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COMMISSIONE NAZIONALE PER LE SOCIETÀ E LA BORSA

QUADERNI DI FINANZA

STUDI E RICERCHE

A QUANTITATIVE RISK-BASED APPROACH TO THE TRANSPARENCY ON NON-EQUITY INVESTMENT PRODUCTS

by the QUANTITATIVE ANALYSIS UNIT

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A quantitative risk-based approach to the transparency on non-equity investment products

by the QUANTITATIVE ANALYSIS UNIT*#

Key Words:
transparency, risk, return, investment time horizon, unbundling, risk-free asset, benchmark, prospectus, product information sheet, volatility, GARCH, stochastic process, probability scenarios, key information document, first passage times, pricing, payoff, fair value, mark-up, mispricing, risk-neutral measure, martingale, migration.

Abstract
The purpose of the transparency on the risk profile of non-equity investment products is to allow investors to take informed investment decisions.
In an international framework characterized by an increasing globalization of markets and a growing integration of banking, asset management and insurance activities, the traditional use of narrative descriptions of the various risks is no longer effective. On the contrary, it would be better to use synthetic indicators which are immediately comprehensible to investors and defined in relation to robust and objective quantitative metrics.
The result is a risk-based approach to transparency built on three pillars: the recommended investment time horizon, that is the investment horizon which is compatible with the investor's liquidity preferences, the potential returns that the non-equity investment product can offer to the investor, and the degree of risk associated with that product. The information obtained from these three pillars allows to determine the essential elements of the financial investment, both for the purposes of the disclosure to be provided in offering prospectuses, and for the suitability tests performed by distributors.
To ensure a concrete implementation of the risk-based approach to transparency, the current fragmentation of the EU regulatory framework should be overcome through the issuing of a single directive on the transparency of non-equity investment products able to concretely realize the levelling the playing field principle.
In this perspective, the Italian law-maker could intervene to align the transparency requirements of class I policies to those of the financial-insurance products already under the Consob supervision.

JEL Codes: C02, C32, C51, C61, G11, G17, G20, G32, G38, K23

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Introduction

Transparency regulation regarding the risk profile of non-equity investment products[(1)] is one of the main competencies of any financial supervisory Authority. The definition of suitable provisions on this issue has been at the centre of EU debate for some time and, also in the light of the recent events involving the international financial markets, such debate is increasingly focused on searching for synthetic risk indicators, based on quantitative metrics which are robust, objective and backward verifiable. The traditional narrative description of the risks associated with a financial product seems inadequate to allow informed investment choices in a context where the integration of markets, products and financial players often makes it difficult to separately analyze the various types of risk, and instead, allows the measurement and monitoring of the overall risk profile of financial products.

Specifically, the integration process of non-equity investment products now seems complete. The result is a situation where products can be classified irrespective of the specific ways and vehicles used to carry out the public offering. In fact, even if the heterogeneity in names of the products, categories of the issuers, distribution channels and labels of the costs applied creates the appearance of actual differences also in the underlying financial engineering, the universe of investment products can effectively be classified into the following three types of financial structures: “risk target” products, “benchmark” products, and “return target” products.

The intermixing of the various risk factors and the fact that all products may be classified within one of the three above listed financial structures are a clear indication for the Authorities competent for risk transparency supervision: the regulation on the matter should be standard and should translate into suitable regulatory provisions a coherent approach to risk measurement and to its correct representation to the potential investors. This will create a context compatible with the concrete realization of a levelled playing field. As a consequence it is necessary to implement a thorough revision of both the European and the Italian regulatory framework, which is currently too much fragmented. European policy-makers, in fact, decided to regulate products having the same financial engineering with different provisions. Moreover, European Union laws and their transposition into national laws and regulations have introduced a partitioning of competencies among the various financial supervisory Authorities, which often ignores the existence of common financial structures among products offered by issuers belonging to different categories and, in some cases, even by issuers belonging to the same category.

This paper illustrates a quantitative approach to the transparency on the risk profile of non-equity investment products. This approach – progressively implemented by Consob into its regulation since the beginning of the decade – is based on three pillars, corresponding to three synthetic indicators defined through the development of specific quantitative methods and the preliminary classification of products as either “risk target”[(2)], “benchmark” or “return target”[(3)], in line with the practices of financial intermediaries.

The first pillar, implemented into Consob regulation since 2004, is represented by a table showing the probability scenarios of the return of the financial investment at the end of the recommended time horizon. The purpose of this indicator is to illustrate the unbundling of the price of a financial product at the subscription date and to provide clear and concise information on the possible results of the investment and on its costs. This information is compared with the results which can be obtained from an investment in a risk-free asset over the same time period, in order to allow a better assessment of the product’s “performance risk”, meant as the product’s likelihood to create added value for the investor, both in absolute terms and in relative terms with respect to the risk-free asset.

[(1)] This expression is used, for instance, to refer to open-ended mutual funds, SICAVs, unit-linked and index-linked financial-insurance products, and financial products issued by banks such as covered warrants, certificates and structured bonds.

[(2)] In the financial-insurance and asset management industries, these products are generally referred as “flexible”.

[(3)] The expression “return target” is a generalization of the more well-known expression “protected portfolios”, which is commonly used for financial-insurance products and for open-ended mutual funds which intend to safeguard the entire financial investment or a part of it, for instance by applying portfolio insurance techniques.

The second pillar, implemented into Consob regulation since 2001, is a synthetic indicator of the degree of risk of the financial product. This may take values within an increasing scale of six qualitative classes: *low*, *medium-low*, *medium*, *medium-high*, *high* and *very high*, which are mapped to specific values of quantitative risk measures based on the volatility of the financial product's returns. For each product, an initial class has to be indicated which is consistent with the risk profile underlying its financial engineering and, where relevant, its investment policy, and hence, taking into account its positioning in terms of the quantitative risk measure adopted. In this perspective, the migration to a class associated with a higher or lower risk than the initial one occurs in relation to the evolution of such measure over time. This work presents a methodological solution which, in order to determine the above risk measure, exploits several well-known results from stochastic limit theory and uses them to calibrate six increasing intervals of annualized volatility of the financial product's daily returns, one for each of the six above listed qualitative risk classes.

In the case of "benchmark" products, as established by Consob regulation since 2007, information deriving from the degree of risk is supplemented by a qualitative indicator of the asset management style, which may be either passive or active. In this second case, the intensity of the active asset management style − and, thus, the amount of deviation from the chosen benchmark − is alternatively classified as: *limited*, *intermediate*, and *considerable*. Each of the three classes, depending on its degree of risk, is mapped into a measure based on the comparison between the volatility of the financial product's returns and that of the benchmark's returns. The initial class of deviation from the benchmark has to be consistent with the planned active management strategy, while any migration to a different class may occur or not depending on the specific time evolution of the risk measure adopted. The methodological solution here suggested to identify this measure is developed according to the same line of reasoning used to quantify the risk class of the product: each of the six volatility intervals is associated with three symmetric intervals − corresponding to the three classes of deviation from the benchmark − of the difference between the annualized volatility of the product's daily returns and that of the benchmark's returns.

The third pillar, implemented into Consob regulation since 2001, is the recommended investment time horizon, meant as an indication of the optimal investment holding period. Clearly, this indication should be formulated on the basis of the product's underlying financial structure and of the related risks and costs. This pillar is important not only for the transparency, but also for the suitability of the investment. In fact, it unequivocally qualifies both the time period for which the investor should give up his cash holdings and the time horizon to take as reference for the calculation of the probability scenarios. In addition, the information regarding the future performance of the financial product takes on significance with respect to the goals that drive the investor's choices only if considered in relation to the optimal investment time period implicit in the underlying financial engineering. At the same time, the recommended investment time horizon is the key to a coherent view of the first two pillars: the interdependence between the degree of risk and the potential returns can be fully appreciated only over this time horizon.

The definition of the pillars and the comprehensive interpretation of the information they contain are strictly linked to the type of the underlying financial structure and, as a consequence, to the concept of liquidability of the non-equity investment product, meant as the possibility of disinvesting at a specific time without incurring a loss and without waiving the benefits offered by the product in terms of extra-returns above those of the risk-free asset.

This paper is organized into three parts.

The first part focuses on the regulatory framework for the transparency of non-equity investment products. In particular, after describing the three financial structures into which all products may be classified, the paper moves on to illustrate how the coexistence of a multitude of EU directives and other provisions on the same subject introduces an unjustified differentiation of rules and favours regulatory arbitrages, thus jeopardizing the effective pursuit of both a levelled playing field between the various categories of issuers and a risk disclosure which can be concretely exploited by investors. This is the reason for the urgent need for a thorough revision of the EU provisions on this issue, which should move towards a single directive on the transparency of non-equity investment products.

The second part outlines the three-pillar approach, highlighting the underlying logic and providing a detailed description of the methods developed in relation to each pillar. These methods provide indications to intermediaries who have to issue a prospectus; at the same time they present possible solutions to map the qualitative risk indicators required in the offering documentation into the corresponding quantitative risk measures. The aim is to develop solutions to ensure that the information given to investors is meaningful and comparable across products and to promptly detect migrations between risk classes or between classes of deviation from the benchmark. This illustration thus presents a risk-based approach to correctly quantify and represent the risk of non-equity investment products, essentially aimed at harmonizing the information presented to investors in the offering documentation with the information produced and analyzed by intermediaries in their own risk management activities, and not at prescribing the use of specific models.

The third part shows the results of an empirical analysis carried out by applying the risk-based transparency approach to a representative sample of European open-ended mutual funds. The decision to limit the analysis to mutual funds is due to the availability of better quality information and panel data large enough for the purposes of the study conducted. In fact, the same analysis could be carried out for other non-equity investment products.

This is because, as has been said, all products may be classified between three types of structures ("risk target", "benchmark" and "return target") which are fully represented in the universe of open-ended mutual funds. In fact, protected funds have the same "return target" management strategy which characterizes structured bonds, index-linked financial-insurance policies and unit-linked financial-insurance policies investing in protected internal funds, and many ETFs are "benchmark products", just like many certificates and passively managed internal insurance funds sold in unit-linked policies.

1 Non-equity investment products: financial structures and regulatory framework

Understanding the financial structure of non-equity investment products is a fundamental step to correctly formalize the transparency regulation and to define a quantitative approach for the measurement and representation of the risk profile.

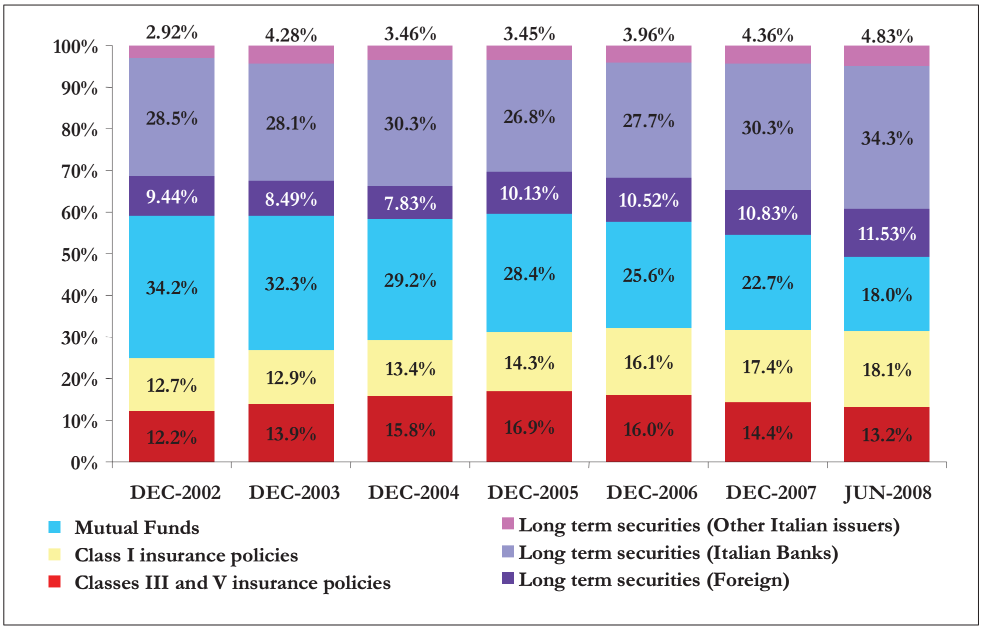
This section illustrates the main characteristics of the three financial structures which are offered to investors, even if through different vehicles. The main reason for these differences is the heterogeneity of the regulatory framework which, at the EU level, attributes the regulation concerning the offering documentation to three distinct directives, even though some sort of standardization in the representation of risk and cost profiles has been introduced by the MiFID, which establishes further disclosure obligations in the distribution of products.

The various regulatory choices made, also recently, by Consob have been carried out in order to align the transparency on products sharing the same financial engineering. These measures have consistently regulated the offering documentation issued by Italian and foreign insurance companies, Italian asset management companies (SGR) and SICAVs, as well as the information that distributors must provide to investors.

Nonetheless, significant differences do remain in the offering documentation of non-equity products issued by banks and in that of class I insurance products.

In this regard, with specific reference to the Italian situation, Figure 1 shows the breakdown of households' wealth invested in non-equity financial products for the period December 2002−June 2008, during which these products represented on average 34.6% of total households' financial wealth[4].

Figure 1. Breakdown of Italian households' portfolios of non-equity investment products (December 2002-June 2008)



Calculations on Consob, Bank of Italy, Isvap and Borsa Italiana S.p.A. data.

As shown by Figure 1, in the last few years there has been a significant drop in investments in

(4) For a more detailed illustration of the data see appendix A.

mutual funds[5] and class III and V insurance products[6], whereas the share of wealth invested in non-equity securities issued by banks[7] and, more recently, in class I insurance products[8] has remarkably increased.

The re-allocation of Italian households' portfolios, occurred over the last years, can be partially explained by the regulatory arbitrages which the fragmentation of the regulatory framework has made possible. It also shows the need for action by the European policy-makers aimed at standardizing the regulation on this matter, in order to ensure a homogeneous representation of the essential information on the risk profile of products sharing the same financial engineering.

1.1 Types of financial structures

The analysis of non-equity investment products offered to the public shows that, despite of the variety of labels, issuers and distribution channels used, it is possible to identify three recurring financial structures: "risk target" products, "benchmark" products, and "return target" products.

"Risk target" products invest in any market and any financial instrument in order to optimize over time a given target in terms of risk exposure. These products pursue specific target returns only as a secondary goal. In other words, within the context of the traditional risk-return approach, where the choice is between maximizing the return for a given level of risk or minimizing the risk given a specific target return, in these products the asset allocation favours the first objective. To this end, *ex ante* minimum and maximum thresholds are typically defined for the values of a risk measure and such thresholds are the reference point for the risk-taking decisions. This financial structure is quite common in the insurance and asset management industries, where one can often observe the marketing of a range of internal insurance funds or mutual funds differentiated by increasing levels of risk.

"Benchmark" products have an investment policy which is anchored to a benchmark, and in relation to this benchmark the asset management style may be either passive or active. In the first case, the product is substantially a replica of the benchmark[9], while in the second case, the composition of the assets' portfolio, differs, to a greater or lesser extent, from that one of the benchmark depending on the specific objectives that the asset manager intends to pursue.

"Return target" products feature a financial engineering (and, in some cases, a consequent investment policy) aimed at pursuing a minimum target return on the financial investment. This type of financial structure includes all products obtained as a static or dynamic combination of one or more risk-free (or low-risk) financial assets and of one or more risky financial assets. The former allow to pursue the target return, while the latter contribute to the generation, at the end of the recommended investment time horizon, of a potential extra-return over the target[10].

It is also possible to have mixed structures, with a double protection mechanism or with the coexistence of both a protection mechanism and a guarantee. In this second case, the protection mechanism typically reduces the role played by the guarantee, and, thus, also its cost, as a smaller number of states of nature needs to be insured against. When the financial guarantee[11] falls

(5) In particular, over the observed period, the share of mutual funds dropped from 34% to 18%.

(6) Between December 2006 and June 2008 − when the obligation to publish a prospectus was extended to class III and V insurance products − the overall weight of these products fell from 16% to approximately 13%.

(7) In the period December 2002−June 2008, the percentage of non-equity investment products issued by banks rose from approximately 28% to over 34%.

(8) Between December 2006 and June 2008, these products rose from approximately 16% to over 18%.

(9) These are usually index funds and many certificates.

(10) Examples of "return target" products are structured bonds, protected unit-linked policies and index-linked policies, as well as all mutual funds which optimize the return over a given time horizon. It follows that this type of products includes also those certificates and index funds (both mutual funds and internal insurance funds) where either the indexation mechanism or the financial structure of the reference index are aimed at achieving the aforementioned target return. A specific case of "return target" products is represented by covered warrants, as they have only a derivative component.

(11) In this paper, the concept of financial guarantee is used to indicate that the non-equity investment product offers, at specific maturities, a financial result either predetermined or whose underlying formulae are known, even without the specific existence of a guarantor as established, in exclusive terms, in the regulations of the Bank of Italy and Isvap (the Italian insurance industry supervisory Authority).

outside the financial engineering of the investment product – such as in the case of some products embedding guarantees provided by a third subject – this may affect the product's overall risk profile, costs regime and recommended investment time horizon, but not the type of underlying structure, which, thus, may also be "risk target" or "benchmark".

1.2 Transparency regulation of non-equity investment products

During the last few years, several EU directives have been issued which define the requirements for the prospectus for the public offering of non-equity investment products. The multitude of directives reflects the intention of European policy-makers to differentiate regulation on prospectuses[12] depending on the category of the issuer, according to the classic distinction between banks, asset management companies/SICAVs and insurance companies.

The UCITS III directive[13] and subsequent recommendations of the European Commission[14] regarding open-ended mutual funds[15] and SICAVs grant Member States the option to require prior authorization for the publication of the prospectus, which is comprised of two documents: the *Simplified Prospectus*, to be mandatorily delivered to investors before the subscription of the contract, and the *Full Prospectus*, to be delivered to investors upon request. In case of a public offering in a host Member State, it is the offeror who must directly file the prospectus with the supervisory Authorities of said State. The UCITS III directive does not specify minimum individual investment thresholds beyond which the offeror is not required to issue a prospectus. As regards the information provided in the prospectus, the UCITS III directive grants Member States the option to define specific templates establishing the format and the minimum content of the prospectus consistently with the indications given in Annex C of the directive and in the aforementioned EC recommendations. Should a Member State opt to exercise this right (as in the case of Italy), it will define the templates for the prospectuses of mutual funds offered by issuers with registered offices in its territory. The Member State's competence is not extended to prospectuses of mutual funds offered within its borders by issuers with registered office in another Member State[16]. The European Commission is currently working on a new directive regarding open-ended mutual funds and SICAVs (so-called UCITS IV directive). Under the new directive the *Simplified Prospectus* would be replaced by a new document called *Key Information Document* (hereinafter KID), organized according to the logic of a *product information sheet* which, similarly to the provisions already implemented into the Italian regulatory framework, has been conceived to illustrate, in a few pages, the essential information on the risk-return profile and on the costs of the investment. The KID shall in fact contain a concise representation based on synthetic risk indicators and on charts and tables which illustrate the costs and returns of the mutual fund.

The directive 2003/71/EC (so-called Prospectus directive) which applies also to non-equity investment products issued by banks (i.e. ordinary and structured bonds, covered warrants and certificates), introduces the obligation of prior approval of the prospectus by the supervisory Authorities of the home Member State through the issue of a specific authorization. The Prospectus directive also specifies minimum individual investment thresholds beyond which the offeror is not required to issue a prospectus[17]. In case of a public offering in a host Member State, the home

(12) This refers to: format and minimum content of prospectus templates, authorization regimes, publication regimes, regimes of transmission to the competent supervisory Authorities of the Member States, and regimes of delivery to investors.

(13) Directive 85/611/EC.

(14) Recommendations 2004/383/EC, 2004/384/EC and 2007/16/EC.

(15) Prospectuses issued by closed-ended mutual funds incorporated in the form of a company are regulated by the Prospectus directive. Transparency regulation of contractual closed-ended funds – which are a typical Italian phenomenon – is established by Consob.

(16) For the sake of simplicity, we avoid an in-depth description of the regulation with reference to the concept of harmonized fund (UCITS). In fact, in derogation of the general principle described above, the offering of non-harmonized funds in a Member State must take place in accordance with the rules on the the prospectus templates provided by said Member State.

(17) Indeed, the prospectus must be issued only for public offerings where the individual investment size is less than

Member State which granted the authorization must send the prospectus to the host Member State. In general[18], the information on the product is contained in two documents, which are not required to be delivered to investors: the *Base Prospectus* and the *Final Terms*. Commission regulation 809/2004/EC, implementing the Prospectus directive and directly applicable in all Member States, defines the format and the minimum content of the *Base Prospectus* according to a narrative logic based on a detailed listing of all costs and risks of the investment. This regulation also defines the format of the *Final Terms*, which, however, may be slightly adjusted by national Authorities, though always in accordance with the standards identified for the *Base Prospectus* by the European policy-maker. In substance, the *Base Prospectus* reveals the issuer's intention to launch the public offering of an investment product, providing a first description of some general information, while the detailed description of the product's characteristics is given in the *Final Terms*. This document is organized according to a *product information sheet* logic and, thus, potential investors base their assessment of the proposed investment on it. It is worth mentioning that the Prospectus directive is currently under revision. Within the scope of this revision, the European Commission is evaluating changes to the format and to the contents of the offering documentation to be provided to retail investors, in order for it to promote a clear understanding of the essential features of financial products, specifically including: potential returns, payoff structure, direct and indirect costs[19], financial risks, and terms and conditions of any capital guarantee. The Commission's analysis focuses on increasing the effectiveness of pre-contractual information in protecting investors, primarily because the current regulation, by penalizing issuers for omitting information, has stimulated the drafting of long and complex prospectuses which are not easily comprehensible to the retail public. For these reasons, the European Commission seems in favour of a simpler offering documentation and of regulatory provisions able to ensure a harmonized approach among various product categories. In this perspective, the European Commission is considering various alternatives, including the introduction of a summary document as an integral part of the prospectus, or the use of an approach similar to that underlying the definition of the KID.

The directive 2002/83/EC (also known as Life Assurance directive) concerning products issued by insurance companies defines general provisions on the offering documentation for such products. It does not require Member States to subject these documents to prior approval nor to systematic controls, and therefore, supervisory Authorities can examine these documents *ex-post*, on a test-basis only. In case of a public offering in a host Member State, the foreign insurance company is required to fulfil the duties related to the publication of the prospectus by directly interacting with the competent supervisory Authorities of said State. The Life Assurance directive grants Member States the option to establish minimum information requirements for the offering documentation by drawing up specific templates. These templates apply to public offerings made within the territory of a Member State, irrespective of the nationality of the offering insurance company, and no exemptions are allowed. Therefore, unlike open-ended mutual funds under the UCITS III directive, EU insurance companies that intend to sell their products in a host Member State must issue prospectuses in compliance with the templates in force in that State. The Life Assurance directive holds the classification of products into various classes[20] as established in previous EU provisions. In the Italian law, this distinction has led to a division of competencies between Consob and Isvap as regards transparency supervision of insurance products[21]. More

50,000 Euros.

[18] For the sake of simplicity, we omit the case of the so-called "tripartite prospectus" and the various cases deriving from the principle of incorporation by reference.

[19] Information on costs must also include the costs which are embedded in the structure of the product as well as the multiple layers of cost resulting from the use of the so-called wrappers.

[20] See Annex I to the Life Assurance directive.

