# <u>Masterclass</u>: Implementing AJD models at the trading desk

by

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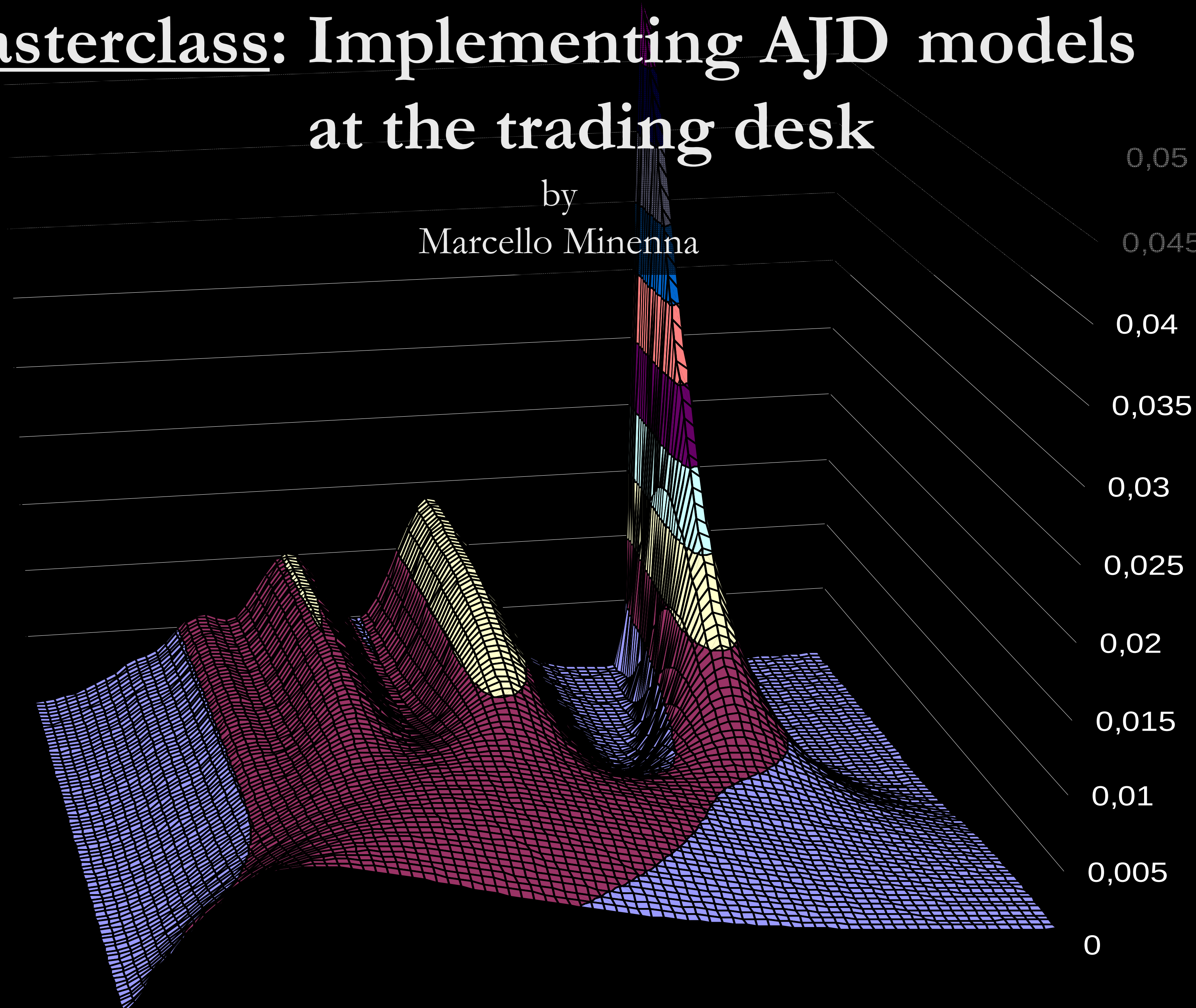

## Syllabus of the presentation

- Review of Fourier Methods in Option Pricing
- Calibration and Performance
- Greek derivation
- Greek Behavior of New FT-Q

**European Call** **Maturity** $T$ **Terminal Spot Price** $S_T$

**In AJD models Call Price can be expressed in a form close to the canonical Black – Scholes - Merton style**

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau} P_2(\Theta)$$

**where**

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln[K])$$

**under different martingale measures**

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln[K])$$

**under different martingale measures**

**can be**

**determined by using the Levy's inversion formula, i.e.:**

$$\Pr(\ln S_T \geq \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi \ln[K]}\widetilde{f}_j(\phi)}{i\phi}\right]d\phi$$

$$\Pr(\ln S_T \geq \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{e^{-i\phi \ln[K]}\tilde{f}_j(\phi)}{i\phi}\right] d\phi$$

**requires**

**a close formula for the Characteristic Function of the log – terminal price, i.e.:**

$$\tilde{f}_T(\phi) = E\left[e^{i\phi \ln S_T}\right]$$

$$\widetilde{f}_T(\phi) = E\left[e^{i\phi \ln S_T}\right]$$

has

**a closed formula for AJD models**

## Example of derivation for Heston Model

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dz_t^{(1)}$$

$$dv_t = \kappa[\theta - v_t]dt + \sigma\sqrt{v_t}dz_t^{(2)}$$

## Example of derivation for Heston Model

### PDE Derivation for portfolio replication

$$f = f(S, v, t)$$

$$df = \frac{\partial f}{\partial S}\left(\mu S dt + \sqrt{v} S dz_1\right) + \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial v}\left[\kappa(\theta - v)\,dt + \sigma\sqrt{v}\,dz_2\right] + \\ + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\left(v S^2 dt\right) + \frac{1}{2}\frac{\partial^2 f}{\partial v^2}\left(\sigma^2 v dt\right) + \frac{\partial^2 f}{\partial S \partial v}\left(S \sigma \rho_{1,2} v\right) dt$$

## Example of derivation for Heston Model

### PDE Derivation for portfolio replication

$$\pi = f_1 - \Delta_1 f_0 - \Delta_0 S$$

the coefficients $\Delta_1, \Delta_0$ are chosen in order to vanish any randomness of the portfolio

$$\begin{aligned} d\pi \quad &= \frac{\partial f_1}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f_1}{\partial S^2}\left(vS^2 dt\right) + \frac{\partial f_1}{\partial v}\left[\kappa\left(\theta - v\right) dt\right] + \frac{1}{2}\frac{\partial^2 f_1}{\partial v^2}\left(\sigma^2 v dt\right) + \\ &+ \frac{\partial^2 f_1}{\partial S \partial v}\left(S\sigma\rho_{1,2} v\right) dt - \frac{\partial f_1/\partial v}{\partial f_0/\partial v}\frac{\partial f_0}{\partial t} dt - \\ &- \frac{\partial f_1/\partial v}{\partial f_0/\partial v}\frac{\partial f_0}{\partial v}\left[\kappa\left(\theta - v\right) dt\right] - \frac{\partial f_1/\partial v}{\partial f_0/\partial v}\frac{1}{2}\frac{\partial^2 f_0}{\partial S^2}\left(vS^2 dt\right) - \frac{\partial f_1/\partial v}{\partial f_0/\partial v}\frac{1}{2}\frac{\partial^2 f_0}{\partial v^2}\left(\sigma^2 v dt\right) - \\ &- \frac{\partial f_1/\partial v}{\partial f_0/\partial v}\frac{\partial^2 f_0}{\partial S \partial v}\left(S\sigma\rho_{1,2} v\right) dt \end{aligned}$$

## Example of derivation for Heston Model

**PDE Derivation for portfolio replication**

no arbitrage hypothesis $d\pi = r\pi dt$

$$\frac{-rf_0 + \frac{\partial f_0}{\partial t} + \frac{\partial f_0}{\partial S} rS + \frac{1}{2}\frac{\partial^2 f_0}{\partial S^2} vS^2 + \frac{1}{2}\frac{\partial^2 f_0}{\partial v^2}\sigma^2 v + \frac{\partial^2 f_0}{\partial S \partial v} S\sigma\rho_{1,2} v - [\kappa(\theta - v)]\partial f_0/\partial v}{\partial f_0/\partial v} =$$

$$= \frac{-rf_1 + \frac{\partial f_1}{\partial t} + \frac{\partial f_1}{\partial S} rS + \frac{1}{2}\frac{\partial^2 f_1}{\partial S^2} vS^2 + \frac{1}{2}\frac{\partial^2 f_1}{\partial v^2}\sigma^2 v + \frac{\partial^2 f_1}{\partial S \partial v} S\sigma\rho_{1,2} v - [\kappa(\theta - v)]\partial f_1/\partial v}{\partial f_1/\partial v}$$

$$\frac{-rf + \frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} rS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2} vS^2 + \frac{1}{2}\frac{\partial^2 f}{\partial v^2}\sigma^2 v + \frac{\partial^2 f}{\partial S \partial v} S\sigma\rho_{1,2} v - [\kappa(\theta - v)]\partial f/\partial v}{\partial f/\partial v} = \lambda^*(S, v, t)$$

## Example of derivation for Heston Model

PDE specification for the pricing of a Call option:

$$-rC+\frac{\partial C}{\partial t}+\frac{\partial C}{\partial S}rS+\frac{1}{2}\frac{\partial^2 C}{\partial S^2}vS^2+\frac{1}{2}\frac{\partial^2 C}{\partial v^2}\sigma^2 v+\frac{\partial C}{\partial S\partial v}S\sigma\rho_{1,2}v+\frac{\partial C}{\partial v}\left[\kappa\left(\theta-v\right)-\lambda^*\left(S,v,t\right)\right]=0$$

$$C(S,v,t=T)=\max(0,S_T-K)$$

## Example of derivation for Heston Model

# PDE Shift into the forward space

$$\tilde{C}(x,v,\tau) = e^{r\tau} C(x,v,\tau) = e^{r(t-t)} C(S,v,t,T)$$

$$-\frac{\partial \tilde{C}}{\partial \tau} + r\frac{\partial \tilde{C}}{\partial x} + \frac{1}{2}\frac{\partial^2 \tilde{C}}{\partial v^2}(\sigma^2 v) + \frac{\partial^2 \tilde{C}}{\partial x \partial v}(v\sigma\rho_{1,2}) + \frac{1}{2}\left(\frac{\partial^2 \tilde{C}}{\partial x^2} - \frac{\partial \tilde{C}}{\partial x}\right)v + \frac{\partial \tilde{C}}{\partial v}\left[\kappa(\theta - v) - \tilde{\lambda} v\right] = 0$$

$$\tilde{C}(x_\tau, v_\tau, \tau = 0) = \max(0, e^{x_{\tau=0}} - K)$$

## Example of derivation for Heston Model

# PDE Shift into Black-Scholes-Merton space

$$C_t(S,v,t,T) = S_t P_1(S,v,t,T) - Ke^{-r(T-t)} P_2(S,v,t,T)$$

$$\tilde{C}_t(x,v,\tau) = e^{x_\tau} P_1(x,v,\tau) - K P_2(x,v,\tau)$$

## Example of derivation for Heston Model

# PDE Shift into Black-Scholes-Merton space

$$-\frac{\partial P_j}{\partial \tau}+\frac{\partial P_j}{\partial x}(r+c_j v)+\frac{1}{2}\frac{\partial^2 P_j}{\partial v^2}(\sigma^2 v)+\frac{\partial^2 P_j}{\partial x \partial v}(v\sigma\rho_{1,2})+\frac{1}{2}\frac{\partial^2 P_j}{\partial x^2}v+\frac{\partial P_j}{\partial v}(a-b_j v)=0$$

$$P_j(x_\tau, v_\tau, \tau=0)=1_{(x_\tau \geq \ln K)}$$

**where** $c_1=\frac{1}{2},\quad c_2=-\frac{1}{2},\quad a=\kappa\theta,\quad b_1=\kappa+\tilde{\lambda}-\rho_{1,2}\sigma,\quad b_2=\kappa+\tilde{\lambda}$

**by using Feynman Cac formula….**

characteristics of the probability measure $P_j$ at a generic time $\tau$:

$$P_j(x_\tau, v_\tau, \tau)=P_j\left(x_{\tau=0}\geq \ln K \mid x_\tau, v_\tau\right)$$

## Example of derivation for Heston Model

### PDE Shift into Fourier space

by using the Levy's inversion formula…

$$P_j\left(x_{\tau=0} \geq \ln K \mid x_\tau, v_\tau\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-i\xi \ln K}}{i\xi} \tilde{f}_j(x_\tau, v_\tau, \tau = 0, \xi \mid x_\tau, v_\tau)\, d\xi$$

$$-\frac{\partial \tilde{f}_j}{\partial \tau} + \frac{\partial \tilde{f}_j}{\partial x}(r + c_j v) + \frac{1}{2}\frac{\partial^2 \tilde{f}_j}{\partial v^2}(\sigma^2 v) + \frac{\partial^2 \tilde{f}_j}{\partial v \partial x}(v\sigma\rho_{1,2}) + \frac{\partial^2 \tilde{f}_j}{\partial x^2} v + \frac{\partial \tilde{f}_j}{\partial v}\left[a - b_j v\right]$$

$$\tilde{f}_j(x_\tau, v_\tau, \tau = 0, \xi) = e^{i\xi x_{\tau=0}}$$

## Example of derivation for Heston Model

# PDE Shift into ODE space

**by using the solution:** $\tilde{f}_j(x_\tau, v_\tau, \tau = 0, \xi | x_\tau, v_\tau) = e^{\left(C_\tau^{(j)} + D_\tau^{(j)} v_t + i\xi x_\tau\right)}$

$$
\begin{aligned}
\frac{\partial C_j}{\partial \tau} &= ri\xi + aD_j \\
\frac{\partial D_j}{\partial \tau} &= c_j i\xi + \frac{1}{2} D_j^2 \sigma^2 + i\xi D_j \sigma \rho_{1,2} - \frac{1}{2}\xi^2 - b_j D_j \\
C_0^{(j)} &= 0 \\
D_0^{(j)} &= 0
\end{aligned}
$$

## Example of derivation for Heston Model

### ODE Solutions

$$C_j = ri\xi(T-t) - \frac{2a}{\sigma^2}\left(\alpha_2(T-t) + \ln\frac{\frac{\alpha_2}{\alpha_1}e^{d(T-t)} - 1}{\frac{\alpha_2}{\alpha_1} - 1}\right)$$

$$D_j = -\frac{2\alpha_2}{\sigma^2}\frac{1-e^{d(T-t)}}{1-\frac{\alpha_2}{\alpha_1}e^{d(T-t)}}$$

$$d = \sqrt{(\rho_{1,2}\sigma\xi i - b_j)^2 - \sigma^2\left(2c_j\xi i - \xi^2\right)}$$

$$\alpha_1 = \frac{\rho_{1,2}\sigma\xi i - b_j + d}{2};$$

$$\alpha_2 = \frac{\rho_{1,2}\sigma\xi i - b_j - d}{2}$$

## Example of derivation for Heston Model

# PRICING

$$C_t = S_t P_1 - K e^{-r(T-t)} P_2$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \Re \left\{ \frac{e^{-i\xi \ln K}}{i\xi} \, e^{\left[ C_T^{(j)} + D_T^{(j)} v_t + i\xi [\ln S_t + r(T-t)] \right]} \right\} d\xi$$

with:

$$C_j = ri\xi(T-t) - \frac{2a}{\sigma^2} \left( \alpha_2 (T-t) + \ln \frac{\frac{\alpha_2}{\alpha_1} e^{d(T-t)} - 1}{\frac{\alpha_2}{\alpha_1} - 1} \right)$$

$$D_j = -\frac{2\alpha_2}{\sigma^2} \frac{1 - e^{d(T-t)}}{1 - \frac{\alpha_2}{\alpha_1} e^{d(T-t)}}$$

$$d = \sqrt{(\rho_{1,2}\sigma\xi i - b_j)^2 - \sigma^2 (2c_j \xi i - \xi^2)}$$

$$\alpha_1 = \frac{\rho_{1,2}\sigma\xi i - b_j + d}{2} \, , \, \alpha_2 = \frac{\rho_{1,2}\sigma\xi i - b_j - d}{2}$$

$$\begin{aligned} c_{1/2} &= \pm\tfrac{1}{2} \\ a &= \kappa\theta \\ b_1 &= \kappa + \tilde{\lambda} - \rho_{1,2}\sigma \\ b_2 &= \kappa + \tilde{\lambda} \end{aligned}$$

## How to compute: $C_t$

**I**

**Quadrature Algorithm**
**FT – Q for**
$P_1(\Theta), P_2(\Theta)$

**a**

**Old FT - Q**

**b**

**New FT - Q**

**II**

**Fast Fourier Transform**
**FFT**

## Algorithms Valuation Criteria

# STABILITY

**The algorithm is defined stable if and only if it "closes" the quadrature scheme**

spanning

the pricing formula on a vast area of the parameters set.

gives

a "reasonable" result different from a NaN value

## Algorithms Valuation Criteria

# ACCURACY

The algorithm is defined accurate **if and only if**

**pricing**

the Call under the Black - Scholes - Merton model via:

- Old FT - Q
- New FT - Q
- FFT

**pricing**

Call via standard Black – Scholes – Merton

up to $10^{-3}$ precision

## Algorithms Valuation Criteria

# SPEED

**The algorithm is defined fast with respect to the results of the FFT algorithm**

⇕

**a set of 4100 prices along the strike**

$P_1(\Theta), P_2(\Theta)$ through **Quadrature Algorithm Old FT - Q**

# High Order Newton Cotes Algorithm

**Up to 8th**

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau} P_2(\Theta)$$

$P_1(\Theta), P_2(\Theta)$ ⟺ through ⟺ **Quadrature Algorithm**
**Old FT - Q**

| **Pros (+)** | **Cons (-)** |
|---|---|
| **ACCURACY** | **STABILITY** |
| | **SPEED** |

$P_1(\Theta), P_2(\Theta)$ ⇔ (through) **Quadrature Algorithm New FT - Q**

**In order to overcome the cited problems of Old FT – Q:**

- **Gauss - Lobatto Quadrature Algorithm**
- **Re-adjustment of** $\tilde{f}_T(\phi) = E\left[e^{i\phi \ln S_T}\right]$

⇕

$$C_t = S_t P_1(\Theta) - K e^{-r\tau} P_2(\Theta)$$

$P_1(\Theta), P_2(\Theta)$ ⟺ through ⟺ Quadrature Algorithm New FT - Q

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- Gauss - Lobatto Quadrature Algorithm
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⇕

$$C_t = S_t P_1(\Theta) - K e^{-r\tau} P_2(\Theta)$$

- **Basic Gauss - Lobatto Quadrature Formula**

$$\int_{-1}^{1} f(x)\,dx \approx w_1 f(-1) + w_N f(1) + \sum_{i=2}^{N-1} w_i f(x_i)$$

$$w_i = \frac{2}{N(N-1)\left[P_{N-1}(x_i)\right]^2}$$

**LIMITED**

**to the interval (-1,1)**

$$w_1 = w_N = \frac{2}{N(N-1)}$$

# The Gautschi - Gander extension (2000)

## ENHANCE

### The Gauss Lobatto formula

They develop a GL recursive adaptive algorithm for a generic interval

# The Gautschi - Gander extension (2000)

$$\int_{\alpha}^{\beta} f(x)\,dx \approx h\left\{ w_1 f(\alpha) + w_N f(\beta) + \sum_{i=2}^{N-1} w_i \left[ f(m + x_i h) \right] \right\}$$

$$w_i = \frac{2}{N(N-1)\left[P_{N-1}(x_i)\right]^2}$$

$$w_1 = w_N = \frac{2}{N(N-1)}$$

$$h = \frac{1}{2}(\beta - \alpha)$$

$$m = \frac{1}{2}(\alpha + \beta)$$

$P_1(\Theta), P_2(\Theta)$ ⟺ through ⟺ **Quadrature Algorithm New FT - Q**

**In order to overcome the cited problems of Old FT – Q:**

- **Gauss - Lobatto Quadrature Algorithm**
- **Re-adjustment of** $\tilde{f}_T(\phi) = E\left[e^{i\phi \ln S_T}\right]$

