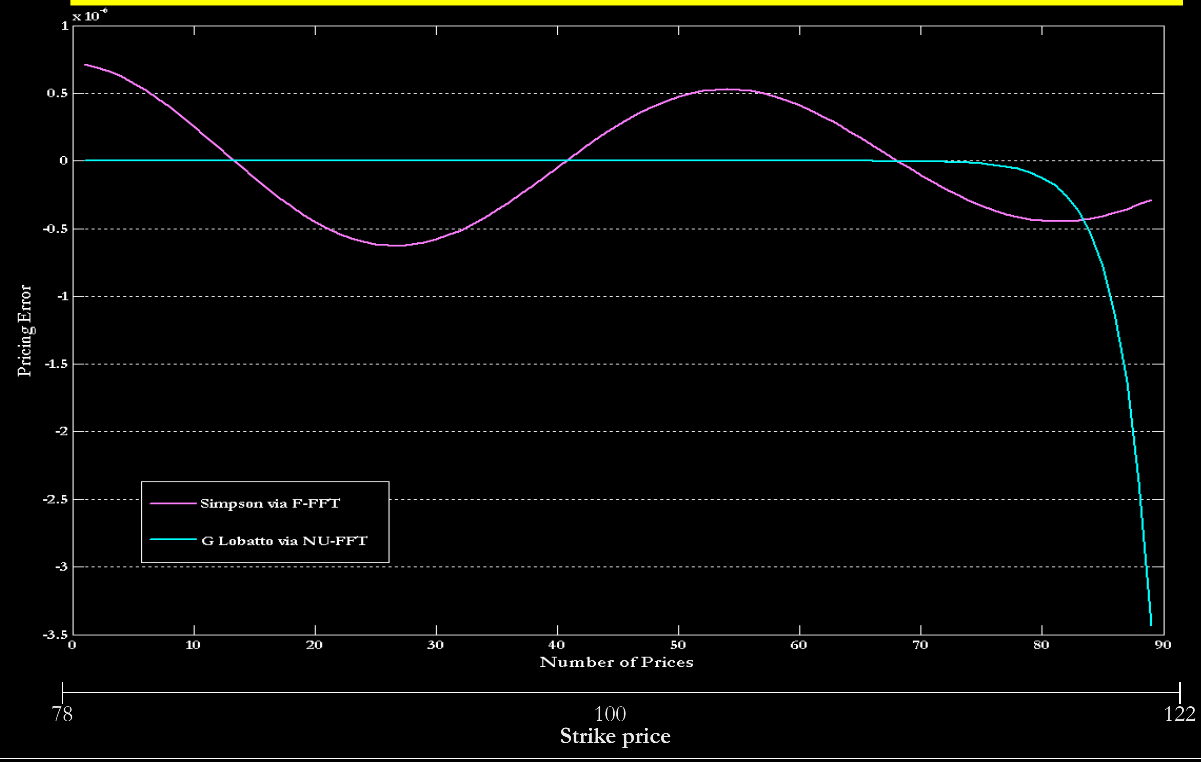
  - Convergence Theorems for Uniform Grids
  - Convergence Theorems for Non Uniform Gaussian Grids
- Fast Derivative Pricing
  - Fractional FFT
  - Non Uniform FFT
    - Gaussian Gridding: a matter of interpolation
  - Fractional vs Non Uniform FFT: Empirical Analsysis
  - Conclusions