[21] Law no. 262 of 28 December 2005, and subsequent amendments pursuant to Legislative Decree no. 303 of 29 December 2006, repealed art. 100, subsection 1 (f) of the Consolidated Law on Finance, thereby extending the obligation to publish a prospectus to financial products issued by insurance companies falling within classes III and V pursuant to Legislative Decree no. 209 of 7 September 2005 (that is, respectively, unit-linked or index-linked policies and capital redemption policies). With these provisions, the Italian regulator extended the obligation to publish a prospectus also to banking products which were previously exempt pursuant to article 100, subsection

specifically, products belonging to classes III and V have been assigned to the transparency supervision of Consob which, consequently, in July 2007 introduced specific prospectus templates. The attribution of competencies to Consob was motivated by the fact that these products feature a strong or exclusive financial component. In fact, class III includes unit-linked and index-linked policies whose performance is linked to that of internal insurance funds or mutual funds, or to financial indices (or other reference values), and which comprise a residual insurance component covering demographic risk. Class V is represented by capital redemption policies, which have a purely financial nature. These products offer a minimum guaranteed return increased by the potential positive performance of internal insurance segregated funds. However, the transparency standard on class I financial-insurance products continues to be defined by the provisions enacted by Isvap and regarding the format and the content of the so-called *Informational Booklet*. This division of competencies seems unjustified, as class I financial-insurance products are linked to internal insurance segregated funds which are identical to those included in capital redemption policies and, therefore, class I and class V are financially equivalent. The only difference is in the insurance component covering demographic risk which is included in class I but not in class V products; moreover, it is worth pointing out that the value of said component is typically marginal compared to the financial component of these products, as it amounts, on average, to 1%-2% of the total premium paid.

The intermixing of the financial structures of products belonging to different classes is not only limited to classes I and V. In effect, the financial assets the performance of capital redemption policies is linked to may also be internal insurance funds or mutual funds or structured financial portfolios, even synthetically packaged. In other words, the revaluation mechanism of class V policies can replicate, in practice, the financial engineering of a unit-linked or index-linked policy. Moreover, in the case of class I products, the similarity to class III policies becomes clear-cut, as both classes include an insurance component covering demographic risk, though the value of this component is minimal. The division of transparency supervisory competencies introduced by the Italian law-maker allows regulatory arbitrages, and introduces an additional problem linked to the offering documentation of the so-called "multi-class" products combining a class I with a class III policy. In fact, in such a case, customized formats are needed, with all the difficulties resulting from the differences in transparency requirements defined by the two Authorities; difficulties which could be overcome by assigning transparency supervisory competencies to just one Authority according to the prevalence of the financial component, that is the same rationale behind the extension of prospectus regulation to class III and V products.

Further transparency requirements and conduct of business obligations are established for the distribution of non-equity financial products by the directive 2004/39/EC (MiFID) and the related second level directive (directive 2006/73/ EC). The provisions of these directives highlight that, for open-ended mutual funds and SICAVs, the offering prospectus is considered as the primary source of information on the risk-return profile and on the costs of the financial investment, requiring distributors to supplement prospectus information where it should be insufficient to comply with the principles of suitability or appropriateness of the investment. In this regard, it is worth pointing out that the MiFID outlines these principles in general terms, and national regulators are required to implement them through the issue of detailed rules also related to the information duties distributors must comply with (so-called Level 3 of the MiFID).

The above outlined framework shows the significant heterogeneity of the regulatory provisions regarding the public offering of financial products which, as seen in section 1.1, feature similar financial engineering and, as a consequence, are exposed to the same risk factors.

In exercising its regulatory powers, Consob has acted on several fronts in order to standardize the regulation of prospectuses for products issued by subjects belonging to different categories, within the limits of the current EC regulation. The choices Consob has made over time represent the regulatory transposition of a specific position which considers the prospectus as the privileged channel for information transparency both in the offering and in the distribution of non-equity

1 (f) of the Consolidated Law on Finance (that is products different to shares or to financial instruments which allow to purchase or to subscribe shares), thus enabling the full applicability of the Prospectus directive and of the regulation 809/2004/EC.

investment products. Consob's provisions are not based on prior authorizations but on timely *ex-post* enforcement measures consistent with a risk-based approach[(22)]. To this end, the contents of the prospectus must be standardized across different categories of issuers, and must provide a clear, synthetic and easily understandable representation of the risks, the recommended investment time horizon and the potential returns, as resulting from measures based on objective quantitative methods[(23)].

In this perspective, regulation regarding the structure of the offering documentation and the related regime of delivery to investors has been issued. More specifically, the prospectuses for open-ended Italian mutual funds and class III and V financial-insurance products offered in Italy are structured in a first document which has the form of a *product information sheet* and must be delivered to investors, and a second document which is more detailed and has to be delivered upon request. The *product information sheet* provides a clear illustration of the indications deriving from the application of objective methods and quantitative analysis tools to the measurement and the monitoring of the product's risk-return profile. This document abandons the traditional narrative approach which consists in listing all the risk factors the investment is exposed to and illustrating them in minute detail. With this document, the reader's attention is drawn to a limited number of key points, specifically selected to provide investors with a concise and meaningful message about the most significant features of a product, and to allow an objective comparison with the available investment alternatives, including the most sophisticated ones[(24)].

The focus on the *product information sheet* allows an immediate match with the indications provided in the MiFID Level 3 document regarding the intermediary's obligation to act in a correct and transparent way when distributing illiquid financial products. In fact, these indications require distributors to use a *product information sheet* containing probability scenarios for the final value of the *invested capital* (also due to the bidirectional relationship of these scenarios with the degree of risk), and drawing the reader's attention to the importance of the recommended investment time horizon. The new provisions on mutual funds and class III and V financial-insurance products offered in Italy established in Consob's Regulation on Issuers, coupled with the MiFID Level 3 implementation measures, have created a regulatory framework which is particularly favourable to distributors. Specifically, distributors may fulfil the majority of their disclosure duties regarding these non-equity investment products by delivering the *product information sheet* drawn up in accordance with the templates annexed to said Regulation.

Moreover, the new regulatory framework will likely contribute to stimulate banking distributors to autonomously adapt the contents of the *Final Terms* provided by regulation 809/2004/EC to the standards of transparency of the *product information sheet*, at least with regard to the products they, or other intermediaries belonging to the same group, issue.

Despite the rationalization of the regulation Consob has recently carried out, significant discrepancies still remain in the offering documentation in Italy as regards:

- class I financial-insurance products, whose offering documentation is regulated by the Isvap provisions on the *Informational Booklet*, and, consequently, also multi-class financial-insurance products;
- European UCITSs, whose offering documentation is regulated by the templates defined in their home Member State regulations;

[(22)] It is worth mentioning that Italian regulation introduced a common exemption policy for financial-insurance and asset management products, which sets the minimum individual investment size at 250,000 Euros; below this amount, it is mandatory to publish the prospectus and to deliver a specific part of it to potential investors.

[(23)] In this regard, it is worth mentioning that information on potential returns based on probability scenarios of the investment payoff at the end of the recommended time horizon is also required by the most recent versions of the draft of the UCITS IV directive.

[(24)] Moreover, this structure of offering documentation has repeatedly received positive feedbacks from consumers' associations, both at the national and European level, and from the academia, precisely because of its greater ease of comprehension compared to descriptions which often are too much dispersive and complex, as well as unsuitable to highlight the distinctive features of the specific product being offered.

- financial products issued by Italian and European banks, whose offering documentation is regulated by the templates contained in the regulation 809/2004/EC. If Italian banking distributors would autonomously adapt their *Final Terms* as described above, these discrepancies would be limited to the prospectuses issued by banks of the other Member States.

While the asymmetry in transparency regulation between financial-insurance products of class I and those of classes III and V could be mitigated by the intervention of the Italian law-maker, the asymmetry concerning European UCITSs and non-equity financial products issued by banks requires a legislative initiative by European policy-makers.

This last point becomes crucial when considering the high household saving rate observed in Italy, which translates into net purchases of financial products offered by issuers with registered offices in other Member States.

Hence, European policy-maker should promptly intervene to realize not only the alignment of the disclosure requirements for products issued by subjects belonging to different categories, but also the effective harmonization of the regulations applicable to Member States, in order to protect investors and to enhance the competition between the various national financial systems.

The concrete implementation of the levelling the playing field principle requires the adoption of transparency standards based on common risk measurement methods, which enable investors to fairly compare the numerous financial products offered in the market and to make investment choices based on the essential elements of such products: the potential returns, the degree of risk and the recommended investment time horizon.

Such objective may be fully achieved only through the revision of the EC regulation in the direction of a single directive on the transparency for non-equity investment products, also preventing any regulatory arbitrage which could arise as a consequence of the choices made by national policy-makers, as happened, for instance, in Italy for financial-insurance products. This would also lay the foundations for a pro-active rethinking of the supervisory activity, assigning a preventive role to transparency rules, and an enforcement role to conduct of business rules.

In fact, effective transparency on potential returns, recommended investment time horizon, risks and costs of the product − together with suitable regulatory provisions − allows a better prevention of episodes of incorrectness whose effects may be just partially mitigated by sanctions.

Should this process of convergence fail to take place, it will result in three undesirable consequences:

1. a considerable room would be left for regulatory arbitrage opportunities, which could be exploited by appropriately choosing the category of the issuer;

2. investors would find it difficult to make a meaningful and fair comparison of the risk-return profile of products sharing the same financial structure;

3. compliance costs borne by financial intermediaries would increase.

The current process of revision of the three directives (i.e., the Prospectus directive, the UCITS III directive and the Life Assurance directive) regarding transparency on non-equity financial products seems the most adequate occasion to promote the above described approach, and to take structural measures on offering documentation regulation[25].

[25] In this perspective, a first useful step could be the introduction, at the European level, of a regulatory framework irrespective of the specific nationality of the issuer and aimed at granting the competent Authorities of each member State the possibility to define the contents of the prospectus for all non-equity investment products being offered in that State, as it is currently established only for financial-insurance products. Such regulatory framework would have the advantage of being more consistent with the fact that the competent Authorities of each member State are charged the surveillance on transparency in order to safeguard investors, even when investors subscribe investment products offered by issuers with registered offices in another member State. In addition, over the long period this regulatory framework would facilitate the natural convergence towards the surveillance approach on transparency more suitable to pursue, at the same time, the safeguard of investors, the improvement of the quality of the products being offered and the increase of competitiveness between the various financial intermediaries.

In particular, the prospectus should have at least a part of it mandatorily delivered to investors and drawn up in accordance with a *product information sheet* template containing the essential information on the risks and costs of the financial investment.

The identification of the specific contents of such information should be based on synthetic indicators, which may take a qualitative form and whose representativeness should, however, be ensured by a robust and objective underlying quantitative methodology. The synthetic indicators would also form a sound information base for *ex-post* enforcement. In this way the need for prior approval of prospectuses would be avoided, as offerors will only be required to send the prospectus to the competent supervisory Authorities. In this regard, the best solution for the notification procedure and for the transmission of the offering documentation would be the adoption, by the European Community, of a direct prospectus filing system: the offeror should directly file the prospectus with the supervisory Authorities of the Member State where the product is intended to be sold. Involving the competent Authorities of the home Member State in this process would generate useless delays and frictions, and would also go against all modern technological protocols of data transmission and information security. Moreover, applying the same risk-based approach under a single directive on the transparency of non-equity financial products would result in the creation of a complete, EC-wide database of products containing homogenous information on risks, potential returns and recommended investment time horizons. Making this database available to investors would allow them to take more informed investment decisions and to choose the most suitable products for their objectives in terms of holding period, costs, risks and potential returns of the investment.

2 A three-pillar approach to the transparency on risks

The risk-based approach to transparency illustrated in this section is organized into three pillars: the representation of probability scenarios on the returns on the financial investment, the degree of risk − supplemented, where necessary, by the level of deviation from the benchmark − and the recommended investment time horizon.

The information provided by the first pillar offers a breakdown of the various cost items and components of capital which constitute the financial investment, and it allows to calculate the probability scenarios of the final value of the *invested capital*. These scenarios, which constitute a return measure, offer a synthetic view of the possible results of the investment, net of costs applied. By comparing them with those of the risk-free asset, it is possible to better appreciate the product's performance risk, meant as its likelihood to create added value for the investors.

The second pillar is the degree of risk of the investment. This indicator summarizes the overall riskiness of the product. Its significance is ensured by the use of metrics defining *ex-ante* risk measurement and monitoring rules which are standard across products. In "benchmark" structures, information deriving from the degree of risk is supplemented by a synthetic indicator of the intensity of the asset management activity, which allows to distinguish between passive and active asset management styles and, in this second case, to represent the manager's positive or negative contribution to the overall risk of the product and to its evolution over time.

The third pillar is the recommended investment time horizon. This indicator expresses a recommendation on the investment holding period, formulated in relation to the product's specific financial structure and costs regime. The information conveyed by this pillar matters for the transparency requirements in the product's public offering and for the suitability tests which must be carried out by distributors. In fact, distributors use this metric to assess whether the product matches client preferences in terms of the period for which he is willing to give up his cash holdings. Thus, it is crucial to identify the recommended time horizon using an objective method, able to reflect the optimal holding period implicit in the financial engineering underlying the non-equity investment product and in the related profiles of risks, costs and potential returns.

The three pillars are closely linked together and this requires them to be read as a whole in order to get an overall valuation of the risk-return characteristics of the financial investment[26].

In products with "risk target" or "benchmark" structures which are not backed by financial guarantees, the degree of risk, together with the costs applied, allows to determine the recommended investment time horizon according to the cost amortization criterion. This horizon, in turn, is used as the reference period to calculate the probability scenarios of the final value of the *invested capital* and, therefore, to illustrate the potential returns on the product[27].

The interdependence of the three pillars takes a different form in "return target" and in guaranteed products. Their financial structure is constrained by the achievement of a target return at a given maturity, which clearly identifies the investment time horizon to recommend to investors, and to use for the suitability tests. A shorter holding period could compromise the liquidability of the product, meant as the possibility of disinvesting at a specific time without incurring a loss and without waiving the benefits offered by the product in terms of extra-returns above those of the risk-free asset. For the same kind of reasons, it is only over the recommended time horizon that the information on the potential returns provided by the probability scenarios takes on significance with respect to the specific final objectives of the investor. Lastly, it is the analysis of the volatility measures implicit in the potential returns throughout the recommended time horizon that makes it possible to determine the degree of risk.

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[26] In this perspective the risk-based approach can be successfully used also to realize a more effective transparency on the risks and costs of products involving financial liabilities. In fact, these products feature a financial engineering mirroring that of investment products, and – like investment products – they typically lead to take a specific risk exposure or to modify an outstanding exposure, also through the embedding of derivative-like components.

[27] An additional factor to consider in the quantitative determinations behind the three indicators is whether the product requires periodic payments which, interacting with the costs of the investment, clearly affect its risk profile.

2.1 The first pillar: costs and potential returns of the investment

In any financial product the price is the first element drawing the attention of the potential investor. In fact, comparing prices of different products having the same financial engineering is the natural tool used by investors to assess the relative cost-effectiveness of the available investment alternatives.

The price of a non-equity investment product is given by the sum of two components: the fair value and the mark-up, the latter meant as the profit margin for the intermediary. The fair value equals the expected value, under the risk-neutral probability measure, of the future cash flows discounted at the risk-free rate, while the mark-up is the total amount of costs applied to the financial investment[(28)].

This approach has the advantage of providing a clear and concise representation of the various cost components of the product, but, in terms of transparency, it fails to show investors how a mispricing of 5, 10 or more percentage points over the recommended investment time horizon would impact on the potential returns of the investment. Moreover, as the price is a synthetic value calculated with reference to the issue date, the level of mispricing may be sensitive to the specific pricing model chosen by the financial intermediary.

For what stated above, the first pillar of the risk-based transparency approach represents the costs and the risk-return profile of non-equity investment products by means of two tables. These tables illustrate:

- the breakdown of the financial investment into its portfolio and cost components (hereinafter also the "*financial investment table*");
- the return scenarios of the financial investment over the recommended time horizon (hereinafter also the "*probability table*").

The first table gives concise information on the mark-up associated with the product, while the second fully illustrates to the investor the impact of this mark-up on the final value of the investment, in terms of potential returns. Moreover, the synthetic representation this measure offers – as it will be explained below – is substantially invariant to the specific quantitative model used.

The adoption of this approach for the universe of non-equity investment products has required the identification of the two concepts of *notional capital* and *invested capital* which, in general terms, correspond to the total price and the fair value, respectively[(29)].

Moreover, in order to achieve a complete representation of the risk profile and of the costs regime, the analysis focuses on two different times: the time of the subscription and the time of the "ideal" exit from the product, meant as the end of the period over which the investment is optimized, that is the recommended time horizon.

The *financial investment table* illustrates the incidence of the various cost items applied (differentiated by the type of service that they are intended to remunerate) and of the values of the *notional capital* and of the *invested capital*.

The *probability table* provides a synthetic representation of the probability distribution of the possible investment's payoffs at the end of the recommended time horizon.

In other words, the item "*invested capital*" of the first table shows the fair value at the initial time (generally indicated as time 0), while the second table contains a synthetic representation of the so-called "pricing at maturity" (where the maturity of the recommended time horizon is generally denoted as time T[(30)]).

Thus, the link between the two tables is evident and ensures the consistency of the information provided to investors. As has been said, the capital which is actually invested in the product (i.e.,

(28) For more details on this approach see [Minenna, D'Agostino, 2001].

(29) Where the product requires periodic payments by the investor, the relationship between the concepts of *notional capital* and *invested capital* and those of total price and fair value must be properly specified.

(30) This notation derives from the bijective relationship between the probability scenarios and the fair value, illustrated in detail hereafter.

the fair value) is the average of the all the possible final values of the investment, calculated by discounting them at the risk-free rate to consider the financial value of time and then by weighting each discounted value for the probability attached to it.

In analytical terms, given the probability space $(\Omega, \Im, \mathbb{P}^*)$, where $\mathbb{P}^*$ is the risk-neutral probability measure, and $\{\Im_t\}_{t\geq 0}$ a filtration defined on this space, denoting by:

- T the maturity of the recommended investment time horizon;
- IC_0 the value of the *invested capital* at time 0;
- $\widetilde{IC}_T$ the random value of the *invested capital* at time T;
- $\{r_t\}_{t\geq 0}$ the stochastic process of the risk-free instantaneous interest rate;

the following equation is obtained[(31)]:

$$IC_0 = E^{\mathbb{P}^*}\left(e^{-\int_0^T r_s ds} \cdot \widetilde{IC}_T \middle| \Im_0\right) \tag{1}$$

In the simplest structures, this expected value can be calculated using closed-form formulae. However, in most financial products, the calculation of the expected value requires the preliminary determination of the distribution of the "pricing at maturity" using simulation procedures.

Thus, the importance of equation (1) for the purposes of determining the fair value of a non-equity investment product is closely linked to the complexity of its financial structure.

In "return target" or guaranteed structures, the *probability table* is a necessary step to obtain the unbundling of the price of the product at time 0. Equation (1) illustrates that IC_0 is equal to the expected value, under the probability measure $\mathbb{P}^*$, of all possible realizations of the random variable $\widetilde{IC}_T$ discounted at the risk-free interest rate.

In "risk target" or "benchmark" structures, equation (1) continues to be satisfied at any time, but the valuation methods of these products do not require the preliminary calculation of the probability distribution of the possible final payoffs. In these products, the calculation of the probability distribution is rather an intermediate step of the process carried out to determine the recommended investment time horizon[(32)].

The explicit requirement of including the table of probability scenarios for the purposes of transparency derives from the higher informativeness of the "pricing at maturity" as compared to the price at time 0. In fact, the fair value, by definition, is a synthetic value which ignores the information provided by moments of order higher than one and it does not allow to appreciate the associated degree of randomness. In fact, the same expected value may be obtained by an infinite number of final payoffs' probability distributions, even with very different shapes. On the contrary, the *probability table* − constructed in accordance with the risk neutrality principle and supplemented, for each scenario, by a synthetic representative value − makes investors aware of the overall performance risk associated with a non-equity investment product.

Sections 2.1.1, 2.1.2 and 2.1.3 provide a detailed illustration of the logic and the methods underlying the determination of the values to report in the two tables.

2.1.1 The financial investment table

At the time of subscription, the financial investment can be broken down, to a first approximation, into three quantities: *invested capital*, costs and *notional capital*.

In general terms, the *invested capital* equals the fair value of the product; the sum of the various cost items constitutes the intermediary's mark-up; and the *notional capital* (equal to the sum of the *invested capital* and of the mark-up) identifies the price actually paid for the product.

[(31)] See [Minenna, 2006].
[(32)] For more details see section 2.3.

According to the logic of the unbundling of the financial investment, the table details the values of the bond and/or derivative components[(33)]. Moreover, in order to make this approach applicable to all non-equity investment products, it is necessary:

1. to ensure a proper representation of deferred costs which should be paid during the recommended investment time horizon;
2. to distinguish between the mark-up charged for financial intermediation services and any profit margin the intermediary receives as compensation for providing non-financial services.

The other point to consider when defining the various items in the table regards the identification of the various types of costs applied. Such identification must be carried out taking into account the nature of the services that the different costs remunerate[(34)].

Due to the presence of ongoing costs (such as asset management fees in the case of mutual funds and internal insurance funds) or one-off costs applied after the subscription date, the format of the table must also include the expected value of these deferred charges, after having discounted them at the present time, in order to preserve transparency on the product's costs and on its fair value[(35)].

Moreover, the proper indication of deferred costs in the table ensures that the information it contains matches that one given in the *probability table*. By construction, costs applied after the subscription date reduce the value of the *invested capital* over time, shifting the probability distribution of the "pricing at maturity" to the left. It follows that the fair value of the product at time 0 must be determined with respect to this distribution, being it the only one which embeds all costs applied[(36)].

Should there be one or more deferred costs, their expected discounted value will be calculated and reported in the *financial investment table*. In this regard, it is worth remarking that these costs are (or can be) expressed as a percentage of the value of the product at the time they are applied. Therefore, as a general rule, the *financial investment table* must include the expected discounted value of the difference between each possible final payoff of the product and the same payoff net of costs applied, taking care to consider any priority of payment between the different types of costs[(37)].

In products which, in addition to a purely financial investment, also offer other types of services (such as demographic risk insurance in class I and III financial-insurance products), the difference between the *notional capital* and the *invested capital* exclusively provides the financial intermediation margin. The remaining costs of the investment are due for other services provided by the

[(33)] It is worth mentioning that, consistently with the portfolio replication principle, the fair value of many non-equity investment products can be split up into the sum of a risk-free component, which is exposed only to the movements of the yield curve and of the associated volatility term structure, and of a component which embeds all others other risk factors affecting the investment. Also notice that the identification of the elementary components can be found, with reference to structured bonds, already in [Minenna, D'Agostino, 2001].

[(34)] In this regard, the MiFID and the related provisions contained in the Italian regulation introduced significant innovations to the general logic underlying the relationship between intermediary and client, assigning a central role to the concept of the service provided.

[(35)] As illustrated in section 1.2, the current revision of EC regulations on offering prospectuses favours a synthetic and schematic layout focused on the essential aspects of the financial investment and, hence, quite similar to that introduced by Consob for class III and V financial-insurance products as well as for open-ended mutual funds. In this context, the exclusion of deferred costs from the *financial investment table* and their illustration in other parts of the prospectus could create the potential for regulatory arbitrages, and, consequently, for the proliferation of no-load products.

[(36)] In order to maintain the information on the value of the *invested capital* at the time of subscription, it could be useful to keep in the prospectus a table showing the unbundling of the financial investment made with reference to the initial costs only. However, it should be pointed out that the numbers in this table have only an algebraic value, and they cannot be interpreted as the time 0 equivalent of the information contained in the *probability table*. This solution has been adopted by Consob, together with a simplified representation of deferred costs in a specific column of the *financial investment table*.