⇕

$$C_t = S_t P_1(\Theta) - K e^{-r\tau} P_2(\Theta)$$

## Example of re-adjstment for Heston Model

$$C_t = S_t P_1 - K e^{-r(T-t)} P_2$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re \left\{ \frac{e^{-i\xi \ln K}}{i\xi} \, e^{\left[C_T^{(j)} + D_T^{(j)} v_t + i\xi[\ln S_t + r(T-t)]\right]} \right\} d\xi$$

with:

$$C_j = ri\xi(T-t) - \frac{2a}{\sigma^2}\left(\alpha_2(T-t) + \ln \frac{\frac{\alpha_2}{\alpha_1} e^{d(T-t)} - 1}{\frac{\alpha_2}{\alpha_1} - 1}\right)$$

$$D_j = -\frac{2\alpha_2}{\sigma^2} \frac{1 - e^{d(T-t)}}{1 - \frac{\alpha_2}{\alpha_1} e^{d(T-t)}}$$

$$d = \sqrt{(\rho_{1,2}\sigma\xi i - b_j)^2 - \sigma^2(2c_j\xi i - \xi^2)}$$

$$\alpha_1 = \frac{\rho_{1,2}\sigma\xi i - b_j + d}{2}, \quad \alpha_2 = \frac{\rho_{1,2}\sigma\xi i - b_j - d}{2}$$

$$\begin{aligned} c_{1/2} &= \pm\tfrac{1}{2} \\ a &= \kappa\theta \\ b_1 &= \kappa + \tilde{\lambda} - \rho_{1,2}\sigma \\ b_2 &= \kappa + \tilde{\lambda} \end{aligned}$$

# Review of Fourier Methods in Option Pricing – practice

## Example of re-adjstment for Heston Model

$$C_j = ri\xi(T-t) - \frac{2a}{\sigma^2}\left(\alpha_2(T-t) + \ln\frac{\frac{\alpha_2}{\alpha_1}e^{d(T-t)} - 1}{\frac{\alpha_2}{\alpha_1} - 1}\right)$$

$$D_j = -\frac{2\alpha_2}{\sigma^2}\frac{1-e^{d(T-t)}}{1-\frac{\alpha_2}{\alpha_1}e^{d(T-t)}}$$

$$C_j = ri\xi\tau - \frac{a}{\sigma^2}\left(\rho_{1,2}\sigma\xi i - b_j + d\right)\tau - \frac{a}{\sigma^2}2\ln\left(1 - \frac{\left(1-e^{-d\tau}\right)\left(\rho_{1,2}\sigma\xi i - b_j + d\right)}{2d}\right)$$

$$D_j = \frac{\left(2c_j\xi i - \xi^2\right)\left(1-e^{-d\tau}\right)}{2d - \left(\rho_{1,2}\sigma\xi i - b_j + d\right)\left(1-e^{-d\tau}\right)}$$

$P_1(\Theta), P_2(\Theta)$ ⇔ through ⇔ **Quadrature Algorithm New FT - Q**

| **Pros (+)** | **Cons (-)** |
|---|---|
| **STABILITY** | **SPEED** |
| **ACCURACY** | |

$C_t$ ⟺ through ⟺ **Fast Fourier Trasform FFT**

# Cooley - Tukey algorithm

$$\omega(n) = \sum_{j=1}^{N} e^{-i\frac{2\pi}{N}(j-1)(n-1)} f_j = \sum_{j=1}^{\frac{N}{2}} e^{-i\frac{2\pi}{N}(2j-1)(n-1)} f_{2j} + \sum_{j=1}^{\frac{N}{2}} e^{-i\frac{2\pi}{N}2j(n-1)} f_{2j+1}$$

$C_t$ ⟺ (through) **Fast Fourier Trasform FFT**

# Cooley - Tukey algorithm

**Applied to the equivalent formula via a recombinant FFT parameters**

$$C_t = \frac{e^{-\alpha \ln K}}{\pi} \int_0^\infty e^{-i\phi \ln K} \widetilde{f}_j(\phi) d\phi$$

**for ATM**

$C_t$ ⟺ through ⟺ **Fast Fourier Trasform FFT**

## Pros (+)

**SPEED**

**FASTER**

**(up to 40 times the quadrature algorithms)**

## Cons (-)

**STABILITY**

*** The formula must be changed arbitrarily according to Option moneyness**

**ACCURACY**

**** the recombinant FFT parameters must be changed according to the choice of the pricing models**

# Syllabus of the presentation

- Review of Fourier Methods in Option Pricing
- Calibration and Performance
- Greek derivation
- Greek Behaviour of New FT-Q

# The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[ C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

**I**

**Quadrature Algorithm**
**FT - Q**

**a**

**Old FT - Q**

**b**

**New FT - Q**

**II**

**Fast Fourier Trasform**
**FFT**

# The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[ C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

through

**Quadrature Algorithm**
**Old FT - Q**

## Pros (+)

## Cons (-)

**STABILITY**

**ACCURACY**

**SPEED**

# The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[ C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

⇔ through ⇔

**Fast Fourier Trasform**
**FFT**

## <u>Pros</u> (+)

**SPEED**

## <u>Cons</u> (-)

**STABILITY ***

**ACCURACY ****

# The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[ C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

⟺ through ⟺

**Quadrature Algorithm**
**New FT - Q**

| **Pros (+)** | **Cons (-)** |
|---|---|
| **STABILITY** | |
| **ACCURACY** | |
| **SPEED** | |

# The Calibration Procedure and Performance

**By keeping in mind that <u>only</u> New FT-Q is stable and accurate, some figures on speed**

**Original Option Pricing Formulas are used**

| | Heston Model | Merton Model | BCC Model |
|---|---|---|---|
| FFT | 7.26 sec. | 10.54 sec. | 18.33 sec. |
| NEW FT - Q | 55.12 sec. | 66.48 sec. | 110.39 sec. |
| OLD FT – Q | 390.41 sec. | 454.76 sec. | 722.1 sec. |

**By now, the speed of Fourier Trasform method is closer than ever to the FFT calibration time**

# Calibration Performances using
# Option Readjusted Pricing Formulas
where available

| | | | |
|---|---|---|---|
| FFT | Heston Model | Merton Model | BCC Model |
| | 7.24 sec. | 10.54 sec. | 18.32 sec. |
| NEW FT - Q | Heston Model | Merton Model | BCC Model |
| | 23.13 sec. | 66.48 sec. | 48.7 sec. |
| OLD FT – Q | Heston Model | Merton Model | BCC Model |
| | 331.6 sec. | 454.76 sec. | 688.5 sec. |

## Syllabus of the presentation

- **Review of Fourier Methods in Option Pricing**
- **Calibration Procedure and Performance**
- **Greek derivation**
- **Greek Behaviour of New FT-Q**

# Greek derivation

**European Call** **Maturity** $T$ **Terminal Spot Price** $S_T$

**In AJD models Greeks can be derived by using the following equivalences**

$$S_t \frac{\partial P_1}{\partial S_t} + K \frac{\partial P_1}{\partial K} = 0$$

$$S_t \frac{\partial P_2}{\partial S_t} + K \frac{\partial P_2}{\partial K} = 0$$

$$\frac{\partial^2 P_1}{\partial S_t \partial K} = \frac{\partial^2 P_1}{\partial K \partial S_t}$$

$$\frac{\partial^2 P_2}{\partial S_t \partial K} = \frac{\partial^2 P_2}{\partial K \partial S_t}$$

$$S_t \frac{\partial P_1}{\partial S_t} - e^{-r(T-t)} K \frac{\partial P_2}{\partial S_t} = 0$$

$$P_1 = \frac{\partial C_t}{\partial S_t}$$

$$\frac{\partial C_t}{\partial K} = -e^{-r(T-t)} P_2$$

# Greek derivation

## Example of derivation for Heston Model

$$\Delta_C = P_1$$

$$\Gamma_C = \frac{\partial P_1}{\partial S_t}$$

$$\mathcal{V}_C = S_t \frac{\partial P_1}{\partial v_t} - K e^{-r\tau} \frac{\partial P_2}{\partial v_t}$$

$$\rho_C = K \tau e^{-r\tau} P_2$$

$$\begin{aligned} \Theta_C = \quad & -\frac{\partial P_1}{\partial S}\left(\frac{1}{2} v S^2\right) - \frac{\partial P_1}{\partial v} S \left[\sigma \rho_{1,2} v + \left[\kappa\left(\theta - v\right) - \lambda v\right]\right] - \frac{\partial^2 P_1}{\partial v^2}\left(\frac{1}{2} S \sigma^2 v\right) - \\ & - K e^{-r\tau}\left[r P_2 - \frac{1}{2}\sigma^2 v \frac{\partial^2 P_2}{\partial v^2} - \frac{\partial P_2}{\partial v}\left[\kappa\left(\theta - v\right) - \lambda v\right]\right] \end{aligned}$$

$$\mathfrak{V}_C = S_t \frac{\partial^2 P_1}{\partial v_t^2} - K e^{-r\tau} \frac{\partial^2 P_2}{\partial v_t^2}$$

## Syllabus of the presentation

- Review of Fourier Methods in Option Pricing
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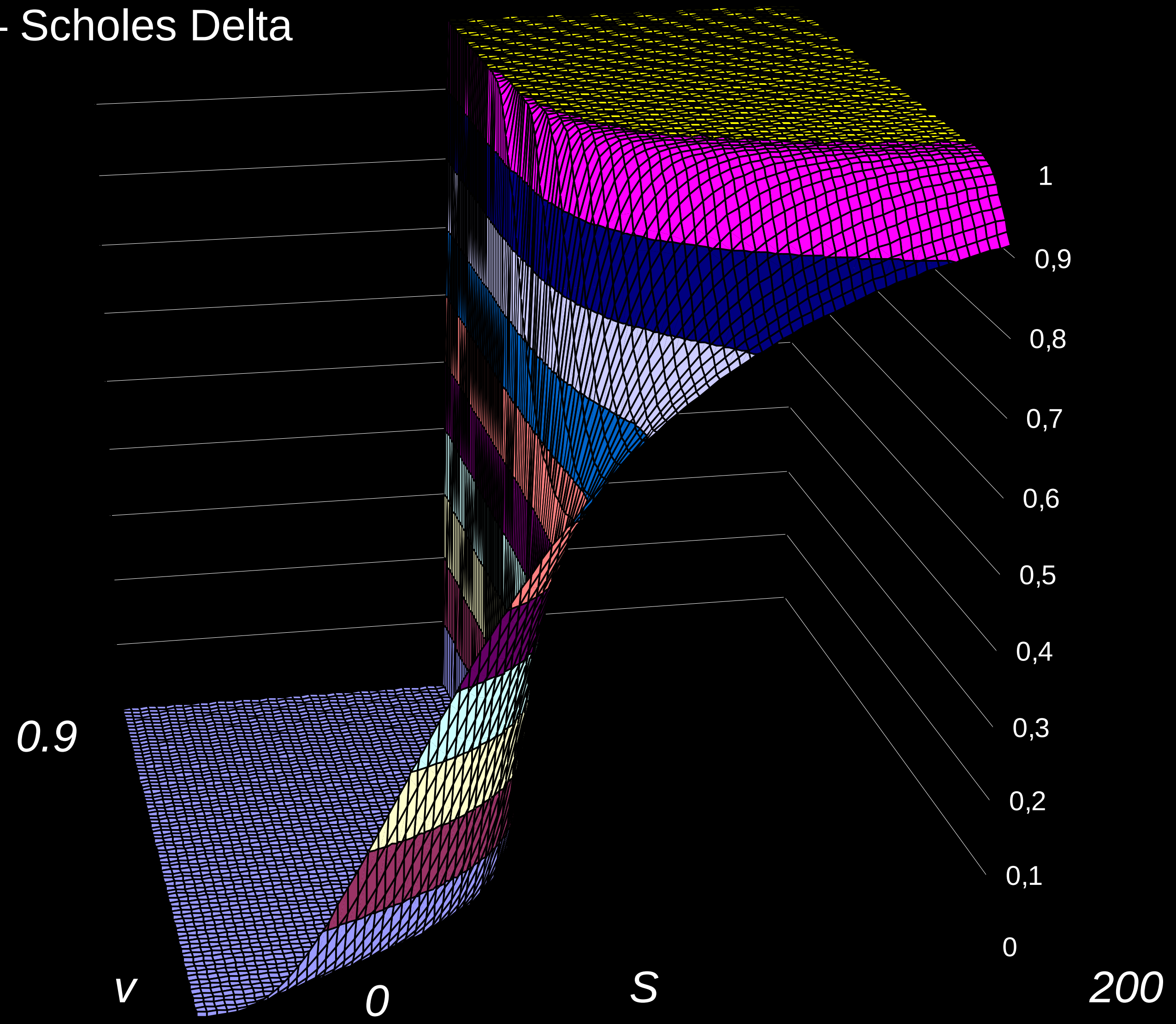
# Greek behaviour of new FT-Q

An impressive methodology to test Stability of the New FT – Quadrature algorithm is to compute Greeks

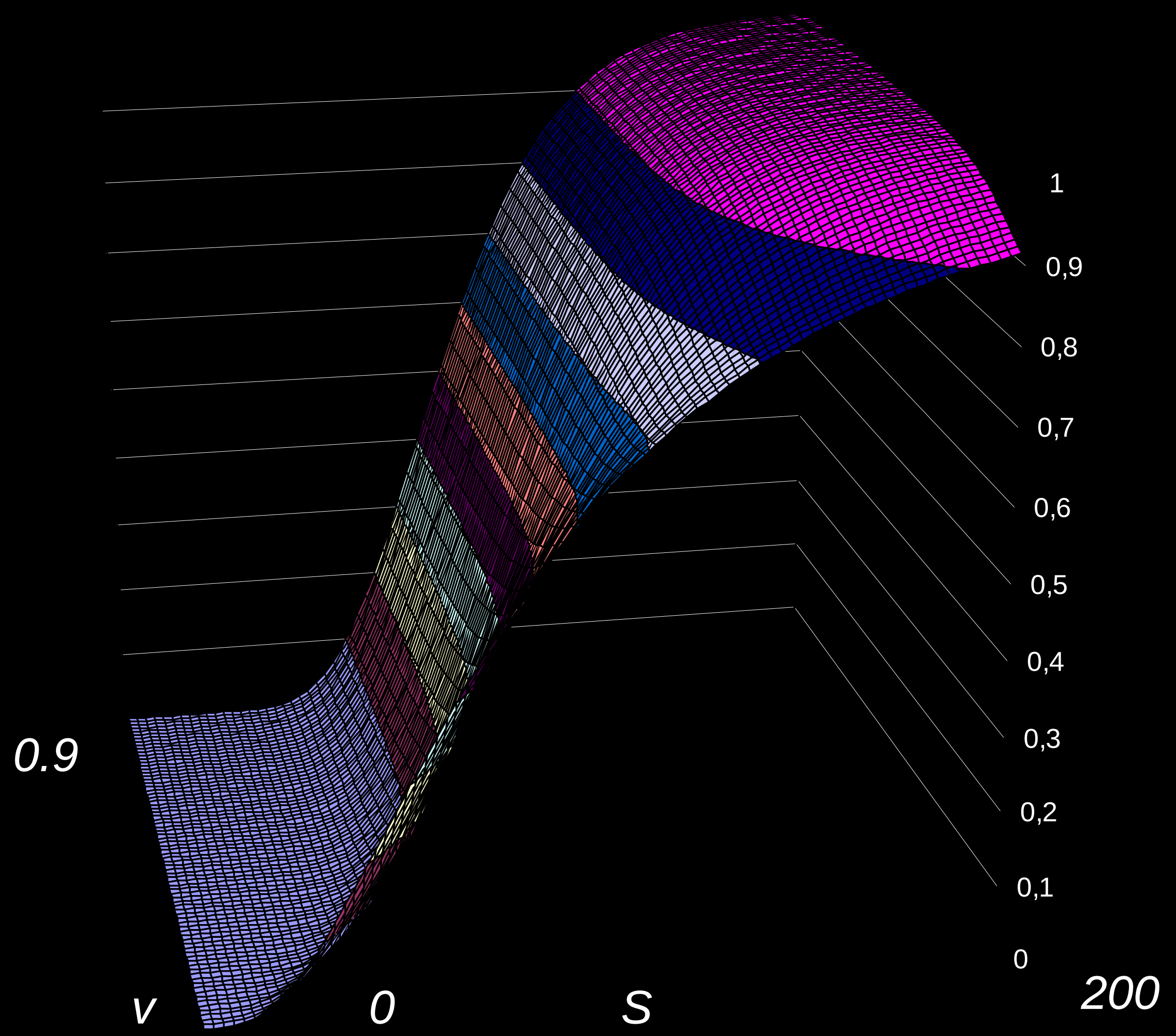
Infact, in an AJD setting the Greeks are available in closed form

So, an extended spanning of the AJD Greeks on the parameters set is useful to assess models and test Stability