ACCURACY

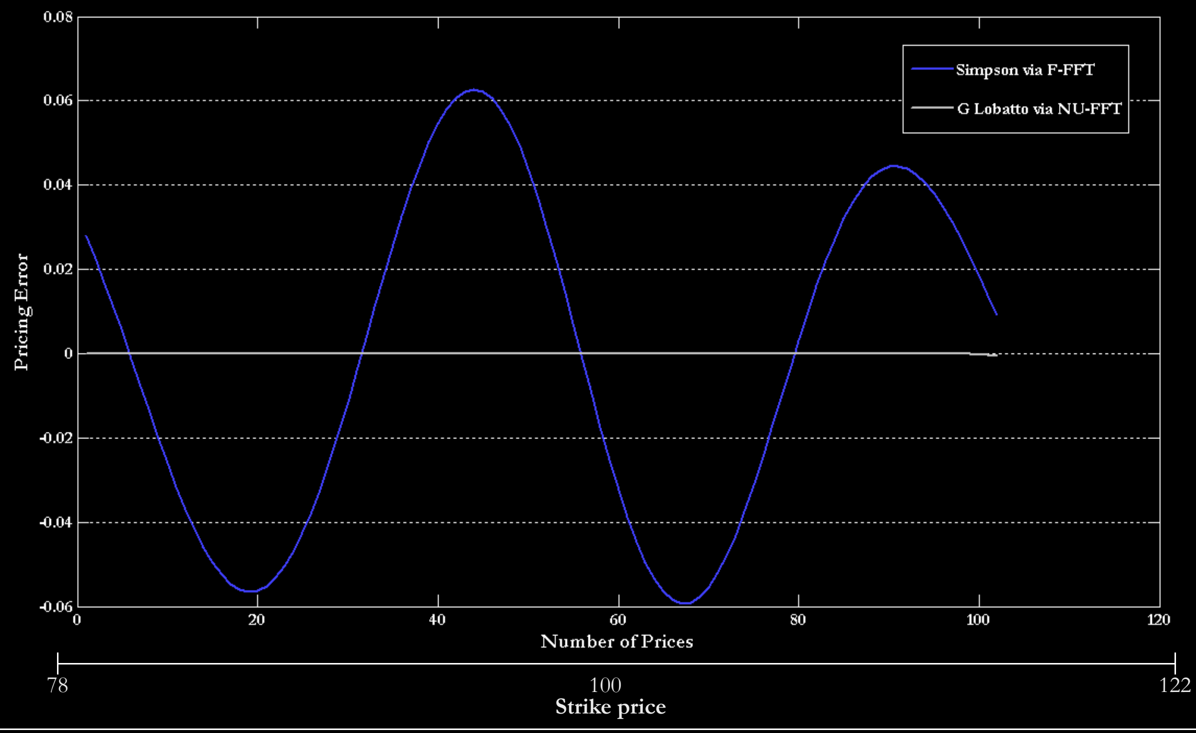
$\sigma = 0.3$

BLACK SCHOLES MODEL



$\sigma = 0.1$

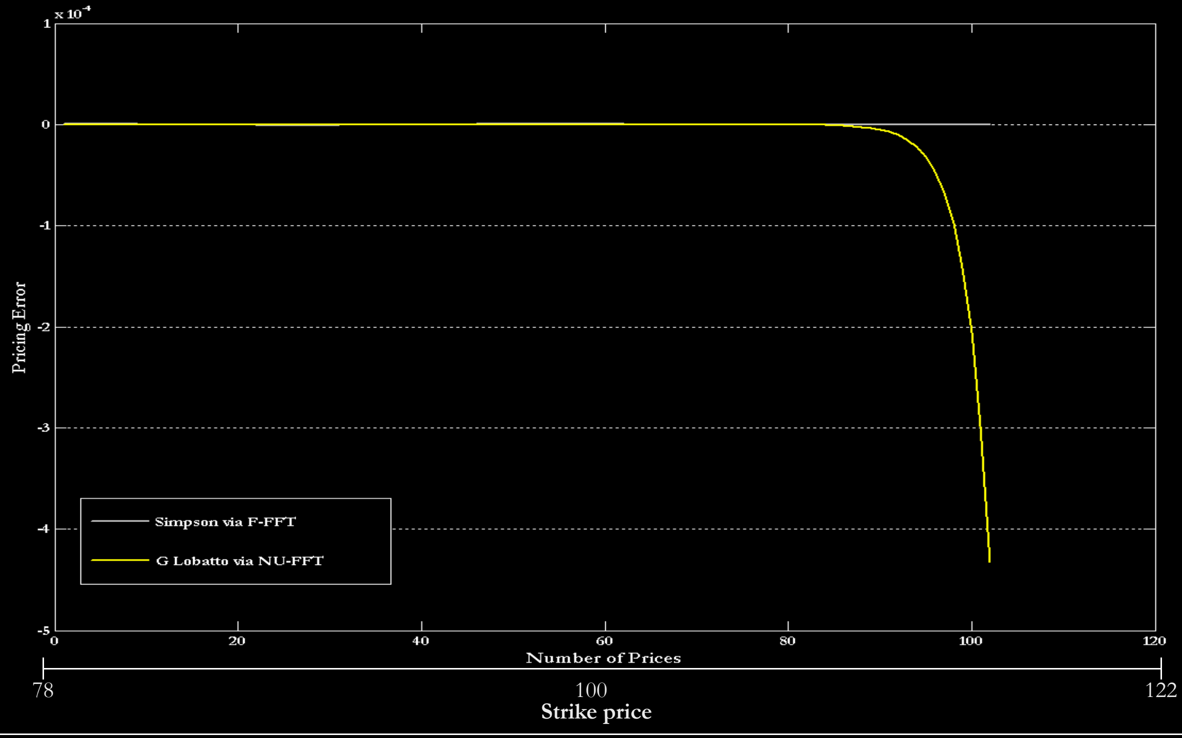
BLACK SCHOLES MODEL



STABILITY

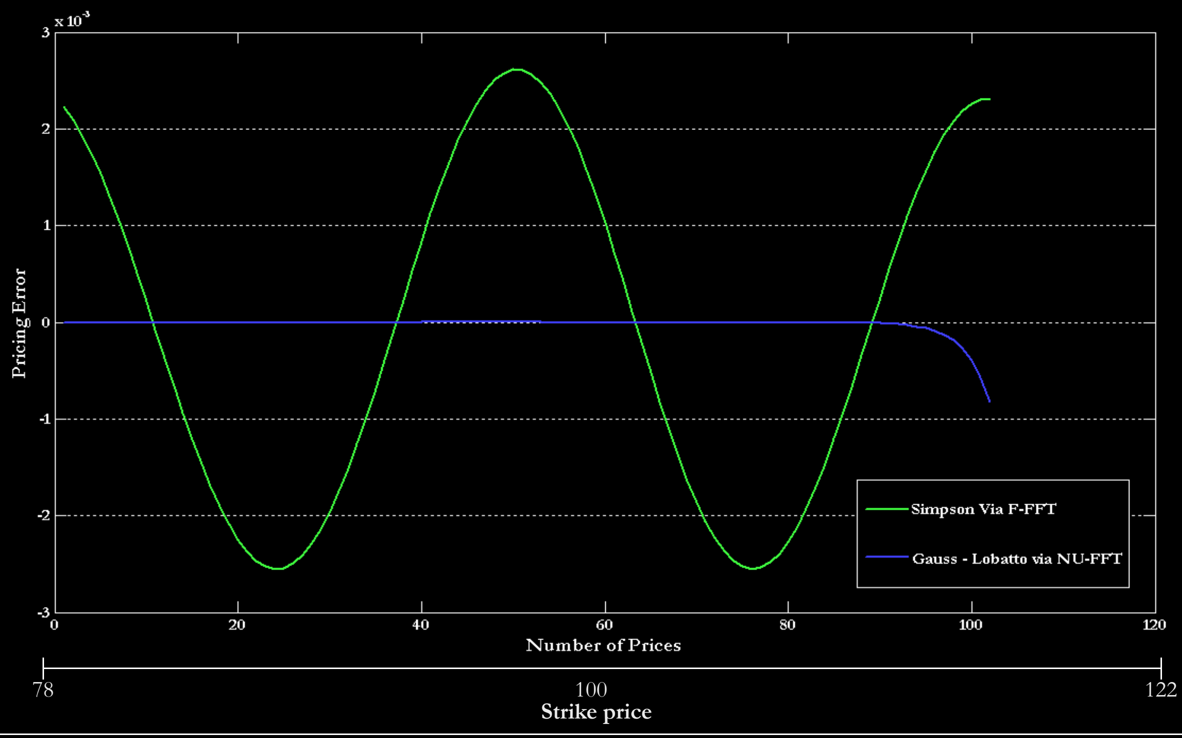
Empirical Analysis

BLACK SCHOLES MODEL

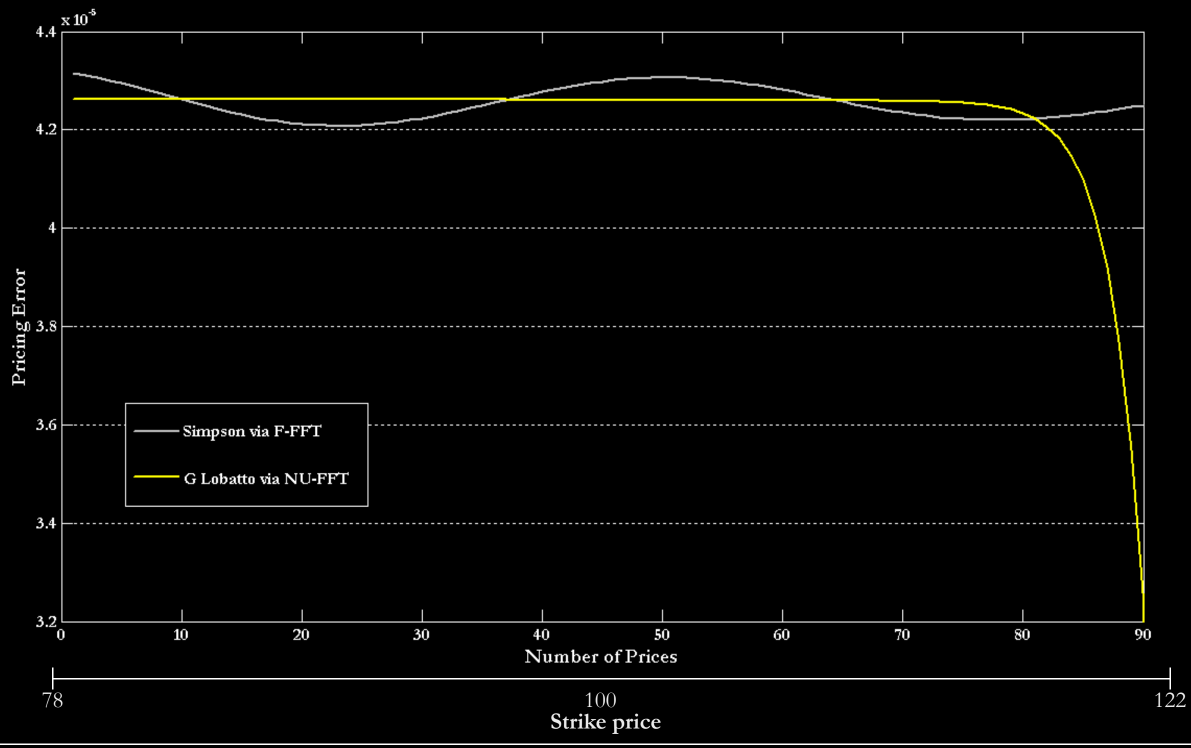


Empirical Analysis