[(37)] For costs whose application is subject to specific conditions (such as performance fees), the corresponding value in the table must be calculated as the expected discounted value of the costs effectively charged over the recommended time horizon, with each cost compounded forward to the end of said horizon.

intermediary. Thus, in this case, the *notional capital* is determined as the difference between the price of the product and the non-financial charges[38].

2.1.2 The probability table

No matter what type of financial structure they may have, non-equity investment products are similar to gambling. The main difference is that in gambling all players share the same *ex ante* information and, in particular, the probabilities of the various possible outcomes. On the contrary, in the case of non-equity investment products, there is an asymmetric information which may create disadvantages for investors. Even though, by using the *financial investment table*, investors can distinguish the fair value from the costs of the product, they usually ignore the probabilities attached to the various possible final payoffs.

The probability scenarios for the final value of the *invested capital*, which may be expressed in terms of returns[39], rebalance the positions of the various subjects involved in a financial investment, allowing investors to appreciate the performance risk, meant as the product's likelihood to create added value for them.

Taken the recommended time horizon as reference period, the probability distribution of the possible values of the *invested capital* at the end of this horizon is partitioned into four alternative scenarios[40][41]:

1. *the final value of the invested capital is lower than the notional capital (so-called negative return scenario);*

2. *the final value of the invested capital is higher than or equal to the notional capital, but lower than the final value resulting from investing the notional capital in the risk-free asset over the same time horizon (so-called scenario where the return is positive or zero but lower than that of the risk-free asset);*

3. *the final value of the invested capital is higher than the notional capital, and in line with the final value resulting from investing the notional capital in the risk-free asset over the same time horizon (so-called scenario where the return is positive and in line with that of the risk-free asset);*

4. *the final value of the invested capital is higher than the notional capital, and higher than the final value resulting from investing the notional capital in the risk-free asset over the same time horizon (so-called scenario where the return is positive and higher than that of the risk-free asset).*

Representing a limited number of scenarios reduces the granularity − often quite considerable − of the probability distribution of the "pricing at maturity", providing both a greater usability to the reader, and a minimization of the so-called "model-risk". In fact, the differences in the results which may be attributed to the choice of different pricing models are mitigated as the probability distribution is partitioned into only four events, and thus, typically many elementary events are aggregated. As a consequence, the differences between the probabilities of the various scenarios calculated using different models are reduced, until these fall within an order of magnitude which is no more significant from the investor's point of view.

The information provided through the *probability table* illustrates the performance risk of the product both in absolute terms and in relative terms with respect to the risk-free asset.

In absolute terms, it is quite evident the high informativeness of the comparison between the likelihood of losing a part of the *notional capital* (first scenario) and that of obtaining a value equal

[38] Should these costs be applied after subscription, they will be determined using the above described procedure to calculate the value of deferred costs which has to be indicated in the *financial investment table*.
[39] This is the solution adopted by Consob since 2004 to improve investors' comprehension of the performance risk.
[40] Where the product requires periodic payments by the investor, the definition of the four scenarios has to be properly specialized.
[41] For the details see section 2.1.3.

to or higher than the *notional capital* at the end of the recommended investment time horizon (second, third and fourth scenario).

In relative terms, the details on the probabilities of the last three scenarios allow an immediate comparison with the possible outcomes of the alternative represented by the investment of the *notional capital* in the "risk-free asset" over the same time horizon. This expression indicates a financial asset whose sole risk comes from the random movements in the yield curve. As a consequence, the comparison with a non-equity investment product highlights the specific risk factors which do characterize such a product. It is for this reason that, in order to describe the time evolution of the risk-free asset, the so-called "*cash account process*" should be used. In fact, its behaviour reproduces, at the end of the recommended time horizon, the impact of the yield curve volatility on the returns of a financial investment. In addition, the cash account process is flexible enough to accommodate the specific features of the payoff profile of any non-equity investment product it is compared with.

The risk-free asset is similar to a government bond whose coupons are indexed to the yield curve (such as floating-rate Treasury bonds), and, thus, to a bond actually traded in the financial markets and whose characteristics are well-known to the average investor[(42)]. On the contrary, in general terms, it is unsuitable to use fixed-rate bonds as their final value is by construction barely sensible to the movements of the yield curve and, therefore, their use would make the results of the probabilistic comparison meaningless. The limit case is represented by zero-coupon bonds, as their final value is not affected at all by the yield curve volatility.

The use of the risk-free asset for the comparison avoids discretionary choices by the intermediaries. In fact, it is likely that, in the lack of explicit guidelines on this subject, each intermediary would have the incentive to compare his own product with the financial alternative which is more convenient to emphasize the characteristics of what he is going to offer.

Moreover, thanks to the direct comparison with the risk-free asset, the *probability table* also allows a comparison across different products, as the risk-free asset is a common reference point and, thus, it can be thought as the numeraire of the risk profile of the different structures existing in the market[(43)].

Another key point of the methodology underlying the *probability table* is the use of the *notional capital* both to define the concept of loss of the financial investment and to perform the probabilistic comparison with the risk-free asset. As far as the concept of loss is concerned, the *notional capital* is used as the threshold to identify the losses, as its value represents the liquidity which the investor actually gives up during the recommended investment time horizon[(44)]. In addition, given that the investment in the risk-free asset has usually a minimum or even negligible mark-up, it can be reasonably assumed that the initial size of the alternative investment in this asset is exactly equal to the *notional capital*. In this way, the probabilities of the four scenarios will implicitly take into account the higher costs that non-equity investment products usually charge.

For each of the four events considered, the information provided by the *probability table* is

[(42)] However, it has to be specified that the direct comparison of the non-equity investment product with floating-rate coupon-bonds, free of any credit risk exposure and currently traded on the markets, such as those issued by the Treasury, would generate an element of discretion associated with the choice of a specific coupon structure, and also an additional computational cost due to the need of modelling such a coupon structure subject to the constraint of the scheduled payoff profile of the financial product. In fact, a perfect match between the coupon structure of a risk-free bond and that of a non-equity investment product is almost exclusively a theoretical case with few chances of occurring in practice. It is understood that when the non-equity product and the risk-free asset are equal in distribution, standard market indicators on the risk-free asset also hold for the product and the probability scenarios are not necessary.

[(43)] If the non-equity product goes to replace a pre-existing product, then the latter becomes the numeraire against which to evaluate punctually the two probabilities that the replacement product will perform better or worse, along with indicators of the difference between the performances of the products.

[(44)] For the same reason, in general terms, the product's return is calculated according to the following formula:

$$\frac{\widetilde{IC}_T - NC_0}{NC_0}$$

where NC_0 denotes the value of the *notional capital* at time 0. Clearly, in products which require periodic payments by the investor, the quantitative principle underlying the above formula must be suitable adapted.

completed by the indication of a representative value for the final payoff. To this end, the representative value of each scenario is calculated as the median of the final payoffs obtained from the simulation[45] and laying within the interval associated with that scenario. These median values allow to highlight the most important information on the shape of the probability distribution of the final value of the *invested capital,* and to associate a synthetic quantification of this final value with the probability of each scenario.

From a methodological point of view, the essential requirement for the calculation of the probabilities and of the median values is the risk neutrality principle: the application of this principle ensures that the figures shown in the table are objective and independent from the higher or lower level of risk aversion of each individual player (so-called "real probabilities"). In this regard, it is worth recalling that, by the Fundamental Theorem of Asset Pricing, the risk-neutral probability measure is − under the complete markets hypothesis − the only one under which the stochastic process of the discounted final payoff of the product is a martingale; and thus, only this measure allows a meaningful comparison of different quantities. More specifically, the risk neutrality ensures the correctness of the probabilistic comparison with the risk-free asset. In fact, only under the risk-neutral measure the expected return of the product and that of the risk-free asset do coincide and, therefore, the *probability table* can highlight the role played by the volatility of the elementary components of the financial investment, by the costs attached to it and by the specific structure of the amounts paid out to the investor over the recommended time horizon.

In addition to representing the performance risk associated with a financial product at the initial time, the *probability table* should also be used to detect the cases when the evolution of the risk factors of the investment over the recommended time horizon requires to update the information previously provided to investors. In fact, the reduction in granularity due to the focus on only four events ignores slight shifts in the probability distribution of the final values of the *invested capital* and, thus, makes it easier to spot the most significant changes in the probability of each scenario, which are those the investor is most interested in.

It should be noticed that the above presented methodology does not prescribe a specific model to use for the quantitative determinations needed to calculate the values in the table. The choice of the model is left to the intermediaries, who are required to use the same solutions internally developed for their pricing and risk management activities. This approach also avoids the costly and useless implementation of "parallel models", one for the internal activities of the intermediaries and the other one for their compliance duties deriving from the transparency regulation.

Within the scope of their modelling autonomy, intermediaries should properly consider and measure the parameters and the variables corresponding to all the risk factors which characterize the product offered, taking care of their consistency with the reality and the complexity of financial markets, and also using, where necessary, suitable stochastic processes to model the various variables involved.

By way of example, the probability of the four scenarios and the related median values should reflect any exposure of the product's elementary components to the credit risk of the issuers or counterparts. The adherence to the effective market conditions is ensured by the use of meaningful indicators when estimating the default probability. In particular, given the intrinsic inertia of credit ratings − also observed in the recent international financial crisis − internal models developed by intermediaries should process the information provided by those market variables whose value promptly embeds the changes in the credit standing of issuers or counterparts, such as CDS spreads or discount margins on bonds issued by said subjects[46].

The globalization of financial markets and the intermixing of activities carried out by various categories of intermediaries often make it difficult to separately assess the individual sources of risk and, instead, signal the convergence towards a sort of mixture of all the different risk factors. In such a framework the *probability table* − with the pair constituted by probabilities and median

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[45] See section 2.1.3 hereinafter.

[46] In order to quantify the default probability using CDS spreads, it has also to be considered that, due also to the lack of a standardized CDS market, the time evolution of such indicator could not reflect in a complete way the credit standing of an issuer (or counterpart), and it would have to be consequently revised.

values − represents the solution suggested by the risk-based transparency approach to successfully synthesize the risk-return profile of non-equity investment products.

2.1.3 Probability calculation methodology

The probabilistic comparison underlying the first pillar of the risk-based approach to transparency requires the numerical simulation of the final value of the *invested capital* and of the final value of the *notional capital* invested in the risk-free asset over a period equal to the recommended investment time horizon[(47)].

As argued in section 2.1.2, both simulations must be carried out according to the models internally developed by intermediaries for their pricing and risk management activities.

The numerical simulation of the final value of the *invested capital* must be compliant with the risk neutrality principle. In particular, to ensure the methodological consistency with the risk-neutral probability measure, interest rates term structure models defined under such a measure should be used[(48)].

Technically, the adoption of the risk-neutral measure requires the calculation of a Radon-Nikodym derivative in order to switch from the real probability measure to the risk-neutral one[(49)]. Specifically, when performing the simulation, risk neutrality requires that, once the time evolution of the risk-free rate has been simulated, the same simulated data also drive the dynamics of the stochastic processes used to obtain the trajectories of the *invested capital*.

The models used to describe the time pattern of the *invested capital* should take into account all the risk factors the financial investment is exposed to, as well as the term structure of the volatility of the underlying financial instruments.

Before running the simulation, it is necessary to calibrate the parameters associated with the different risk factors by means of estimates based on current market data. Once again the parametric calibration should be in line with that adopted by the intermediary to carry out his proprietary pricing and risk management activities, also because, as said in section 2.1, the risk-neutral simulation of the possible final payoffs of a product is an intermediate step to determine its current value.

The starting point of the simulation has to be set equal to the difference between the *notional capital* and the up-front costs, where existing[(50)]. The simulation should also consider the size and the time schedule of any deferred cost applied over the recommended time horizon[(51)], as well as those of any periodic or one-off amount paid out to the investor or invested in other financial assets[(52)]. The former reduce the *invested capital*, while the latter increase it.

In products with two or more elementary components, the intermediary identifies the specific methods to carry out the simulation paying attention to the features of the different financial structures which may be engineered. This is because sometimes these structures can require quantitative determinations directly referred to the entire *invested capital*, while, in other cases, separate simulations of the various elementary components of the *invested capital* − or of only a subset of these components − may be necessary.

Where one of the elementary components of the product is a bond[(53)], or in the equivalent case of a synthetic bond-like component backed by a financial guarantee, and this component is

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(47) For a detailed illustration of the methodogy to determine the recommended investment time horizon see paragraph 2.3.

(48) As far as the discretization step to use in the simulation is concerned, it is suitable to refer to the overnight rate or similar variables, while the parametric calibration should be done in line with the specific features of the model chosen and with the actual market conditions.

(49) See [Minenna, 2006].

(50) If there are deferred costs owed in exchange for some non-financial service, the starting point of the simulation has to be set equal to the difference between the price of the product and the total initial costs.

(51) Similarly, in products with periodic payments, the simulation should reflect the discontinuities in the value of the *invested capital* occurring at each payment date.

(52) In both cases (i.e., distribution or accumulation), in general terms, these amounts should be compounded − at the risk-free rate − forward to the end of the recommended investment time horizon.

(53) This refers to a bond-like component that corresponds to a bond which has the same notional value and is held in the treasury over all the investment time horizon.

free of any credit risk exposure, its final value does not need to be simulated as it is equal to the redemption value of the bond (or of the guaranteed synthetic bond-like component). However, it is still necessary to compound any bond coupon payment, at the risk-free rate, forward to the end of the recommended investment time horizon. Nonetheless, should market indicators, such as the aforementioned CDS spreads and discount margins, signal a specific exposure to the credit risk of the issuer (or guarantor), the simulation will have to include also the final value of the above types of bond-like components, and it will have to be based on models able to properly quantify the impact of this risk factor.

The numerical simulation of the final value of the *notional capital* invested in the risk-free asset must be compliant with the risk neutrality principle and it should be based on models able to consider the characteristics of the stochastic term structure of interest rates and consistent with the approach adopted to simulate the final value of the *invested capital*[(54)]. In fact, these requirements ensure the correctness of the probabilistic comparison, as the use of a probability measure different to the risk-neutral one or the use of different models in the two simulations would determine the comparison of quantities defined according to heterogeneous criteria and, hence, would make it completely meaningless.

As seen in section 2.1.2, the simulation of the final value of the *notional capital* is based on the use of the "*cash account process*". The simulated values of this stochastic process are determined in correspondence with those obtained from the simulation of the time evolution of the risk-free rate, given the well-known functional relationship between the two financial variables[(55)]. In particular, it is necessary to use the same values of the risk-free rate obtained for the risk-neutral simulation of the final value of the *invested capital*. This would also provide a simple solution to the problem of the correlation between the various components of the *invested capital* for products engineered as bundle of two or more elementary components. In fact, such procedure results in a structure of implicit correlation between the dynamics of the risk-free rate and those of the risky components of the financial investment[(56)].

The two above described simulations provide the probability distributions of the final values of the *invested capital* and of the *notional capital* invested in the risk-free asset. In order to determine the probabilities of the four scenarios listed in section 2.1.2, first of all it is necessary to identify a probability mass representative of the distribution of the final value of the *notional capital* invested in the risk-free asset. This is done by truncating this distribution in a symmetric way. Denoting by $NC_{\min}$ and $NC_{\max}$ the values corresponding to the two cut-off points, the probabilities of the four scenarios are determined as follows[(57)]:

1. *negative return scenario*: the probability mass of the distribution of the final value of the *invested capital* which is to the left of the value NC_0[(58)] is calculated;

2. *scenario where the return is positive or zero but lower than that of the risk-free asset*: the probability mass of the distribution of the final value of the *invested capital* which is distributed between the value of NC_0 (included) and the value of $NC_{\min}$ (excluded) is calculated;

3. *scenario where the return is positive and in line with that of the risk-free asset*: the probability mass of the distribution of the final value of the *invested capital* which is distributed between the values of $NC_{\min}$ (included) and $NC_{\max}$ (included) is calculated;

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(54) This requires that the parameters which are common to the two simulations should be assigned the same values, exception made for those adjustments needed to consider the credit risk of those components of the *invested capital* which are exposed to this risk factor.

(55) Denoting by $\{r_t\}_{t\geq 0}$ the stochastic process of the risk-free rate and by B_t^T the value of the stochastic process of the cash account over the period from t to T, the following equality holds:
$B_t^T = e^{\int_t^T r_s ds}$.
See [Minenna, 2006].

(56) For example, in some index-linked financial-insurance products, which are synthetically indexed to a reference value, this procedure allows to establish a precise correlation between the dynamics of the bond component and those of the derivative component of the structured financial portfolio.

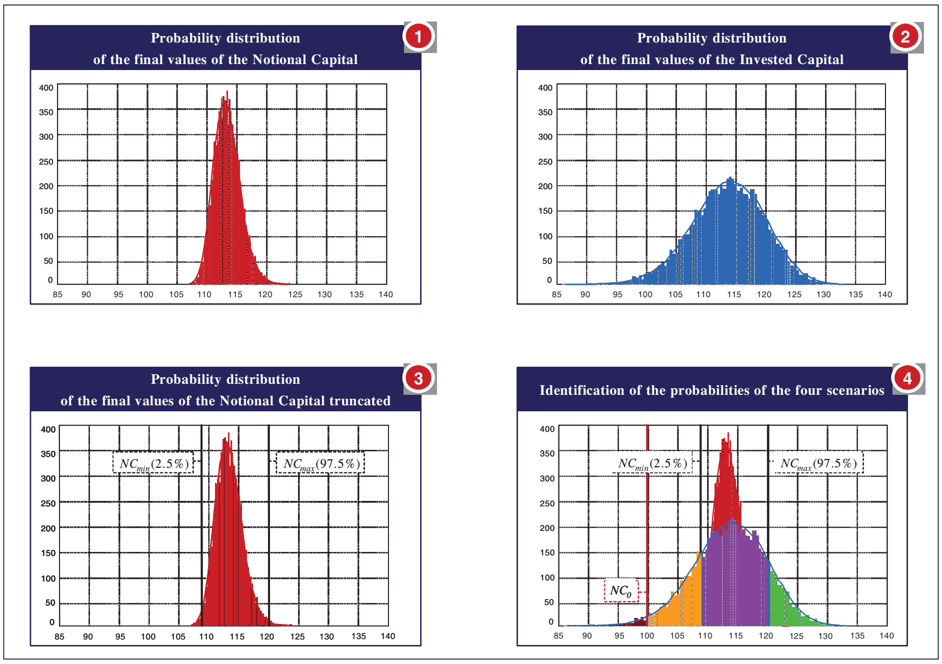
(57) For a more detailed description of the scenarios see section 2.1.2.

(58) This notation indicates the value of the *notional capital* at time 0, meant as the time of subscription.

4. *scenario where the return is positive and higher than that of the risk-free asset*: the probability mass of the distribution of the final value of the *invested capital* which is to the right of the value $NC_{\max}$ is calculated.

A graphical representation of the comparative procedure is illustrated in the charts of Figure 2.

Figure 2. Comparative procedure to determine scenarios



2.2 The second pillar: degree of risk

The information produced by the first pillar synthesizes the risk-return profile and the costs implicit in the price of the non-equity investment product at two times: the initial time and the end of the recommended investment time horizon. In particular, over this horizon, the probability scenarios and the associated median values – calculated according to the proprietary models of intermediaries – implicitly express the level of riskiness of the product through the illustration of the potential returns on the financial investment.

The second pillar completes this information by providing an explicit representation of the product's degree of risk at the time of subscription and during the recommended investment time horizon. The degree of risk is determined using synthetic quantitative indicators which process the information contained in the probability scenarios of the first pillar through suitable volatility measures of the potential returns. It follows that also these quantitative analyses are based on the intermediaries' proprietary pricing and risk management models.

The degree of risk is then disclosed to the investor through the translation of the quantitative results of said indicators into a qualitative representation which is easily understandable to the reader**.**

In this perspective, the qualitative representation has to combine the need for enough detailed information with the simple illustration of the results provided by the synthetic indicators based on quantitative determinations. The solution is offered by a set of risk classes sorted in ascending order according to the above mentioned quantitative results and defined in a clear and univocal manner. Six classes named according to the following scheme: *low, medium-low, medium, medium-high, high* and *very high*, constitute a valid compromise between the complexity of the phenomenon to be described and the need for the investors' univocal comprehension. In fact, the choice of too few or, alternatively, two many classes could reduce the effectiveness of the representation of the degree of risk. In the case of "benchmark" products the six risk classes are supplemented by four classes (so-called "management classes") which signal the intensity of the asset management activity, in terms of deviation from the chosen benchmark, as: *passive, limited, intermediate,* and *considerable*[(59)].

In particular, for "benchmark" products, the indication of the four management classes helps to qualify the specific contribution of the asset management activity to the product's overall riskiness with respect to the exogenous risk source due to the stochastic evolution of the benchmark over time. Passive asset management strategies substantially replicate the benchmark and thus, inherit its degree of risk. Conversely, in active asset management strategies, asset allocation choices reflect the autonomous investment decisions of the manager, and the associated more or less wide deviations from the benchmark constitute an additional, endogenous risk component.

The degree of risk of the non-equity investment product is initially identified by the intermediary choosing the risk class which he deems to better match the specific features of the product's financial engineering over the recommended investment time horizon. During this horizon, the intermediary uses suitable volatility measures of the potential returns, defined consistently with his internal models, to monitor any possible migration of the degree of risk to a different risk class or, for "benchmark" products, to a different management class. In this way, any significant change in the risk-return profile of the non-equity investment product can be promptly detected and consequently used to update the prospectus. In fact, migrations of the degree of risk do likely affect the potential returns of the product and also the recommended investment time horizon for a given costs regime.

The following sections present a methodological approach to define the initial degree of risk and to model the migration risk by means of several well-known results from stochastic limit theory. They also present some operative solutions to estimate volatility intervals of the product's returns for each of the above listed qualitative risk classes and to detect any migration and the related time of occurrence.

2.2.1 Volatility metrics for the degree of risk

The classification of all possible degrees of risk into a limited number of qualitative classes requires to partition the full range of the possible values of the adopted volatility metric (whatever it may be) into increasing intervals, one for each risk class.

When calibrating these intervals by using proprietary intermediaries' models, two technical conditions become crucial:

1. the identification of a suitable width for each interval;
2. the sampling frequency of the values of the metric.

As far as the width of the intervals of the metric is concerned, on the one hand it must be carefully calibrated to ensure the stability of the product's degree of risk when the changes in its value are due to its normal dynamics under different market trends. Similarly, the intervals must be wide enough to accommodate the normal asset management activity aimed at achieving the stated targets, or, in some products, the working of the quantitative algorithms underlying their financial engineering. Otherwise, the metric would be oversensitive to micro-movements in the markets or − as in the case of financial products such as open-ended mutual funds and unit-linked policies − to re-allocations in the assets' portfolio due to specific decisions of the asset manager, even though he has a commitment in terms of risk-taking or in terms of maximum departure from

(59) This approach has been adopted by Consob's transparency regulation on prospectuses of mutual funds and class III and V financial-insurance products, and it is also the basis of the method underlying the revision of EU regulation regarding the *Simplified Prospectus.*

the benchmark. On the other hand, too wide intervals could lead to level out products which may have quite different risk profiles into too few qualitative classes.

Regarding the second technical condition, by construction, the sampling frequency of the volatility metric affects the variability associated with such metric. In fact, a higher frequency typically implies a higher instability of the metric used (and vice versa), thus affecting the outcomes of the calibration procedure and, consequently, the detection of migrations between different classes.