## Black – Scholes Delta



## Heston Delta




Lambda = -2

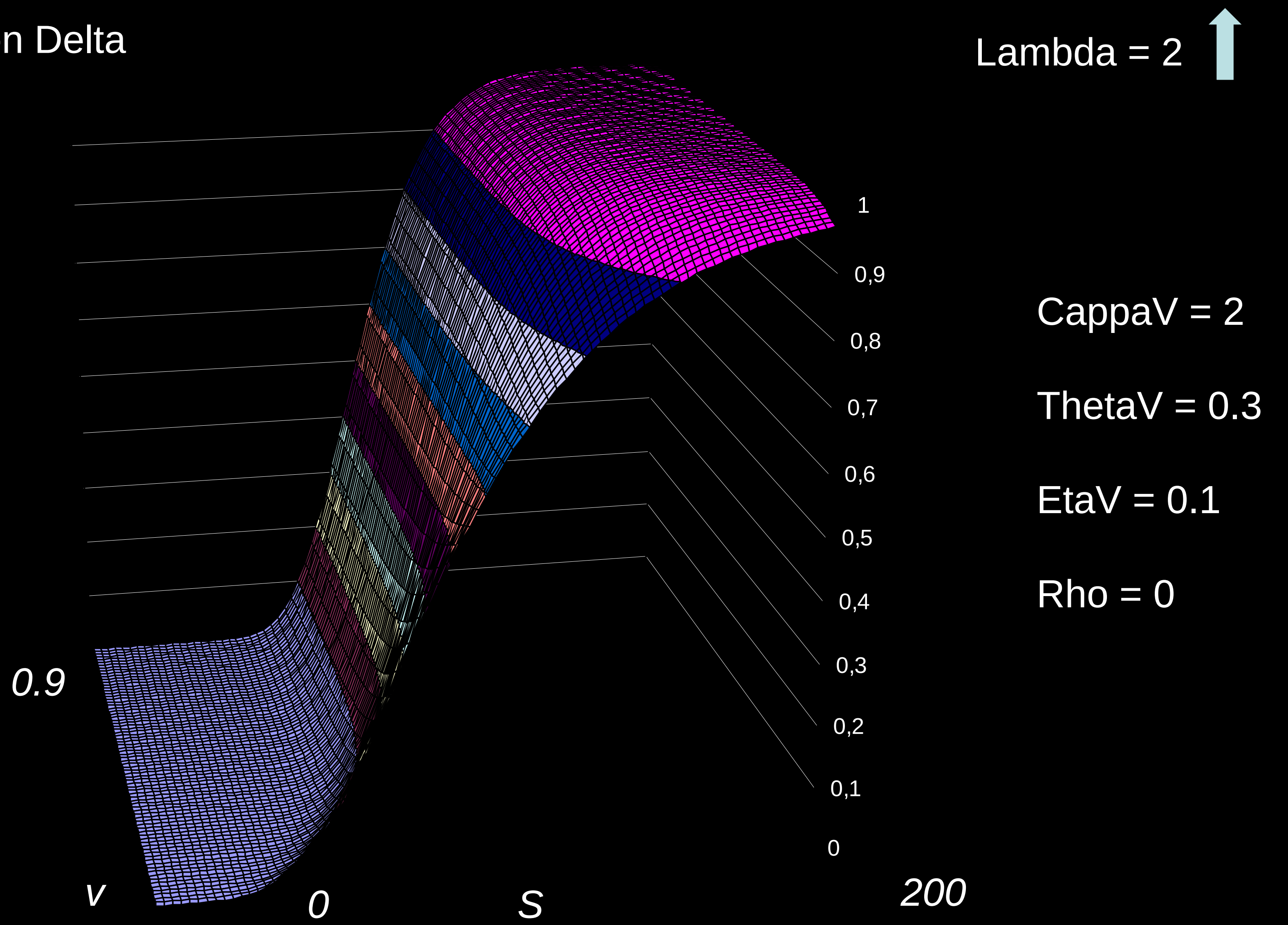
CappaV = 2

ThetaV = 0.3

EtaV = 0.1

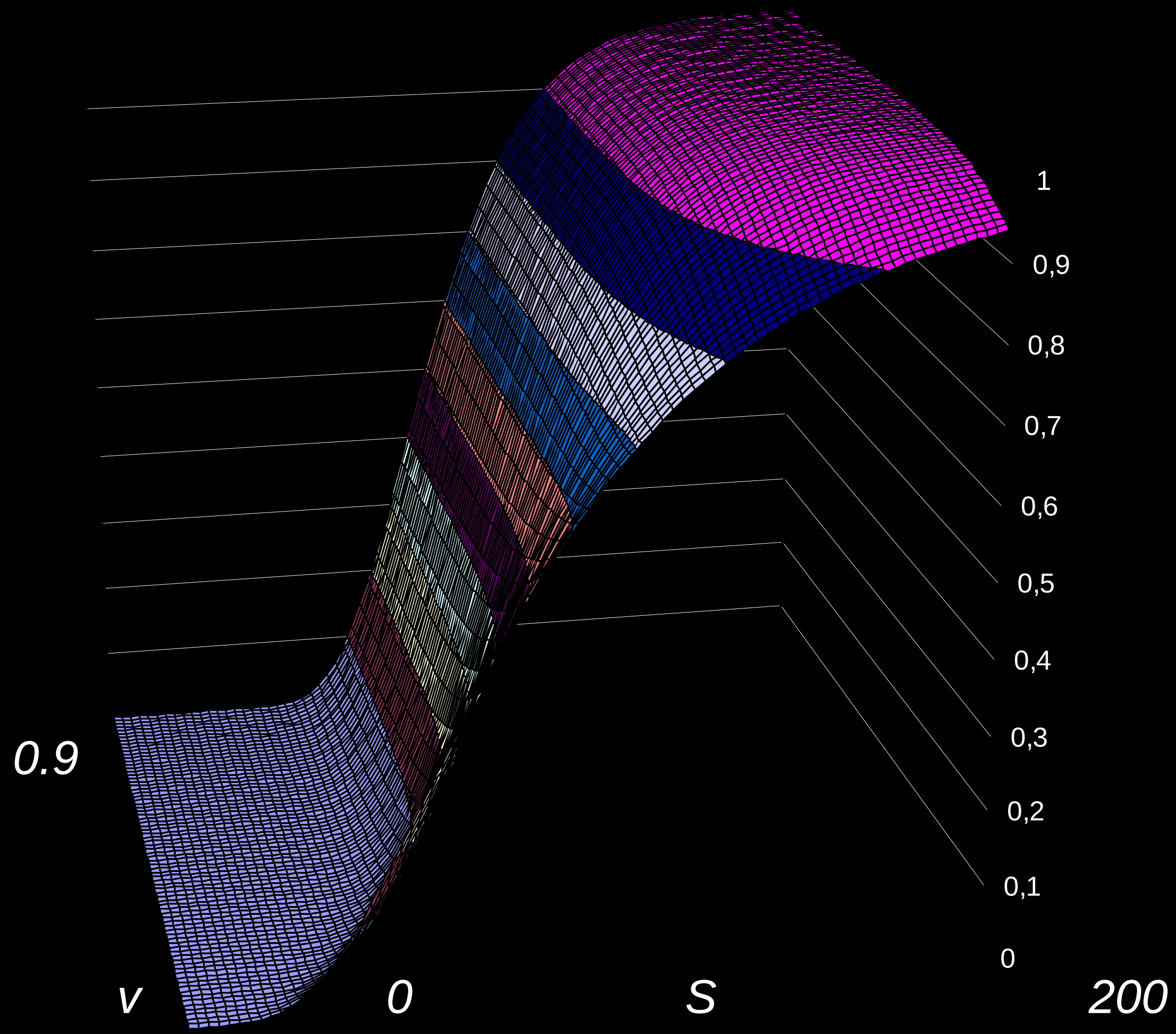
Rho = 0

Heston Delta



# Heston Delta

Rho = -1




CappaV = 2

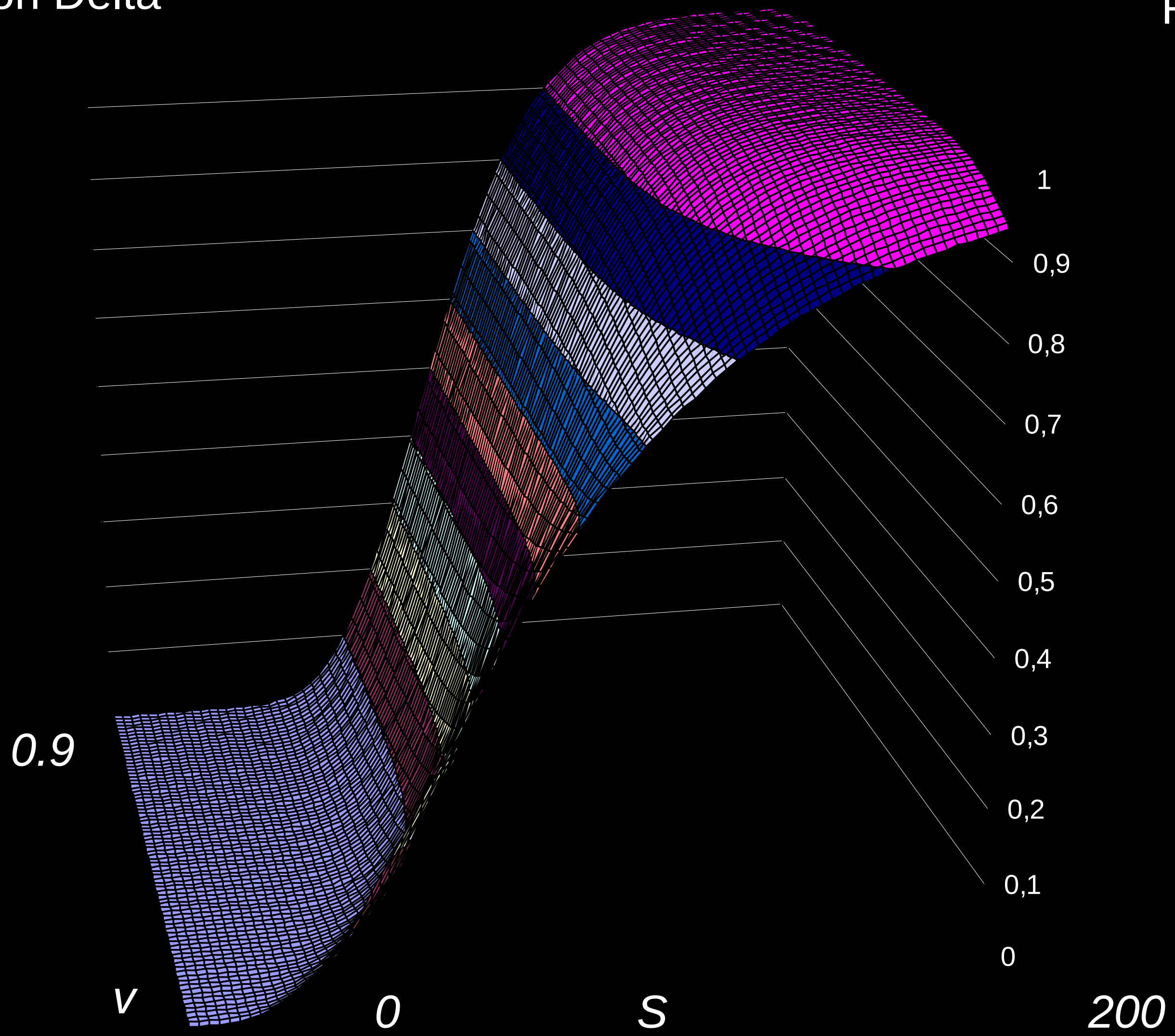
ThetaV = 0.3

EtaV = 0.1

Lambda = 0

# Heston Delta

Rho = 1




CappaV = 2

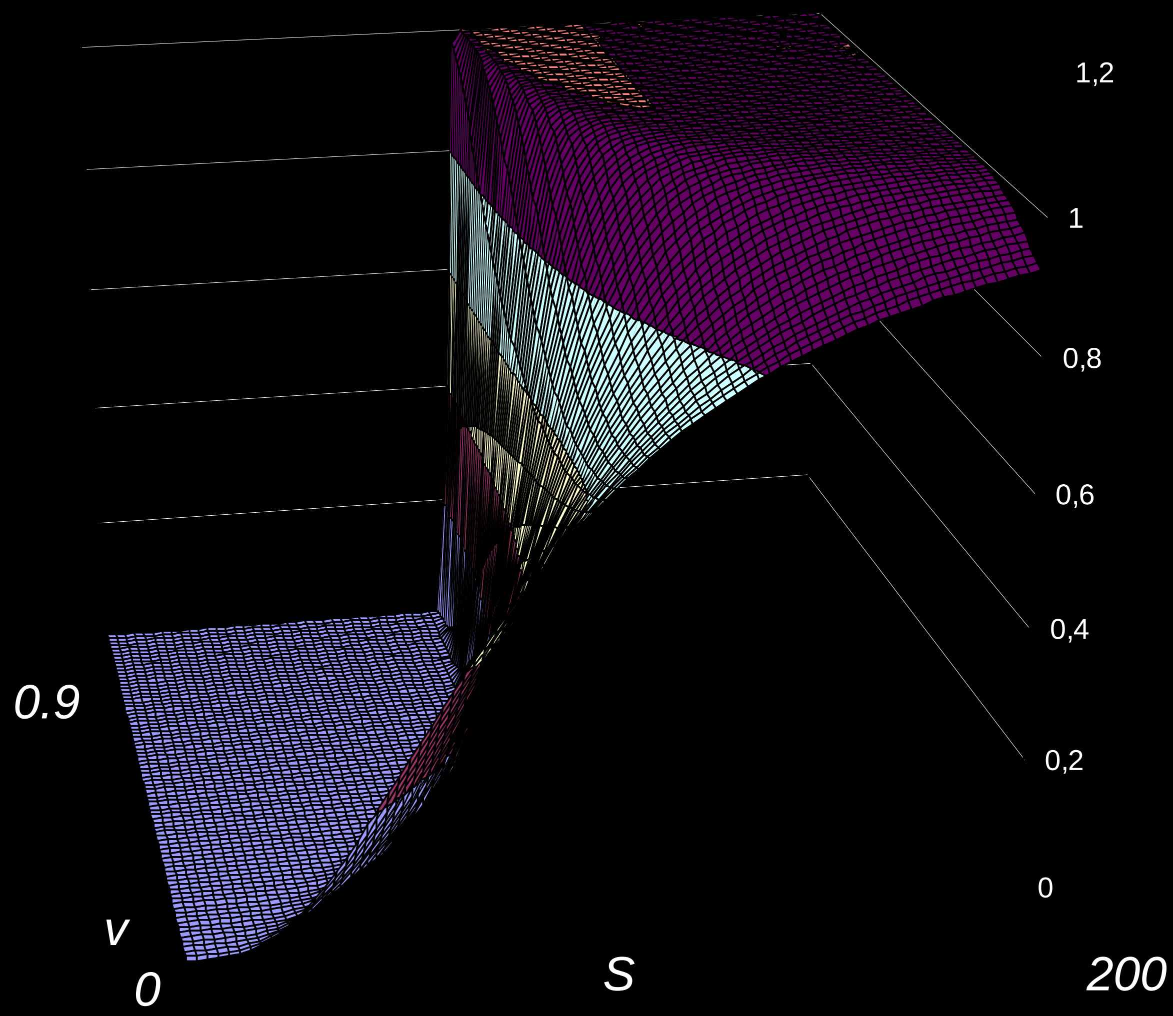
ThetaV = 0.3

EtaV = 0.1

Lambda = 0

Merton Delta

LambdaJ = 0




EtaJ = 0

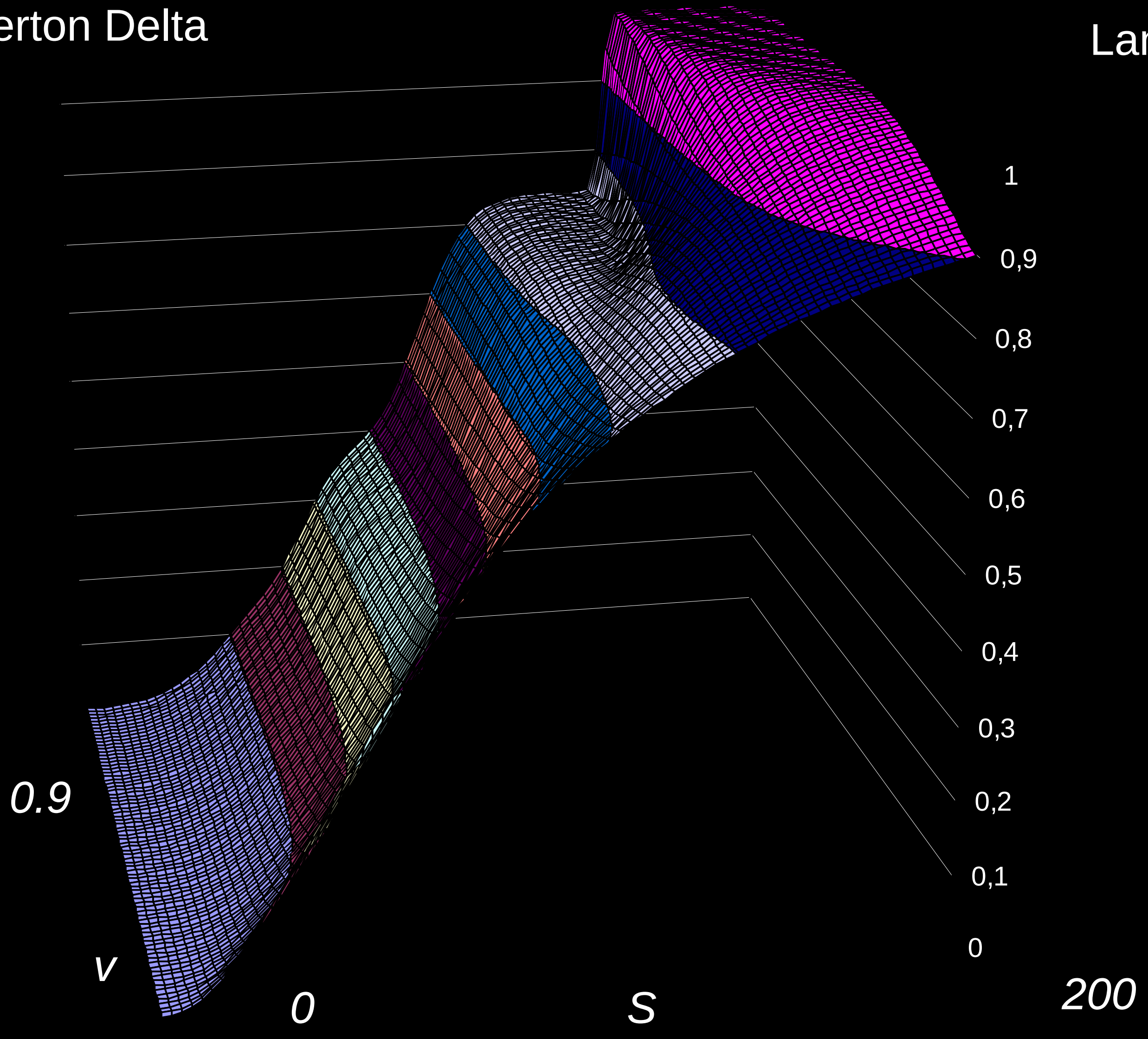
MuJ= 0

