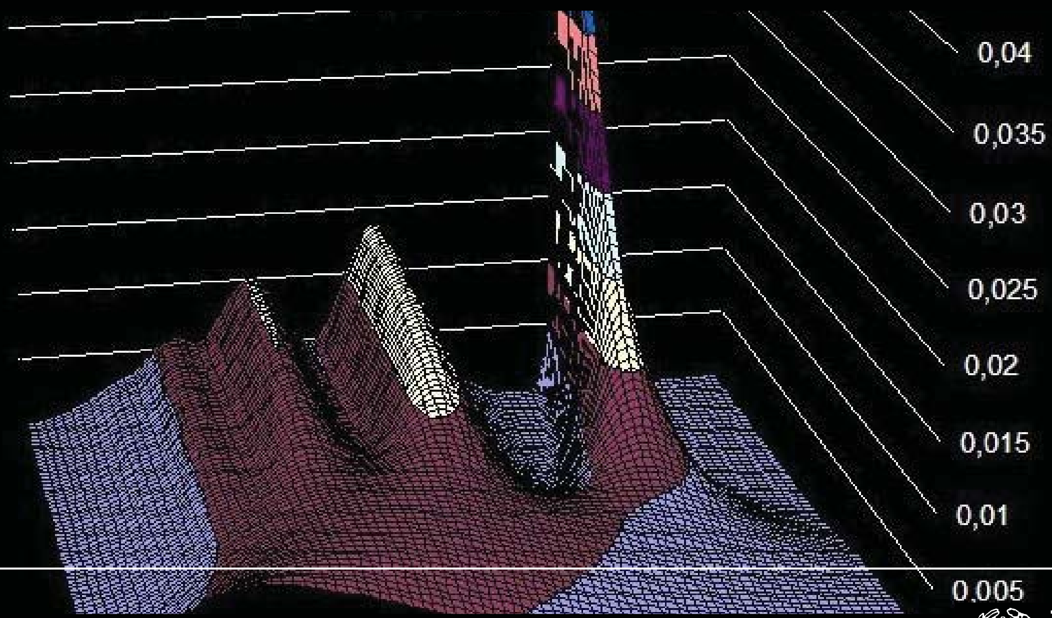
Towards a feasible calibration of Affine Jump Diffusion Models

Greeks Behavior and Inverse Fourier Transform Stability



UNIVERSITA' DEGLI STUDI DI MILANO BICOCCA

Marcello Minenna - Paolo Verzella

XXIX AMASES Conference – Sept. 15, 2005 - Palermo

CONSOB

Syllabus of the presentation

- Review of Fourier Methods in Option Pricing
- Calibration and Performance
- Greek Behaviour of New FT-Q

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Review of Fourier Methods in Option Pricing – theory

European Call   Maturity T Terminal Spot Price S_T

In AJD models Call Price can be expressed in a form close to the canonical Black – Scholes - Merton style

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau} P_2(\Theta)$$

where

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln[K])$$

under different martingale measures

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \geq \ln[K])$$

under different martingale measures

can be

determined by using the Levy's inversion formula, i.e.:

$$\Pr(\ln S_T \geq \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi\ln[K]}\tilde{f}_j(\phi)}{i\phi}\right]d\phi$$

$$\Pr(\ln S_T \geq \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi\ln[K]}\tilde{f}_j(\phi)}{i\phi}\right]d\phi$$

requires

a close formula for the Characteristic Function of the log – terminal price, i.e.:

$$\tilde{f}_T(\phi) = E\left[e^{i\phi\ln S_T}\right]$$

$$\tilde{f}_T(\phi) = E\left[e^{i\phi \ln S_T}\right]$$

has

a closed formula for AJD models

How to compute: $P_1(\Theta), P_2(\Theta)$

I

Quadrature Algorithm
FT - Q

a

Old FT - Q

b

New FT - Q

II

Fast Fourier Trasform
FFT

$P_1(\Theta), P_2(\Theta)$ through **Quadrature Algorithm**
Old FT - Q

Newton Rapson Algorithm

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau} P_2(\Theta)$$

$P_1(\Theta), P_2(\Theta)$ through **Quadrature Algorithm**
Old FT - Q

Pros (+)	**Cons (-)**
ACCURACY	STABILITY
	SPEED

$P_1(\Theta), P_2(\Theta)$ $\xleftrightarrow{\text{through}}$ **Quadrature Algorithm New FT - Q**

In order to overcome the cited problems of Old FT – Q:

$P_1(\Theta), P_2(\Theta)$ $\xleftrightarrow{\text{through}}$ **Quadrature Algorithm New FT - Q**

In order to overcome the cited problems of Old FT – Q:

- **Gauss - Legendre Quadrature Algorithm**
- **Re-adjustment of** $\widetilde{f}_T(\phi) = E\left[e^{i\phi \ln S_T}\right]$

$$\Updownarrow$$

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau} P_2(\Theta)$$

$P_1(\Theta), P_2(\Theta)$ through **Quadrature Algorithm New FT - Q**

Pros (+)

STABILITY

ACCURACY

Cons (-)

SPEED

Review of Fourier Methods in Option Pricing – practice

$P_1(\Theta), P_2(\Theta)$ through **Fast Fourier Trasform FFT**

Simpson algorithm

Applied to the equivalent formula via a recombinant FFT parameters

$$C_t = \frac{e^{-\alpha k}}{\pi} \int_0^\infty e^{-i\phi \ln[K]} \tilde{f}_j(\phi) d\phi$$

Review of Fourier Methods in Option Pricing – practice

$P_1(\Theta), P_2(\Theta)$ ⟺ through ⟺ Fast Fourier Trasform FFT

Pros (+)

SPEED

FASTER

(up to 20 times the quadrature algorithms)

Cons (-)

STABILITY

EXPLOSION

for OTM options

ACCURACY

ARBITRARY

choice of the recombinant FFT parameters

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The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

I

Quadrature Algorithm
FT - Q

a

Old FT - Q

b

New FT - Q

II

Fast Fourier Trasform
FFT

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The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

through

Quadrature Algorithm
Old FT - Q

Pros (+)

Cons (-)

STABILITY

ACCURACY

SPEED

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The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

through

Fast Fourier Trasform
FFT

Pros (+)

SPEED

Cons (-)

STABILITY

ACCURACY

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The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

through

Quadrature Algorithm
New FT - Q

Pros (+)

STABILITY

ACCURACY

SPEED

Cons (-)

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The Calibration Procedure and Performance

By keeping in mind that only New FT-Q is stable and accurate, some figures on speed

FFT	Heston Model 7.2 sec.	Merton Model 10.6 sec.	BCC Model 18.4 sec.
NEW FT - Q	Heston Model 23.1 sec.	Merton Model 29.5 sec.	BCC Model 48.7 sec.
OLD FT – Q	Heston Model 331.6 sec.	Merton Model 410.9 sec.	BCC Model 688.5 sec.

By now, the speed of Fourier Trasform method is closer than ever to the FFT calibration times