Among the many volatility metrics commonly used in the financial literature and in market practices, the solution illustrated hereafter – for the specific purposes above described – directly takes the volatility of the returns of the non-equity investment product as input. In this way, the solution reconciles the needs for representativeness and simplicity, and it minimizes subjective assumptions as well as computational difficulties. In fact, volatility combines ease of calculation, ability to express the actual risk of a product and a strong affinity with the other existing metrics, including, for instance, the *Drawdown*, the *Maximum Drawdown*, the *Value-at-Risk* and the *Expected Shortfall*. Ultimately, the theoretical simplicity of the volatility indicator makes it the best tool to ensure an objective risk measurement and a fair comparison across products, and it maintains these suitable properties also when quite peculiar and complex financial structures are involved.

More specifically, the annualized volatility of the financial product's daily returns has been adopted, and the sampling frequency of the volatility has been set daily. Clearly, if intermediaries do implement risk measurement and monitoring models based on the annualized volatility of returns observed at a lower frequency (for example, weekly or monthly) or on other metrics than the volatility *tout court*, they should define procedures to calibrate intervals and to detect migrations in accordance with the assumptions which characterize their models.

Once the volatility metric and its sampling frequency have been chosen, the definition of the six qualitative risk classes (*low, medium-low, medium, medium-high, high* and *very high*) according to suitable quantitative determinations requires to map these classes into six increasing and non-overlapping intervals of annualized volatility of the product's daily returns.

In "benchmark" products, the identification of the four management classes (*passive, limited, intermediate,* and *considerable*) is clearly based on quantitative criteria consistent with those used to define the six risk classes and, thus, it is still built on the concept of annualized volatility of daily returns. Given the specific features of "benchmark" financial structures, this metric must also be defined according to a methodology able:

- to verify the significance of the stated benchmark;
- to distinguish between passive and active management styles;
- to measure the intensity and the direction of active asset management styles.

In this type of financial structures, the need to verify whether the manager observes the mandate granted by the investor involves assessing whether asset management results are coherent with respect to the stated benchmark. In this perspective, the metric should allow to monitor the consistency between the asset management style – passive or active – which concretely characterizes the product, and the benchmark. In particular, for actively managed products, the volatility metric should be able to distinguish anomalies, which could signal that the benchmark has become inadequate, from changes in the value of the product due to the fact that the asset manager is exploiting the margin of discretion he has with respect to the benchmark. In fact, for a given qualitative risk class, depending on his targets, an asset manager may place the financial investment at volatility levels which are higher or lower than those of the benchmark, or he may also take decisions regarding portfolio's composition aimed at speeding up or slowing down any migration induced by the exogenous riskiness of the benchmark chosen. *A priori*, active managements are, therefore, a potential risk management tool which the asset manager can use to create risk overexposure or underexposure with respect to the benchmark. It follows that to correctly represent the existence and scope of this phenomenon, it is necessary to consider the size of the deviation from the benchmark as a random variable with a positive or negative sign.

The actual identification of the four management classes makes thus necessary to compare the volatility of the product's returns to that of the benchmark's returns. To this end, a new metric is defined as the difference between the two volatilities. This metric is named *delta-vol* and it is indicated with the notation $\Delta\sigma$.

The *delta-vol* takes on near-zero values under passive management styles, while under active management styles the three classes – *limited, intermediate,* and *considerable* – are associated to *delta-vol* intervals which are increasing and symmetric with respect to zero in order to quantify both the intensity and the direction of the asset management activity.

Clearly, the analysis of the *delta-vol* allows an objective valuation of the (either positive or negative) contribution of the asset manager to the product's overall risk exposure. In fact, the total volatility of the product is equal to the algebraic sum of the *delta-vol* and of the benchmark's volatility. Moreover, the observation of extreme values (in both possible directions) with respect to the maximum thresholds of the *delta-vol* intervals corresponding to the *considerable* class constitutes, under some conditions, an alert of a misalignment from the benchmark and thus, a call for a more in-depth analysis.

Sections 2.2.2, 2.2.2.1 and 2.2.2.2 provide a detailed illustration of the calibration of the volatility intervals; section 2.2.3 shows the calibration procedure of the *delta-vol* intervals; and, eventually, section 2.2.4 describes the criteria to detect migrations in the degree of risk.

2.2.2 The grid of volatility intervals

Volatility intervals allow the identification of fixed thresholds to which the realized volatility of a non-equity investment product has to be compared in order to assign a qualitative risk class to the product. It is evident that the definition of these intervals requires the prior identification of a model to forecast the future behaviour of the volatility.

In other words, it is necessary to find a model able to ensure that the possible trajectories of the returns' volatility of a product belonging to a given risk class lie, with a reasonably high confidence level, in the volatility interval associated with that class.

The tool used to obtain volatility intervals compliant with this requirement is the diffusion limit of a GARCH (*Generalized Autoregressive Conditional Heteroskedasticity*) model, suitably inserted into an iterative procedure of non-linear stochastic programming.

The starting point of the procedure is a grid of initial intervals of annualized volatility obtained from corresponding initial intervals of annual percentage loss.

The risk neutrality principle requires to define the concept of loss on a financial investment over a given time horizon in relation to the returns obtained by the risk-free asset over the same time horizon.

In particular, also considered that most of non-equity investment products are required to annually update the information on their risk-return profile and costs, the probability distribution of the one-year risk-free rate is used. Denoting the expected value of this distribution by $\overline{r}_{rf,1y}$, the range of values of the annual percentage loss falls within -100% and $\overline{r}_{rf,1y}$.

Intuitively, the riskier the product, the higher the loss which it could realize in one year. This suggests to partition the range $(-100\%, \overline{r}_{rf,1y}]$ into six increasing intervals of annual percentage loss, one for each of the six qualitative risk classes presented in section 2.2. More precisely, the bounds of each loss interval are calculated as increasing multiples of $\overline{r}_{rf,1y}$, so that the direct relationship between the riskiness of a product and its potential loss is immediately expressed by the values and the width of the intervals.

By exploiting the functional relationship between volatility and loss measures, it is then possible to derive the annualized volatility intervals from the corresponding annual percentage loss intervals.

2.2.2.1 Construction of volatility prediction intervals

The six initial volatility intervals obtained from the six loss intervals must be revised to obtain a final grid of intervals with the following properties:

- ability to express in a robust and meaningful way the “typical” risk level of the corresponding qualitative class;
- stability over time with respect to both normal asset management decisions and to fluctuations in the value of a product due to movements in the reference markets, even when such movements are quite considerable.

The identification of intervals with these properties requires, first of all, to use a model to forecast the future values of the volatility. As mentioned in section 2.2.2, the solution presented in this work relies on GARCH models. Specifically, among the various specifications of GARCHs, the M-GARCH(1,1) model has been selected because of its weak convergence properties[60].

The model describes the dynamics of volatility in discrete time by means of the following stochastic difference equation:

$$\ln \sigma_{k+1}^2 = \beta_0^{(k)} + \beta_1^{(k)} \ln \sigma_k^2 + \beta_1^{(k)} \ln (Z_k)^2 \tag{2}$$

where $\beta_0^{(k)}$ and $\beta_1^{(k)}$ are deterministic functions of time, $\ln \sigma_0^2 = l_0$, and $\{Z_k\}_{k\in\mathbb{N}}$ is a sequence of i.i.d. standard normal random variables on $\mathbb{R}$.

The model represented by equation (2) allows consistent forecasts of the future volatility, provided that the number of available observations is sufficiently high. Relying on a limited number of daily observations of annualized volatility (e.g. considering a monthly or even shorter time horizon) would lead to a loss of statistical significance or to computational difficulties. It follows that the calibration of volatility intervals able to promptly detect the occurrence of migrations between different qualitative risk classes requires the prior analysis of the distributive properties of the continuous-time version of equation (2).

This last equation weakly converges to a stochastic differential equation whose solution is known in terms of its distributive properties. Hence, it is possible to construct a confidence interval for the prediction of the variable described by the diffusion process. The conjunction of the predictive intervals thus obtained defines over time a band for the future volatility values. The dynamic thresholds of this band allow to assess the adequacy of the fixed bounds of the volatility interval associated with a given qualitative risk class. And indeed, as better shown in section 2.2.2.2, the progressive adjustment of the bounds of each volatility interval is driven by the comparison of a series of annualized volatility trajectories − obtained, via simulation, from the initial interval − with the predictive band produced by the diffusion limit of the M-GARCH(1,1) model.

The continuous-time version of equation (2) is obtained by applying the theorem of Weak Convergence of discrete Markov chains to diffusion processes[61][62]. In fact, this theorem allows to prove[63] that equation (2) weakly converges to the diffusion process $\{\ln \sigma_t^2\}_{t\geq 0}$, driven by the following stochastic differential equation[64]:

$$d \ln \sigma_t^2 = \left(\beta_0 + 2\beta_1 E\left(\ln |Z_t|\right) + (\beta_1 - 1) \ln \sigma_t^2\right) dt + 2\left|\beta_1\right| \sqrt{Var\left(\ln |Z_t|\right)} dW_t \tag{3}$$

where β_0 and β_1 are deterministic functions of time, Z_t is a standard normal random variable and W_t is a one-dimensional standard Brownian motion.

(60) This is a multiplicative model for the conditional variance, introduced by Geweke, Pantula and Mihoj. See [Geweke, 1986], [Pantula, 1986] and [Mihoj, 1987].
(61) See appendix B, section B.1.
(62) See [Ethier e Kurtz, 1986] and [Stroock e Varadhan, 1979].
(63) The main contributions on the weak convergence of GARCH models are those of Nelson and Duan. See [Nelson, 1990] and [Duan, 1997].
(64) See appendix B, section B.2.

The stochastic differential equation (3) is an Ornstein-Uhlenbeck arithmetic diffusion process and, therefore[65], its probability distribution − given any constant initial condition $\ln \sigma_s^2$ at time s (with $s < t$) − is:

$$\ln \sigma_t^2 \sim N\left(\begin{array}{c}\left(\ln \sigma_s^2 + \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right) e^{(\beta_1-1)(t-s)} - \frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}; \\ \sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)(t-s)}-1\right)}{2(\beta_1-1)}}\end{array}\right) \tag{4}$$

The knowledge of the distributive properties of the solution of equation (3) allows to construct a volatility prediction interval with a given confidence level, say α. More precisely, for $s = t-1$, the bounds of the one-day volatility prediction interval with a confidence level equal to α are respectively[66]:

$$\sigma_{t,\min}^G = e^{\frac{-z_{\frac{1+\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}}+\left(\ln \sigma_{t-1}^2+\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}{2}} \tag{5}$$

and:

$$\sigma_{t,\max}^G = e^{\frac{z_{\frac{1+\alpha}{2}}\sqrt{\frac{\left(2|\beta_1|\sqrt{Var(\ln|Z_t|)}\right)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}}+\left(\ln \sigma_{t-1}^2+\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0+2\beta_1 E(\ln|Z_t|)}{(\beta_1-1)}}{2}} \tag{6}$$

and, thus, substituting for the explicit values of $E(\ln|Z_t|)$ and $Var(\ln|Z_t|)$[67]:

$$\sigma_{t,\min}^G = e^{\frac{-z_{\frac{1+\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}}+\left(\ln \sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}} \tag{7}$$

and:

$$\sigma_{t,\max}^G = e^{\frac{z_{\frac{1+\alpha}{2}}\sqrt{\frac{(2.2214|\beta_1|)^2\left(e^{2(\beta_1-1)}-1\right)}{2(\beta_1-1)}}+\left(\ln \sigma_{t-1}^2+\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}\right)e^{(\beta_1-1)}-\frac{\beta_0-1.2704\beta_1}{(\beta_1-1)}}{2}} \tag{8}$$

The estimate of the parameters β_0 and β_1 appearing in (7) and in (8) is performed by exploiting the discrete-continuous relationship existing between equations (2) and (3) and it requires the

[65] See [Minenna, 2003].
[66] The superscript G indicates that these are prediction intervals obtained via the diffusion limit of a GARCH model.
[67] The values of $E(\ln|Z_t|)$ and $V(\ln|Z_t|)$ are deterministic functions of the Eulero-Mascheroni constant, also known as *EulerGamma*, whose value is the result of the following limit:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{k}-\ln n\right)$$

and it is approximately equal to 0.5772. See [Abramowitz, Steygun, 1964].

maximization – via numerical methods – of the logarithm of the following likelihood function, where: $Y_k = \ln \sigma_k^2 - \ln \sigma_{k-1}^2$[(68)]:

$$\begin{aligned} L(Y;\beta_0,\beta_1) = \; & \prod_{k=2}^{K} \Bigg[\frac{1}{|\beta_1|\sqrt{2\pi}} \sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}} \cdot \exp\Bigg(\frac{1}{2|\beta_1|} \sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}} \cdot \\ & \cdot \Bigg(Y_k - \frac{(\beta_0 - 1{,}2704\beta_1)\big(e^{(\beta_1-1)}-1\big)}{\beta_1-1} - \\ & -1{,}2704\,|\beta_1| \sqrt{\frac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} - \big(e^{(\beta_1-1)}-1\big)\ln\sigma_{k-1}^2 \Bigg)\Bigg) \\ & \cdot \exp\Bigg(-\frac{1}{2} \exp\Bigg(\frac{1}{|\beta_1|} \sqrt{\frac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}} \cdot \\ & \cdot \Bigg(Y_k - \frac{(\beta_0 - 1{,}2704\beta_1)\big(e^{(\beta_1-1)}-1\big)}{\beta_1-1} - \\ & -1{,}2704\,|\beta_1| \sqrt{\frac{e^{2(\beta_1-1)}-1}{2\big(\beta_1-1\big)}} - \big(e^{(\beta_1-1)}-1\big)\ln\sigma_{k-1}^2 \Bigg)\Bigg)\Bigg)\Bigg] \end{aligned} \quad (9)$$

2.2.2.2 The calibration of the volatility intervals

This section describes the procedure to calibrate the final grid of the volatility intervals. For the sake of simplicity the description is limited to a single interval, with the understanding that the same procedure holds also for the other intervals.

Once selected the initial volatility interval $[{}_0^n\sigma_{\min}, {}_0^n\sigma_{\max}]$[(69)] associated with the n^{th} class ($n = 1, 2, ..., 6$), the calibration procedure involves the simulation of m, ($m \in \mathbb{N}$) trajectories of the process V_t, which represents the value at the generic time t of a hypothetical non-equity investment product belonging to this risk class.

The simulation is performed by discretizing the stochastic differential equation that describes the dynamics of the process V_t[(70)]. The drift and diffusion coefficients of this equation are modelled according to criteria able to ensure the observance of the risk neutrality principle and the robustness of the final volatility intervals, also with respect to considerable movements of the yield curve. In addition, the diffusive component of the stochastic differential equation of V_t is modelled also taking into account the initial volatility interval. Indeed, this interval is the only quantitative information available *ex ante* on the riskiness of the n^{th} qualitative risk class and, therefore, it is the only information at disposal to ensure the representativeness of the final volatility intervals and, at the same time, to minimize the likelihood of obtaining overlapping intervals.

Each trajectory is made up of N realizations of the process V_t and it univocally identifies a corresponding trajectory of daily returns, constituted by $N-1$ values.

Given τ (the width of the returns' time window defined in order to compute the volatility), it is possible to determine m annualized volatility trajectories; hence, each trajectory has $H = N - \tau$ values.

Each of these trajectories – denoted by $\{{}_1^n\sigma_{i,j}\}_{i=1,2,\dots,m;\; j=1,\dots,H}$[(71)] – is modelled through the stochastic difference equation (2) of section 2.2.2.1. Setting a window of K volatility observations in order to estimate the parameters β_0 e β_1 and applying the equalities (7) and (8) of section 2.2.2.1, the one-day α–confident prediction interval for the annualized volatility is calculated. The overall number of prediction intervals obtained for each trajectory is therefore equal to $(H-K)$. The

(68) For an illustration of the estimation procedure see appendix B, section B.3.

(69) The superscript n preceding the symbol of the volatility indicates that these are the bounds of the initial volatility interval associated with the n^{th} risk class, while the subscript 0 indicates that these are the bounds obtained from the initial loss intervals and, therefore, before performing the calibration procedure.

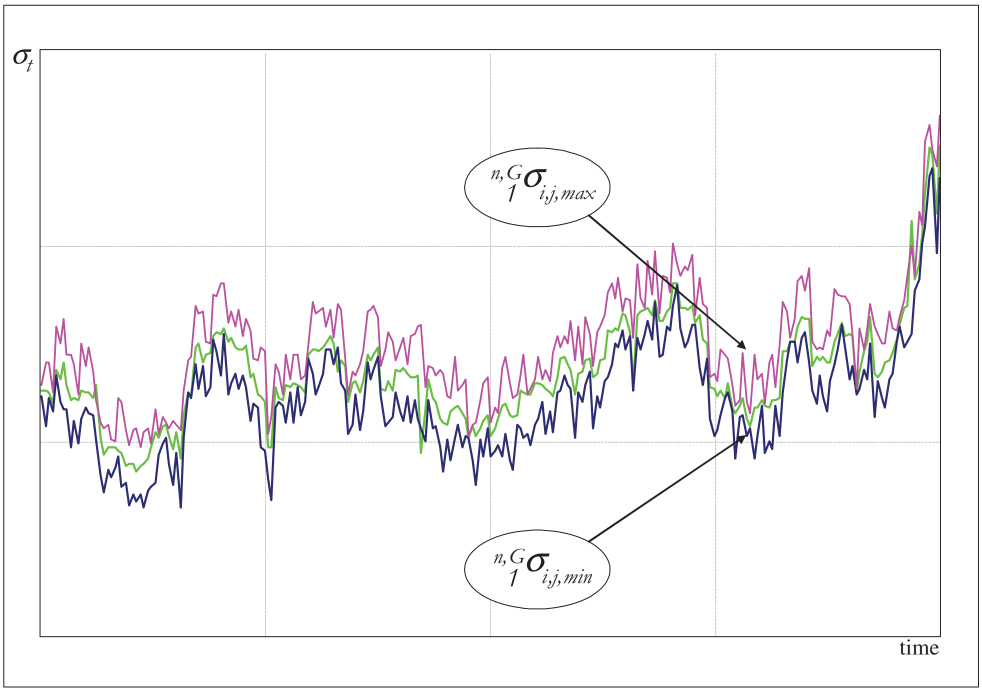
(70) The discretization step used in the simulation is daily.

(71) The subscript 1 preceding the symbol of the volatility indicates that these are the values of this financial quantity obtained from the first iteration of the calibration procedure.

conjunction of these intervals defines a fluctuation band with dynamic thresholds for the values of the annualized volatility.

Figure 3 offers a qualitative representation of the fluctuation band[72].

Figure 3. The volatility fluctuation band



As suggested by Figure 3, the fluctuation band is *adaptive*, meaning that it immediately embeds past information and uses it to dynamically update its bounds. In other words, the way the band is determined ensures that, if on a given day the realized volatility breaches the dynamic thresholds, this information implies an immediate widening of the band itself. Therefore, the volatility forecast for the next day is performed with a filtration that contains all the information on recent breaches, and it is not affected by echoes that may show up in the days after the first breach.

By considering the fluctuation bands for all the m simulated trajectories of the annualized volatility, the following three quantities can be calculated:

1. the percentage of out-of-band observations, denoted by Δ, i.e.:

$$\Delta = \frac{\sum_{i=1}^{m} \sum_{j=K+1}^{H} \mathbf{1}\left(\sigma_{i,j} \ > \ {}^{n,G}_{1}\sigma_{i,j,\max}\right) + \mathbf{1}\left(\sigma_{i,j} \ < \ {}^{n,G}_{1}\sigma_{i,j,\min}\right)}{m \cdot (H-K)}$$

2. the percentage of observations above the upper bound of the band, denoted by Δ_{up}, i.e.:

$$\Delta_{up} = \frac{\sum_{i=1}^{m} \sum_{j=K+1}^{H} \mathbf{1}\left(\sigma_{i,j} \ > \ {}^{n,G}_{1}\sigma_{i,j,\max}\right)}{m \cdot (H-K)}$$

[72] The superscript G denotes that this is the volatility fluctuation band obtained via the diffusion limit of a GARCH model.

3. the percentage of observations below the lower bound of the band, denoted by Δ_{down}, i.e.:

$$\Delta_{down} = \frac{\sum_{i=1}^{m}\sum_{j=K+1}^{H}\mathbf{1}\left(\sigma_{i,j} < {}_{1}^{n,G}\sigma_{i,j,\min}\right)}{m\cdot(H-K)}$$

Depending on the values of these quantities, there are three possible cases.

CASE 1: $\Delta > \alpha$ and $\Delta_{down} < \Delta_{up}$
In this case the initial volatility interval $[{}_{0}^{n}\sigma_{\min}, {}_{0}^{n}\sigma_{\max}]$ is updated. The lower bound remains unchanged, while the upper bound is revised downwards, so that the resulting interval is:

$$[{}_{0}^{n}\sigma_{\min}, {}_{1}^{n}\sigma_{\max}]$$

where the subscript 1 placed before the upper bound indicates the first update with respect to the initial interval.

CASE 2: $\Delta > \alpha$ and $\Delta_{down} > \Delta_{up}$
Also in this case, the initial volatility interval $[{}_{0}^{n}\sigma_{\min}, {}_{0}^{n}\sigma_{\max}]$ is updated. The upper bound remains unchanged, while the lower bound is revised upwards, so that the resulting interval is:

$$[{}_{1}^{n}\sigma_{\min}, {}_{0}^{n}\sigma_{\max}]$$

where the subscript 1 placed before the lower bound indicates the first update with respect to the initial interval.

CASE 3: $\Delta \leq \alpha$
In this case the initial interval $[{}_{0}^{n}\sigma_{\min}, {}_{0}^{n}\sigma_{\max}]$ is not updated.

If cases **1** or **2** occur, the updated interval becomes the new initial volatility interval associated with the n^{th} class and it constitutes the starting point for the next simulation of m trajectories of the process V_t. The entire procedure is then repeated to obtain the new values of Δ, Δ_{down} and Δ_{up} which allow to determine which of the three above described possible cases has occurred.

This iterative scheme represents the implementation of a non-linear stochastic programming technique. The iteration ends the first time that the initial volatility interval of the last iteration coincides with the final volatility interval for the n^{th} qualitative risk class. This interval is indicated with the notation $[{}_{k}^{n}\sigma_{\min}, {}_{k}^{n}\sigma_{\max}]$, where the subscript k denotes the number of iterations executed.

The remaining five volatility intervals are calibrated simultaneously with the n^{th} and according to the same procedure.

Finally, at each successive iteration the intervals so obtained are subject to a further fine-tuning intervention aimed at ensuring that they have substantially the same significance level and at avoiding any overlapping.

The result of the calibration is shown in table 1.

Table 1. Risk Classes and Volatility Intervals *

Risk Class	Final Volatility Interval σ_{min} σ_{max}
low	[0.01%, 0.49%]
medium-low	[0.5%, 1.59%]
medium	[1.6%, 3.99%]
medium-high	[4%, 9.99%]
high	[10%, 24.99%]
vey high	[25%, over 25%)

* The values in the table have been rounded.

The univocal determination of the volatility intervals ensures the transparent and explicit representation of the risk profile of non-equity investment products. In fact, the risk exposure of each product can be assigned to one of the six qualitative classes by simply identifying the interval where the volatility of the potential returns falls into.

In "risk target" or "benchmark" structures, this volatility has to be inferred, at the initial time, from the characteristics of the underlying financial engineering and, where relevant, of the asset management style associated with the product. In this perspective, the impact of the various risk sources on the uncertainty of the potential returns at the end of the recommended investment time horizon deserves a particular attention.

In "return target" structures or in those backed by financial guarantees, it is necessary to consider the volatility of the potential returns associated with the evolution of the value of the *invested capital* throughout the entire recommended time horizon. In fact, the assignment to a given qualitative risk class should be done by comparing this volatility (properly annualized) with the grid in table 1.