# Merton Delta

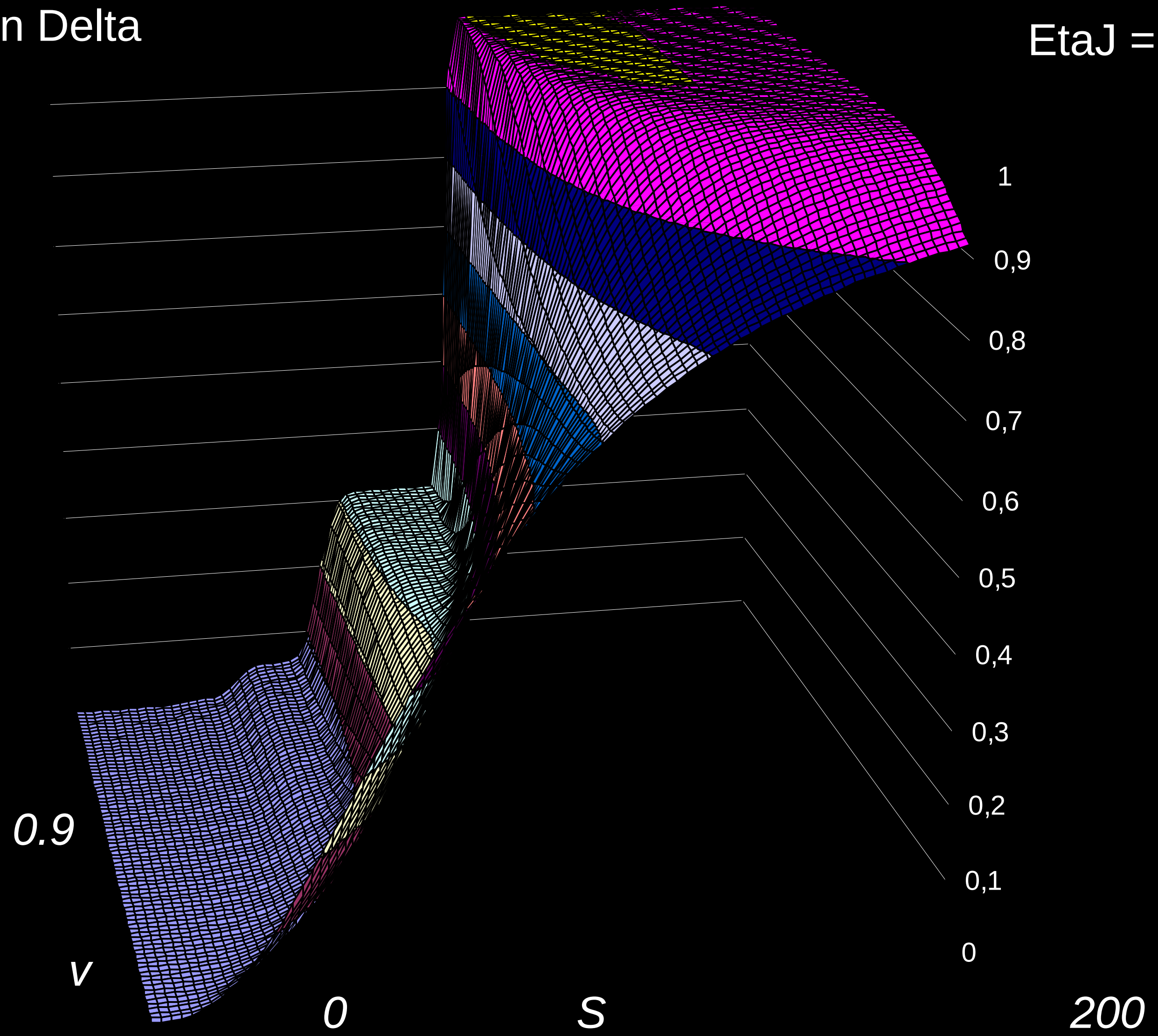
LambdaJ = 1.8

EtaJ = 0.1

MuJ= 0.5



# Merton Delta



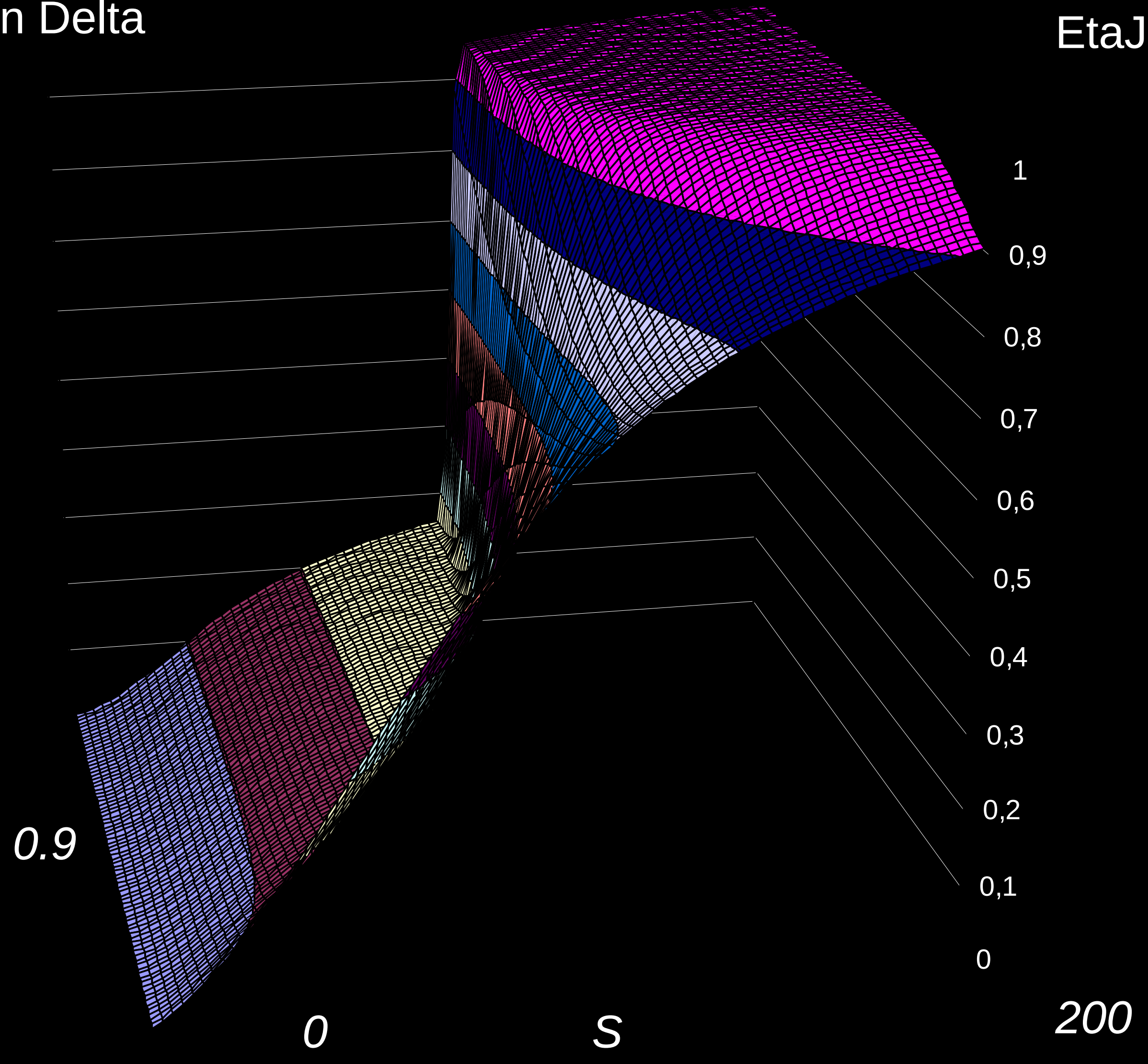

EtaJ = 0.1

LambdaJ = 0.5

MuJ= 0.5

Merton Delta

EtaJ = 0.75




LambdaJ = 0.5

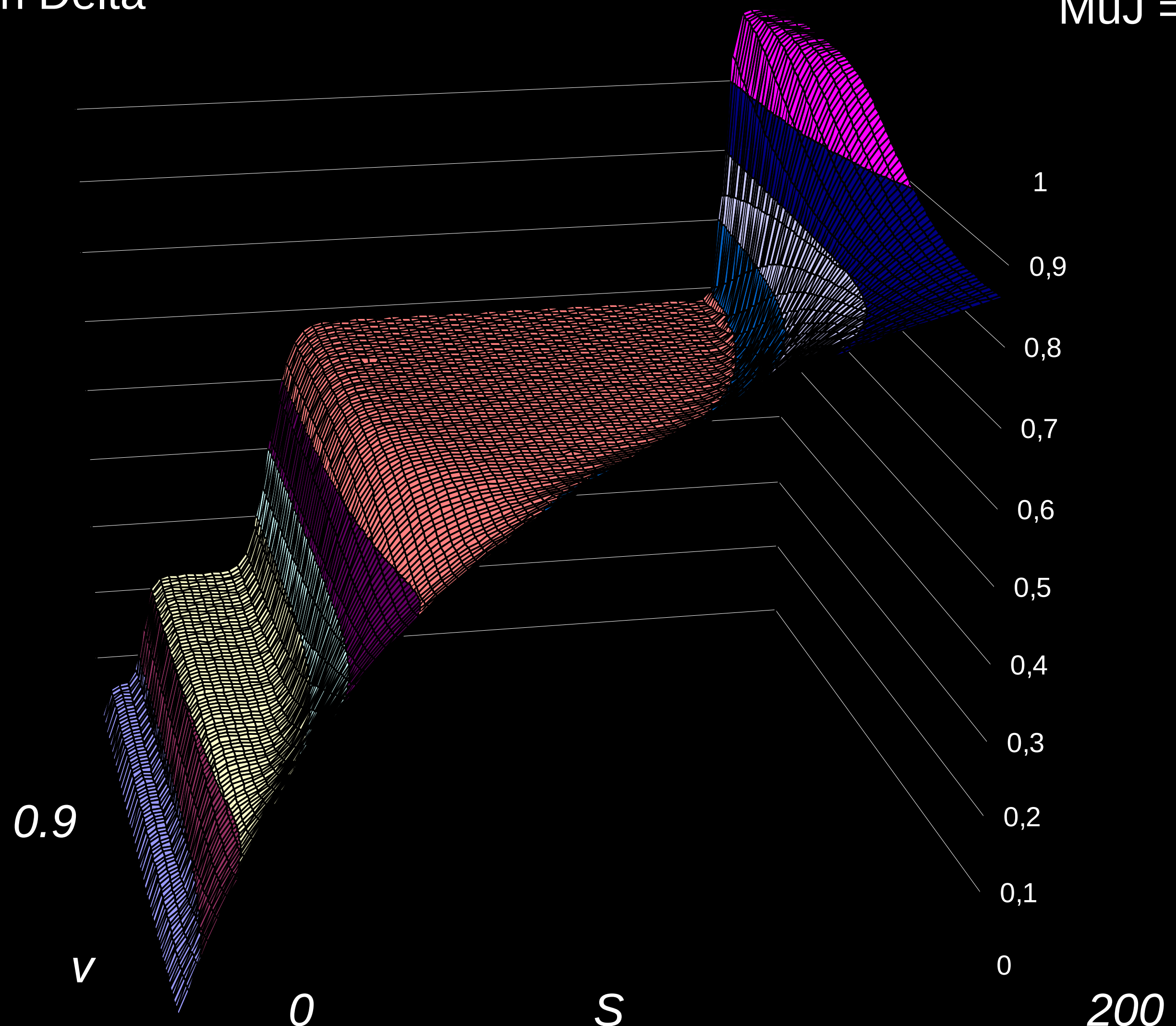
MuJ= 0.5

# Merton Delta

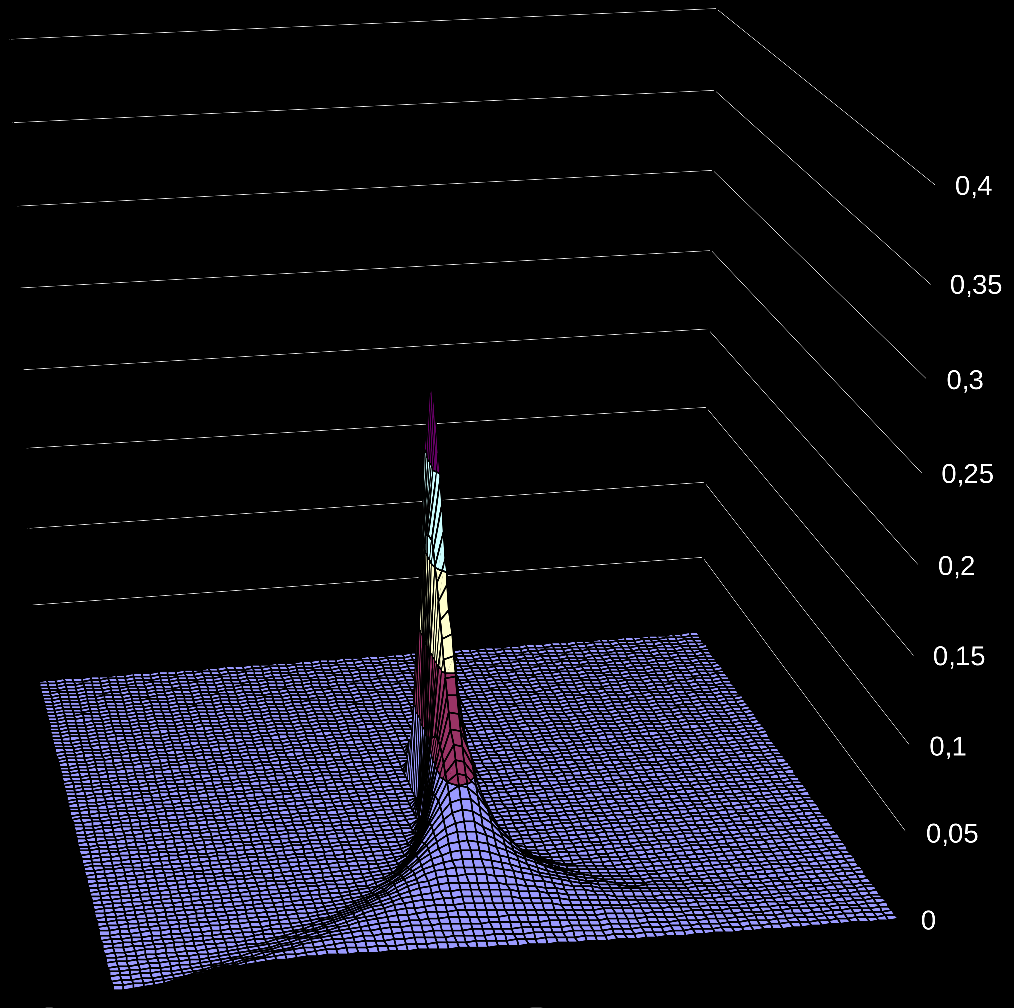
MuJ = 2.5

LambdaJ = 0.5

Eta= 0.1

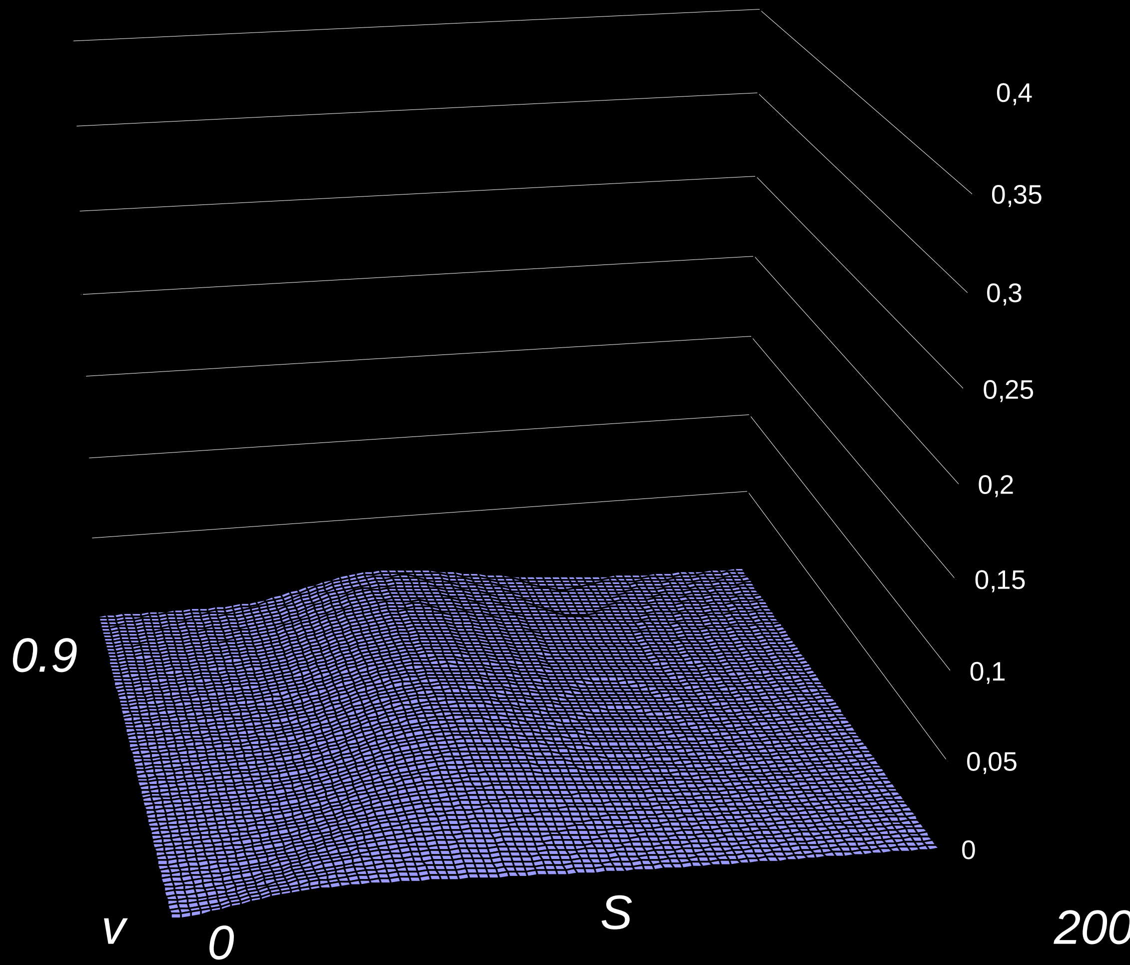


# Black – Scholes Gamma



# Heston Gamma

Lambda = -2



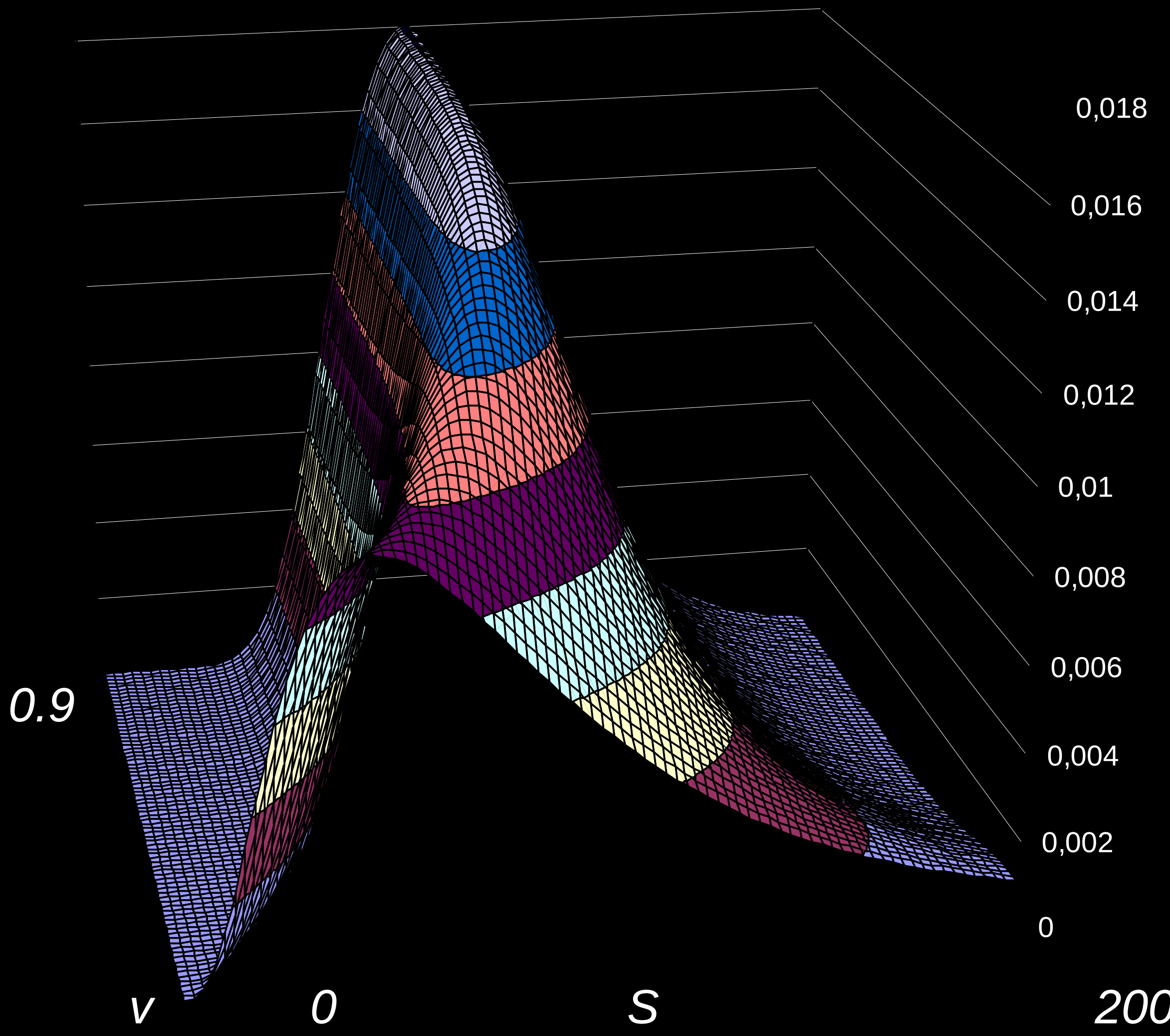

CappaV = 2

ThetaV = 0.3

EtaV = 0.1

Rho = 0