Syllabus of the presentation

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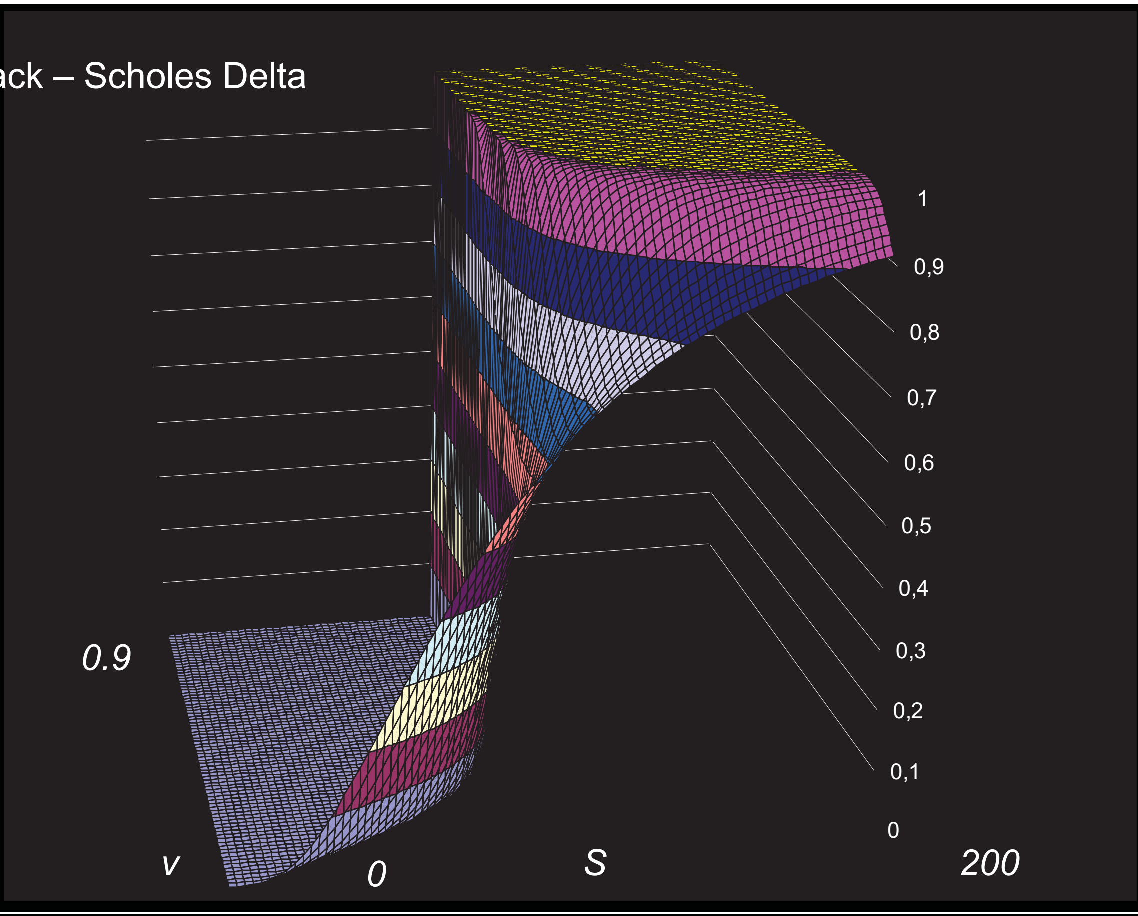
Greek behaviour of new FT-Q

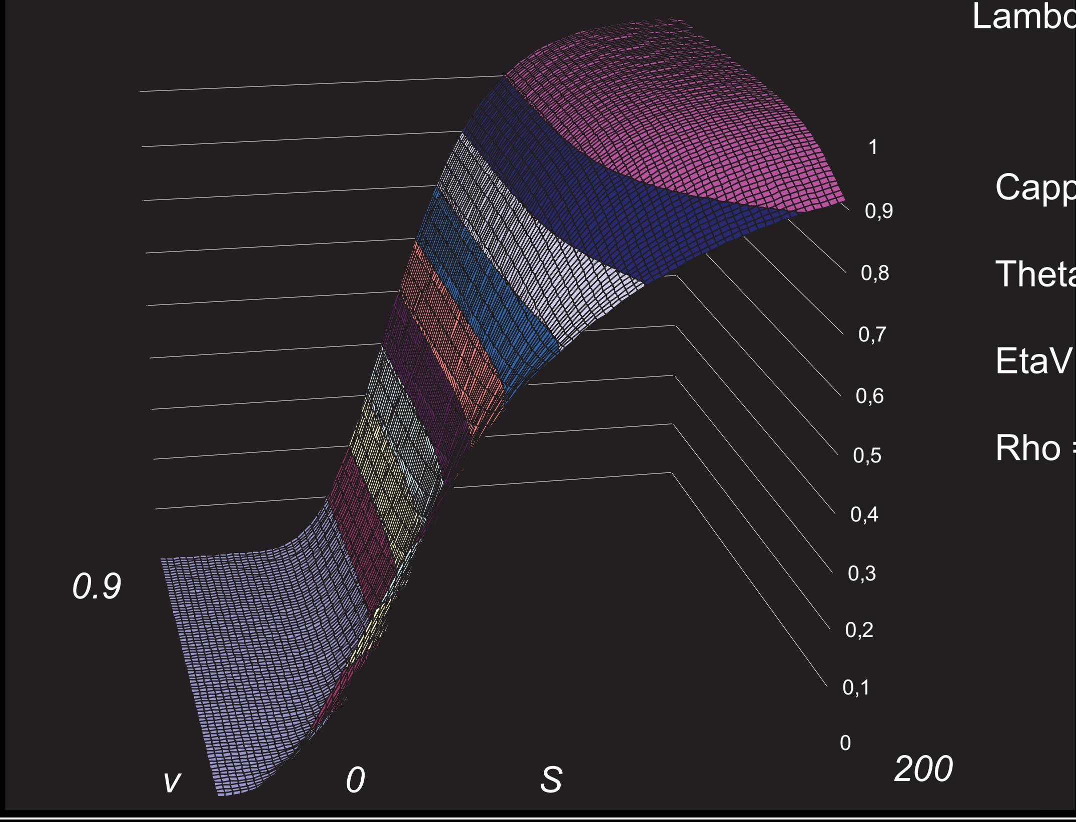
An impressive methodology to test Stability of the New FT – Quadrature algorithm is to compute Greeks

Infact, in an AJD setting the Greeks are available in closed form

So, an extended spanning of the AJD Greeks on the parameters set is useful to assess models and test Stability