HESTON MODEL

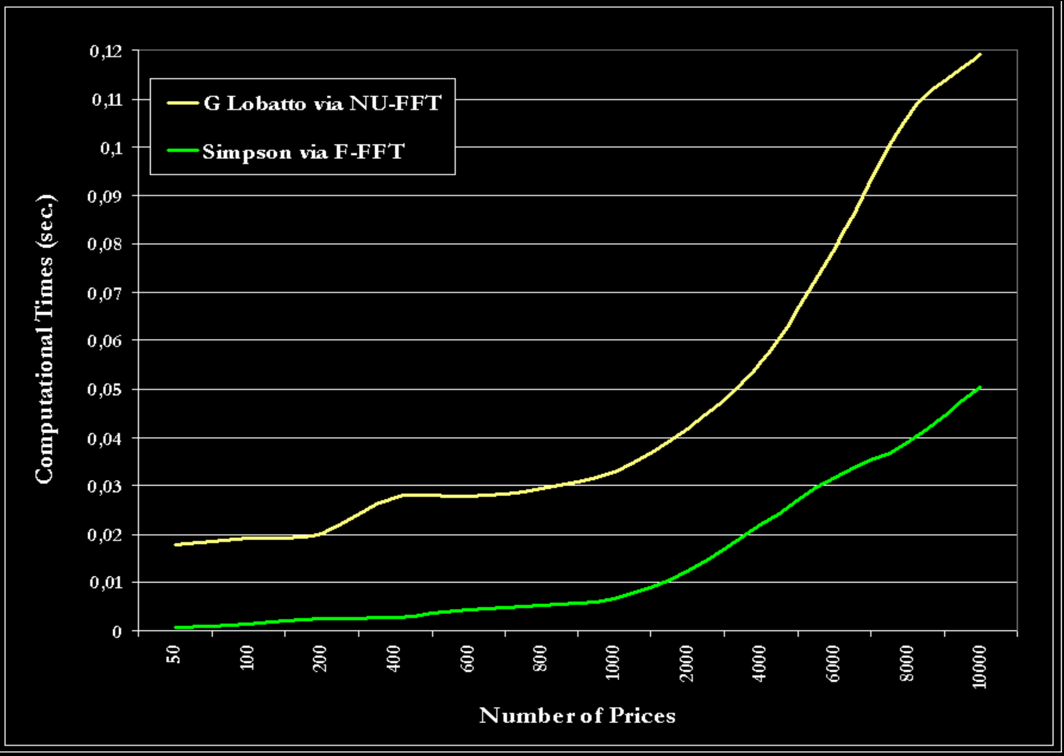


MERTON MODEL



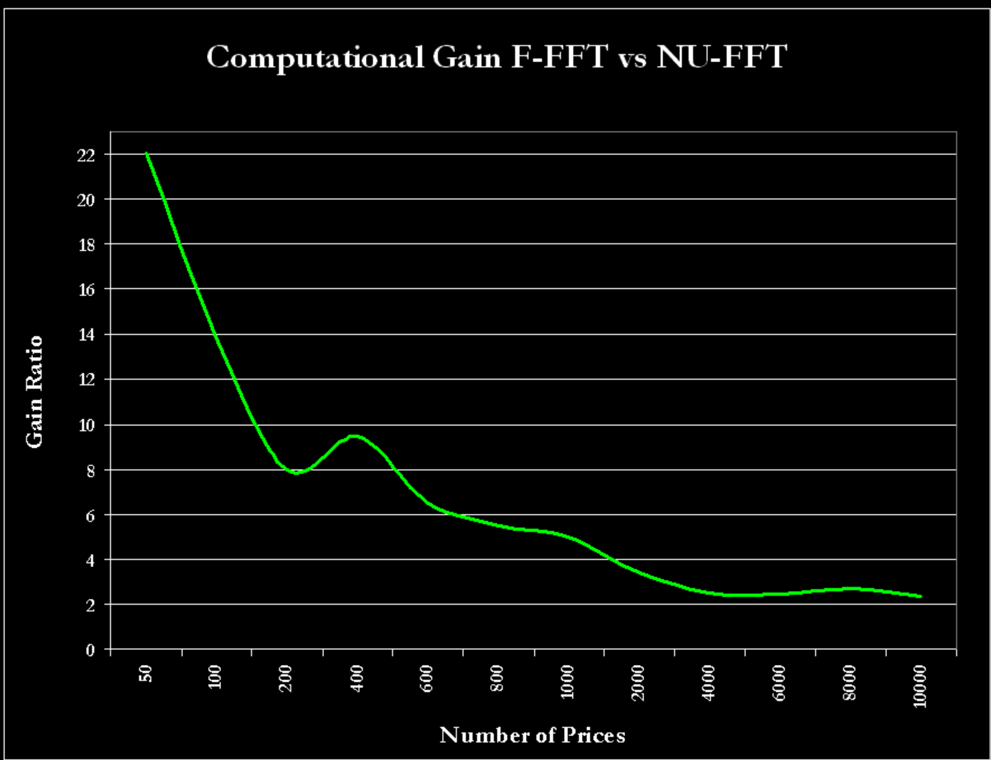
SPEED

Empirical Analysis



Centrino 1600Mhz – 1gb RAM
Mean Value over 1000 runs

The Computational Framework



Centrino 1600Mhz – 1gb RAM
Mean Value over 1000 runs

At very low time scales, the differences are negligible

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Use of Gaussian Grids

F-FFT	NO
NU – FFT	YES

Indifference to Nyquist-Shannon Limit

F-FFT	YES
NU – FFT	YES

Indipendent Price Grids

F-FFT	YES
NU – FFT	YES

FFT's like - Accuracy

F-FFT	YES
NU – FFT	YES

Stability of Pricing

F-FFT	NO
NU – FFT	YES

Speed of Pricing

F-FFT	YES
NU – FFT	YES

Conclusions

	F-FFT	NU – FFT
Gaussian Grids		■
NS Limit	■	■
Indipendent Grids	■	■
Accuracy	■	■
Stability		■
Speed	■	■

Numerical Methods in Semianalytical Derivatives Pricing

Efficient Solutions for Standard, Fractional and Non Uniform Discrete Transforms