## Heston Gamma




Lambda = -2

CappaV = 2

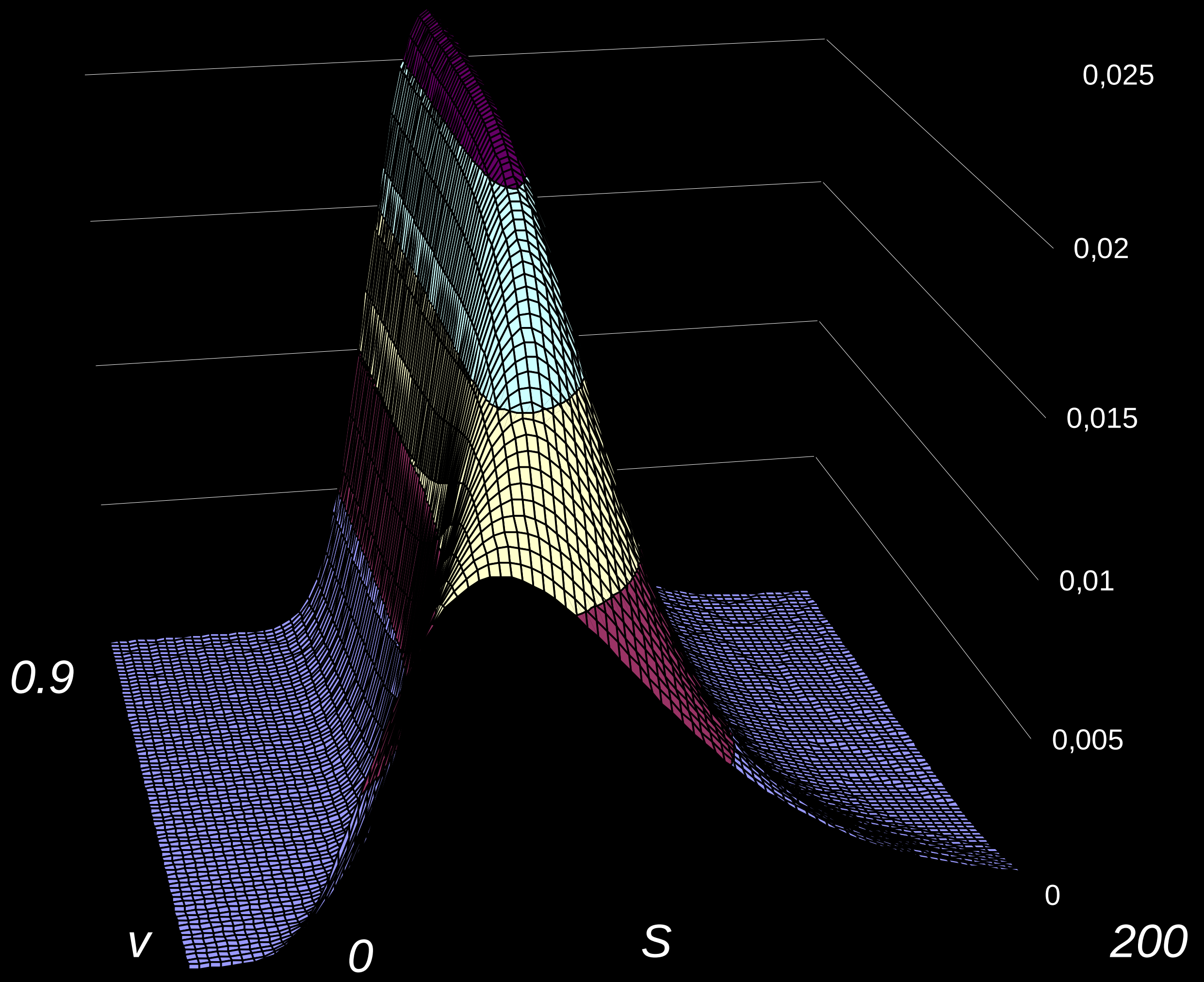
ThetaV = 0.3

EtaV = 0.1

Rho = 0

## Heston Gamma

Lambda = 2




CappaV = 2

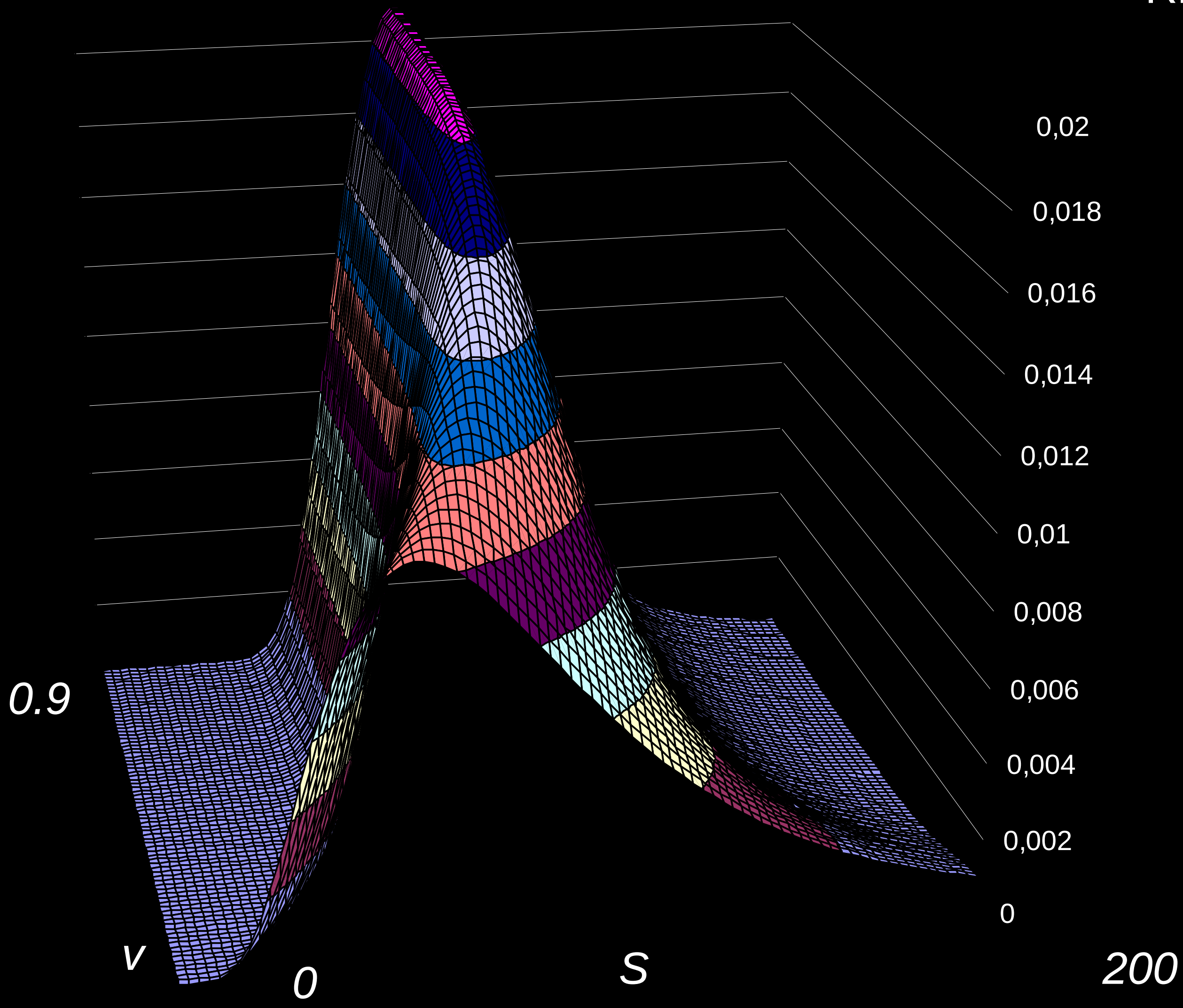
ThetaV = 0.3

EtaV = 0.1

Rho = 0

## Heston Gamma

Rho = -1




CappaV = 2

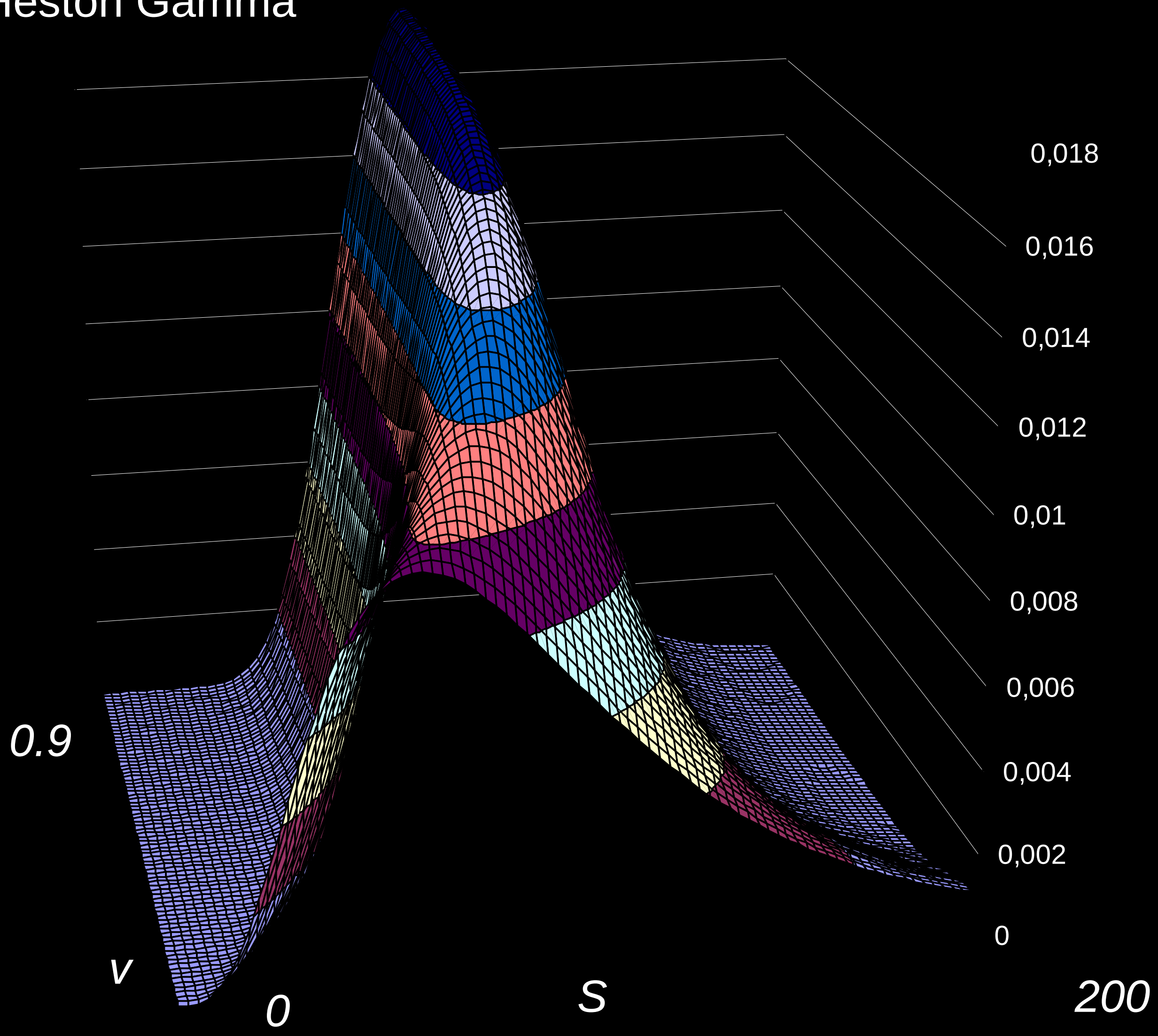
ThetaV = 0.3

EtaV = 0.1

Lambda = 0

Heston Gamma

Rho = 1




CappaV = 2

ThetaV = 0.3

EtaV = 0.1

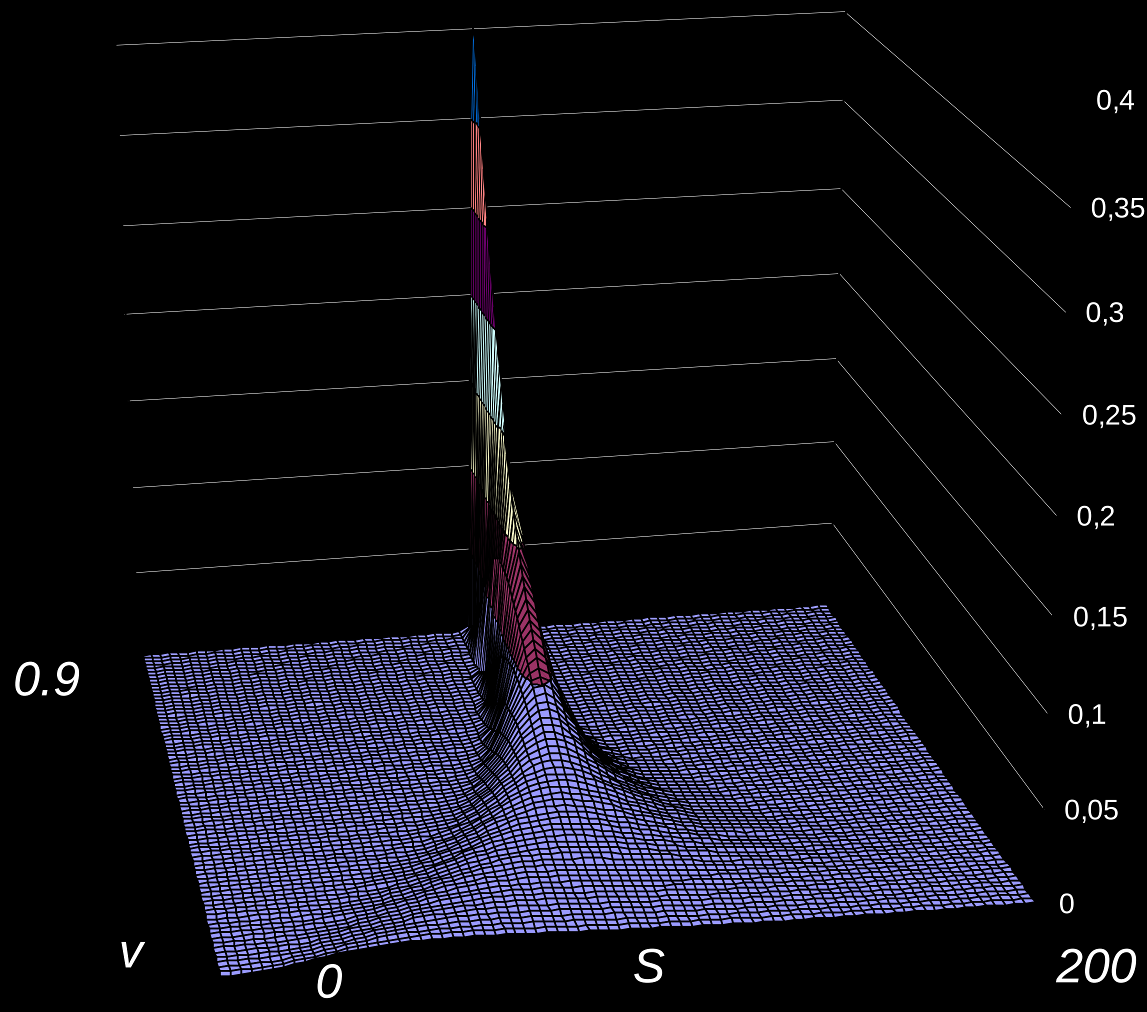
Lambda = 0

# Merton Gamma

LambdaJ = 0

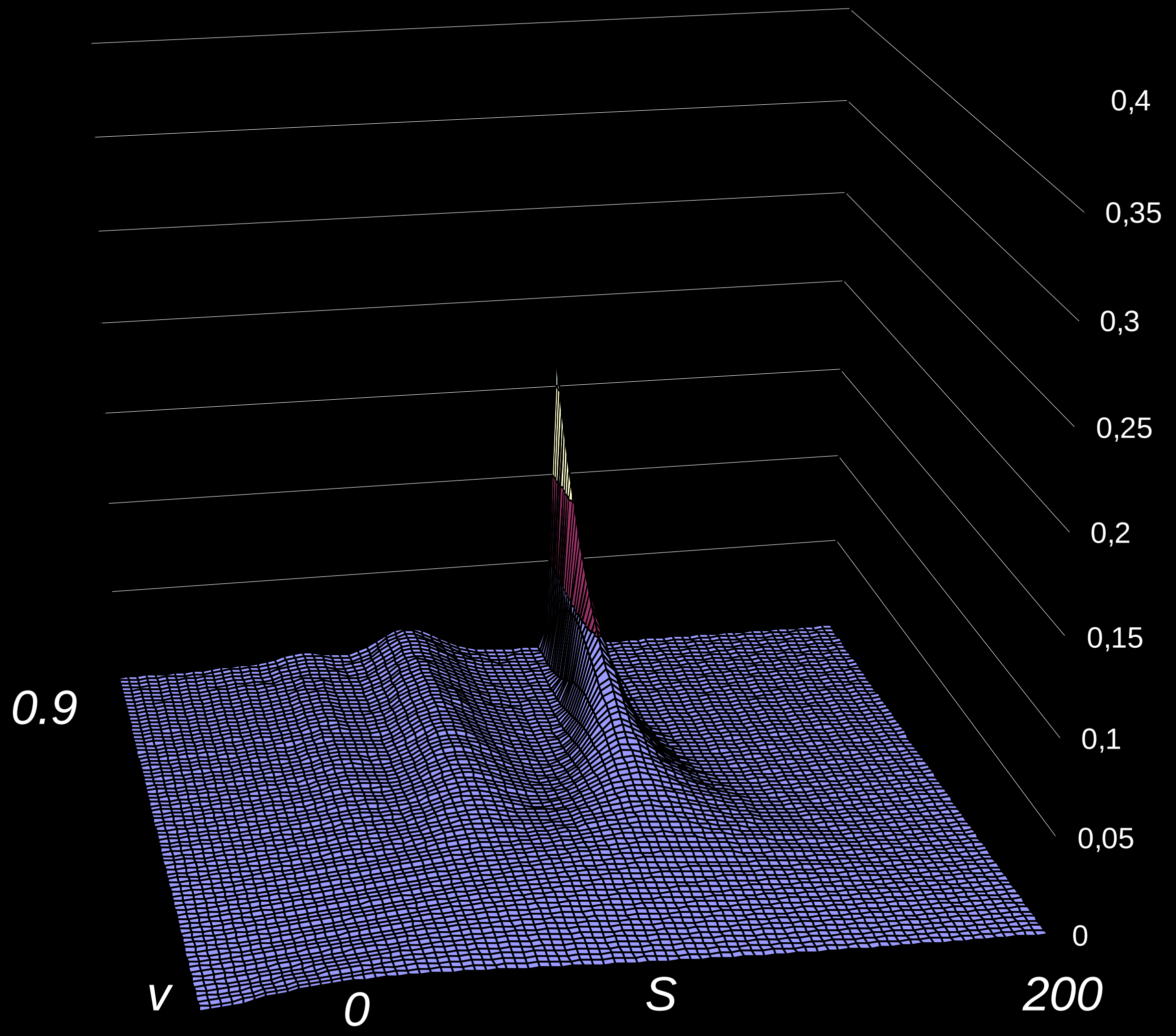