Heston Delta



Lambda = -2

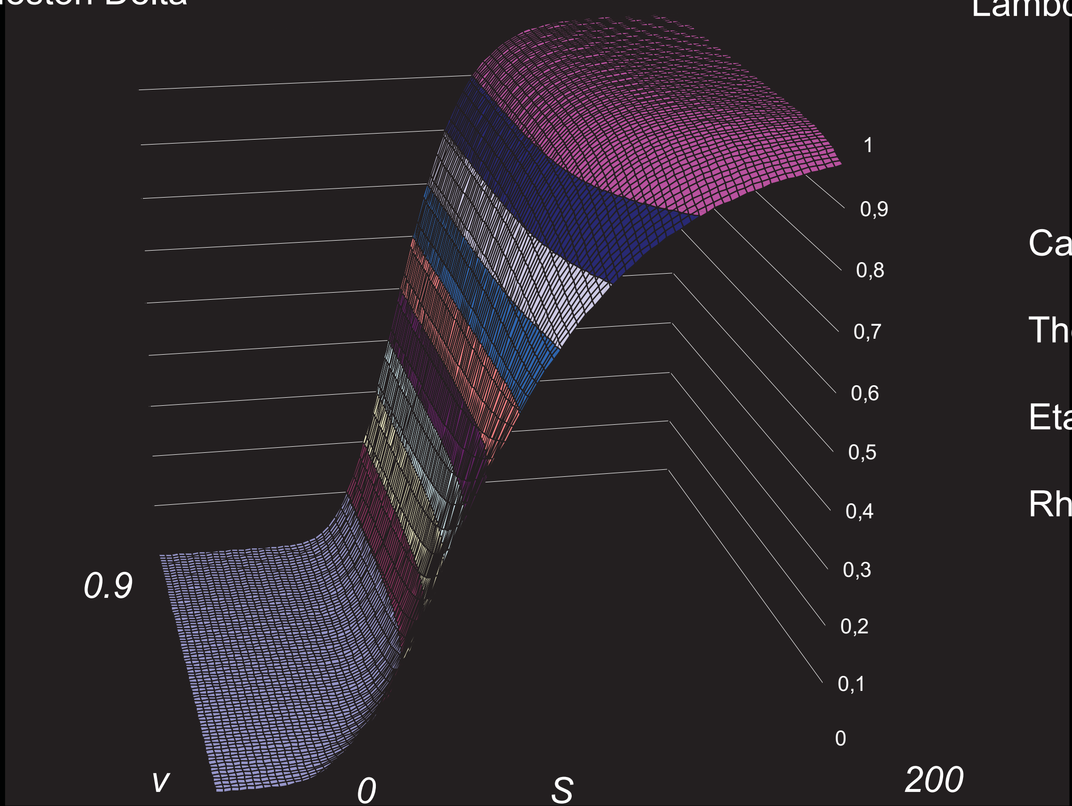
CappaV = 2

ThetaV = 0.3

EtaV = 0.1

Rho = 0

Heston Delta



Lambda = 2

CappaV = 2

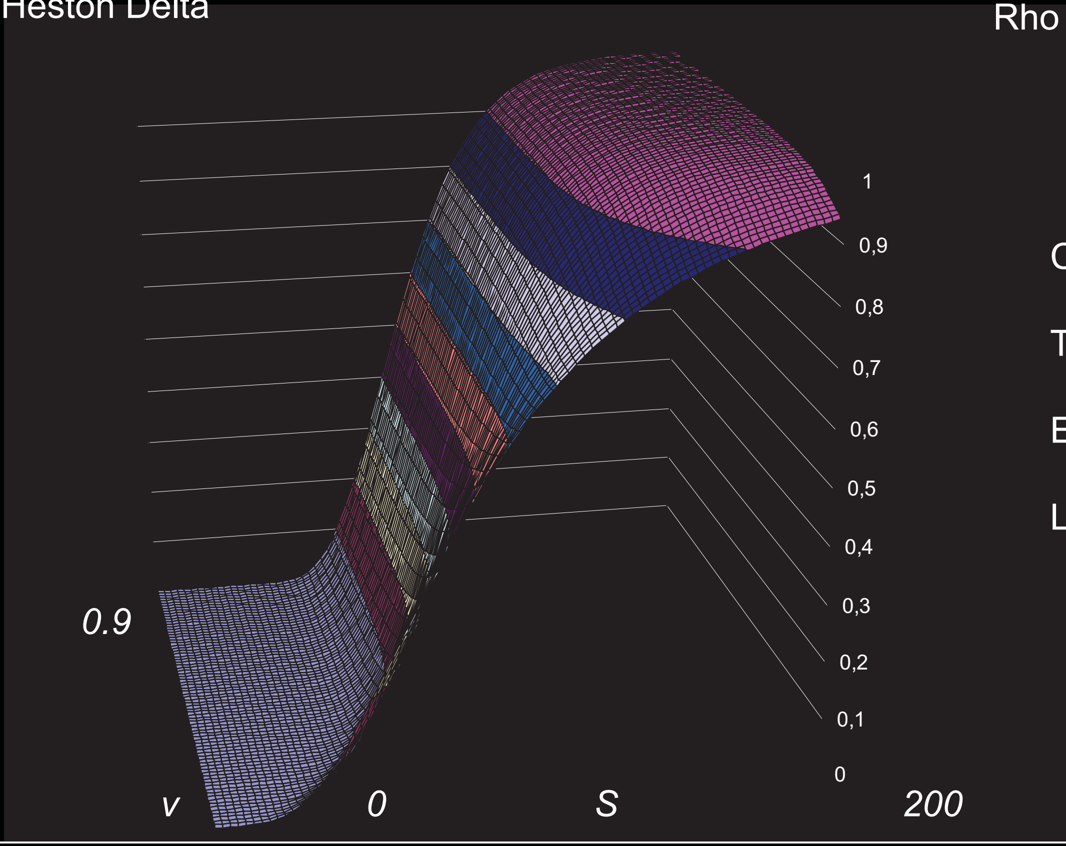
ThetaV = 0.3

EtaV = 0.1

Rho = 0

Heston Delta

Rho = -1



CappaV = 2

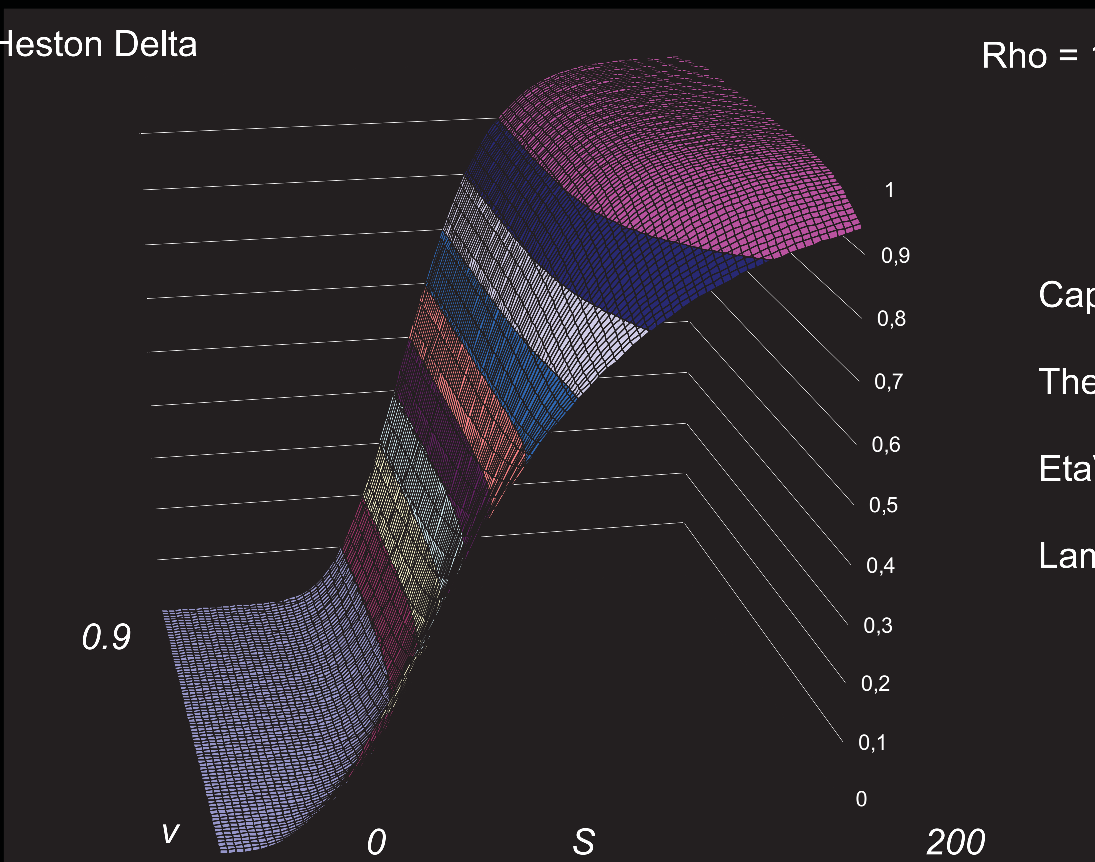