Whatever the financial structure of the product, it is evident that there is a close connection between the information on the riskiness conveyed by the qualitative risk class and that on the probability scenarios conveyed by the first pillar. This connection is partially explained by considering that both indicators are developed having regard to the concept of potential losses. Therefore, reading the information provided by the two indicators leads to interpret the grid in table 1 as the annualized equivalent of the volatilities which, once projected over the entire recommended time horizon, allow the effective definition of the probability scenarios associated with a specific financial investment.

2.2.3 The grid of *delta-vol* intervals

In line with the methodological solution presented in this work, the calibration procedure of the *delta-vol* intervals is developed according to quantitative determinations which inherit the same assumptions used to calibrate the volatility intervals.

That said, it is worth noticing that for passively managed products, there is no need to calibrate different *delta-vol* intervals in relation to the specific product's risk class. In fact, it is clear that, no matter what metric is used to measure the deviation from the benchmark − including, therefore, the *delta-vol* − its values will tend to be zero, so signalling, as expected, the close relationship, or even the identity, with the benchmark.

Conversely, in products with an active asset management style, the multiplicity of classes of deviation allows to distinguish the different risk exposure of products sharing the same benchmark. In particular, for each of the three classes (*limited, intermediate* and *considerable*) there is a corresponding *delta-vol* interval, whose width is proportional to the volatility interval associated with the risk class of the product. A smaller deviation from the benchmark translates into a narrower *delta-vol* interval and vice versa.

In practice, the definition of the *delta-vol* intervals starting from the grid of volatility intervals reported in table 1 of paragraph 2.2.2.2 is carried out in three computational steps[73]:

[73] In some cases, the percentages used in the above computational steps required small adjustments to ensure that, irrespective of the management class adopted, the maximum deviation from the benchmark (considered in absolute terms) is higher for the products having more risk than for those products with a lower overall riskiness, as signalled by the corresponding qualitative risk class.

1. *calculation of the intervals associated with the considerable class*: for each possible qualitative risk class, these intervals are determined by taking a fixed percentage of one of the bounds of the volatility interval corresponding to that class.
   The specific percentages and the bounds used are shown below:

Risk Class	Percentage	Bounds of the Volatility Interval
low	50.00%	Upper
medium-low	30.00%	Upper
medium	30.00%	Upper
medium-high	25.00%	Upper
high	25.00%	Upper
vey high	50.00%	Lower

2. *calculation of the intervals associated with the intermediate class*: for each possible qualitative risk class, the bounds of these intervals are equal to 75% of the bounds of the intervals associated with the *considerable* class;

3. *calculation of the intervals associated with the limited class*: for each possible qualitative risk class, the bounds of these intervals are equal to 50% of the bounds of the intervals associated with the *considerable* class.

The apparent simplicity of the above described procedure is due to the fact that, as has been said, the *delta-vol* is a metric specifically defined to measure the contribution of the endogenous risk source represented by the specific asset management activity to the overall risk of the product, as summarized by the corresponding qualitative risk class. In analytical terms, denoting the returns' volatility of the product and that of its benchmark respectively by σ_P and σ_B, the following equation holds:

$$\sigma_P = \sigma_B + \Delta\sigma \tag{10}$$

The additive logic linking the quantities in equation (10) shows that the relationship between the two types of intervals has to be read like that existing between a part and its entirety. In this perspective, as far as "benchmark" products are concerned, the calibration of the volatility intervals appearing in table 1 of paragraph 2.2.2.2 allows to implicitly quantify proportional intervals for the metric used to measure the departure from the benchmark. Moreover, while the volatility is an absolute risk metric, the *delta-vol* is a relative metric, as it is anchored to the specific dynamics of the benchmark. For this reason, the relationship of proportionality between the intervals of this metric and the volatility intervals is defined in a decreasing way: for a given management class, the greater the total risk of the product, the lesser the overexposure (or the underexposure) assumed with respect to the benchmark is likely to be.

Table 2 shows the grid of the *delta-vol* intervals obtained at the end of the calibration procedure.

Table 2. Management Classes and *Delta-vol* Intervals

Risk Class	Limited		Intermediate		Considerable	
	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$	$\Delta\sigma_{min}$	$\Delta\sigma_{max}$
low	-0.118%	0.118%	-0.176%	0.176%	-0.235%	0.235%
medium-low	-0.239%	0.239%	-0.358%	0.358%	-0.477%	0.477%
medium	-0.600%	0.600%	-0.900%	0.900%	-1.200%	1.200%
medium-high	-1.250%	1.250%	-1.875%	1.875%	-2.500%	2.500%
high	-3.125%	3.125%	-4.688%	4.688%	-6.249%	6.249%
vey high	-6.250%	6.250%	-9.375%	9.375%	-12.500%	12.500%

As has been said, the peculiarity of this metric compared to volatility is that the *delta-vol* intervals are symmetric with respect to zero. As a result, narrower intervals are fully contained within wider intervals. This characteristic differentiates the *delta-vol* from the tracking error volatility (hereinafter *TEV*), the latter being the volatility metric commonly used in finance as a measure of alignment with the benchmark. The *TEV* is calculated as the volatility of the difference between the returns of the product and those of the benchmark and it provides an information very similar to that conveyed by the *delta-vol*. However, compared to the *delta-vol*, the *TEV* has the drawback of ignoring the direction of the deviations from the benchmark due to an active asset management style and, thus, it does not reveal whether the management activity is aimed at mitigating the risk induced by the benchmark or at increasing it.

On the contrary, the *delta-vol* allows to analyze active management strategies not only in terms of intensity, but also in terms of direction[(74)]: intuitively, this metric is similar to a potentiometer which may be used to regulate the risk exposure of the product with respect to that of the benchmark.

For example, once the level of deviation from the benchmark has been chosen, a manager willing to maintain unchanged the qualitative risk class of the product over time should monitor on a continuous basis the riskiness of the benchmark, assess its impact on the short-term dynamics of the returns' volatility and, consequently, decide whether and how much to use the flexibility offered by the *delta-vol* interval of the product. Depending on the importance assigned to this target as well as on the contingent behaviour of the benchmark and on his own views, the asset manager could preserve his *delta-vol* margin by accepting a possible change in the qualitative risk class of the product or he could alternatively expand this margin by intensifying his asset management activity, and possibly moving to a wider *delta-vol* interval. The second option is more feasible where the original deviation from the benchmark is either *limited* or *intermediate*. Otherwise, the problem of exceeding the maximum thresholds defined by the bounds of the interval associated to the *considerable* class could arise, with possible consequences on the significance of the chosen benchmark.

2.2.4 The migration of the degree of risk

Information transparency on the degree of risk of non-equity investment products requires the analysis of the time evolution of the related synthetic quantitative indicators to promptly revise, if necessary, the original qualitative class according to robust and objective criteria.

In order to correctly detect migrations, the width of both the volatility and the *delta-vol* intervals must be adequately set with respect to the period taken as a reference to assess the occurrence of these phenomena. Too wide intervals could result in an artificial reduction in the number of migrations detected, and therefore the synthetic indicators would lose their significance. Too narrow intervals could result in an excessive number of migrations, many of them being spurious.

As said in section 2.2.1, the formalization presented in this work has been based on the above considerations. Specifically, the time period used as reference to detect migrations in the degree of risk has been determined through a backtesting analysis performed on the grid of the six volatility intervals. For each interval, the time evolution of the annualized volatility of daily returns has been simulated for a hypothetical non-equity investment product belonging to the risk class associated with that interval. Then, for each trajectory, the length of the periods outside the calibrated interval has been calculated, in order to determine its empirical probability distribution. The observation of the distributions associated with the six volatility intervals has highlighted that, for

[(74)] For this reason, at least theoretically, a greater intensity of the asset management activity should justify the application of higher management fees.

each interval, a probability mass of approximately $(1-\alpha)$ was concentrated around values lower than or equal to three months, and therefore this time period has been adopted as the reference time period to detect migrations. Therefore, a migration to a different qualitative risk class (or, for "benchmark" products, to a different management class) occurs when the values of the related volatility metric remain in one or more classes different to the original one for more than three consecutive months.

Intuitively, the three-month rule seems consistent with the characteristics of the iterative procedure used to calibrate the volatility intervals and, thus, also the *delta-vol* intervals.

It is worth recalling that, for a product which stably remains in a given qualitative risk class, this procedure ensures that the number of times that the fluctuation band obtained via the GARCH diffusion approach is breached does not exceed a percentage α of the quantity $(H-1)$. Moreover, being the band adaptive (i.e. with dynamic thresholds), it is reasonable to assume that these breaches would not be consecutive, but more or less randomly distributed over the time horizon used for the calibration. In fact, considered that a migration is a persistent breach of an interval having a constant width, the confidence level used for the calibration is intrinsically prudential with respect to the three-month rule[75].

Once established the three-month rule, the detection of migration events becomes quite easy. Given the initial risk class − determined by the intermediary in relation to the specific features of the product's financial engineering over the recommended investment time horizon − a check whether the annualized volatility of daily returns has breached the interval corresponding to the original class is performed on a daily basis. The migration occurs when the outcome of this control is always positive for more than three months. In this case, the risk class assigned to the product should be updated to be consistent with the new interval which the volatility has moved to. If the volatility remains for more than three consecutive months in two or more classes different to the original one, the new qualitative risk class is assigned according to a prevalence criterion. In fact, the product is assigned to the class associated with the interval where the volatility has remained more frequently over the previous three months.

In "benchmark" products, migrations from the original management class are detected on the basis of the outcome of a similar check, which is specialized in order to take into account the fact that the classes with a lower deviation are proper subsets of those with higher *delta-vol*. In general, for a given qualitative risk class, the migration to a new management class occurs when, for three consecutive months[76] the *delta-vol* lays:

- on values falling within the intersection of the original class and one or more other classes with a lower intensity of the management activity, in the case the original class is either *intermediate* or *considerable*;
- outside the bounds of the interval associated with the original class, if this is either the *limited* or the *intermediate* class.

The migration criteria developed in this work and illustrated in detail hereafter reflect the assumption that the statement of a specific asset management style has to be confirmed by the actual behaviour of the product compared to its benchmark and, thus, by the values of the *delta-vol*. Even though the four management classes partially overlap, the key feature of products with a more intense asset management activity − meaning those belonging to either the *intermediate*

[75] It has been used a value of α in line with those commonly used in practice to ensure a reasonable level of acceptability to the valuations of phenomena which are anyway subject to residual areas of uncertainty, and hence not completely explained by whatever statistical model.

[76] Obviously, also in this case, the three-month rule is a quantitative determination based on the key assumptions stated above and underlying the entire methodological solution behind the second pillar.

or the *considerable* class – corresponds to the areas where there is no overlapping between these two classes and the other ones.

It follows that in order to identify any significant change in the class of deviation from the benchmark, the *delta-vol* intervals shown in table 3 hereafter[77] have to be considered.

Table 3. *Delta-vol* intervals to detect migrations in Management Classes *

Risk Class	Limited		Intermediate		Considerable	
	L_{min}	L_{max}	I_{min}	I_{max}	C_{min}	C_{max}
low	0^+	0.118%	0.1181%	0.176%	0.1761%	0.235%
medium-low	0^+	0.239%	0.2391%	0.358%	0.3581%	0.477%
medium	0^+	0.600%	0.6001%	0.900%	0.9001%	1.200%
medium-high	0^+	1.250%	1.2501%	1.875%	1.8751%	2.500%
high	0^+	3.125%	3.1251%	4.688%	4.6881%	6.249%
vey high	0^+	6.250%	6.2510%	9.375%	9.3751%	12.500%

* The lower bound of the "*limited*" class, denoted by L_{min}, equals 0^+, meaning a positive value reasonably close to zero, but not zero. This allows to detect the changes from an active management style to a passive management style and vice versa.

Given the qualitative risk class of the product, all possible cases (including migration to and from the passive asset management style) are summarized in table 4, where M denotes the total number of observations of the *delta-vol* over three months.

Table 4. Criteria to assign the Management Class*

Case	Class of Deviation from the Benchmark
$\{C_{max} \geq \Delta\sigma_i \geq C_{min}\}_{i=1,\dots,M}$	Considerable
$\{I_{max} \geq \Delta\sigma_i \geq I_{min}\}_{i=1,\dots,M}$	Intermediate
$\{L_{max} \geq \Delta\sigma_i \geq L_{min}\}_{i=1,\dots,M}$	Limited
$\{L_{min} > \Delta\sigma_i \geq 0\}_{i=1,\dots,M}$	Passive

* For the meaning of the notation used in this table see table 3.

[77] Given the symmetry of active management classes with respect to zero, the migration rules are illustrated, for the sake of simplicity, only in relation to the real non-negative half-line, without prejudice to the fact that they do equivalently apply to the real non-positive half-line. Cases of joint migration of both the management class and the sign of the *delta-vol* are excluded from the analysis, given that any change in sign occurs without leaving the *limited* class.

The formalization of the migration rules illustrated in table 4 is limited only to the case when the *delta-vol* lays in only one of the intervals shown in table 3 – or, for the *passive* class, in the interval $[0, L_{\min})$ – for three consecutive months. Otherwise, a prevalence criterion, similar to that used to detect migrations between different qualitative risk classes, applies. This formalization also excludes movements in the *delta-vol* that are not consistent with the evolution of the benchmark. Indeed, such movements occur when most of the values of the annualized *delta-vol* observed over three months fall outside the interval associated with the *considerable* class.

2.3 The third pillar: recommended investment time horizon

The recommended investment time horizon completes the representation of the risk-return profile of non-equity investment products[78]. The reason is that, in general terms, out of the range of products available on the market, first the investor chooses those whose recommended time horizon matches his liquidity preferences; then, he assesses the consistency between his risk appetite and the degree of risk of the products which passed the first stage of the selection process; and, finally, he chooses the product with the highest potential returns among those which have been identified according to the second selection criterion.

The close connection between the above mentioned variables is adequately represented by the methodological approach adopted in this work. In this perspective, the quantitative determinations required to identify and synthesize the risk-return characteristics of the non-equity investment product in terms of the probability scenarios and of the degree of risk have to be completed by those needed to identify the recommended investment time horizon.

In this regard, it is worth mentioning that, within the above described integrated approach, information on the risk-return profile of the product and on its recommended investment time horizon cannot disregard the various cost components of the financial investment and the specific features of the three types of financial structures.

For “return target” products and for products backed by a financial guarantee (whether they have “risk target” or “benchmark” structures), if the simplicity of the protection technique or that of the underlying financial engineering makes it possible to univocally identify the exact time when the returns are optimized, the recommended investment time horizon clearly coincides with the reference horizon of the target return. The engineering of these products and, in some cases, also their asset management techniques, are aimed at achieving, at a given maturity, a predetermined result or a result which is dynamically updated over time. Closing the position before this maturity would be inconsistent with the characteristics of the product and unprofitable for a rational investor, who would be giving up the chance, or even the certainty, of obtaining at least the target return declared by the intermediary; moreover, by abandoning too early the product without taking into account the costs paid, the investor would also risk to experience a loss. Vice versa, holding the investment beyond the maturity which characterizes the product and its asset management style, in some cases, is not feasible because the product ceases to exist, while in other cases, it could cause a change in the overall risk exposure of the outstanding position or the transformation of the position into an investment substantially similar to the risk-free asset.

In more complex structures, where the actual target return of the product results from the overlapping of two or more elementary protection or guarantee mechanisms which operate over different time horizons or which obey heterogeneous conditions, the identification of the recommended investment time horizon requires a careful analysis of the product’s elementary components in order to get a clear picture of how any single protection or guarantee mechanism actually works. For example, if the analysis highlights that one of these elementary mechanisms stochastically

[78] Information on this variable has been included by Consob in its prospectus templates since 2001.

dominates the others[79], then the technical time horizon associated with the former identifies the recommended investment time horizon of the product.

In "risk target" or "benchmark" products, the absence of a target return or a financial guarantee that indicates, at least *ex ante,* the best investment time horizon for the investor, requires to determine the recommended investment time horizon according to the criterion of the costs break-even, given the degree of risk of the product. In fact, from the investor's point of view, the recommended investment time horizon of these types of products should express a recommendation on the minimum time period within which the costs incurred may be amortized, taking into account the risks embedded in their financial engineering.

This concept requires to adopt a methodology where the said horizon is identified as the first year T within which the probability of amortizing the costs of the financial investment − calculated under the risk-neutral probability measure − reaches a predetermined threshold at least once.

Given that, usually, the difference between the initial values of the *notional capital* and the *invested capital* is exactly equal to the costs of the product[80][81], the calculation of T involves the concept of "first passage time" of the stochastic process of the *invested capital* through a barrier set equal to the value of the *notional capital* at time 0[82].

Formally, given the probability space $(\Omega, \Im, \mathbb{P}^*)$, where $\mathbb{P}^*$ is the risk-neutral probability measure, and denoting by NC_0 the value of the *notional capital* at time 0, and by $\left\{\widetilde{IC}_t\right\}_{t\geq 0}$ the stochastic process of the *invested capital* on $(\Omega, \Im, \mathbb{P}^*)$, the following random variable $t^*$is defined:

$$t^* = \inf\left\{t \geq 0 : \widetilde{IC}_t \geq NC_0\right\} \tag{11}$$

Once the internal model used to describe the random dynamics of the *invested capital* is known, it is possible to determine the probability density function and the cumulative distribution function of t^*. The recommended investment time horizon is then given by the year T which satisfies the following equation:

$$\Pr\left(t^* \leq T\right) = x \tag{12}$$

where x is the given threshold that, considering also the level of risk, quantifies the probability of the product amortizing its costs by the year T[83]. In other words, the recommended investment time horizon is the first year within which the value of the *invested capital* equals the initial value of the *notional capital* with a probability equal to x[84].

Irrespective of the specific modelling choices, this approach ensures that the recommended investment time horizon T is an increasing function of the product's costs and degree of risk[85].

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(79) This is the case where it is possible to identify a protection or guarantee mechanism that ensures better performances than the others in every possible state of nature.

(80) See section 2.1.1.

(81) The only exception occurs when there are non-financial deferred costs, because they are excluded from the definition of *notional capital*. See section 2.1.1.

(82) The identification of the recommended investment time horizon according to the criterion just described is in line with the information provided by the first pillar of the risk-based approach to transparency in terms of the "absolute performance risk" of the financial investment. Such a risk is indeed assessed taking as reference value the *notional capital* at time 0.

(83) If the result of (12) is not an integer number of years, T is rounded by excess.

(84) Where the product requires periodic payments by the investor, the procedure to determine T needs to be adapted in accordance with the above described methodology.

(85) In "benchmark" products, the relationship between the recommended investment time horizon and the management class is increasing if the asset management strategy is aimed at taking more risk than the benchmark ($\Delta\sigma > 0$); otherwise, it is decreasing ($\Delta\sigma < 0$).

The logic underlying the described methodology is intrinsically prudential. To better understand this point, imagine a product for which the probability of the event $\{t^* \leq 1\}$ is equal to 95% of x. It is clear that at the end of the first year of the financial investment, there is a high probability that the investor will have amortized the costs of the product at least once. Should the occurrence of the said event not coincide with the exit from the product, the sole fact of having reached the barrier (equal to the value of the *notional capital* at time 0) at least once will give the investor a higher probability of amortizing again the costs in less than one year. This property follows from the fact that $\left\{\widetilde{IC}_t\right\}_{t\geq 0}$ is a Markov process. Therefore, if at time $t = 1$ this process will have reached the barrier NC_0, its future values will no longer be affected by its past history, but they will continue to evolve according to the stochastic model used by the intermediary, starting from an initial value equal to the value reached at that date.

In order to suitably represent the degree of risk, the diffusive component of the stochastic equation used to model the dynamics of the *invested capital* needs to be consistent with the volatility interval corresponding to the qualitative risk class of the product. The intermediary's choice of a diffusive component which weights the different volatility values of the product's risk class in a more or less balanced way essentially depends on the specific features of the product. For a given costs regime, this choice has a modest impact on the recommended investment time horizon in the case of either a *low* or *medium-low* qualitative risk class, as the volatility intervals associated with these classes are relatively narrow. Conversely, for classes with higher levels of risk, the specific features of the model chosen are likely to impact on the recommended investment time horizon. In fact, being the volatility interval wider, the choice of a more or less symmetric model determines the clustering in a specific area of the said interval.

In "benchmark" products, the above described analysis requires a further specialization in relation to the asset management style adopted. In this perspective, the time evolution of the *invested capital* can be described by stochastic volatility models calibrated on the benchmark's volatility term structure, such as the implied volatility of options written on it and expiring at increasing maturities.

In particular, when considering actively managed products, the correct representation of the random dynamics of the *invested capital* requires to add some noise to the benchmark's stochastic volatility model in order to make the definition of the recommended time horizon sensitive also to the effect of possible departures from the benchmark due to specific decisions of the asset manager. This can be done, for instance, by drawing from a random noise component whose size is consistent with the *delta-vol* interval corresponding to the level of intensity of the active asset management style.

The methodology presented in this section to identify the recommended investment time horizon completes the information conveyed by the first two pillars of the risk-based approach to transparency, ensuring the objectivity of the recommendation given to investors, and the significance of said recommendation with respect to the concept of liquidability of a non-equity investment product. In fact, in "risk target" or "benchmark" structures, the recommended time horizon is identified by applying a criterion aimed at minimizing the likelihood of incurring a loss when disinvesting from the product; while in "return target" or guaranteed products, this time horizon suggests to hold the investment for the period required to benefit from the extra-returns over the risk-free asset that the product is likely (or even certain) to offer at a given maturity.

3 Empirical analysis

This section presents the results of an empirical analysis which − according to the classification criteria and the migration rules of the second pillar of the risk-based transparency approach presented in this paper − examined the degree of risk and its time evolution for a sample of 544 open-ended mutual funds offered in the European market during the period from 1 January 2006 to 31 December 2008.

As has been said, the decision to limit the analysis to mutual funds is due to the availability of better quality information and panel data large enough for the purposes of the study conducted, and the same analysis can be carried out for other non-equity investment products.

Moreover, given the close connection between the three pillars (due also to the strong integration of the methodologies underlying each pillar), the analysis of the degree of risk allows to test the effectiveness of the approach proposed also with reference to the other two pillars. And this because the degree of risk is determined in relation to volatility metrics which are actually used also in the quantitative determinations behind both the probability scenarios of the potential returns and the definition of the recommended investment time horizon.

The empirical analysis presented hereafter is thus a concrete way to assess the soundness of the three-pillar approach and its ability to effectively monitor the behaviour of the risk-return profile of a non-equity investment product over time.

After a brief description of the sample provided in section 3.1, section 3.2 offers an overview of the entire market through the study of the volatilities realized by several financial indices which are representative of the assets held in the portfolios of the funds examined.

Section 3.3 illustrates the distribution of the funds across the six qualitative risk classes, and section 3.4 provides an analysis of the migrations occurred between these classes. A similar representation is given in sections 3.5 and 3.6 with reference to the distribution of actively managed "benchmark" products across the various management classes.

3.1 The sample of data

The sample of 544 funds is the result of a selection performed over the main European markets of open-ended mutual funds (Luxembourg, United Kingdom, France, Italy, Germany and Spain), as shown in table 5[(86)]:

Table 5. Reference Area: Net assets under management as of the 31 December 2008 (Values in millions of Euros)

Domicile	Net AUM	Market Share	Cumulative Market Share	No. Funds
Luxembourg	796,228.2	33.24%	33.24%	13,472.0
France	594,300.5	24.81%	58.05%	5,271.0
United Kingdom	377,859.4	15.78%	73.83%	3,051.0
Germany	253,494.0	10.58%	84.41%	1,808.0
Italy	200,830.7	8.38%	92.80%	811.0
Spain	172,511.0	7.20%	100.00%	2,831.0
Total	**2,395,224**	**100.00%**		**27,244**

The sample was selected considering the main *Lipper Global* categories which are representative of the three types of financial structures ("risk target", "benchmark" and "return target").