EtaJ = 0

MuJ= 0



Merton Gamma

LambdaJ = 0.9




EtaJ = 0.1

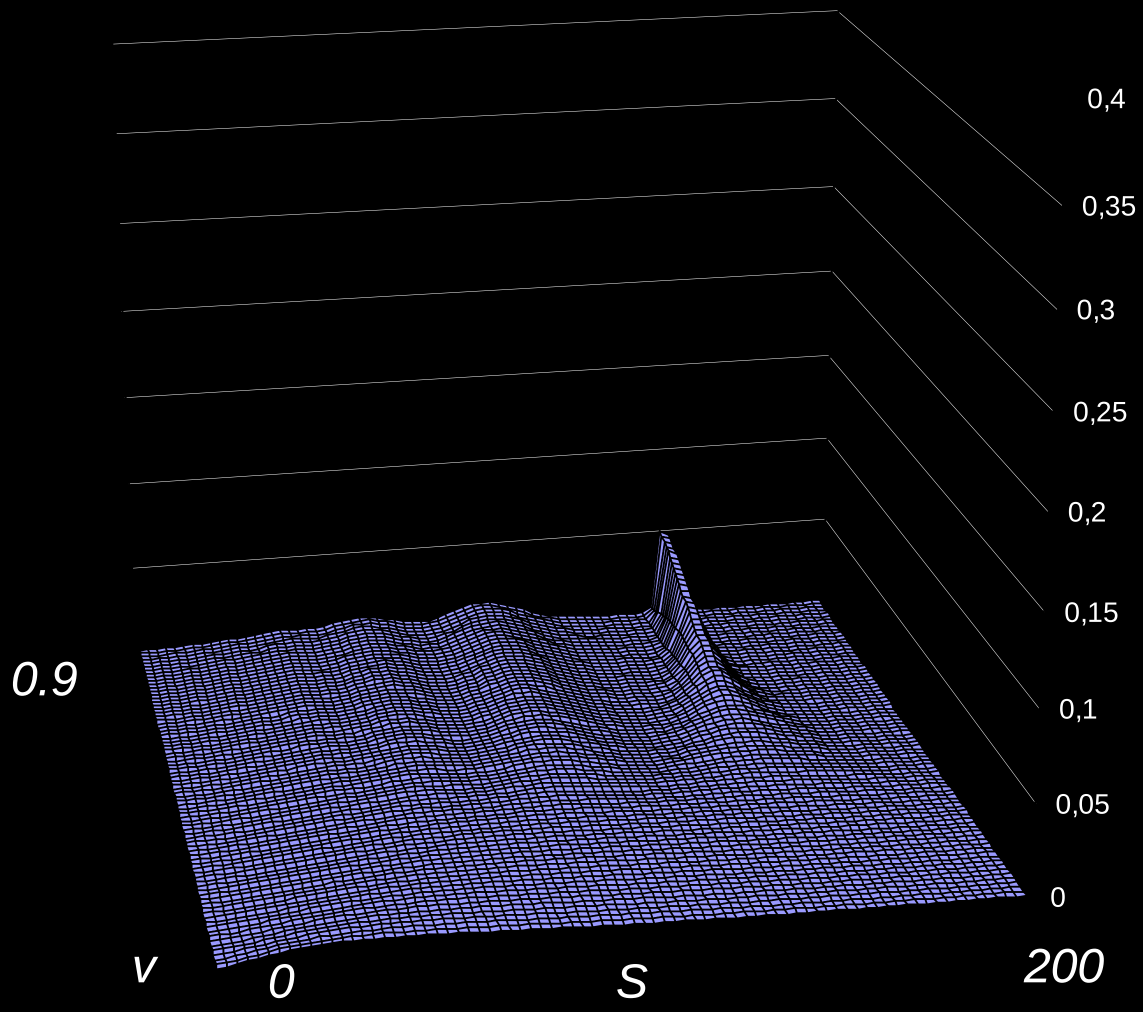
MuJ = 0.5

# Merton Gamma

LambdaJ = 1.8

EtaJ = 0.1

MuJ = 0.5

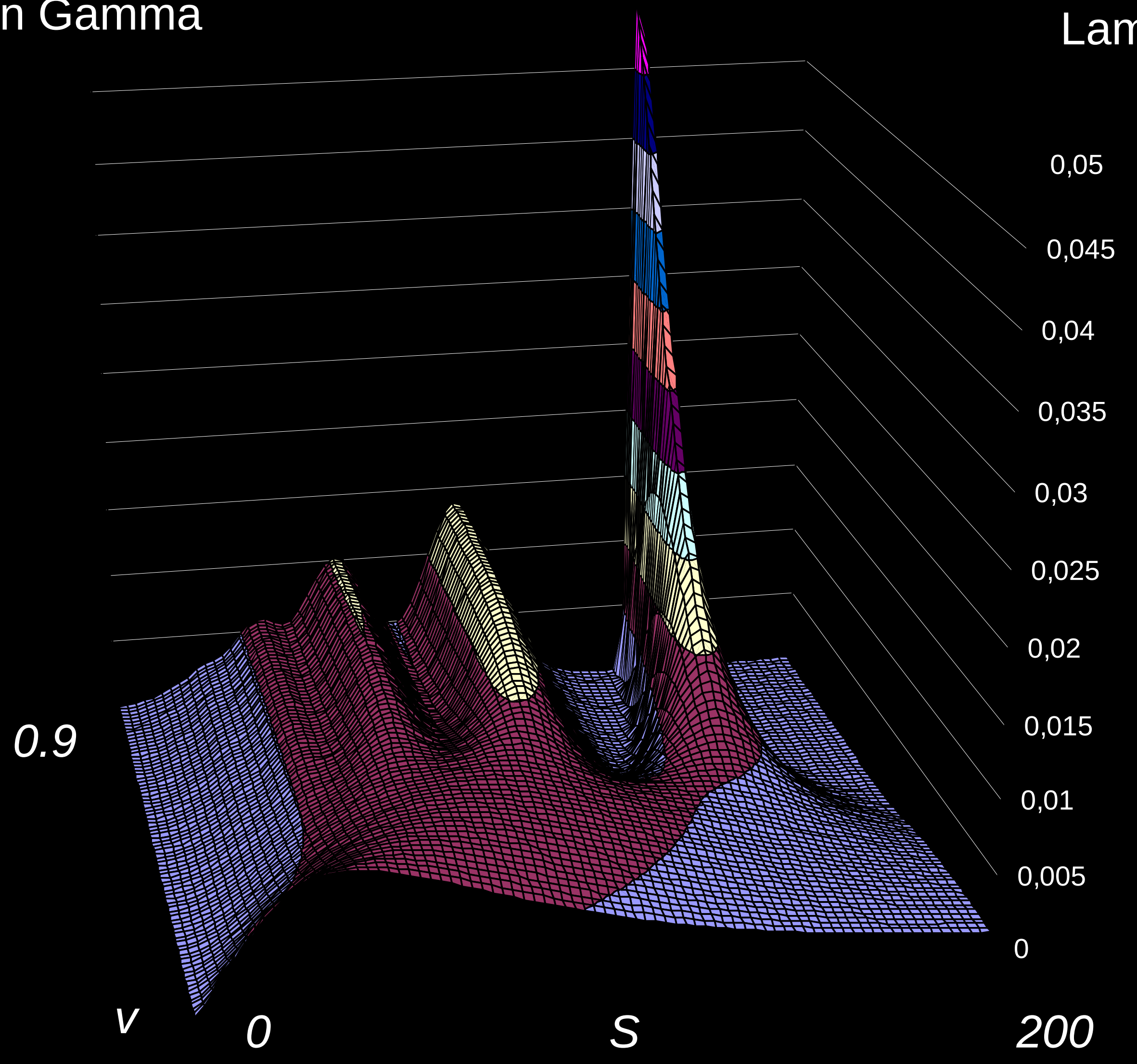


# Merton Gamma

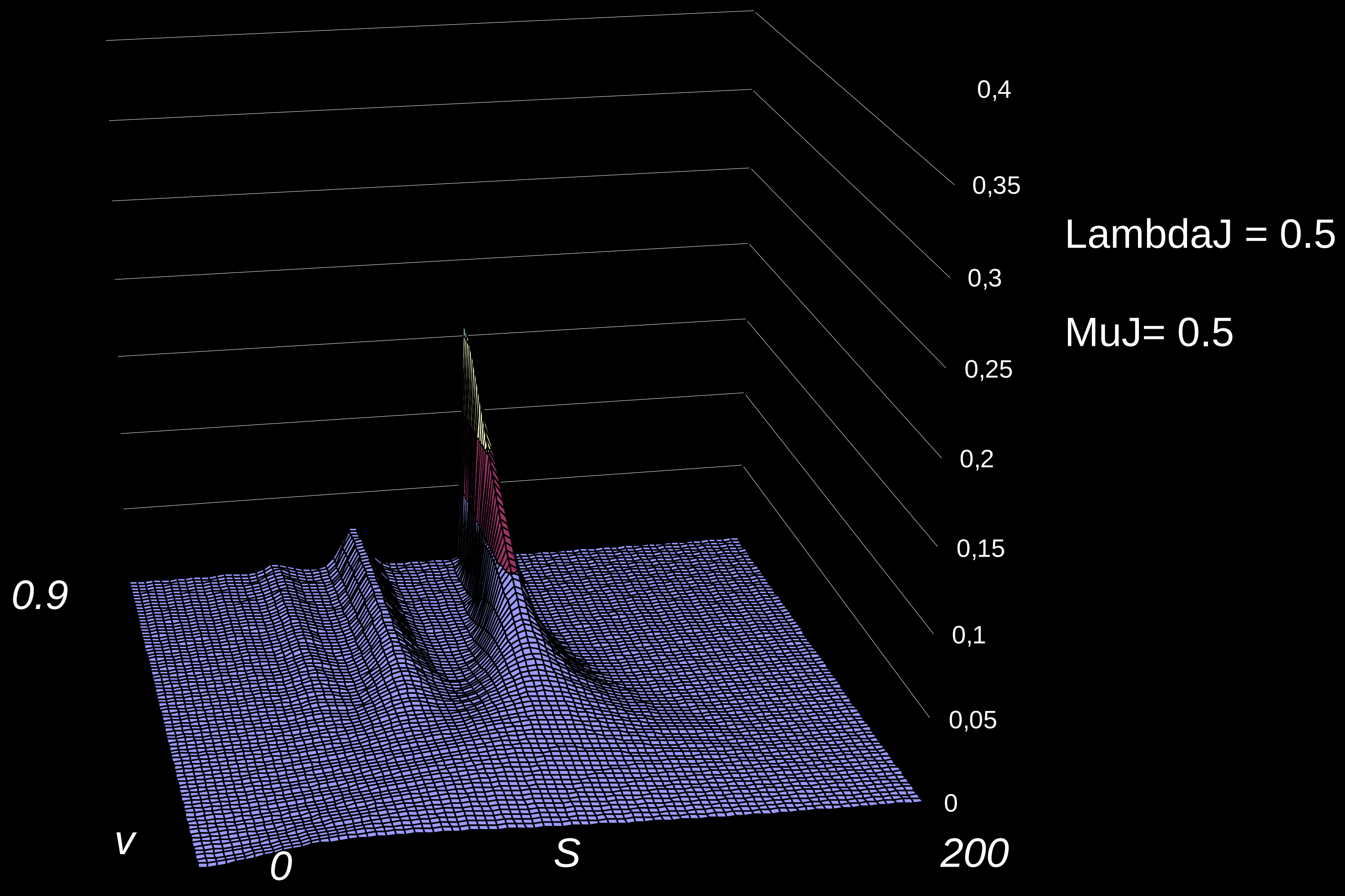
LambdaJ = 1.8

EtaJ = 0.1

MuJ= 0.5

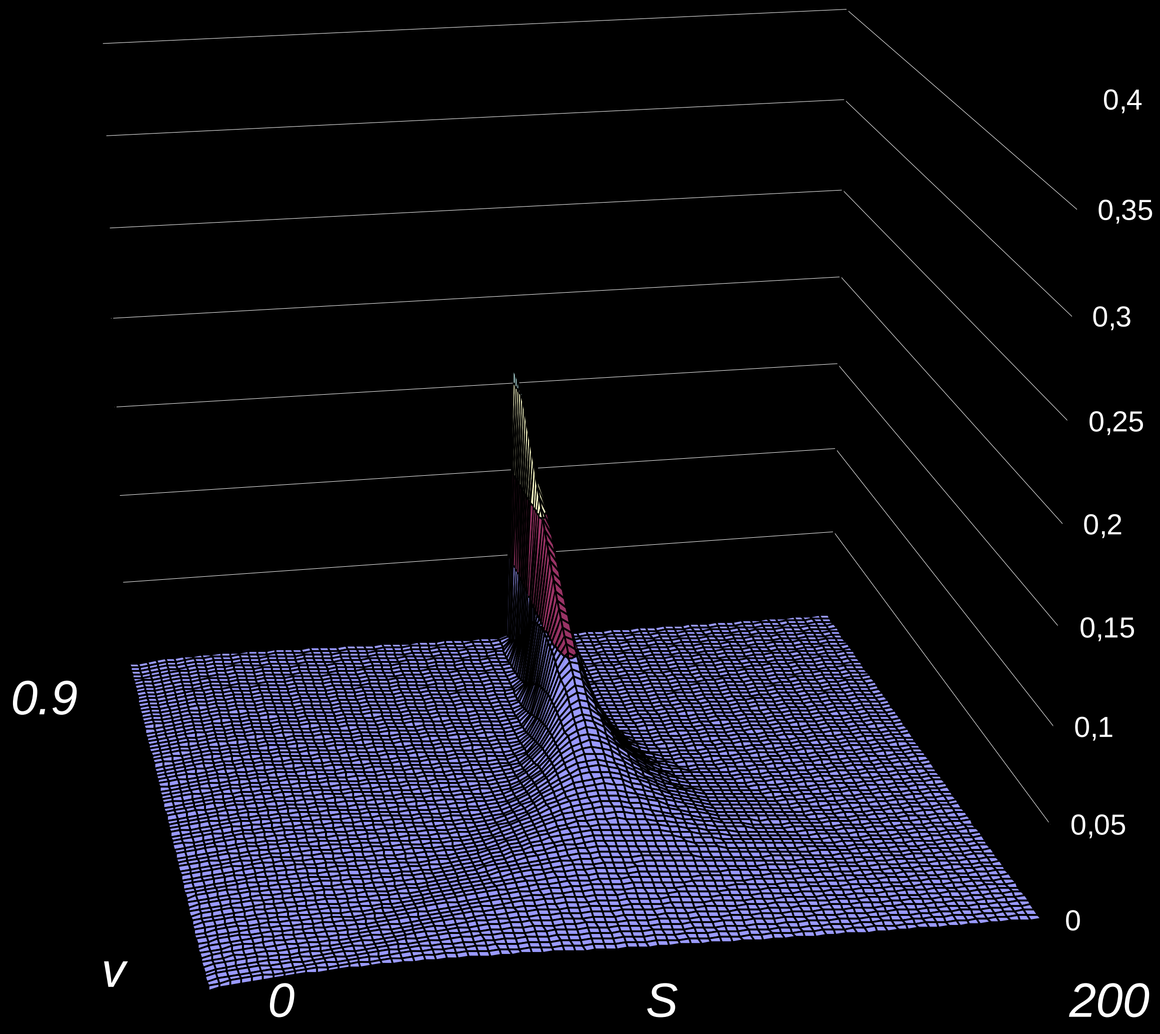


# Merton Gamma



# Merton Gamma

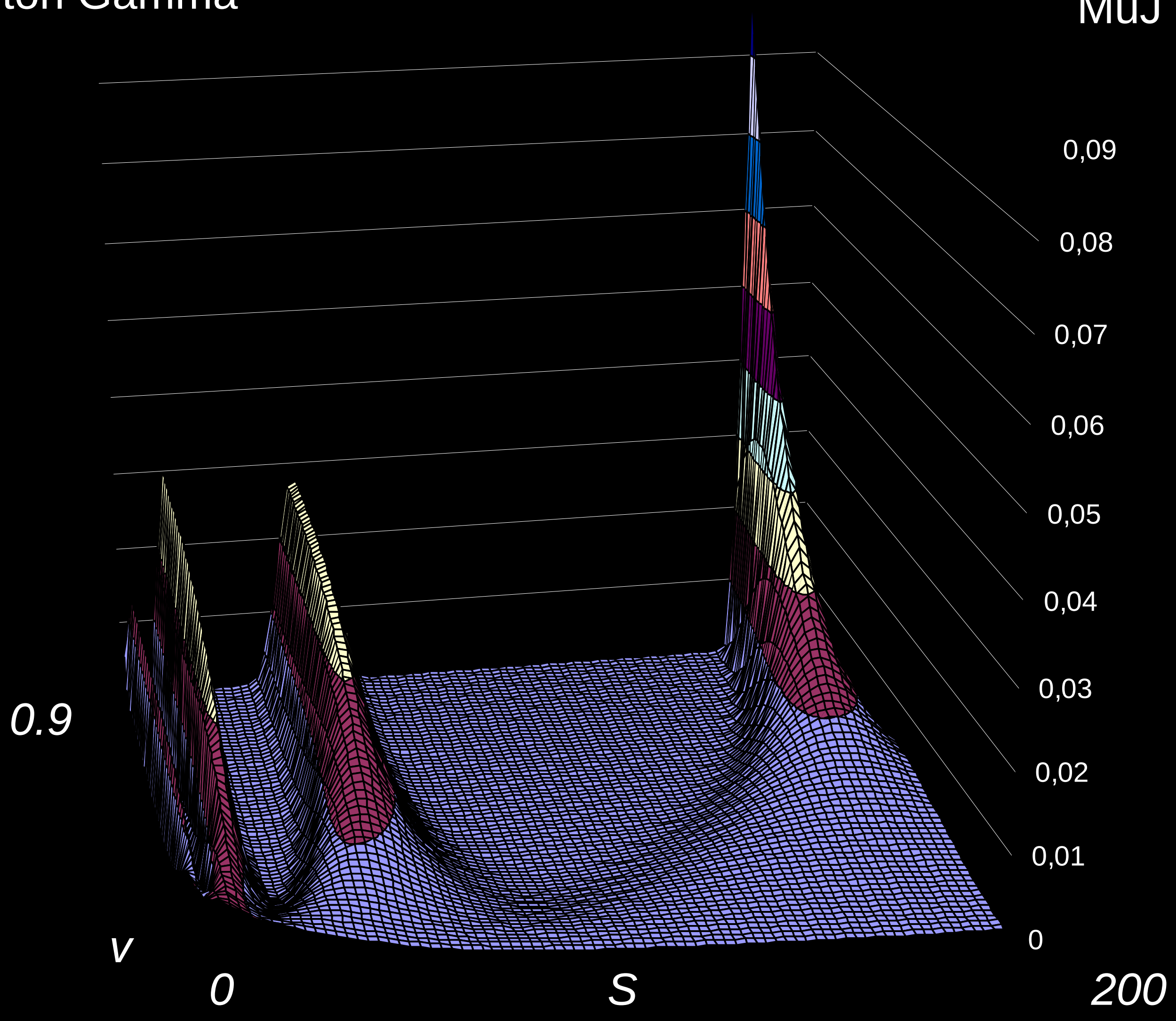
EtaJ = 0.75



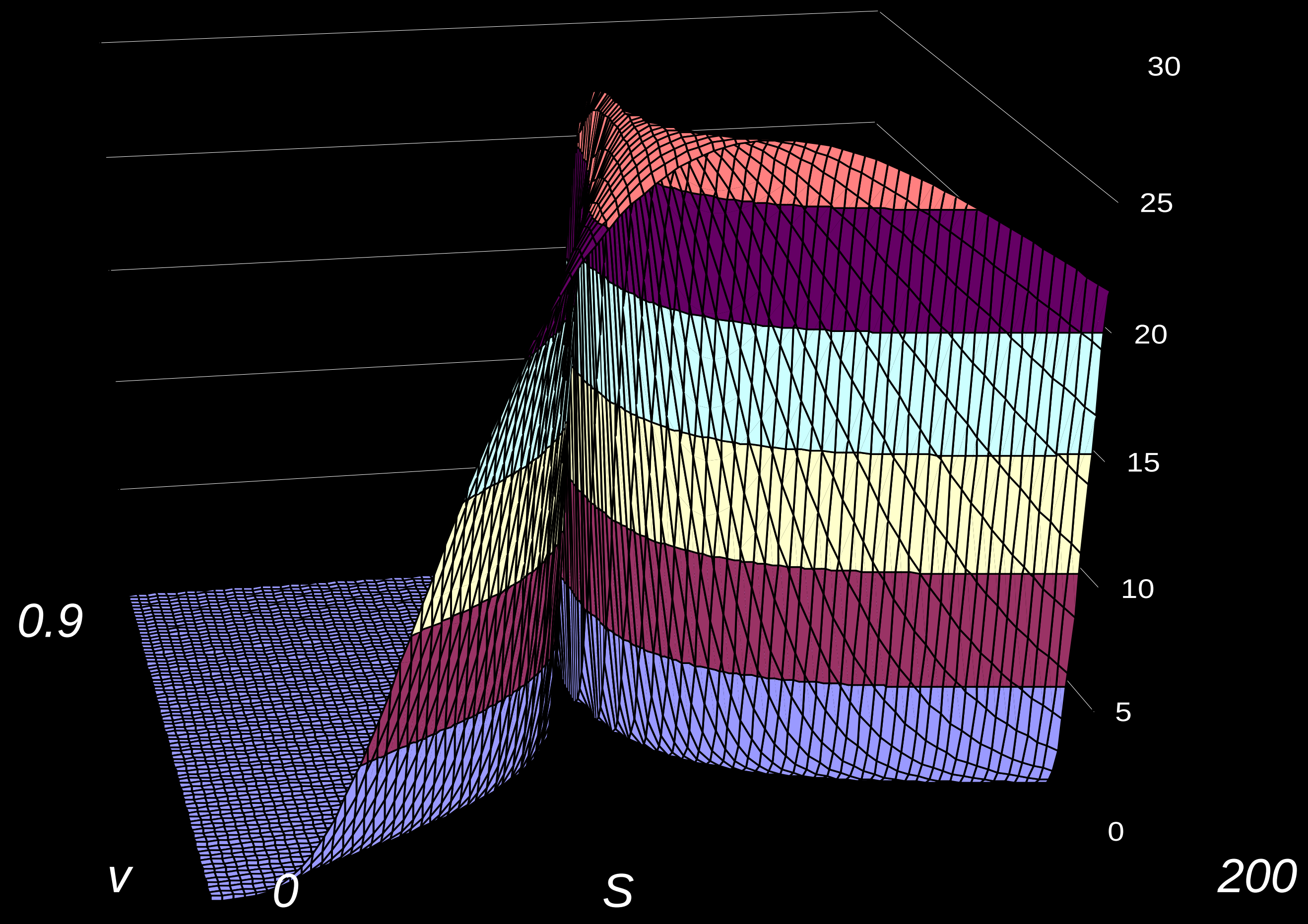

LambdaJ = 0.5

MuJ= 0.5

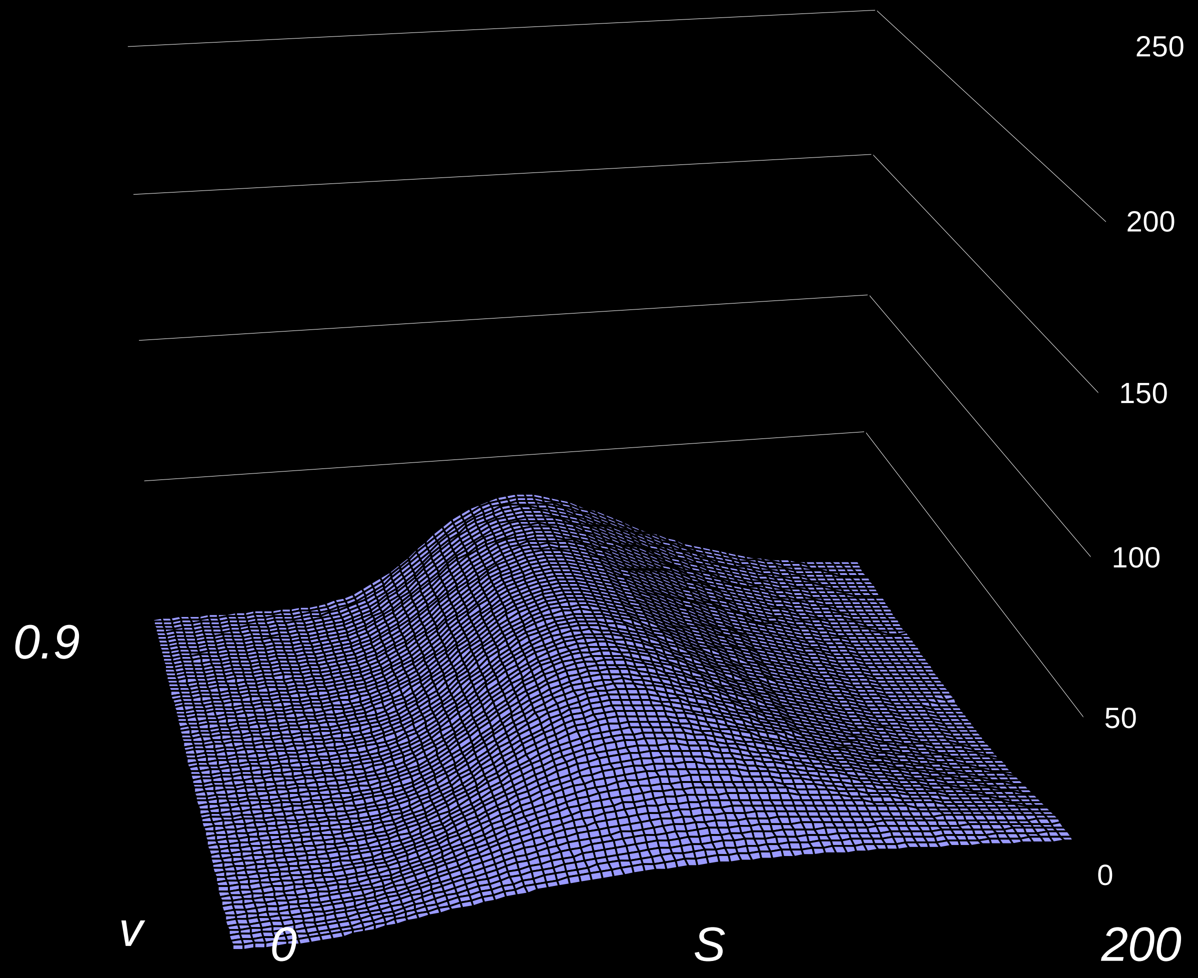