ThetaV = 0.3

EtaV = 0.1

Lambda = 0

Heston Delta

Rho = 1



CappaV = 2

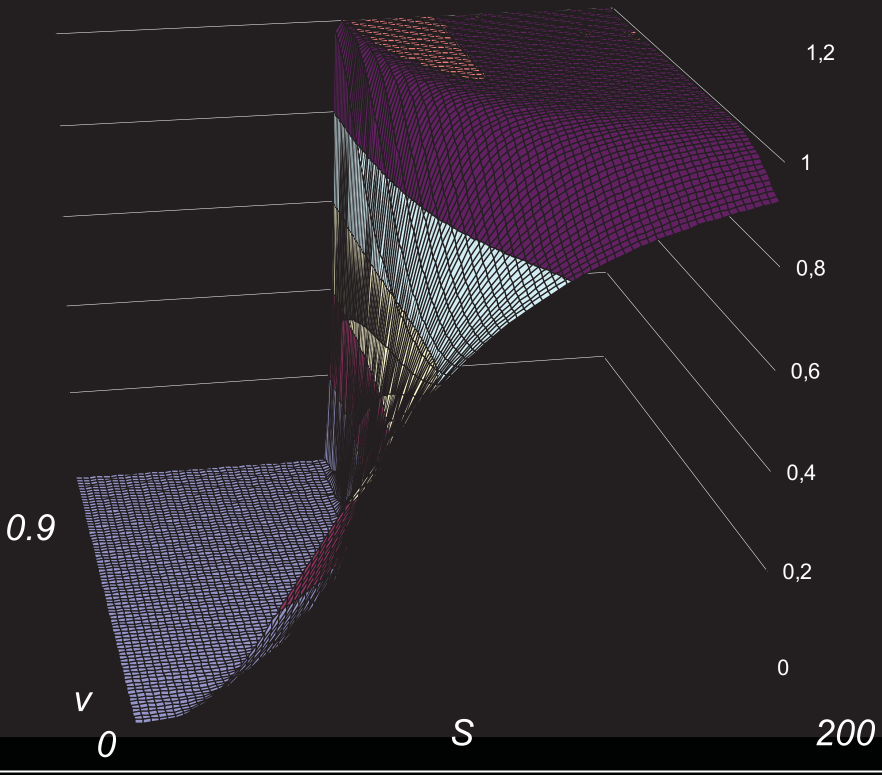
ThetaV = 0.3

EtaV = 0.1

Lambda = 0

Merton Delta

LambdaJ = 0

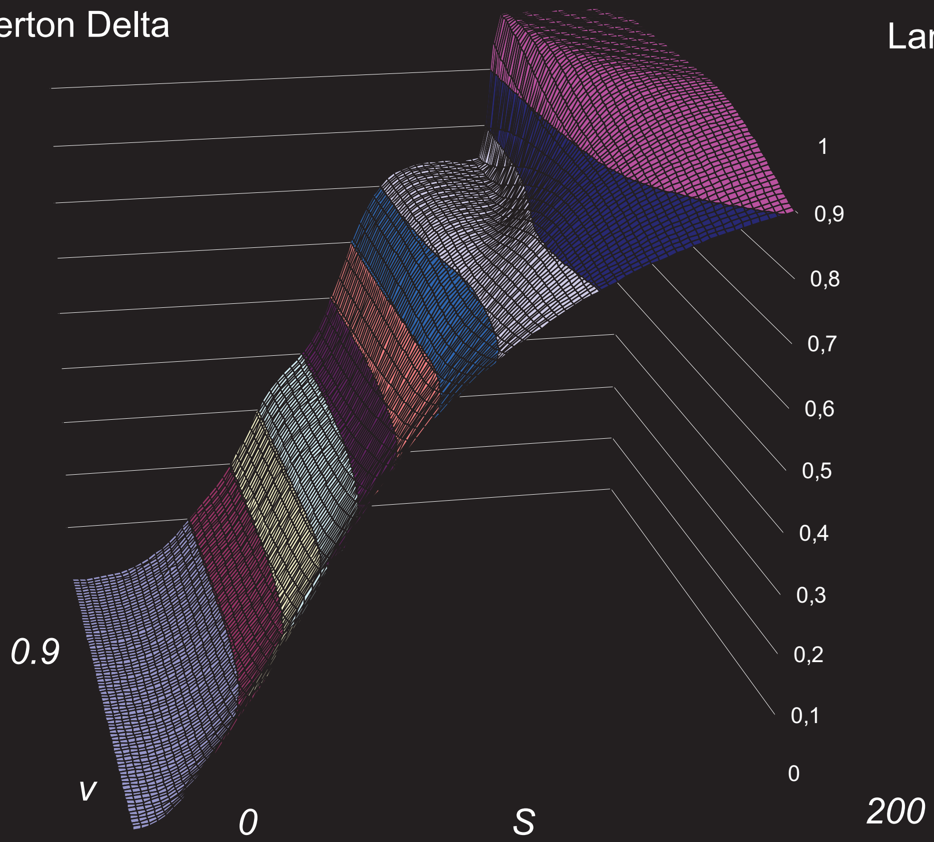


EtaJ = 0

MuJ= 0

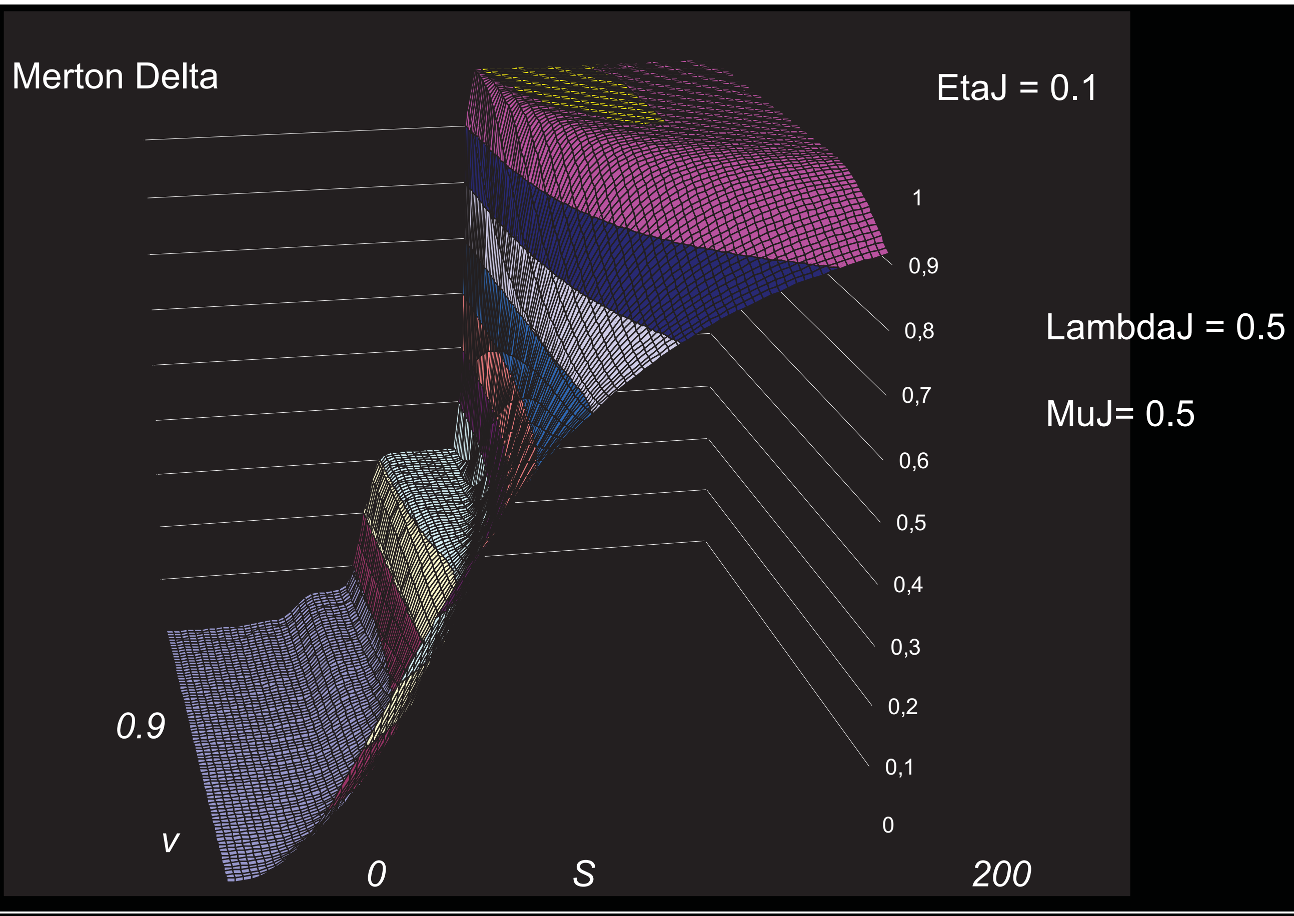