(86) The six countries represent 76.31% of the assets under management in the European area at the end of December 2008.

The choice of the *Lipper* classification comes from the need to ensure the significance of the comparison between funds belonging to the same category. In fact, *Lipper* aggregates the funds into homogenous groups according to the analysis of multiple information sources: investment policies, annual and semi-annual reports and historical balance sheet data[87].

The *Lipper Global* categories were grouped as follows in terms of their corresponding financial structure:

- "risk target" funds: only one category named *Mixed Asset EUR Flex-Global*;
- "benchmark" funds: twenty-three categories, broken down into one "monetary" category, seven "bond" categories, twelve "equity" categories and three "balanced" categories;
- "return target" funds: two categories named *Protected* and *Guaranteed*, respectively.

Then, for each category and each country, the analysis considered, where present, the top eight funds in terms of average assets under management whose category did not change over the last six years. In this way the selection picked up the funds characterized by a greater stability of the category over time and, as a consequence, also by a greater stability of the type of financial structure. This last step of the selection process led to the final sample of 544 funds, equal to 10.58% of the total assets under management of the universe examined.

Tables 6 and 7 summarize the characteristics of the sample by category, net assets under management, country and type of financial structure.

[87] The use of the categories stated by asset management companies in the prospectuses does not allow to obtain homogeneous groups, due to the differences (also in the labels adopted) between countries.

Table 6. Representativeness of the *Lipper* Global categories and the sample of selected funds: Net assets under management as of 31 December 2008 (Values in millions of Euros)

Selected Lipper Global	Net AUM	Net AUM (%)
Bond Emerging Markets Global	8,318	0.35%
Bond EUR	57,887	2.42%
Bond EUR Corporates	19,947	0.83%
Bond EUR Long Term	12,069	0.50%
Bond EUR Short Term	41,634	1.74%
Bond Global	39,959	1.67%
Bond Global High Yield	7,762	0.32%
Equity Emerging Mkts Europe	5,193	0.22%
Equity Emerging Mkts Global	27,487	1.15%
Equity Europe	87,029	3.63%
Equity Europe Sm&Mid Cap	8,076	0.34%
Equity France	35,105	1.47%
Equity Germany	22,703	0.95%
Equity Global	105,135	4.39%
Equity Italy	5,152	0.22%
Equity North America	48,260	2.01%
Equity Sector Information Tech	3,396	0.14%
Equity Spain	3,132	0.13%
Equity UK	69,283	2.89%
Guaranteed	124,865	5.21%
Mixed Asset EUR Agg - Global	19,904	0.83%
Mixed Asset EUR Bal - Global	41,568	1.74%
Mixed Asset EUR Cons - Global	38,988	1.63%
Mixed Asset EUR Flex - Global	56,665	2.37%
Money Market EUR	519,567	21.69%
Protected	14,917	0.62%
Selected Lipper Global	**1,424,000**	**59.45%**
Non Selected Lipper Global	**971,248**	**40.55%**
Total	**2,395,249**	**100.00%**

Funds:	Net AUM	Net AUM (%)
- selected	253,385.76	10.58%
- non selected	2,141,862.96	89.42%
Total	**2,395,249**	**100.00%**

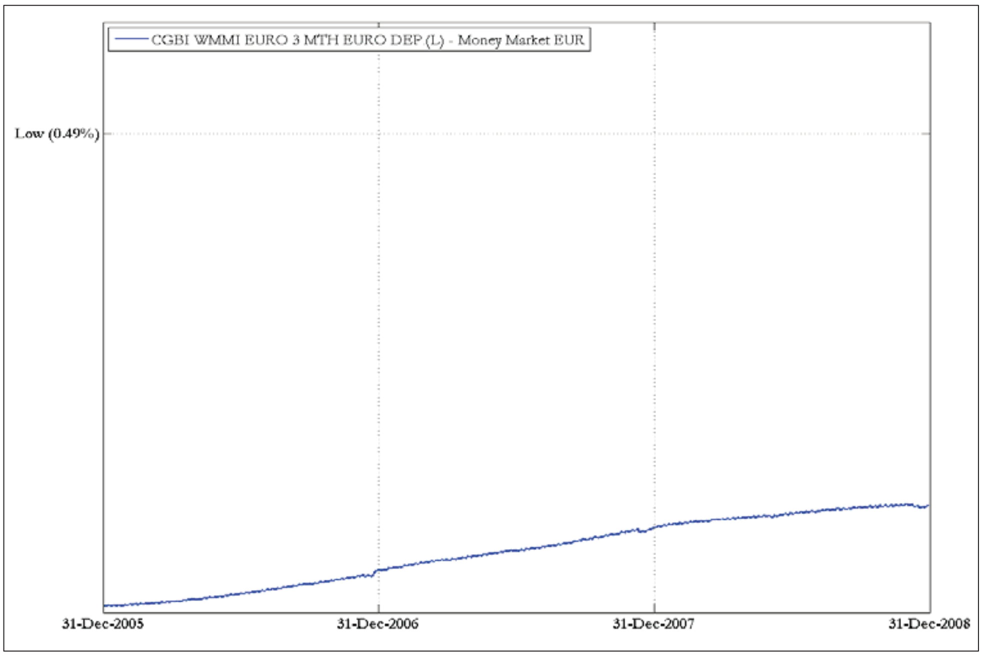
Table 7. Number of funds by type of financial structure, Country and *Lipper* Global category

Type of structure	Lipper Global		FR	DE	IT	LU	ES	UK	Total
Risk Target	Mixed Asset EUR Flex - Global		7	7	7	8	6		**35**
Return Target	Guaranteed		5				1		**6**
	Protected		5		1	3		1	**10**
Benchmark	Bond Emerging Markets Global	Monetary/Bond	2	1	5	4		1	**13**
	Bond EUR		5	6	5	6	5		**27**
	Bond EUR Corporates		2	2	3	5			**12**
	Bond EUR Long Term		2	3		5			**10**
	Bond EUR Short Term		4	5	6	8	3		**26**
	Bond Global		5	6	5	5	6	3	**30**
	Bond Global High Yield		1	1	1	6	1	3	**13**
	Money Market EUR		6	8	6	8			**28**
	Equity Emerging Mkts Europe	Equity	3	1		7		2	**13**
	Equity Emerging Mkts Global		2		5	6	1	6	**20**
	Equity Europe		4	6	5	7	6	3	**31**
	Equity Europe Sm&Mid Cap		6	6	2	7	1	6	**28**
	Equity France		6	2		2			**10**
	Equity Germany		2	8		7			**17**
	Equity Global		7	8	6	8	7	7	**43**
	Equity Italy		1		5	7			**13**
	Equity North America		5	6	4	6	2	4	**27**
	Equity Sector Information Tech		4	6	1	5	2	3	**21**
	Equity Spain		1	2		2	6		**11**
	Equity UK		3	1		8		3	**15**
	Mixed Asset EUR Agg - Global	Balanced	8	7	6	7	4		**32**
	Mixed Asset EUR Bal - Global		4	7	7	7	6		**31**
	Mixed Asset EUR Cons - Global		4	4	6	5	3		**22**
Total			**104**	**103**	**86**	**149**	**60**	**42**	**544**

3.2 The evolution of European financial markets over the period of the analysis

The evolution of European financial markets over the period from January 2006 to December 2008 is represented through the analysis of the annualized returns' volatilities of twenty-three financial indices[88]. Seven of these indices can be classified in the "bond" macro-category (see Figure 5); twelve in the "equity" macro-category (see Figure 7); three in the "balanced" macro-category (see Figure 6) and one in the "monetary" macro-category (see Figure 4).

Figure 4. Annualized volatility of funds within the "monetary" macro-category (1 January 2006 - 31 December 2008)



[88] Each value of the annualized volatility was calculated using 250 observations of daily returns. Hence, by applying a daily rolling window, the calculation of these data led to a three-year time series.

Figure 5. Annualized volatility of funds within the “bond” macro-category (1 January 2006 - 31 December 2008)

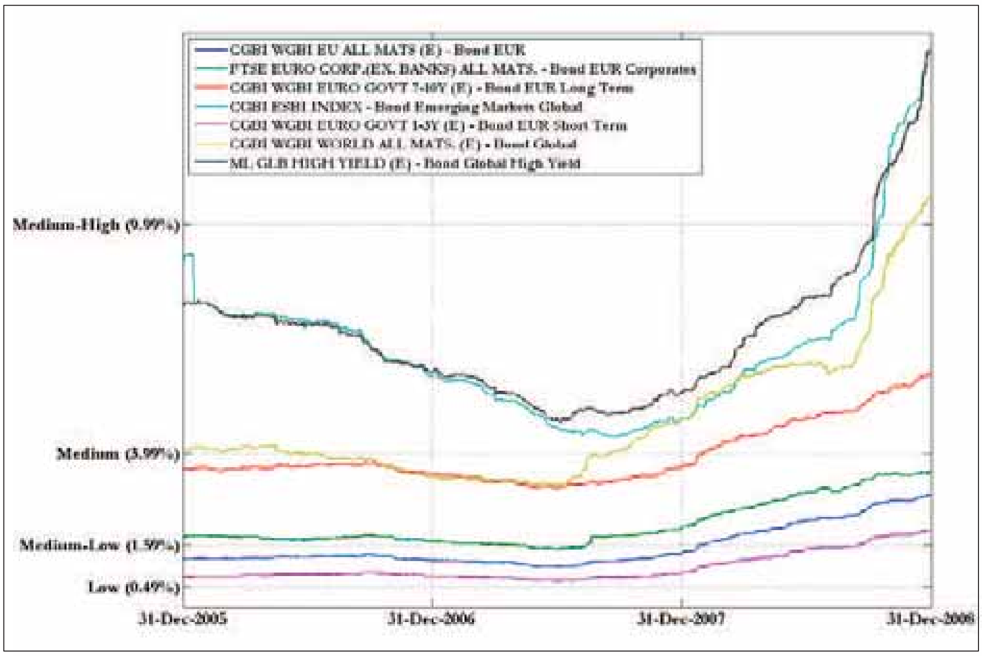


Figure 6. Annualized volatility of funds within the “balanced” macro-category (1 January 2006 - 31 December 2008)

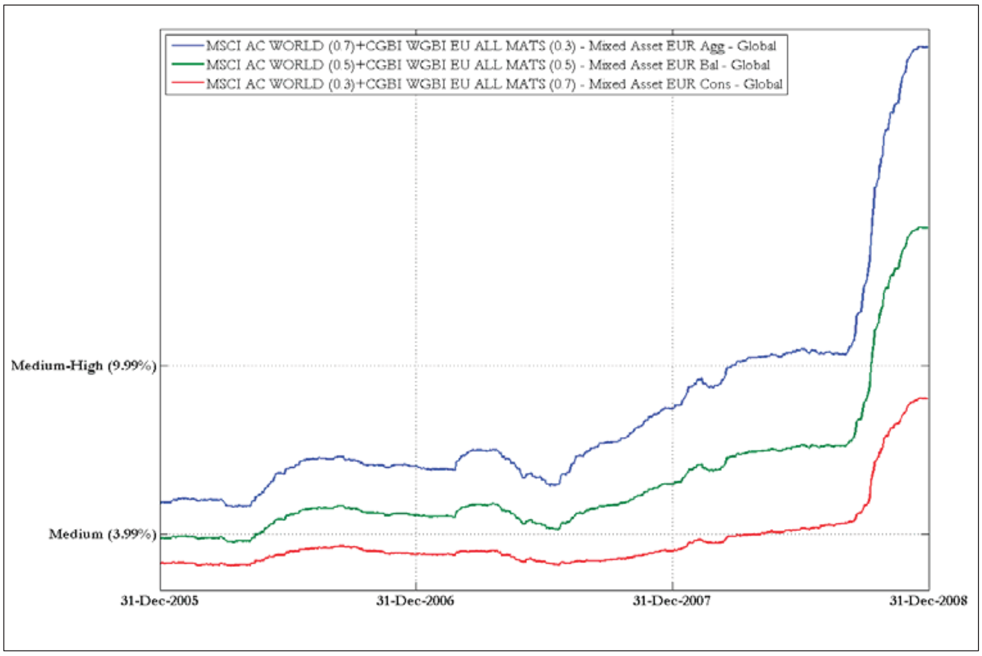
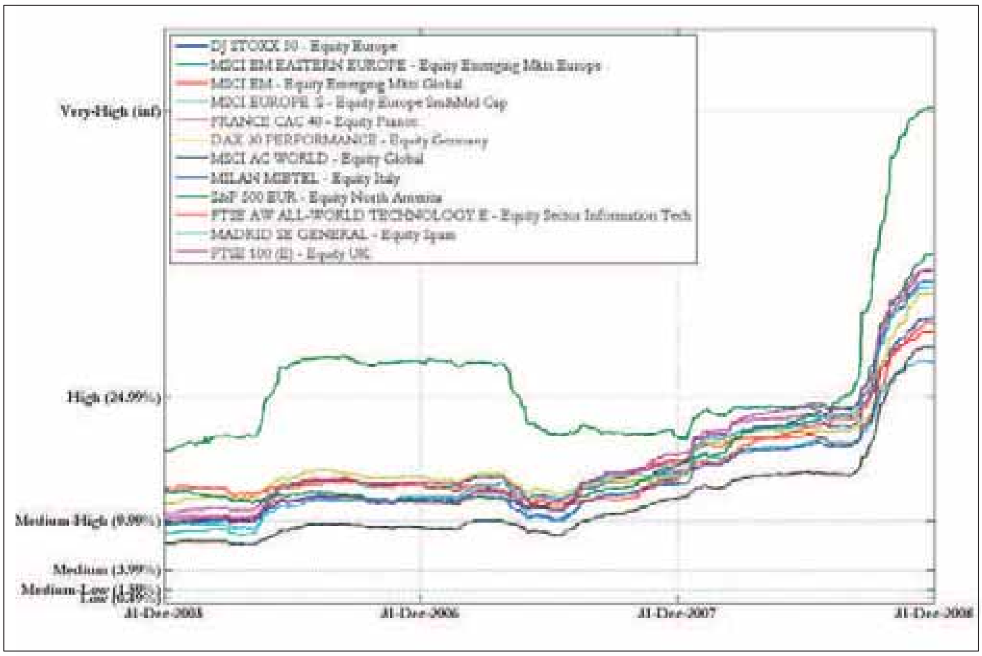


Figure 7. Annualized volatility of funds within the "equity" macro-category (1 January 2006 - 31 December 2008)



The analysis of the volatility for each macro-category highlighted an overall increasing trend. More in detail, a first increase in the growth rate of this financial variable was observed starting from the second half of 2007, simultaneously with the liquidity crisis that global financial markets experienced following to the credit events connected to the phenomenon of subprime mortgages. The worsening of the crisis − happened in September 2008 with the technical defaults of several international financial institutions (*Lehman Brothers, Fannie Mae, Freddie Mac, AIG*) and the freezing of the interbank market − was associated with an unprecedented increase of the volatility across all categories of mutual funds over the last quarter of the period examined.

This trend had relevant consequences in terms of migration of the funds to riskier classes. Looking at each single category, it is clear that, exception made for the index representing the "monetary" macro-category, almost all the financial indices underwent at least one upwards migration during the period 2006−2008. Moreover, at the end of 2008, almost all indices within the "balanced" and the "equity" macro-categories, and many of those within the "bond" macro-category, were at least in the *high* risk class.

3.3 The distribution of the funds across the six qualitative risk classes

The concrete implementation of the methodology underlying the second pillar allowed to classify the time series of the annualized volatility of each fund according to the grid of volatility intervals shown in table 1 of section 2.2.2.2. Table 8 breaks down the results (in percentage terms) of this classification by *Lipper* category and by type of financial structure.

Table 8. Funds distribution across qualitative risk classes over the period 1 January 2006 – 31 December 2008 (Percentage values)

Type of structure	Lipper Global		Total					
			Low	Medium Low	Medium	Medium High	High	Very High
Risk Target	Mixed Asset EUR Flex - Global		2.9%	7.5%	14.9%	53.4%	20.5%	0.8%
Return Target	Guaranteed		12.2%	32.2%	48.7%	6.9%	0.0%	0.0%
	Protected		0.0%	14.4%	31.0%	50.7%	4.0%	0.0%
Benchmark	Money Market EUR	Monetary/Bond	94.8%	5.0%	0.1%	0.1%	0.0%	0.0%
	Bond EUR		3.7%	16.6%	73.3%	6.3%	0.0%	0.0%
	Bond EUR Corporates		0.0%	11.5%	83.9%	4.7%	0.0%	0.0%
	Bond EUR Long Term		0.0%	0.0%	65.5%	34.5%	0.0%	0.0%
	Bond Emerging Markets Global		0.0%	0.2%	45.2%	49.4%	5.2%	0.0%
	Bond EUR Short Term		30.0%	61.4%	8.7%	0.0%	0.0%	0.0%
	Bond Global		3.0%	11.0%	39.9%	44.4%	1.7%	0.0%
	Bond Global High Yield		0.0%	15.9%	30.5%	49.5%	4.1%	0.0%
	Equity Europe	Equity	0.3%	0.1%	0.1%	14.7%	77.7%	7.2%
	Equity Emerging Mkts Europe		0.0%	0.0%	0.0%	0.0%	73.3%	26.7%
	Equity Emerging Mkts Global		0.0%	0.0%	0.0%	1.4%	88.3%	10.3%
	Equity Europe Sm&Mid Cap		0.0%	0.0%	0.0%	13.4%	79.9%	6.7%
	Equity France		0.0%	0.0%	0.0%	2.1%	86.5%	11.4%
	Equity Germany		0.0%	0.0%	0.0%	1.5%	91.2%	7.3%
	Equity Global		0.0%	0.0%	0.0%	24.7%	69.7%	5.6%
	Equity Italy		0.0%	0.0%	0.0%	20.1%	73.5%	6.4%
	Equity North America		0.0%	0.0%	0.0%	5.8%	85.9%	8.2%
	Equity Sector Information Tech		0.0%	0.0%	0.0%	1.3%	89.8%	8.9%
	Equity Spain		0.0%	0.0%	0.0%	8.7%	83.3%	8.0%
	Equity UK		0.0%	0.0%	0.0%	6.2%	85.1%	8.7%
	Mixed Asset EUR Agg - Global	Balanced	0.0%	0.0%	0.1%	72.1%	27.5%	0.3%
	Mixed Asset EUR Bal - Global		0.0%	0.0%	15.0%	79.1%	5.8%	0.0%
	Mixed Asset EUR Cons - Global		0.0%	11.7%	74.4%	13.7%	0.2%	0.0%

Table 8 confirms the strength of the methodology developed to classify the risk profile according to suitably calibrated increasing volatility intervals. In fact, for all "benchmark" funds belonging to macro-categories which typically invest in low-risk assets (such as those which include monetary funds or some bond funds), the historical volatility time series was assigned to the *low*, *medium-low* and *medium* classes. Similarly, macro-categories including funds which mainly invest in riskier assets (such as balanced funds and equity funds) were assigned to the *medium-high*, *high* and *very-high* classes.

"Risk target" funds exhibited on average a medium-high risk exposure together with a considerable dispersion between the six qualitative risk classes, most likely explained by the variety of the specific asset management techniques used for funds with this financial engineering. "Return target" funds were classified as bearing medium risk; in particular, protected-guaranteed products (i.e. those in the *Guaranteed* category) featured a lower risk than protected-only products (i.e. those in the *Protected* category), being the latter characterized only by asset management techniques aimed at protecting a given target return.

3.4 The migrations between qualitative risk classes

Tables 9 and 10 below summarize the results of the analysis on the migrations occurred between the various risk classes. The analysis covered both the entire three-year period (from 2006 to 2008) and each single year. These tables, like those in section 3.3, are broken down by *Lipper* category and by type of financial structure. Specifically, table 9 refers to the entire period of the survey, while table 10 refers to year 2008 only[(89)]. As far as "benchmark" structures are concerned, in order to verify the relationship existing between the evolution of the funds' risk *vis-a-vis* that of their benchmarks, both table 9 and 10 show, for each *Lipper* category belonging to the said type of financial structure, an additional row where the number of migrations occurred for the benchmark is marked by an "x" appearing in the corresponding column.

As shown by table 9, during the period considered by the analysis, no fund category experienced more than four migrations. "Benchmark" funds typically exhibited a positive correlation between the risk exposure (as signalled by their category) and the frequency of the migrations occurred. A possible interpretation of this evidence is that the so-called *vol-of-vol* (that is the volatility of the volatility) is an increasing function of the volatility itself. The same kind of evidence appears more clearly in the other two financial structures. Specifically, "risk target" funds showed a greater number of migrations than "return target" funds, consistently with the positioning of the former in riskier classes than the latter[(90)]. "Return target" funds are usually less exposed to the migration risk, as their portfolios are often invested in low-volatility assets in order to pursue the target return. Table 9 also indicates that, for all the examined categories, most of the migrations were to riskier classes than the original ones, coherently with the increasing trend of the volatility during the period of the analysis[(91)].

With specific reference to "benchmark" funds, a strong co-movement between the changes of the risk class of the funds and those of their benchmark could be observed. In fact, in table 9, for most of the *Lipper* categories belonging to this structure, funds are clustered in the column whose number of migrations is equal to that of the benchmark, the latter marked by the "x".

Data for the year 2008, reported in table 10, essentially confirm the evidence of a positive correlation between the risk taken and the instability of said risk, and open to further interesting considerations about the transparency regulation of non-equity investment products. In fact, table 10 shows that, over the year 2008, all funds experienced one migration at most. Thus, the phenomenon of the migration between different risk classes seems to be fully consistent with the requirement to update the prospectus at least on a yearly basis, as such requirement allows to detect most of the changes in the degree of risk.

(89) For the sake of simplicity, the results of the migration analysis for the years 2006 and 2007 are not shown, as they exhibited trends similar to those observed in 2008.
(90) On this point see section 3.3.
(91) On this point see section 3.2.