# Merton Gamma



# Black – Scholes Vega



## Heston Vega




Lambda = -2

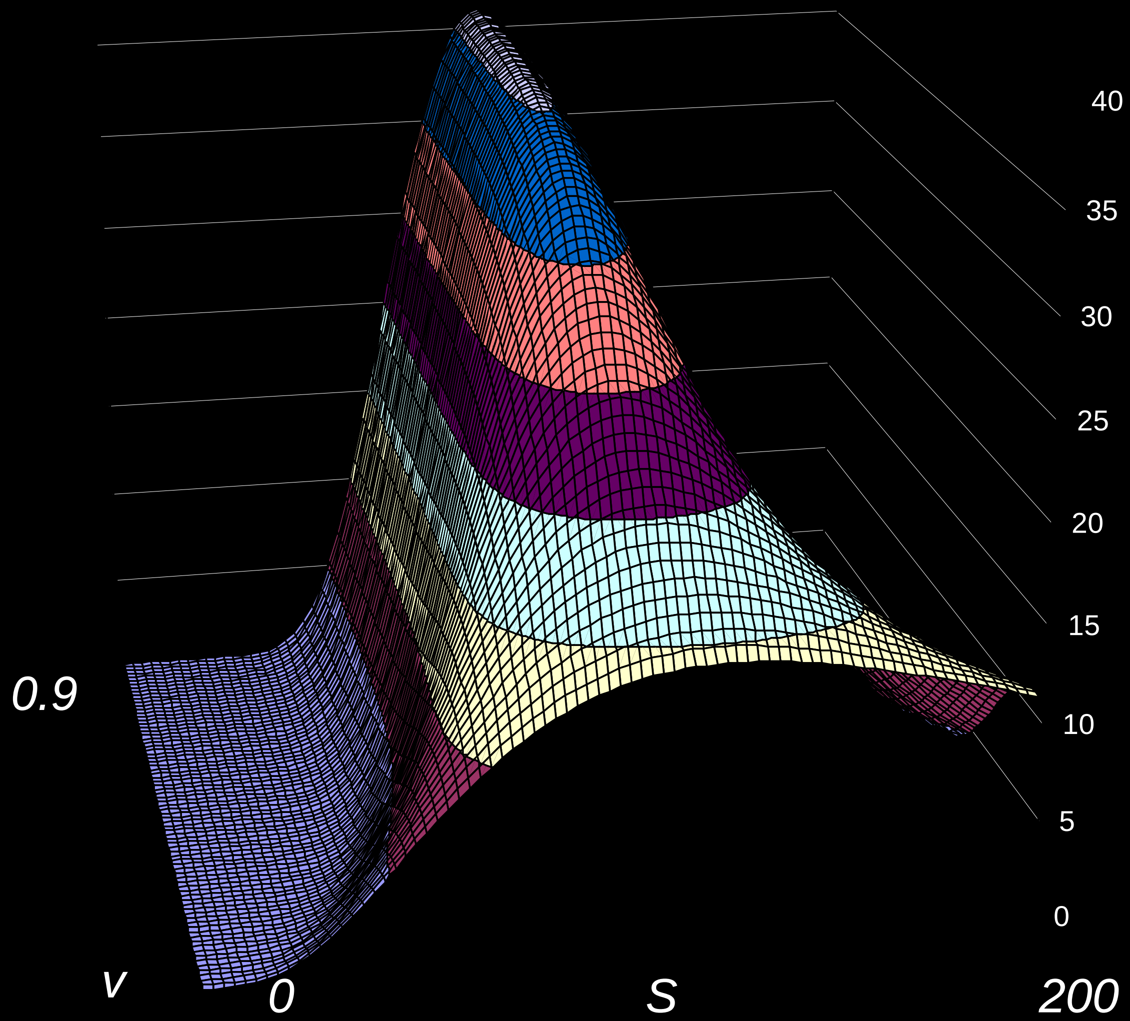
CappaV = 2

ThetaV = 0.3

EtaV = 0.1

Rho = 0

# Heston Vega




Lambda = -2

CappaV = 2

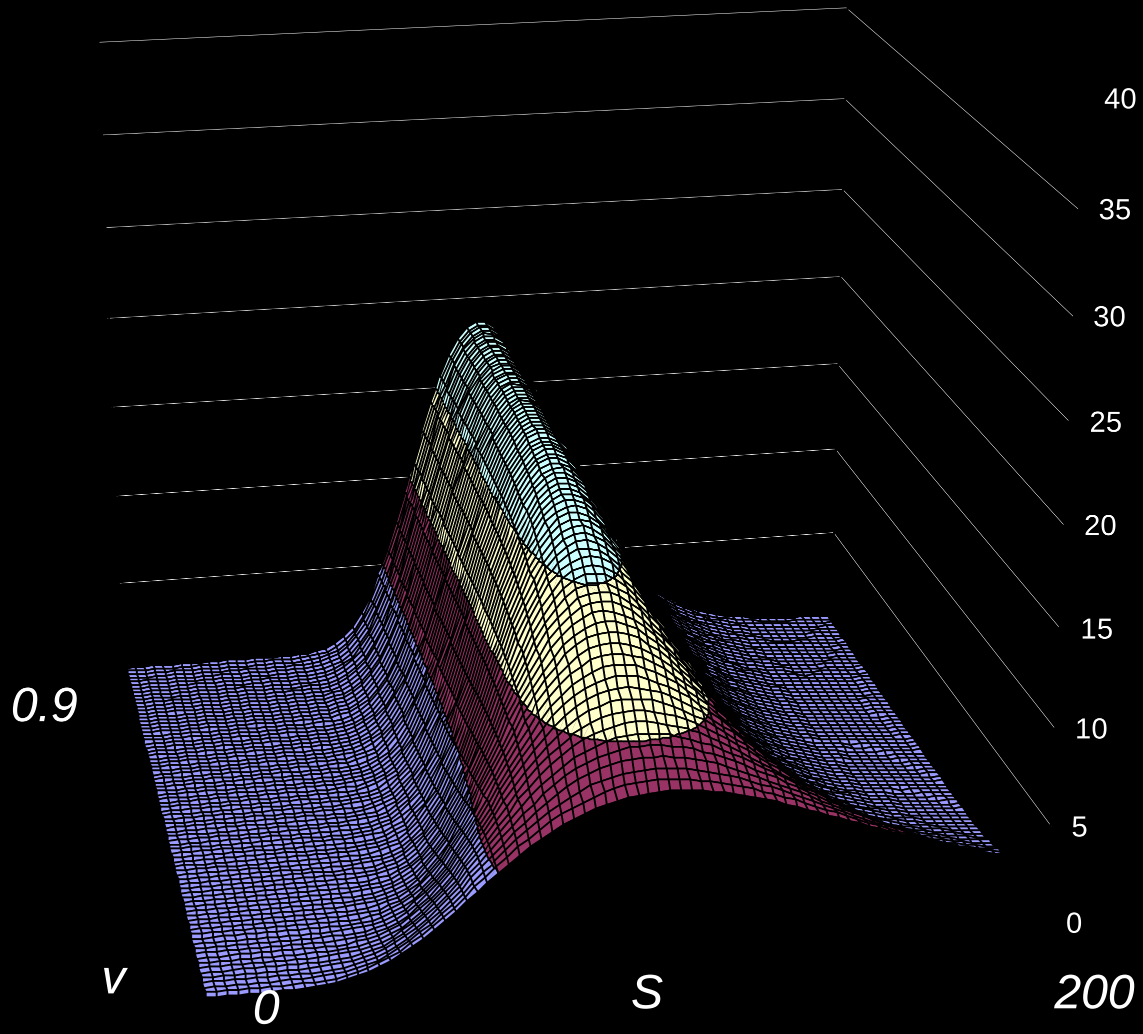
ThetaV = 0.3

EtaV = 0.1

Rho = 0

# Heston Vega

Lambda = 2



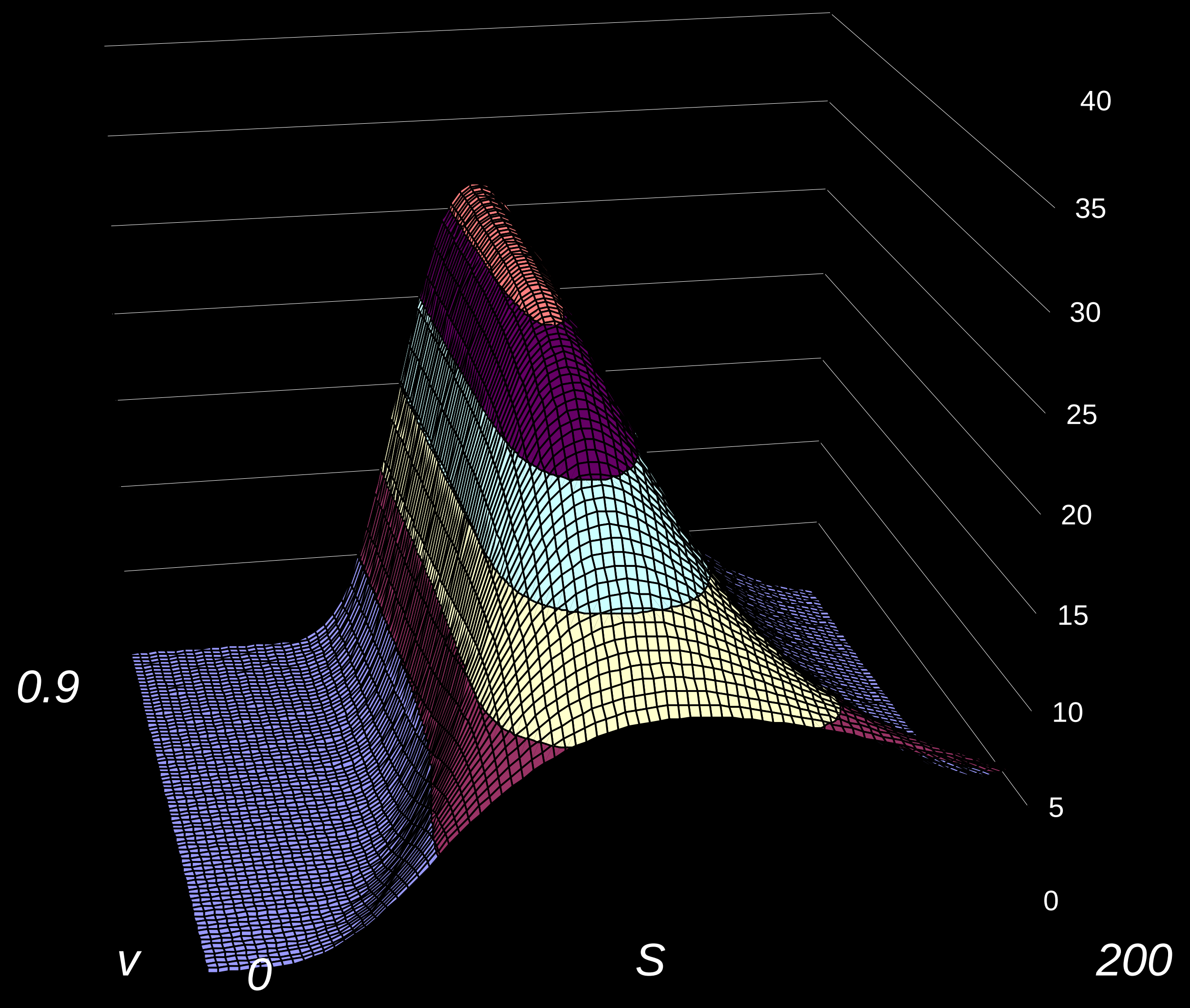

CappaV = 2

ThetaV = 0.3

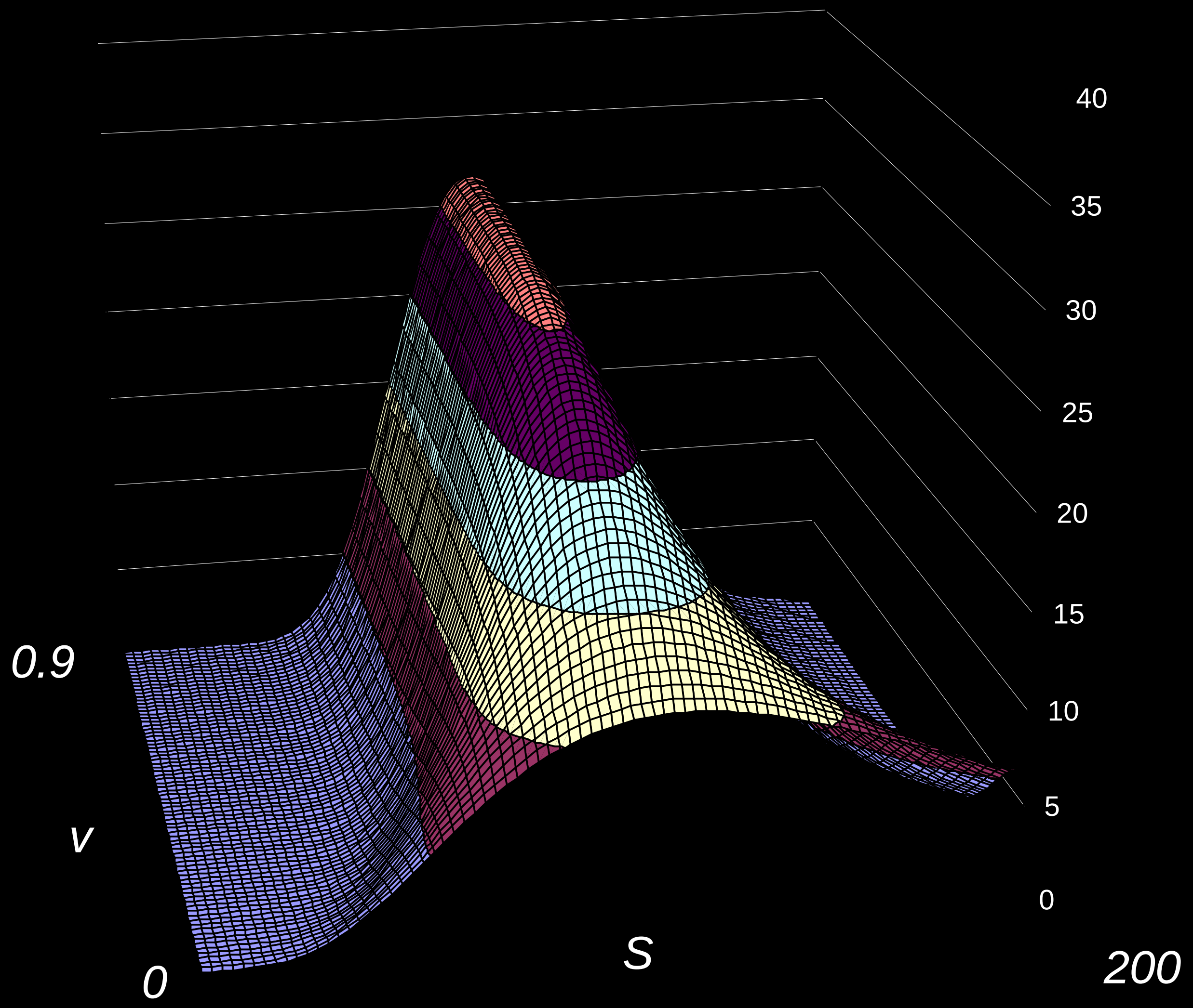
EtaV = 0.1

Rho = 0

# Heston Vega


Rho = -1
40
35
30
25
20
15
10
5
0
CappaV = 2
ThetaV = 0.3
EtaV = 0.1
Lambda = 0
0.9
v
0
S
200