Merton Delta

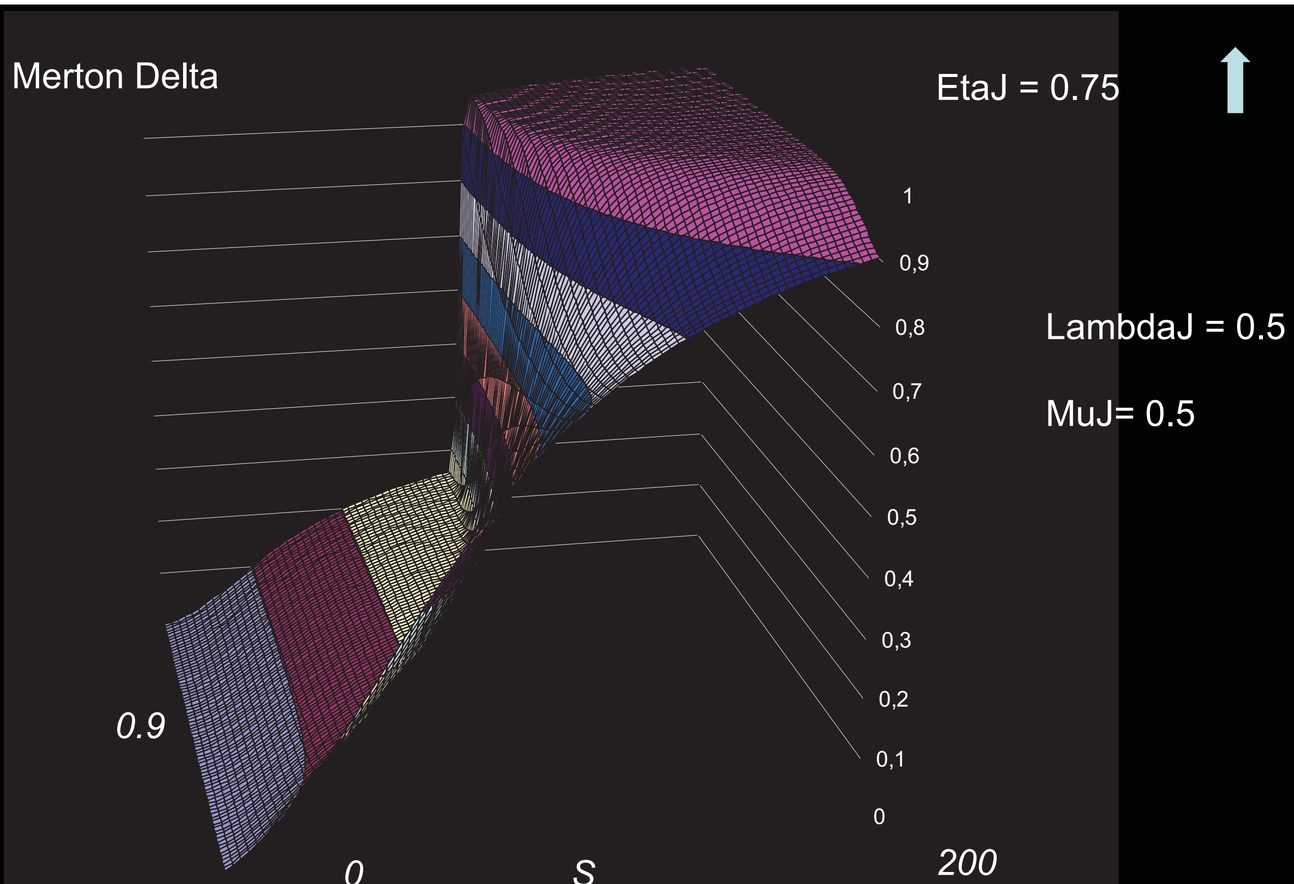
LambdaJ = 1.8

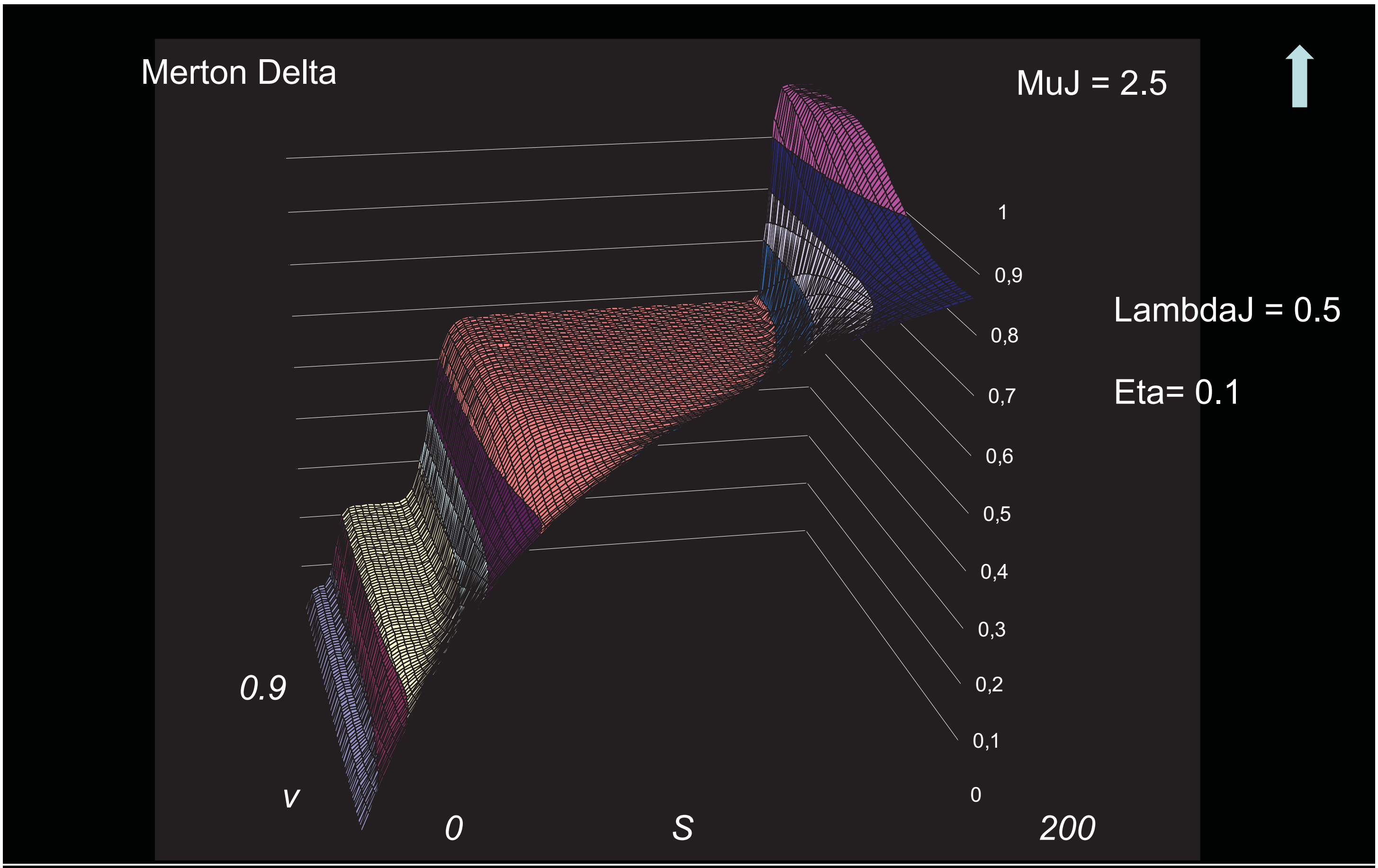


EtaJ = 0.1

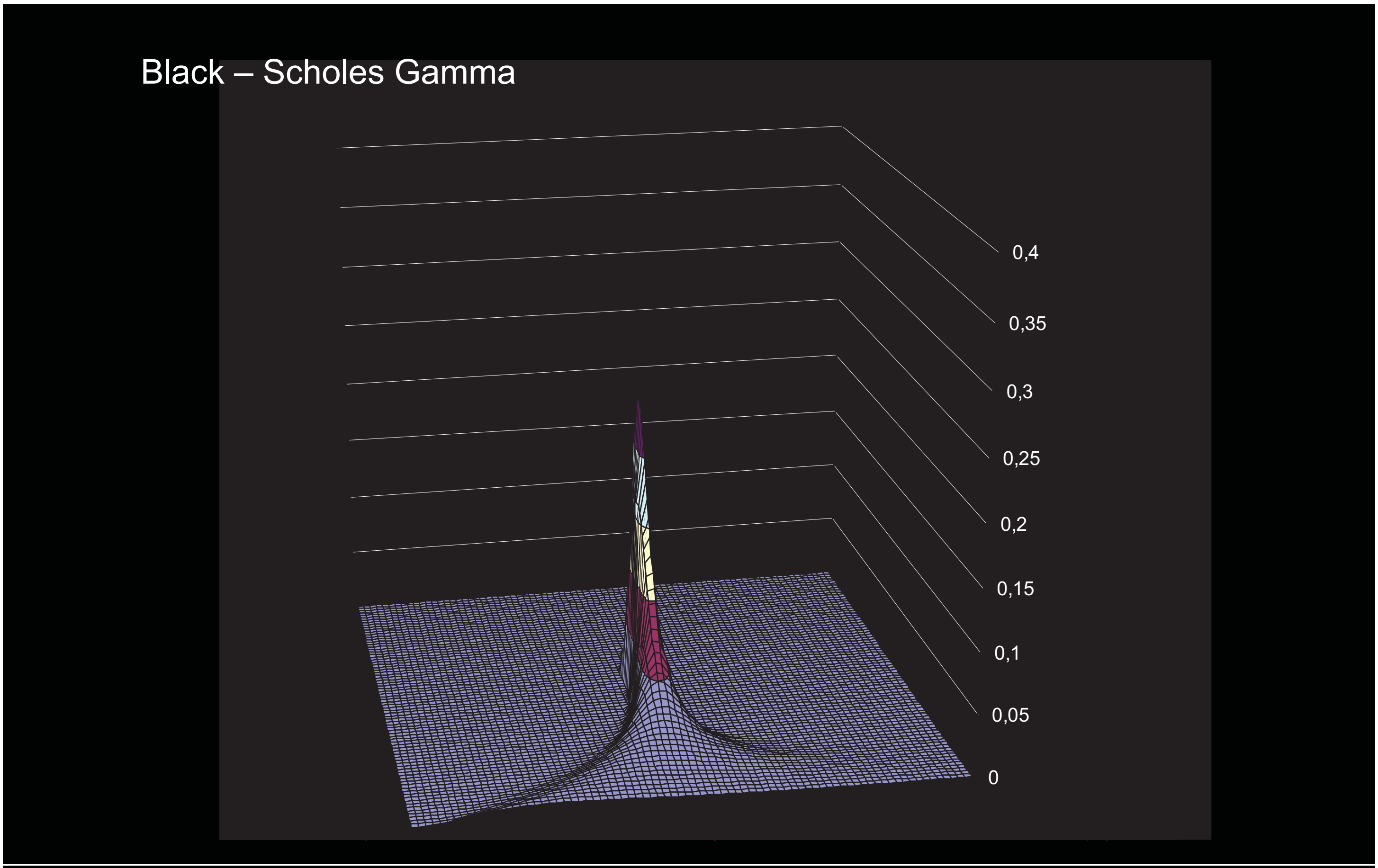
MuJ= 0.5


Merton Delta
EtaJ = 0.1
LambdaJ = 0.5
MuJ= 0.5
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0.9
v
0
S
200