Table 9. Number of migrations between different risk classes over the period 1 January 2006 – 31 December 2008 (Percentage values)

Type of structure	Lipper Global		Total no. of Migrations					
			0	1	2	3	4	5
Risk Target	Mixed Asset EUR Flex - Global		20%	49%	17%	14%	-	-
Return Target	Guaranteed		17%	67%	-	17%	-	-
	Protected		40%	50%	10%	-	-	-
Benchmark	Money Market EUR	Monetary/Bond	79%	18%	-	4%	-	-
			x	-	-	-	-	-
	Bond EUR		30%	63%	4%	4%	-	-
			-	x	-	-	-	-
	Bond EUR Corporates		42%	42%	-	17%	-	-
			-	-	x	-	-	-
	Bond EUR Long Term		10%	90%	-	-	-	-
			-	x	-	-	-	-
	Bond Emerging Markets Global		23%	54%	15%	-	8%	-
			x	-	-	-	-	-
	Bond EUR Short Term		31%	54%	8%	8%	-	-
			-	x	-	-	-	-
	Bond Global		17%	50%	20%	13%	-	-
			-	-	-	x	-	-
	Bond Global High Yield		23%	31%	8%	31%	8%	-
			x	-	-	-	-	-
	Equity Europe	Equity	-	84%	13%	-	3%	-
			-	-	x	-	-	-
	Equity Emerging Mkts Europe		-	54%	-	46%	-	-
			-	-	-	x	-	-
	Equity Emerging Mkts Global		20%	75%	-	5%	-	-
			-	x	-	-	-	-
	Equity Europe Sm&Mid Cap		-	75%	18%	7%	-	-
			-	x	-	-	-	-
	Equity France		-	80%	20%	-	-	-
			-	x	-	-	-	-
	Equity Germany		53%	41%	6%	-	-	-
			x	-	-	-	-	-
	Equity Global		21%	58%	2%	19%	-	-
			-	x	-	-	-	-
	Equity Italy		-	69%	-	31%	-	-
			-	x	-	-	-	-
	Equity North America		22%	67%	11%	-	-	-
			-	x	-	-	-	-
	Equity Sector Information Tech		38%	57%	5%	-	-	-
			-	x	-	-	-	-
	Equity Spain		-	45%	55%	-	-	-
			-	-	x	-	-	-
	Equity UK		-	33%	53%	7%	7%	-
			-	-	x	-	-	-
	Mixed Asset EUR Agg - Global	Balanced	6%	91%	-	3%	-	-
			-	x	-	-	-	-
	Mixed Asset EUR Bal - Global		16%	65%	13%	6%	-	-
			-	x	-	-	-	-
	Mixed Asset EUR Cons - Global		23%	59%	9%	9%	-	-
			-	x	-	-	-	-

Table 10. Number of migrations between different risk classes over the period 1 January 2008 – 31 December 2008 (Percentage values)

Type of structure	Lipper Global		Total no. of Migrations - 2008					
			0	1	2	3	4	5
Risk Target	Mixed Asset EUR Flex - Global		49%	49%	3%	-	-	-
Return Target	Guaranteed		67%	33%	-	-	-	-
	Protected		60%	40%	-	-	-	-
Benchmark	Money Market EUR	Monetary/Bond	79%	21%	-	-	-	-
			x	-	-	-	-	-
	Bond EUR		48%	52%	-	-	-	-
			-	x	-	-	-	-
	Bond EUR Corporates		67%	33%	-	-	-	-
			x	-	-	-	-	-
	Bond EUR Long Term		10%	90%	-	-	-	-
			-	x	-	-	-	-
	Bond Emerging Markets Global		31%	69%	-	-	-	-
			x	-	-	-	-	-
	Bond EUR Short Term		35%	62%	4%	-	-	-
			-	x	-	-	-	-
	Bond Global		30%	70%	-	-	-	-
			x	-	-	-	-	-
	Bond Global High Yield		31%	62%	8%	-	-	-
			x	-	-	-	-	-
	Equity Europe	Equity	77%	23%	-	-	-	-
			-	x	-	-	-	-
	Equity Emerging Mkts Europe		100%	-	-	-	-	-
			-	x	-	-	-	-
	Equity Emerging Mkts Global		45%	55%	-	-	-	-
			x	-	-	-	-	-
	Equity Europe Sm&Mid Cap		75%	25%	-	-	-	-
			x	-	-	-	-	-
	Equity France		20%	80%	-	-	-	-
			-	x	-	-	-	-
	Equity Germany		76%	24%	-	-	-	-
			x	-	-	-	-	-
	Equity Global		74%	26%	-	-	-	-
			x	-	-	-	-	-
	Equity Italy		100%	-	-	-	-	-
			x	-	-	-	-	-
	Equity North America		37%	63%	-	-	-	-
			-	x	-	-	-	-
	Equity Sector Information Tech		38%	62%	-	-	-	-
			-	x	-	-	-	-
	Equity Spain		45%	55%	-	-	-	-
			-	x	-	-	-	-
	Equity UK		33%	60%	7%	-	-	-
			-	x	-	-	-	-
	Mixed Asset EUR Agg - Global	Balanced	13%	88%	-	-	-	-
			-	x	-	-	-	-
	Mixed Asset EUR Bal - Global		45%	55%	-	-	-	-
			x	-	-	-	-	-
	Mixed Asset EUR Cons - Global		32%	68%	-	-	-	-
			-	x	-	-	-	-

3.5 The distribution of actively managed "benchmark" funds across management classes

Table 11 shows the distribution (broken down by *Lipper* category and by type of financial structure) of actively managed "benchmark" funds across the three management classes: *limited*, *intermediate* and *considerable*, according to the *delta-vol* intervals shown in table 2 of section 2.2.3. This analysis used the benchmarks identified by *Lipper* for its own categories. This choice was driven, firstly, by the above mentioned need to ensure the significance of the comparison between funds belonging to the same category and, secondly, by the unavailability of data on the proprietary benchmarks of several funds within the sample, as these funds are offered by issuers belonging to countries whose current regulation does not require to specify the benchmark in the offering documentation.

Table 11. "Benchmark" funds distribution across management classes over the period 1 January 2006 – 31 December 2008 (Percentage values)

Lipper Global		Total for the period			
		Limited	Interm.	Cons.	Breaching DeltaVol Interval
Money Market EUR	Monetary/Bond	84%	4%	4%	7%
Bond EUR	Monetary/Bond	30%	20%	19%	31%
Bond EUR Corporates	Monetary/Bond	78%	15%	1%	7%
Bond EUR Long Term	Monetary/Bond	67%	14%	15%	4%
Bond Emerging Markets Global	Monetary/Bond	18%	19%	9%	54%
Bond EUR Short Term	Monetary/Bond	43%	10%	5%	42%
Bond Global	Monetary/Bond	42%	19%	8%	31%
Bond Global High Yield	Monetary/Bond	36%	7%	4%	53%
Equity Europe	Equity	78%	11%	4%	7%
Equity Emerging Mkts Europe	Equity	63%	13%	17%	7%
Equity Emerging Mkts Global	Equity	61%	19%	14%	6%
Equity Europe Sm&Mid Cap	Equity	77%	12%	6%	6%
Equity France	Equity	95%	1%	2%	2%
Equity Germany	Equity	95%	3%	1%	1%
Equity Global	Equity	77%	17%	5%	2%
Equity Italy	Equity	79%	14%	7%	0%
Equity North America	Equity	81%	8%	5%	7%
Equity Sector Information Tech	Equity	86%	9%	2%	3%
Equity Spain	Equity	98%	1%	0%	0%
Equity UK	Equity	91%	4%	2%	3%
Mixed Asset EUR Agg - Global	Balanced	55%	20%	15%	10%
Mixed Asset EUR Bal - Global	Balanced	62%	16%	9%	13%
Mixed Asset EUR Cons - Global	Balanced	47%	14%	5%	34%

In the period of the survey, most of the funds examined were concentrated in the *limited* class, which reveals a modest intensity of the management activity, very similar in practice to the simple replication of the benchmark and, thus, to the *passive* management style.

Moreover, at a first glance some funds seemed inconsistent with their benchmark, showing an anomalous percentage of products which significantly breached the *delta-vol* intervals associated with the three above mentioned management classes[92][93].

A more in-depth analysis of the cases of benchmark's inadequacy was conducted by making a consistency test on the entire sample of "benchmark" funds (and for each *Lipper* category). The test was organized into two consecutive steps. In the first step it was checked whether, for each fund, at least 70% of the volatility observations fell within the *delta-vol* intervals associated with the corresponding management class. All funds which did not pass this step were *a priori* declared inconsistent with the benchmark. In the second step data were broken down according to the following criteria: coherence between the risk class of the benchmark and that of the fund, consistency with the bounds of the *delta-vol* intervals associated with the fund's management class and, finally, direction of the departure from the benchmark.

Following to this further analysis, whose results are summarized in table 12[94], it came out that the categories initially classified as inconsistent with the benchmark had a high percentage of funds (over 40%) which did not pass the first step of the test. Over 80% of the funds which passed the first step had the same risk class of their benchmark and fell in the *delta-vol* interval associated with the management class assigned to them. The study of the direction of the deviation from the benchmark did not provide any significant additional information. Finally, the above results were essentially confirmed by those obtained from the same analysis performed on each single year of the sample.

[92] These were the funds belonging to the following categories: *Bond EUR*, *Bond Emerging Markets Global*, *Bond Eur Short Term*, *Bond Global*, *Bond Global High Yield* and *Mixed Asset EUR Cons - Global*.

[93] Data in table 11, which refers to the European market, were split up and classified according to the country of the issuer to detect any differences, in terms of classification, across different countries. However, this further analysis did not provide any significant information, thus confirming a relatively homogeneous framework across different countries.

[94] In particular, the following notation is used in the table:

- the coherence or not between the risk class of the benchmark and that of the fund is denoted respectively by $SRI_f = SRI_b$ and $SRI_f \neq SRI_b$;
- the consistency or not with the bounds of the *delta-vol* intervals associated with the fund's management class is denoted respectively by ${}_{in}\Delta vol$ and ${}_{out}\Delta vol$;
- the direction, either positive or negative, of the departure from the benchmark is denoted respectively by $\Delta vol+$ and $\Delta vol-$.

Table 12. Distribution of actively managed "benchmark" funds according to their consistency w.r.t. the reference management class over the period 1 January 2006 - 31 December 2008 (Percentage values)

Lipper Global		No. Funds	Consistent		SRI_f=SRI_b & $_{in}\Delta Vol$ $\Delta Vol+$	SRI_f=SRI_b & $_{in}\Delta Vol$ $\Delta Vol-$	SRI_f=SRI_b & $_{out}\Delta Vol$ $\Delta Vol+$	SRI_f=SRI_b & $_{out}\Delta Vol$ $\Delta Vol-$	$SRI_f \neq SRI_b$ & $_{in}\Delta Vol$ $\Delta Vol+$	$SRI_f \neq SRI_b$ & $_{in}\Delta Vol$ $\Delta Vol-$	$SRI_f \neq SRI_b$ & $_{out}\Delta Vol$ $\Delta Vol+$	$SRI_f \neq SRI_b$ & $_{out}\Delta Vol$ $\Delta Vol-$
			NO	YES								
Money Market EUR	Monetary/Bond	28	2	26	72%	23%	2%	0%	1%	0%	2%	0%
Bond EUR	Monetary/Bond	27	10	17	32%	17%	3%	1%	45%	1%	1%	0%
Bond EUR Corporates	Monetary/Bond	12	1	11	66%	19%	0%	0%	11%	3%	0%	1%
Bond EUR Long Term	Monetary/Bond	10	0	10	40%	43%	1%	3%	9%	4%	0%	1%
Bond Emerging Markets Global	Monetary/Bond	13	7	6	60%	27%	3%	3%	2%	5%	0%	1%
Bond EUR Short Term	Monetary/Bond	26	11	15	24%	61%	2%	1%	7%	2%	0%	2%
Bond Global	Monetary/Bond	30	10	20	30%	39%	0%	1%	16%	10%	0%	4%
Bond Global High Yield	Monetary/Bond	13	7	6	59%	39%	0%	0%	1%	1%	0%	0%
Equity Europe	Equity	31	4	27	29%	63%	0%	0%	1%	8%	0%	0%
Equity Emerging Mkts Europe	Equity	13	2	11	9%	69%	0%	1%	2%	17%	0%	1%
Equity Emerging Mkts Global	Equity	20	3	17	69%	24%	0%	0%	6%	0%	0%	0%
Equity Europe Sm&Mid Cap	Equity	28	3	25	56%	36%	0%	0%	4%	2%	0%	1%
Equity France	Equity	10	0	10	28%	65%	1%	0%	3%	2%	0%	1%
Equity Germany	Equity	17	0	17	39%	59%	0%	0%	0%	0%	0%	2%
Equity Global	Equity	43	0	43	41%	25%	0%	0%	30%	4%	0%	1%
Equity Italy	Equity	13	0	13	15%	71%	0%	0%	1%	13%	0%	0%
Equity North America	Equity	27	2	25	27%	69%	0%	0%	0%	2%	0%	1%
Equity Sector Information Tech	Equity	21	0	21	52%	42%	1%	0%	2%	1%	0%	2%
Equity Spain	Equity	11	0	11	40%	54%	0%	0%	4%	1%	0%	0%
Equity UK	Equity	15	1	14	39%	55%	0%	0%	0%	5%	0%	0%
Mixed Asset EUR Agg - Global	Balanced	32	5	27	59%	24%	5%	0%	7%	5%	0%	0%
Mixed Asset EUR Bal - Global	Balanced	31	5	26	45%	36%	1%	1%	9%	4%	0%	3%
Mixed Asset EUR Cons - Global	Balanced	22	9	13	38%	46%	0%	2%	7%	5%	0%	2%
Total funds		**493**	**82**	**411**	**43%**	**42%**	**1%**	**0%**	**9%**	**4%**	**0%**	**1%**

3.6 The migrations between management classes

The main results of the analysis of the migrations between the various management classes for actively managed "benchmark" funds are shown in tables 13 and 14, broken down by *Lipper* categories belonging to this type of financial structure[95][96].

Table 13. Number of migrations between different management classes over the period 1 January 2006 – 31 December 2008 (Percentage values)

Lipper Global		Total for the period						
		No. of migrations occurred in consistent funds						
		0	1	2	3	4	>=5	Migr.
Money Market EUR	Monetary/Bond	62%	23%	12%	4%	0%	0%	15
Bond EUR		13%	13%	25%	13%	38%	0%	40
Bond EUR Corporates		20%	20%	50%	0%	10%	0%	16
Bond EUR Long Term		50%	10%	0%	10%	10%	20%	20
Bond Emerging Markets Global		0%	0%	0%	60%	0%	40%	19
Bond EUR Short Term		36%	0%	36%	21%	7%	0%	23
Bond Global		26%	5%	21%	16%	11%	21%	48
Bond Global High Yield		17%	33%	50%	0%	0%	0%	8
Equity Europe	Equity	56%	19%	11%	11%	4%	0%	24
Equity Emerging Mkts Europe		27%	9%	9%	18%	36%	0%	25
Equity Emerging Mkts Global		41%	18%	18%	12%	12%	0%	23
Equity Europe Sm&Mid Cap		44%	12%	20%	4%	12%	8%	38
Equity France		89%	11%	0%	0%	0%	0%	1
Equity Germany		80%	20%	0%	0%	0%	0%	3
Equity Global		13%	23%	35%	23%	5%	3%	77
Equity Italy		38%	15%	8%	8%	23%	8%	24
Equity North America		67%	4%	17%	4%	8%	0%	20
Equity Sector Information Tech		71%	5%	5%	5%	14%	0%	18
Equity Spain		100%	0%	0%	0%	0%	0%	-
Equity UK		86%	0%	0%	0%	14%	0%	8
Mixed Asset EUR Agg - Global	Balanced	4%	4%	56%	12%	4%	20%	69
Mixed Asset EUR Bal - Global		44%	4%	16%	16%	8%	12%	47
Mixed Asset EUR Cons - Global		17%	25%	42%	0%	8%	8%	24

(95) Also in this case, the first table refers to the entire period of the survey, while the second one is focused only on year 2008.
(96) The analysis was limited only to those funds which, according to the results of the test described in section 3.5, were consistent with their benchmarks.

Table 14. Number of migrations between different management classes over the period 1 January 2008 – 31 December 2008 (Percentage values)

Lipper Global		2008						
		No. of the Migration in consistent funds						
		0	1	2	3	4	>=5	Migr.
Money Market EUR	Monetary/Bond	62%	27%	12%	0%	0%	0%	13
Bond EUR	Monetary/Bond	44%	25%	31%	0%	0%	0%	14
Bond EUR Corporates	Monetary/Bond	60%	30%	10%	0%	0%	0%	5
Bond EUR Long Term	Monetary/Bond	70%	0%	30%	0%	0%	0%	6
Bond Emerging Markets Global	Monetary/Bond	20%	40%	40%	0%	0%	0%	6
Bond EUR Short Term	Monetary/Bond	50%	14%	21%	14%	0%	0%	14
Bond Global	Monetary/Bond	42%	16%	32%	11%	0%	0%	21
Bond Global High Yield	Monetary/Bond	100%	0%	0%	0%	0%	0%	-
Equity Europe	Equity	59%	30%	11%	0%	0%	0%	14
Equity Emerging Mkts Europe	Equity	36%	55%	9%	0%	0%	0%	8
Equity Emerging Mkts Global	Equity	65%	24%	12%	0%	0%	0%	8
Equity Europe Sm&Mid Cap	Equity	72%	12%	12%	0%	4%	0%	13
Equity France	Equity	89%	11%	0%	0%	0%	0%	1
Equity Germany	Equity	100%	0%	0%	0%	0%	0%	-
Equity Global	Equity	75%	18%	8%	0%	0%	0%	13
Equity Italy	Equity	77%	23%	0%	0%	0%	0%	3
Equity North America	Equity	71%	4%	17%	8%	0%	0%	15
Equity Sector Information Tech	Equity	76%	5%	14%	5%	0%	0%	10
Equity Spain	Equity	100%	0%	0%	0%	0%	0%	-
Equity UK	Equity	86%	7%	7%	0%	0%	0%	3
Mixed Asset EUR Agg - Global	Balanced	60%	20%	12%	8%	0%	0%	17
Mixed Asset EUR Bal - Global	Balanced	60%	12%	16%	12%	0%	0%	20
Mixed Asset EUR Cons - Global	Balanced	58%	17%	25%	0%	0%	0%	8

Over the period 2006−2008 most of the categories of "benchmark" funds rarely experienced more than three migrations, and, on average, 74% of the funds changed management class at most twice. More in detail, on average, 43% of the funds considered did not move from the original class, 12% migrated only once, and 19% migrated twice. Similarly to what observed in section 3.4 about the migrations between risk classes, also the data on the evolution of the *delta-vol* over the period of the analysis exhibited a significant positive correlation between the risk exposure (as signalled by the category of the funds) and the number of migrations occurred.

Data in table 14 do not convey further key informations: in 2008, on average, over 90% of the funds examined did not experience more than two migrations, and almost 70% of the funds maintained the original class.

4 Conclusions

Transparency on the risk-return profile of non-equity investment products is crucial to safeguard investors' confidence in the financial system. In fact, at the G-20 summit held in November 2008, the leaders of the twenty countries recognized its key role for a positive solution of the current international financial crisis.

The regulation on the matter is quite heterogeneous both at the European and national level, with multiple provisions differentiated mainly in relation to the category of the issuer, and often unable to shed light on the risk-return characteristics of the products offered. This regulatory fragmentation ignores the strong similarities in the financial engineering of products issued by subjects belonging to different categories, thus jeopardizing the effective pursuit of the levelling the playing field principle.

As has been said, non-equity investment products can be systematically classified into three fundamental types of financial structures: "risk target", "benchmark" and "return target", each bearing specific characteristics in terms of exposure to the various risk factors.

In exercising its regulatory powers, Consob has acted on several fronts in order to standardize the regulation of prospectuses for products issued by subjects belonging to different categories, and, within the limits of the current EC regulation, it has successfully implemented measures regarding Italian mutual funds and class III and V financial-insurance products offered in Italy.

Despite the rationalization of the regulation Consob has recently carried out, significant discrepancies still remain in the offering documentation in Italy as regards:

- class I financial-insurance products, whose offering documentation is regulated by the Isvap provisions on the *Informational Booklet*, and, consequently, also multi-class financial-insurance products;
- European UCITSs, whose offering documentation is regulated by the templates defined in their home Member State regulations;
- financial products issued by Italian and European banks, whose offering documentation is regulated by the templates contained in the regulation 809/2004/EC.

While the asymmetry in transparency regulation between financial-insurance products of class I and those of classes III and V could be mitigated by the intervention of the Italian law-maker, the asymmetry concerning European UCITSs and non-equity financial products issued by banks requires a legislative initiative by European policy-makers.

Such legislative initiative should answer the need for a thorough revision of the EU regulatory framework in the direction of a single directive on the transparency of non-equity investment products, specifically conceived to provide investors with the information they need to take informed investment decisions.

In this perspective and in support of the said revision of the regulatory framework, this work illustrates the three-pillar approach for the risk-based transparency of non-equity investment products, which has been implemented by Consob for Italian mutual funds and for class III and V financial-insurance products offered in Italy.

The first pillar, by representing the various components of the financial investment at the time of subscription as well as the probability scenarios of the product's potential performances at the end of the recommended investment time horizon, allows to assess both the incidence of the various cost items applied and the product's likelihood to create added value for the investors as compared to the alternative investment represented by the risk-free asset.

The second pillar supplements the first pillar information on the performance risk, by offering a qualitative representation of the product's degree of risk and its evolution over time, both in absolute terms and – for "benchmark" structures – in terms of relative risk taken by the asset management style with respect to that associated with the benchmark.

The third pillar completes and specializes the information provided by the probability scenarios and the degree of risk by identifying, as reference time horizon, the recommended investment time horizon, which expresses a recommendation on the optimal investment holding period, determined with regard to the specific features of the product's financial engineering, costs, risk profile and potential returns.

Hence, the three pillars fully define the contents of a *product information sheet* intended to effectively support investors in the selection process of a non-equity investment product.

This process typically involves three consecutive steps which intersect the information from the three pillars with the investor's preferences.

Specifically, the first step of the selection process is based on the information about the recommended investment time horizon. This indicator allows the investor to identify the products which match his own *holding period*, meant as the period for which he is willing to give up his cash holdings. In the next level of the process, the focus is on the selection of those products bearing a degree of risk consistent with the investor's risk appetite. Finally, using probability scenarios, the investor identifies the product whose final payoff structure best matches his expectations on the investment's returns.

This risk-based approach to transparency relies on quantitative analysis tools and methodologies specifically aimed at ensuring the significance and the objectivity of the information provided to investors and which do not require the use of specific models. In other words, the proposed approach represents a methodological solution aimed at producing clear, meaningful, comparable and backward verifiable information. The aspects concerning the practical implementation of the approach are left to the proprietary models used by intermediaries to carry out their pricing and risk management activities. In fact, such activities are clearly preliminary to the launch of the public offering of a financial product and are necessarily linked to (and often coincide with) the above mentioned qualitative and quantitative information to be provided in the prospectuses of non-equity products.

This solution also avoids the costly and useless implementation of "parallel models", and favours the convergence towards virtuous market practices to the benefit of both investors and financial intermediaries.

In fact, on the demand side, investors may rely on a coherent and standardized information which summarizes the overall exposure of non-equity financial products to the various risk factors. In this way, thanks to easy and meaningful indicators, investors can autonomously assess even highly sophisticated products; while on the supply side, intermediaries can extend the use of proprietary models to the production of the information required by transparency regulation, with clear advantages for their compliance offices. In addition, compliance with risk-based transparency regulations provides a concrete opportunity to minimize reputational risk by offering a clear and objective representation of the key features underlying the products' financial engineering.

It is hoped that this work may contribute to the ongoing EC work regarding the representation of risk-return profiles in prospectuses, enhance the attention on the need for a single EU directive concerning the offering documentation of non-equity investment products to retail, and provide useful suggestions for the solution of the current international financial crisis.

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A Appendix A

Calculations on Consob, Bank of Italy, Isvap and Borsa Italiana S.p.A. data.

Table 1. Breakdown of Italian households' portfolios of non-equity investment products in the period December 2002 - June 2008 (Values in millions of Euros) *

	31/12/02	**31/12/03**	**31/12/04**	**31/12/05**	**31/12/06**	**31/12/07**	**30/06/08**
Mutual Funds Shares	329,833	342,241	323,846	334,211	304,675	266,690	206,018
Class I insurance policies	122,492	136,523	148,884	168,099	191,723	204,347	207,430
Classes III and V insurance policies	117,723	147,545	175,585	198,272	190,447	168,302	151,744
Long term securities (Italian Banks)	274,527	298,263	336,945	315,051	329,654	355,095	393,451
Long term securities (Other Italian issuers)	28,168	45,357	38,485	40,600	47,123	51,155	55,384
Long term securities (Foreign)	91,000	90,000	87,000	119,000	125,000	127,000	132,138
Tot. non-equity investment products	963,743	1,059,929	1,110,745	1,175,233	1,188,622	1,172,589	1,146,166
Tot. Financial Assets	**2,865,058**	**2,956,031**	**3,147,649**	**3,340,417**	**3,479,456**	**3,494,997**	**3,308,861**

* The last row of the table reports the total wealth of Italian households invested in financial assets.