Heston Vega




Rho = 1

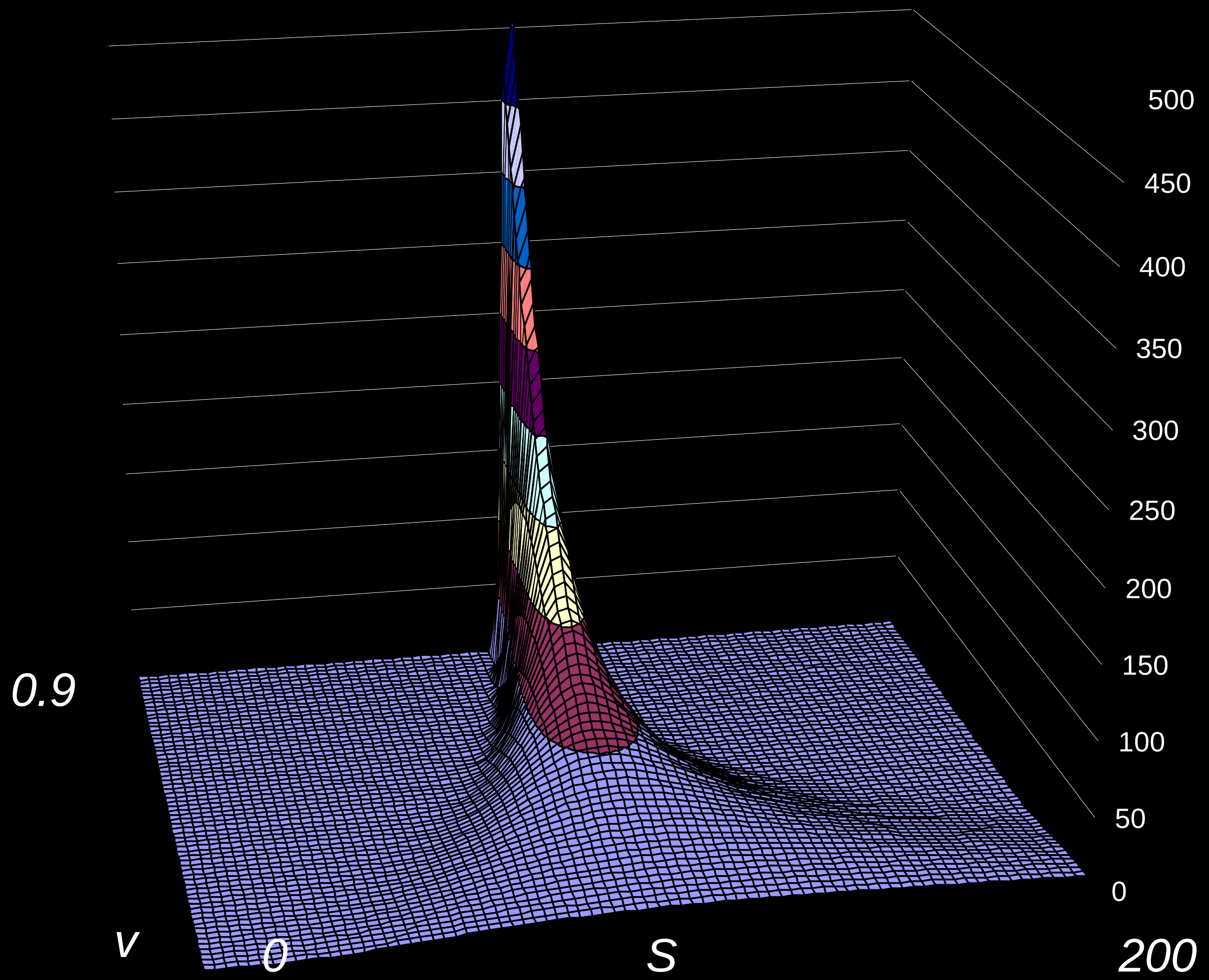
CappaV = 2

ThetaV = 0.3

EtaV = 0.1

Lambda = 0

# Merton Vega



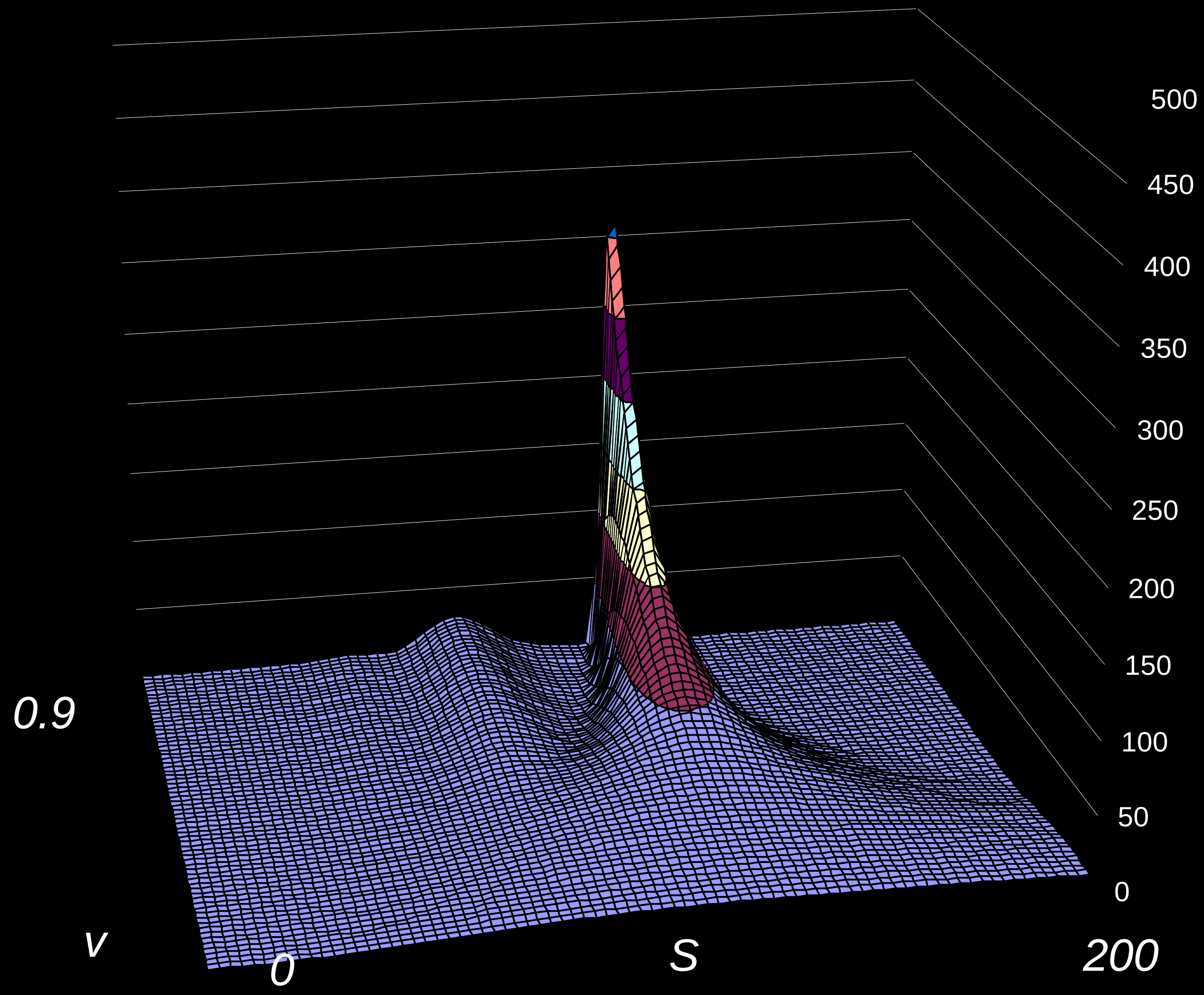

LambdaJ = 0

EtaJ = 0

MuJ= 0

Merton Vega

LambdaJ = 0.9




EtaJ = 0.1

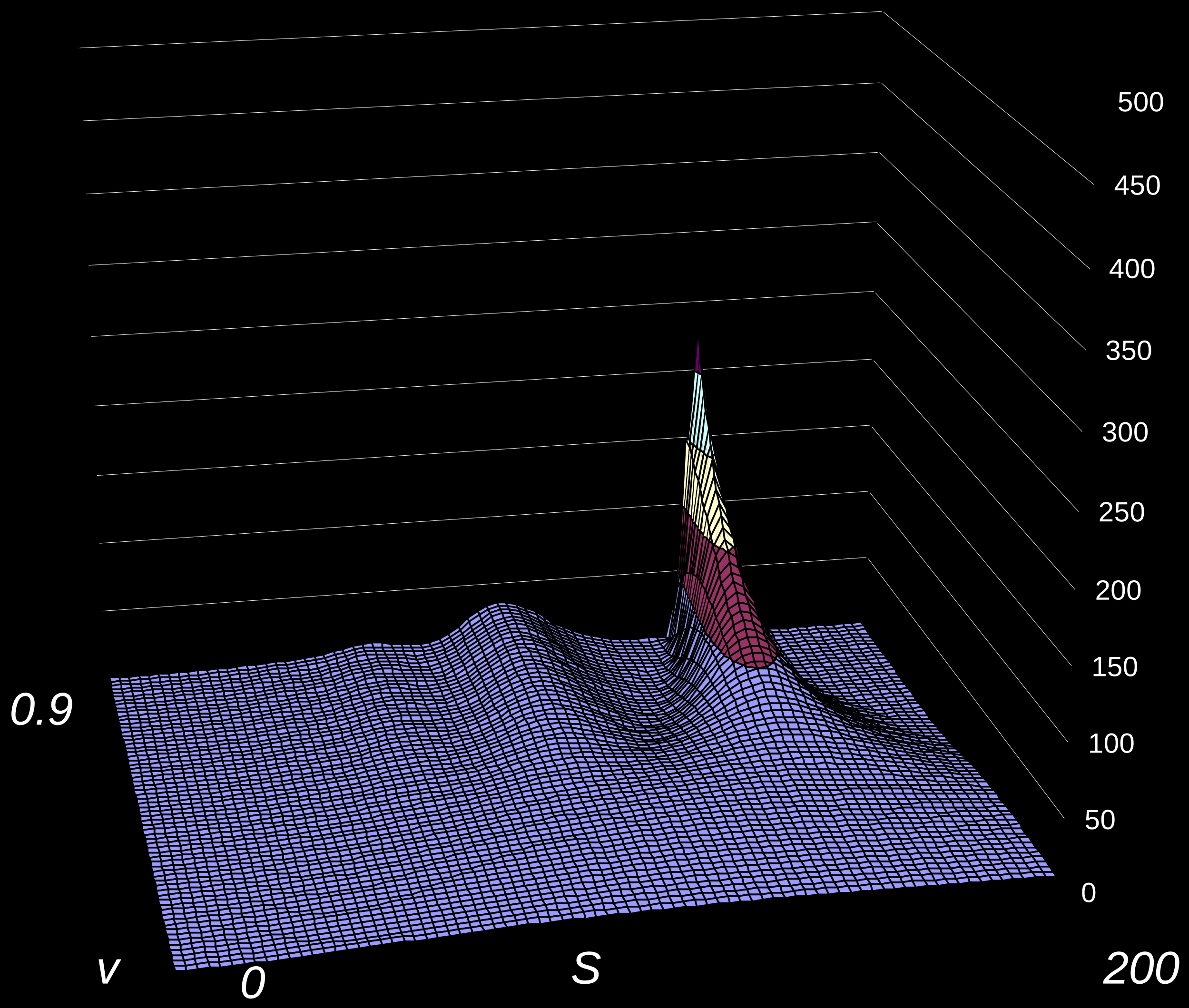
MuJ = 0.5

Merton Vega

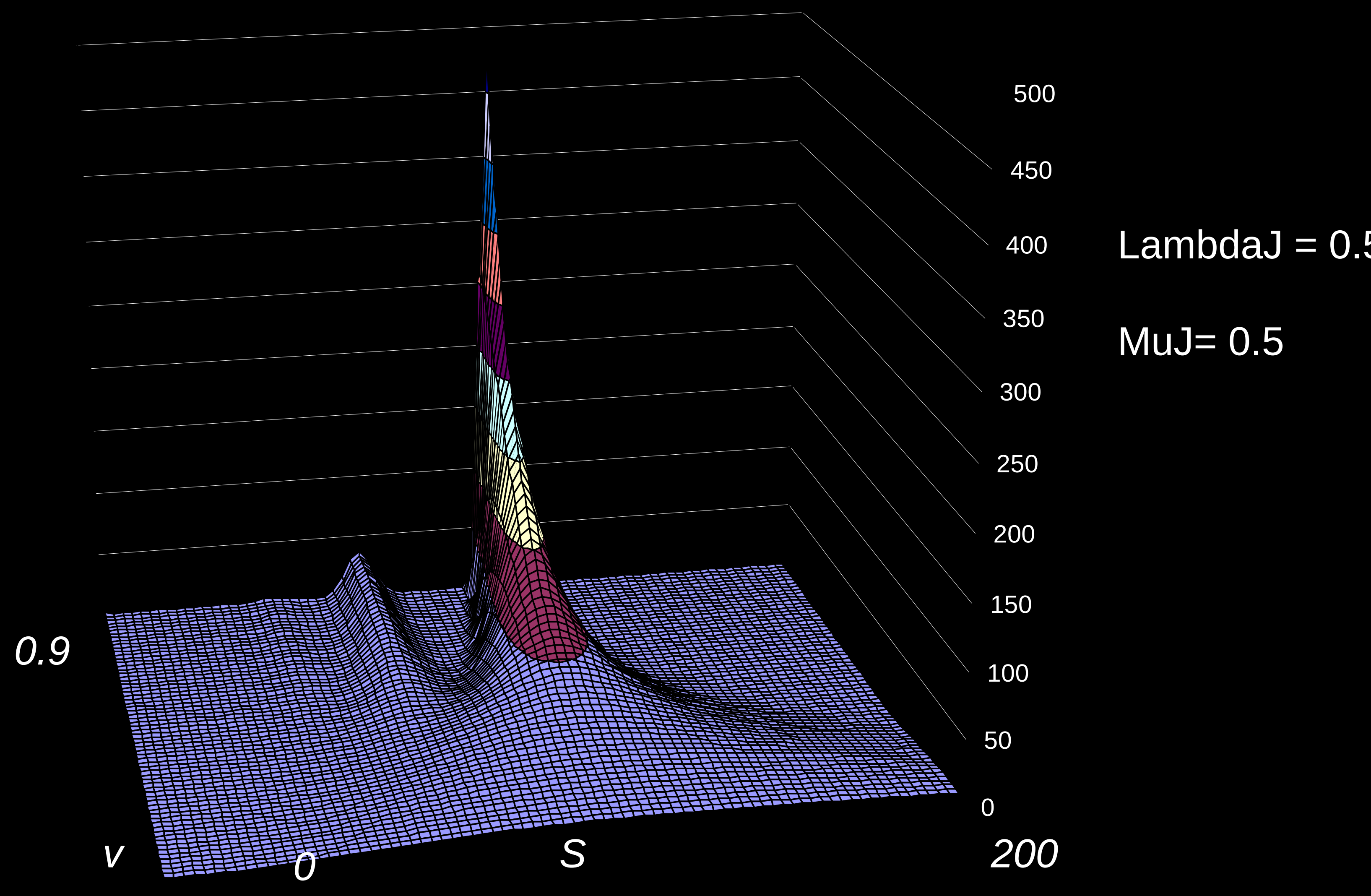
LambdaJ = 1.8

EtaJ = 0.1

MuJ= 0.5



# Merton Vega



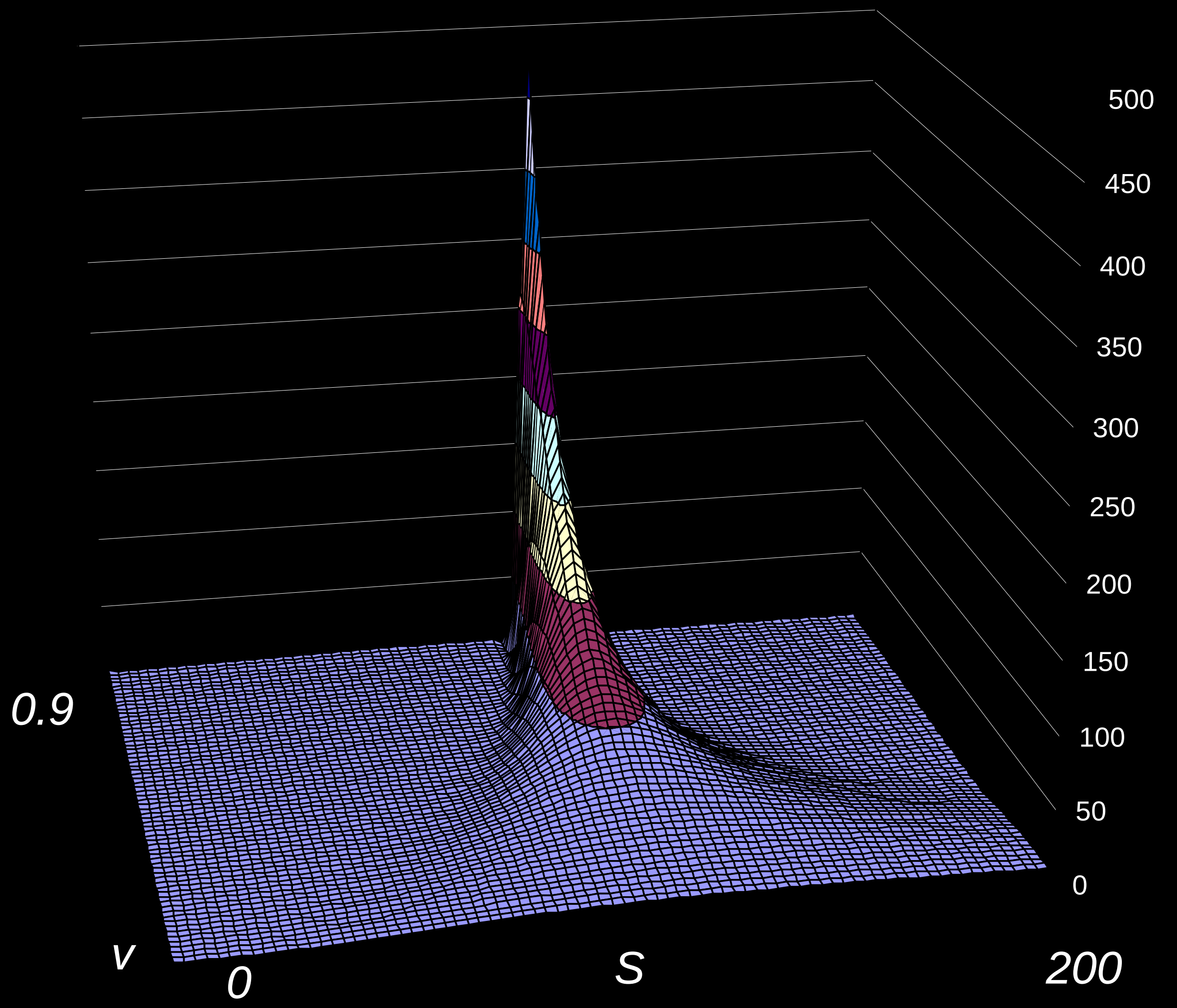
EtaJ = 0.1
LambdaJ = 0.5
MuJ= 0.5
500
450
400
350
300
250
200
150
100
50
0
0.9
v
0
S
200

# Merton Vega

EtaJ = 0.75

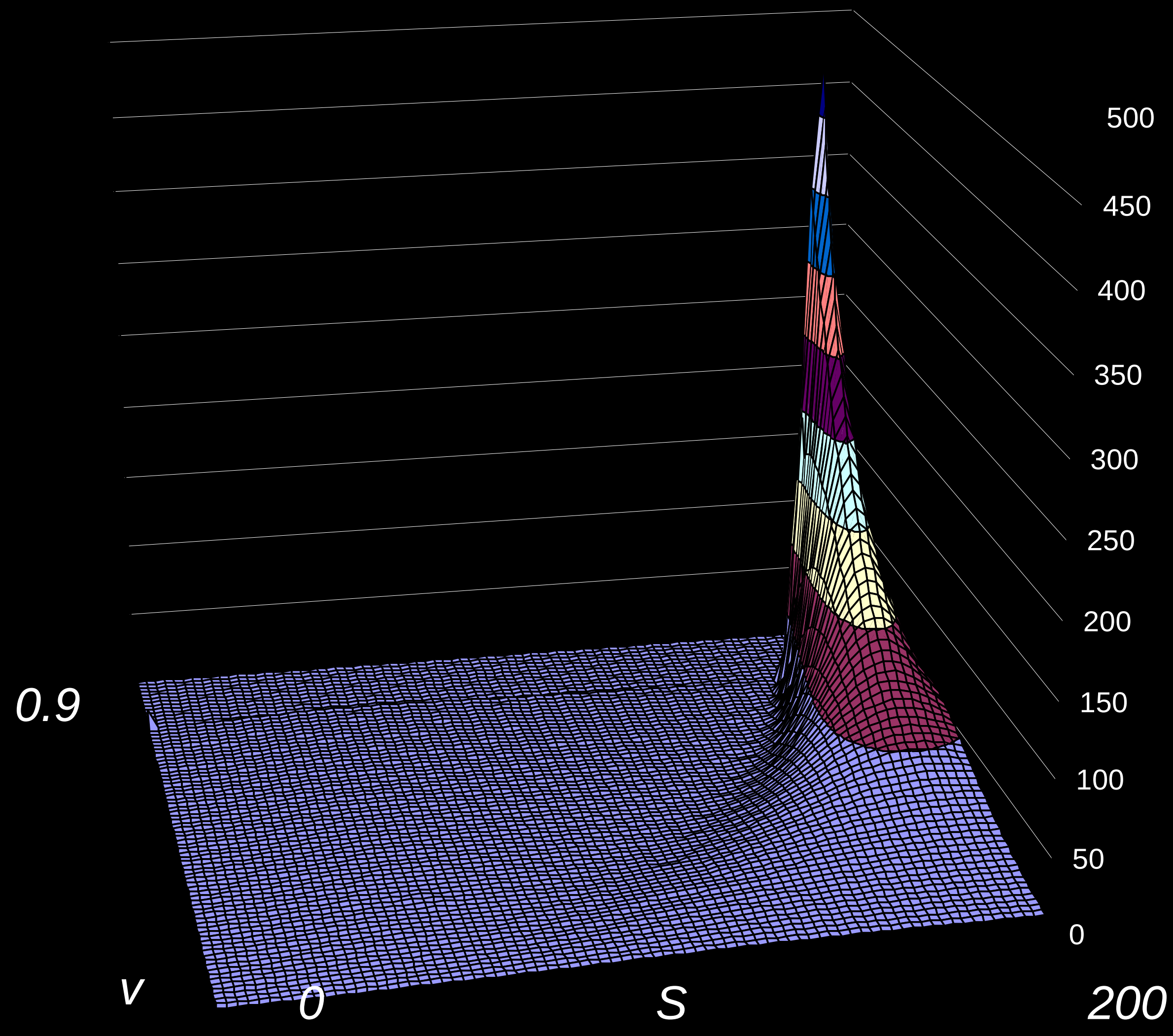
LambdaJ = 0.5

MuJ= 0.5



# Merton Vega

MuJ = 2.5




LambdaJ = 0.5

EtaJ= 0.5