Merton Delta
EtaJ = 0.75
LambdaJ = 0.5
MuJ= 0.5
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0.9
0
S
200

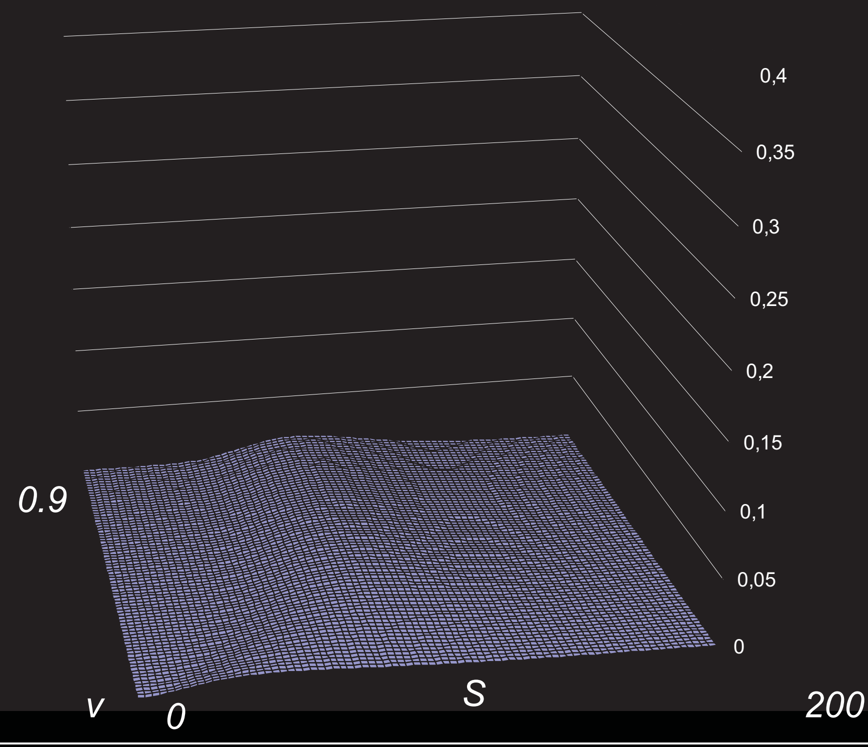


Merton Delta
MuJ = 2.5
LambdaJ = 0.5
Eta= 0.1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0.9
v
0
S
200