Table 2. Breakdown of Italian households' portfolios of non-equity investment products in the period December 2002 - June 2008 (Percentage values) **

	31/12/02	**31/12/03**	**31/12/04**	**31/12/05**	**31/12/06**	**31/12/07**	**30/06/08**
Mutual Funds Shares	34.2%	32.3%	29.2%	28.4%	25.6%	22.7%	18.0%
Class I insurance policies	12.7%	12.9%	13.4%	14.3%	16.1%	17.4%	18.1%
Classes III and V insurance policies	12.2%	13.9%	15.8%	16.9%	16.0%	14.4%	13.2%
Long term securities (Italian Banks)	28.5%	28.1%	30.3%	26.8%	27.7%	30.3%	34.3%
Long term securities (Other Italian issuers)	2.9%	4.3%	3.5%	3.5%	4.0%	4.4%	4.8%
Long term securities (Foreign)	9.4%	8.5%	7.8%	10.1%	10.5%	10.8%	11.5%
Tot. non-equity investment products	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
Tot. Financial Assets	**33.6%**	**35.9%**	**35.3%**	**35.2%**	**34.2%**	**33.6%**	**34.6%**

** The last row of the table shows the percentage weight of non-equity investment products on total Italian households' wealth invested in financial assets.

Table 3. Breakdown of Italian households' portfolios of non-equity investment products in the period December 2002 - June 2008: detail by type of banking and financial-insurance products (Values in millions of Euros)

	31/12/02	**31/12/03**	**31/12/04**	**31/12/05**	**31/12/06**	**31/12/07**	**30/06/08**
Mutual Funds Shares	329,833	342,241	323,846	334,211	304,675	266,690	206,018
Class I insurance policies	122,492	136,523	148,884	168,099	191,723	204,347	207,430
Classes III and V insurance policies	117,723	147,545	175,585	198,272	190,447	168,302	151,744
of which Class V	*33,746*	*41,074*	*52,203*	*62,991*	*52,333*	*33,620*	*21,932*
of which Class III	*83,977*	*106,471*	*123,382*	*135,281*	*138,114*	*134,682*	*129,812*
of which Unit-Linked	*47,989*	*58,325*	*65,065*	*71,988*	*73,895*	*71,575*	*68,987*
of which Index-Linked	*35,988*	*48,145*	*58,318*	*63,294*	*64,218*	*63,108*	*60,826*
Long term securities (Italian Banks)	274,527	298,263	336,945	315,051	329,654	355,095	393,451
of which Ordinary Bonds	*61,285*	*73,346*	*94,095*	*82,160*	*90,026*	*103,918*	*128,894*
of which Structured Bonds	*210,425*	*222,802*	*240,983*	*230,995*	*237,746*	*248,673*	*263,140*
of which Covered Warrant and Certificates	*2,817*	*2,115*	*1,866*	*1,896*	*1,882*	*2,504*	*1,417*
Long term securities (Other Italian issuers)	28,168	45,357	38,485	40,600	47,123	51,155	55,384
Long term securities (Foreign)	91,000	90,000	87,000	119,000	125,000	127,000	132,138
Tot. non-equity investment products	**963,743**	**1,059,929**	**1,110,745**	**1,175,233**	**1,188,622**	**1,172,589**	**1,146,166**

Table 4. Breakdown of Italian households' portfolios of non-equity investment products in the period December 2002 - June 2008: detail by type of banking and financial-insurance products (Percentage values)

	31/12/02	**31/12/03**	**31/12/04**	**31/12/05**	**31/12/06**	**31/12/07**	**30/06/08**
Mutual Funds Shares	34.2%	32.3%	29.2%	28.4%	25.6%	22.7%	18.0%
Class I insurance policies	12.7%	12.9%	13.4%	14.3%	16.1%	17.4%	18.1%
Classes III and V insurance policies	12.2%	13.9%	15.8%	16.9%	16.0%	14.4%	13.2%
of which Class V	*3.5%*	*3.9%*	*4.7%*	*5.4%*	*4.4%*	*2.9%*	*1.9%*
of which Class III	*8.7%*	*10.0%*	*11.1%*	*11.5%*	*11.6%*	*11.5%*	*11.3%*
of which Unit-Linked	*5.0%*	*5.5%*	*5.9%*	*6.1%*	*6.2%*	*6.1%*	*6.0%*
of which Index-Linked	*3.7%*	*4.5%*	*5.3%*	*5.4%*	*5.4%*	*5.4%*	*5.3%*
Long term securities (Italian Banks)	28.5%	28.1%	30.3%	26.8%	27.7%	30.3%	34.3%
of which Ordinary Bonds	*6.4%*	*6.9%*	*8.5%*	*7.0%*	*7.6%*	*8.9%*	*11.2%*
of which Structured Bonds	*21.8%*	*21.0%*	*21.7%*	*19.7%*	*20.0%*	*21.2%*	*23.0%*
of which Covered Warrant and Certificates	*0.3%*	*0.2%*	*0.2%*	*0.2%*	*0.2%*	*0.2%*	*0.1%*
Long term securities (Other Italian issuers)	2.9%	4.3%	3.5%	3.5%	4.0%	4.4%	4.8%
Long term securities (Foreign)	9.4%	8.5%	7.8%	10.1%	10.5%	10.8%	11.5%
Tot. non-equity investment products	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

Table 5. Breakdown of Italian households' portfolios of non-equity investment products by type of financial structure as of 30 June 2008 (Values in millions of Euros and Percentage Values) ***

	Values	Percentages
(1) "Risk Target" structures		
Mutual Funds Shares	22,868	2.39%
Class III insurance policies - Unit-linked	7,451	0.78%
Total Structures (1)	**30,319**	**3.16%**
(2) "*Benchmark*" structures		
Mutual Funds Shares	177,175	18.48%
Class III insurance policies - Unit-linked	55,486	5.79%
Total Structures (2)	**232,661**	**24.27%**
(3) "Return Target" structures		
Mutual Funds Shares	5,975	0.62%
Class III insurance policies - Unit-linked	6,050	0.63%
Class III insurance policies - Index-linked	60,826	6.34%
Structured Bonds	263,140	27.45%
Covered Warrant and *Certificates*	1,417	0.15%
Class I insurance policies	207,430	21.64%
Class V insurance policies	21,932	2.29%
Ordinary Bonds	128,894	13.45%
Total Structures (3)	**695,664**	**72.57%**
Total Structures (1)+(2)+(3)	**958,644**	**100%**

*** The analysis refers to products issued by Italian banks, insurance companies and asset management companies.

B Appendix B

B.1 The Weak Convergence Theorem on $\mathbb{R}$

Theorem 1 *Let $\left\{X_t^h\right\}_{t\geq 0}$ be the jump-continuous Markov process whose measurable space is $(\mathbb{R}, \mathbb{B}(\mathbb{R}))$ and whose conditional first, second and n^{th} ($n > 2$) moments are represented by the following equations:*

$$b_h(x,t) = \frac{1}{h}\int_{\mathbb{R}} (y-x)\Pi_{h,[\frac{t}{h}]h}(x,dy)$$

$$a_h(x,t) = \frac{1}{h}\int_{\mathbb{R}} (y-x)^2\Pi_{h,[\frac{t}{h}]h}(x,dy)$$

$$c_{h,\delta}(x,t) = \frac{1}{h}\int_{\mathbb{R}^1} (y-x)^{2+\delta}\Pi_{h,[\frac{t}{h}]h}(x,dy)$$

Then, if the four conditions presented below are satisfied, $\left\{X_t^h\right\}_{t\geq 0}$ converges weakly for $h \downarrow 0$ to the continuous-time process $\{X_t\}_{t\geq 0}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x,t)dt + \sigma(x,t)dW_t \tag{13}$$

where W_t is a one-dimensional standard Brownian motion independent of the initial random condition X_0.

Condition 1 *If there exists a $\delta > 0$ such that $\forall T > 0$ and $\forall R \in \mathbb{R}^+$:*

$$\lim_{h\downarrow 0} \sup_{\|x\|\leq R, t\in[0,T]} c_{h,\delta}(x,t) = 0 \tag{14}$$

then there exists $a(x,t)$, a continuous function mapping from $\mathbb{R}\times[0,\infty)$ to $\mathbb{R}^+$, and there exists $b(x,t)$, a continuous function mapping from $\mathbb{R}\times[0,\infty)$ to $\mathbb{R}$, such that, $\forall T > 0$ and $\forall R \in \mathbb{R}^+$:

$$\lim_{h\downarrow 0} \sup_{\|x\|\leq R, t\in[0,T]} \|b_h(x,t) - b(x,t)\| = 0 \tag{15}$$

$$\lim_{h\downarrow 0} \sup_{\|x\|\leq R, t\in[0,T]} \|a_h(x,t) - a(x,t)\| = 0 \tag{16}$$

Condition 2 *There exists $\sigma(x,t)$,a continuous function mapping from $\mathbb{R}\times[0,\infty)$ to $\mathbb{R}^+$, such that $\forall x \in \mathbb{R}$ and $\forall t \in [0,T]$ where $T \in \mathbb{R}^+ \cup \{0\}$:*

$$\sigma(x,t) = \sqrt{a(x,t)} \tag{17}$$

Condition 3 *For $h \downarrow 0$, the initial probability of the process $\left\{X_t^h\right\}_{t\geq 0}$ converges in distribution to that of the process $\{X_t\}_{t\geq 0}$, i.e.:*

$$\lim_{h\downarrow 0} P_0^h(X_0^h \in \Gamma) = v_0(\Gamma), \quad \forall\Gamma \in \mathbb{B}(\mathbb{R}) \tag{18}$$

where $P(X_0 \in \Gamma) \equiv v_0(\Gamma)$, $\forall\Gamma \in \mathbb{B}(\mathbb{R})$.

Condition 4 *$v_0(\cdot)$, $a(x,t)$ and $b(x,t)$ uniquely specify the distribution of the process $\{X_t\}_{t\geq 0}$, characterized by an initial distribution $v_0(\cdot)$, a conditional second moment $a(x,t)$ and a conditional first moment $b(x,t)$.*

B.2 The convergence of a M-GARCH(1,1) process

The model identified by:

$$\ln\sigma_{k+1}^2 = \beta_0^{(k)} + \beta_1^{(k)}\ln\sigma_k^2 + \beta_1^{(k)}\ln\left(Z_k\right)^2 \tag{2}$$

or, equivalently, by:

$$\ln\sigma_{k+1}^2 = \beta_0^{(k)} + \beta_1^{(k)}\ln\sigma_k^2 + 2\beta_1^{(k)}\ln|Z_k| \tag{19}$$

(where $k \in \mathbb{N}$ is the discrete-time indicator) belongs to the family of M-GARCH models. Equation (19) can be expressed in differential terms as follows:

$$\ln\sigma_{k+1}^2 - \ln\sigma_k^2 = \beta_0^{(k)} + \left(\beta_1^{(k)} - 1\right)\ln\sigma_k^2 + 2\beta_1^{(k)}\ln|Z_k| \tag{20}$$

We then rescale the discrete-time Markov process $\left\{\ln\sigma_k^2\right\}_{k\in\mathbb{N}}$ by defining for every $h > 0$ a new discrete-time Markov process $\left\{\ln\sigma_{kh}^2\right\}_{kh\geq 0}$ with respect to the filtration $\{\Im_{kh}\}_{kh\geq 0}$, generated by the sequence of i.i.d. random variables $(\ln|Z|)_0$,$(\ln|Z|)_h$, $(\ln|Z|)_{2h}$, . . . , $(\ln|Z|)_{kh}$, where kh is the new discrete-time indicator.

To this end we recall some well-known results of the stochastic limit theory.

Lemma 1 *Let $\{Z_k\}_{k\in\mathbb{N}}$ be a sequence of independent and identically distributed continuous random variables, where k is the indicator of the generic variable Z_k, whose probability density function is denoted by $f_{Z_k}(z_k)$. Moreover, let $\{Z_{kh}\}_{kh\geq 0}$ be the sequence of random variables, where kh is the indicator of the generic variable Z_{kh}, obtained by dividing each interval of unitary width − i.e. $[k-(k-1)]$ − into $\frac{1}{h}$ sub-intervals of width h, $h \geq 0$. Then, $\forall k \in \mathbb{N}$, $\forall h > 0$, the following equality holds:*

$$f_{Z_k}(z_k) = \sqrt{h}f_{Z_{kh}}(z_{kh}) \tag{21}$$

if and only if Z_{kh} is defined as:

$$Z_{kh} = \sqrt{h}Z_k + \left(h - \sqrt{h}\right)E\left(Z_k\right) \tag{22}$$

Corollary 1 *Let $\{Z_k^*\}_{k\in\mathbb{N}}$ be the sequence of continuous random variables obtained multiplying by the constant γ each term of $\{Z_k\}_{k\in\mathbb{N}}$, a sequence of independent and identically-distributed continuous random variables, where k is the indicator of the generic random variable of the sequence. For each random variable $Z_k^* = \gamma Z_k$, let $f_{Z_k^*}(z_k^*)$ denote the corresponding probability density function. Moreover, let $\{Z_{kh}^*\}_{kh\geq 0}$ be the sequence of random variables, where kh is the indicator of the generic variable Z_{kh}^*, obtained by dividing each interval of unitary width − i.e. $[k-(k-1)]$ − into $\frac{1}{h}$ sub-intervals of width h, $h \geq 0$. Then, $\forall k \in \mathbb{N}$, $\forall h > 0$, the following equality holds:*

$$f_{Z_k^*}(z_k^*) = \sqrt{h}f_{Z_{kh}^*}(z_{kh}^*) \tag{23}$$

if and only if Z_{kh}^* is defined as:

$$Z_{kh}^* = \sqrt{h}Z_k^* + \left(h - \sqrt{h}\right)E\left(Z_k^*\right) \tag{24}$$

or, equivalently, as:

$$Z_{kh}^* = \gamma\left[\sqrt{h}Z_k + \left(h - \sqrt{h}\right)E\left(Z_k\right)\right] \tag{25}$$

Theorem 2 *Let $\left\{\ln\sigma_k^2\right\}_{k\in\mathbb{N}}$ be a discrete-time stochastic process as specified by (2) or, equivalently, by (19). Moreover, let $\left\{\ln\sigma_{kh}^2\right\}_{kh\geq 0}$ be a new discrete-time stochastic process obtained by rescaling $\left\{\ln\sigma_k^2\right\}_{k\in\mathbb{N}}$ via the division of each interval of unitary width − i.e. $[k-(k-1)]$ − into $\frac{1}{h}$ sub-intervals of width h, $h > 0$. By applying equation (22) of Lemma 1, to this specific case, the following stochastic difference equation holds for $\left\{\ln\sigma_{kh}^2\right\}_{kh\geq 0}$:*

$$\begin{aligned}&\ln\sigma_{(k+1)h}^2 - \ln\sigma_{kh}^2\\ &=\\ &\beta_{0h} + \left(\beta_{1h} - h\right)\ln\sigma_{kh}^2 + 2\beta_{1h}h^{-1}\left(\sqrt{h}\ln|Z_k| + \left(h - \sqrt{h}\right)E\left(\ln|Z_k|\right)\right)\end{aligned} \tag{26}$$

where $Z_k \sim N(0,1)$.
Therefore, necessary and sufficient conditions for the two processes, $\{\ln\sigma_k^2\}_{k\in\mathbb{N}}$ *and* $\{\ln\sigma_{kh}^2\}_{kh\geq 0}$ *to be linked by the following equality:*

$$\ln\sigma_{k+1}^2 - \ln\sigma_k^2 = \sum_{j=1}^{\frac{1}{h}} \left(\ln\sigma_{k+jh}^2 - \ln\sigma_{k+jh-h}^2\right) \tag{27}$$

or, equivalently:

$$\begin{aligned}
&\beta_0^{(k)} + \left(\beta_1^{(k)} - 1\right)\ln\sigma_k^2 + 2\beta_1^{(k)}\ln|Z_k| \\
&= \\
&\sum_{j=1}^{\frac{1}{h}} \beta_{0h} + (\beta_{1h} - h)\ln\sigma_{k+jh-h}^2 + 2\beta_{1h}h^{-1}\left(\sqrt{h}\ln|Z_k| + \left(h-\sqrt{h}\right)E\left(\ln|Z_k|\right)\right)
\end{aligned} \tag{28}$$

are that:

- *the* β_{0h} *parameter is equal to* $\beta_0^{(k)}\cdot h$*;*
- *the* β_{1h} *parameter is determined as a solution to the following equation of* $\frac{1}{h}$ *degree:*

$$\begin{aligned}
0 &= \left(\beta_0^{(k)} + \beta_1^{(k)}\ln\sigma_k^2 + 2\beta_1^{(k)}\ln|Z_k|\right) - \beta_{1h}^{\frac{1}{h}}\ln\sigma_k^2 - \\
&- \left(\beta_{0h} + 2\beta_{1h}h^{-1}\left(\sqrt{h}\ln|Z_k| + \left(h-\sqrt{h}\right)E\left(\ln|Z_k|\right)\right)\right)\sum_{j=0}^{\frac{1}{h}-1}\beta_{1h}^j
\end{aligned} \tag{29}$$

Let $D([0,\infty),\mathbb{R}) \stackrel{def}{=} \{f:[0,\infty)\to\mathbb{R} : \forall t\geq 0, f(t^+) = f(t) \text{ and } f(t^-) \text{ exist}\}$ be the Skorokhod space. We rescale the new discrete-time Markov process $\{\ln\sigma_{kh}^2\}_{kh\geq 0}$ on this space by defining for each $h>0$ a new jump-continuous Markov process $\left\{\ln\sigma_t^{2^h}\right\}_{t\geq 0}$ with respect to the filtration $\{\Im_t^h\}_{t\geq 0}$, generated by the sequence of i.i.d. random variables $\left\{\ln Z_t^h\right\}_{t\in[kh,(k+1)\cdot h)}$. Then, it is possible to state the following theorem which governs the sought convergence of a M-GARCH(1,1) process.

Theorem 3 *Let* $\left\{\ln\sigma_t^{2^h}\right\}_{t\geq 0}$ *be a jump-continuous Markov process with respect to the filtration* $\{\Im_t^h\}_{t\geq 0}$*, generated by the sequence of i.i.d. random variables* $\left\{\ln Z_t^h\right\}_{t\in[kh,(k+1)\cdot h)}$ *on the Skorokhod space. This process is defined by the equation:*

$$\begin{aligned}
&\ln\sigma_{t+1}^{2^h} - \ln\sigma_t^{2^h} \\
&= \\
&\beta_{0h} + (\beta_{1h} - h)\ln\sigma_t^{2^h} + 2\beta_{1h}h^{-1}\left\{\sqrt{h}\ln\left|Z_t^h\right| + \left(h-\sqrt{h}\right)E\left(\ln\left|Z_t^h\right|\right)\right\}
\end{aligned} \tag{30}$$

where $Z_t^h \sim N(0,1)$*. It follows that equation (30) converges weakly for* $h\downarrow 0$ *to the following stochastic differential equation:*

$$d\ln\sigma_t^2 = \left(\beta_0 + 2\beta_1 E(\ln|Z_t|) + (\beta_1 - 1)\ln\sigma_t^2\right)dt + 2|\beta_1|\sqrt{Var(\ln|Z_t|)}dW_t \tag{3}$$

where β_0 *and* β_1 *are deterministic functions of time,* Z_t *is a standard normal random variable and* W_t *is a one-dimensional standard Brownian motion.*

B.3 The estimate of the parameters

Equations (5) and (6) of section 2.2.2.1 formally define the bounds of the prediction interval for the volatility with a confidence level equal to α. To calculate the actual values of the two bounds, it is necessary to identify a method to estimate the parameters that characterize the stochastic differential equation (3) by using the data observed in discrete time and defined in equation (2).

To this end we state the following theorem.

Theorem 4 *Let* $\{\ln \sigma_k^2\}_{k \in \mathbb{N}}$ *be the discrete-time stochastic process defined in (2) of section B.2, i.e.:*

$$\ln \sigma_k^2 = \beta_0^{(k)} + \beta_1^{(k)} \ln \sigma_{k-1}^2 + \beta_1^{(k)} \ln Z_{k-1}^2 \tag{2}$$

or, equivalently, in (19) of the same section, i.e.:

$$\ln \sigma_k^2 = \beta_0^{(k)} + \beta_1^{(k)} \ln \sigma_{k-1}^2 + 2\beta_1^{(k)} \ln |Z_{k-1}| \tag{19}$$

Moreover, let $\{\ln \sigma_t^2\}_{t \geq 0}$ *be the diffusion process defined in (3) of section B.2, i.e.:*

$$d \ln \sigma_t^2 = \left(\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2\right) dt + 2 |\beta_1| \sqrt{Var(\ln |Z_t|)} dW_t \tag{3}$$

Then, the parameters of the two processes are linked by the following relations:

$$\left|\beta_1^{(k)}\right| = |\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)} - 1}{2(\beta_1 - 1)}} \tag{31}$$

$$\begin{aligned} \beta_0^{(k)} = & -2|\beta_1| \sqrt{\tfrac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} E(\ln |Z_{k-1}|) - |\beta_1| \sqrt{\tfrac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} \ln \sigma_{k-1}^2 + \\ & + e^{(\beta_1-1)} \ln \sigma_{k-1}^2 + \tfrac{[\beta_0 + 2\beta_1 E(\ln|Z_{k-1}|)]\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1} \end{aligned} \tag{32}$$

Equations (31) and (32) of theorem 4 establish an univocal relationship between the parameters of the discrete-time process and those of the continuous-time process. Exploiting this relationship, β_0 and β_1 can be estimated by maximizing the logarithm of the likelihood function evaluated with reference to the data observed in discrete-time; the expression of the likelihood function is given in the following proposition.

Proposition 1 *The likelihood function for the estimate of the parameters* β_0 *and* β_1 *has the following expression:*

$$\begin{aligned} L(Y; \beta_0, \beta_1) = & \prod_{k=2}^{K} \left[\tfrac{1}{|\beta_1|\sqrt{2\pi}} \sqrt{\tfrac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}} \cdot \exp\left(\tfrac{1}{2|\beta_1|} \sqrt{\tfrac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}} \cdot \right.\right. \\ & \cdot \left(Y_k - \tfrac{(\beta_0 - 1{,}2704\beta_1)\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1} - \right. \\ & \left.\left. -1{,}2704 |\beta_1| \sqrt{\tfrac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} - \left(e^{(\beta_1-1)} - 1\right) \ln \sigma_{k-1}^2 \right)\right) \\ & \cdot \exp\left(-\tfrac{1}{2} \exp\left(\tfrac{1}{|\beta_1|} \sqrt{\tfrac{2(\beta_1-1)}{e^{2(\beta_1-1)}-1}} \cdot \right.\right. \\ & \cdot \left(Y_k - \tfrac{(\beta_0 - 1{,}2704\beta_1)\left(e^{(\beta_1-1)}-1\right)}{\beta_1 - 1} - \right. \\ & \left.\left.\left.\left. -1{,}2704 |\beta_1| \sqrt{\tfrac{e^{2(\beta_1-1)}-1}{2(\beta_1-1)}} - \left(e^{(\beta_1-1)} - 1\right) \ln \sigma_{k-1}^2 \right)\right)\right)\right] \end{aligned} \tag{9}$$

where $Y_k = \ln \sigma_k^2 - \ln \sigma_{k-1}^2$.

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