Black – Scholes Gamma
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0

Heston Gamma



Lambda = -2

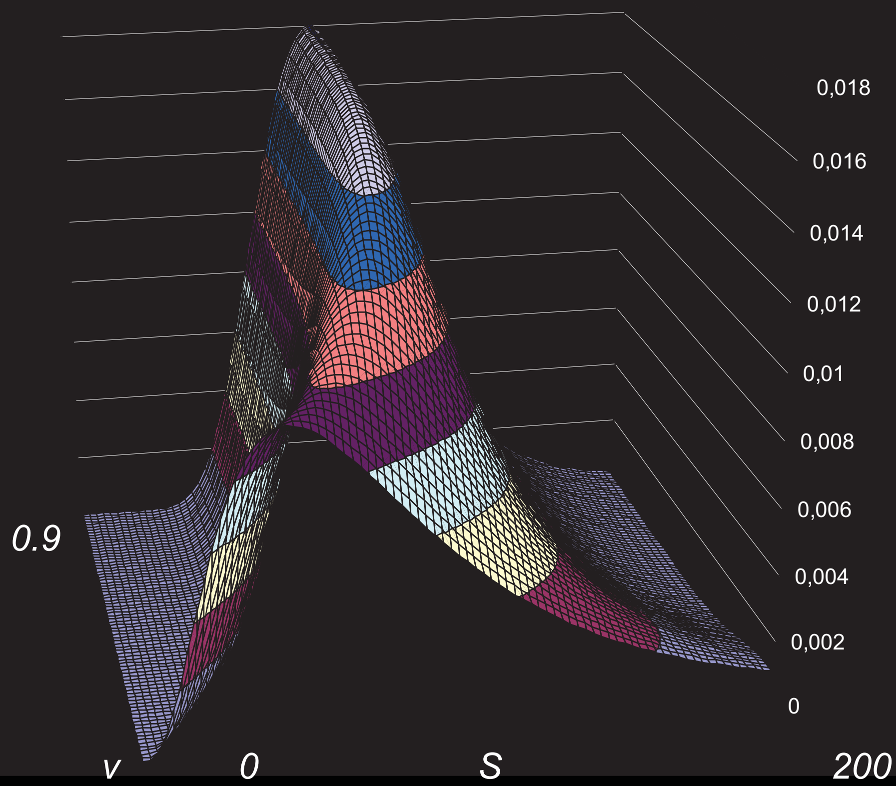
CappaV = 2

ThetaV = 0.3

EtaV = 0.1

Rho = 0

Heston Gamma



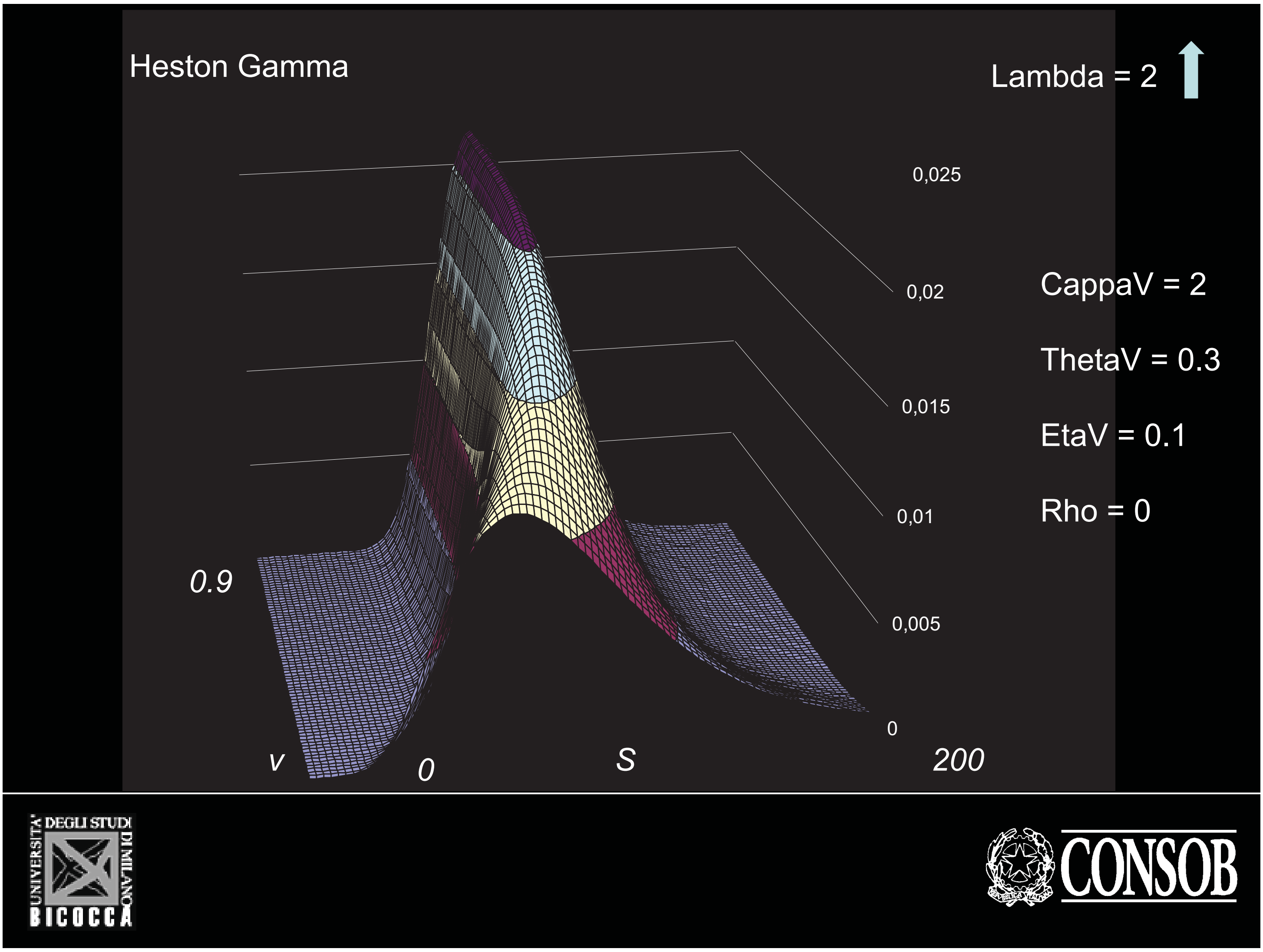
Lambda = -2

CappaV = 2

ThetaV = 0.3

EtaV = 0.1

Rho = 0


Heston Gamma
Lambda = 2
CappaV = 2
ThetaV = 0.3
EtaV = 0.1
Rho = 0
0,025
0,02
0,015
0,01
0,005
0
0.9
v
0
S
200
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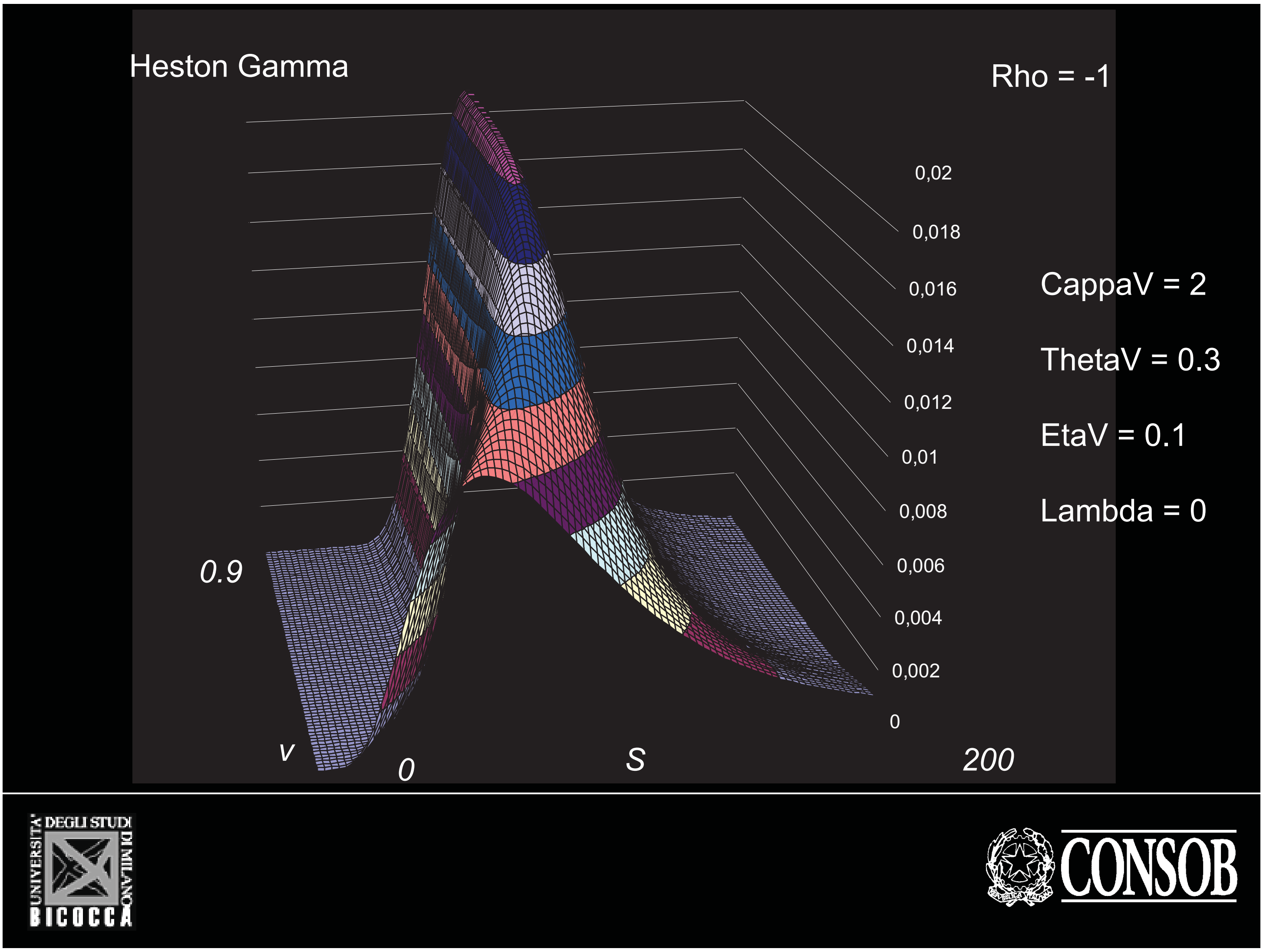
Heston Gamma
Rho = -1
CappaV = 2
ThetaV = 0.3
EtaV = 0.1
Lambda = 0
0,02
0,018
0,016
0,014
0,012
0,01
0,008
0,006
0,004
0,002
0
0.9
v
0
S
200
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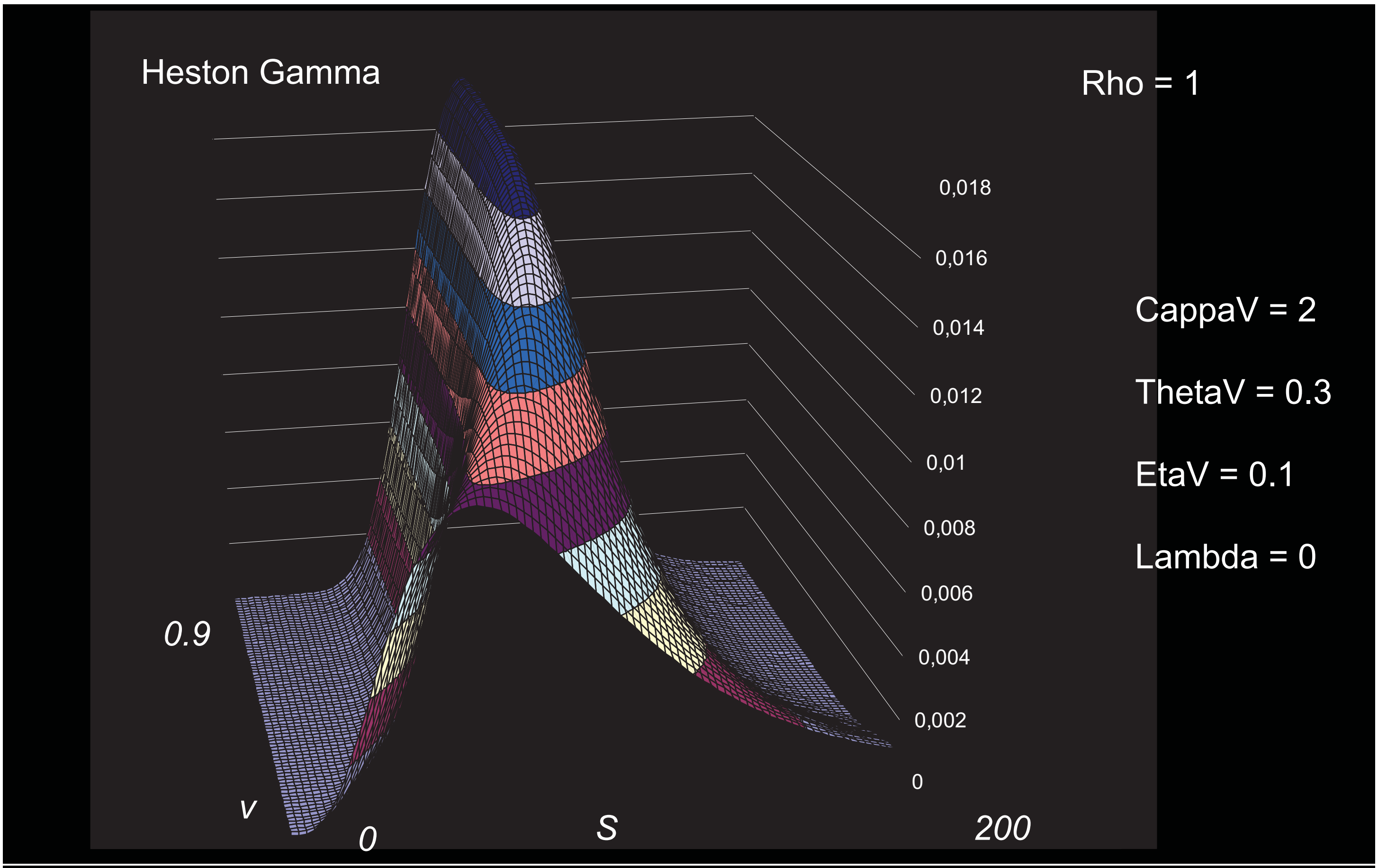
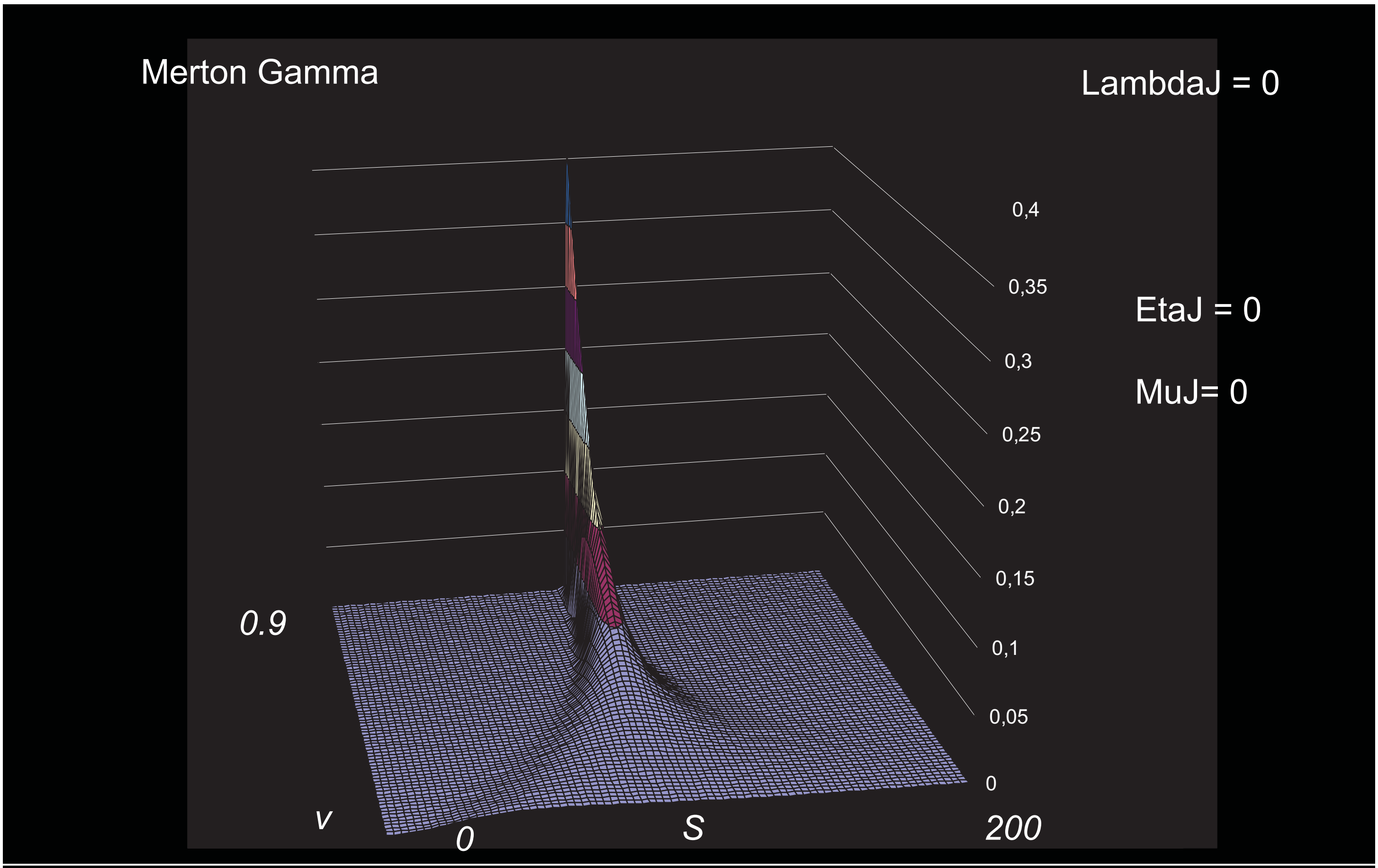


Heston Gamma
Rho = 1
CappaV = 2
ThetaV = 0.3
EtaV = 0.1
Lambda = 0
0,018
0,016
0,014
0,012
0,01
0,008
0,006
0,004
0,002
0
0.9
v
0
S
200



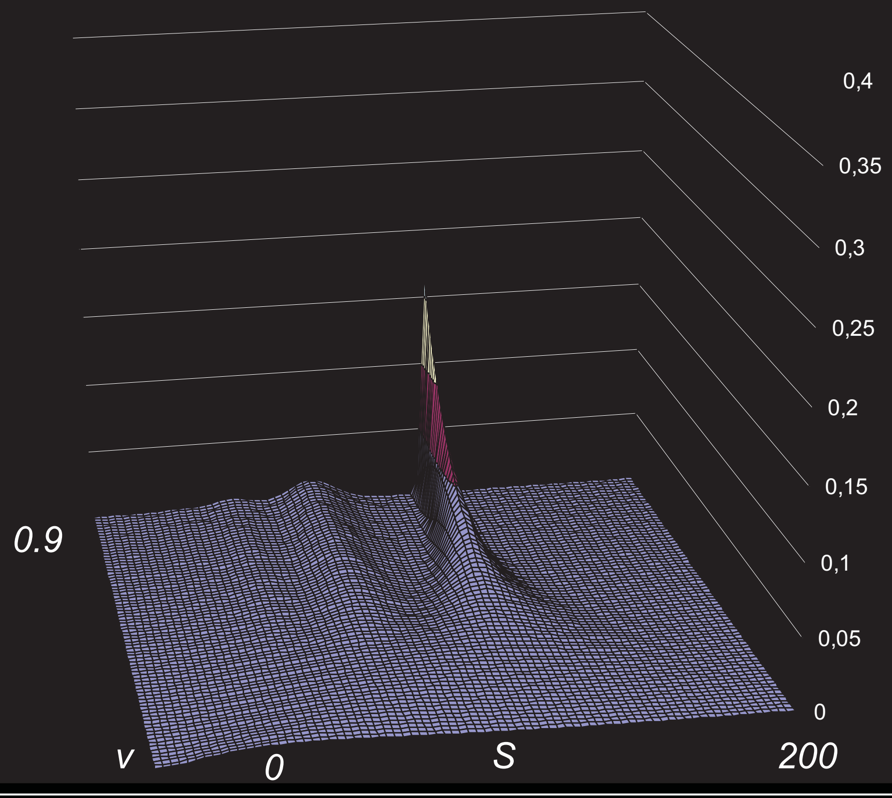
Merton Gamma
LambdaJ = 0
EtaJ = 0
MuJ= 0
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
0.9
v
0
S
200

Merton Gamma

LambdaJ = 0.9

EtaJ = 0.1

MuJ = 0.5

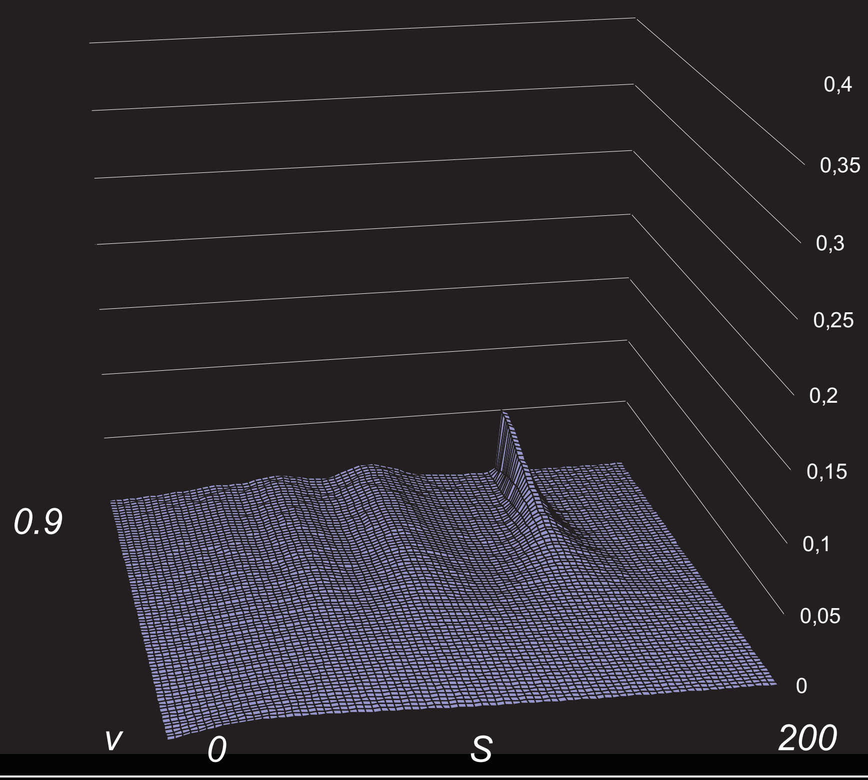


Merton Gamma

LambdaJ = 1.8

EtaJ = 0.1

MuJ = 0.5

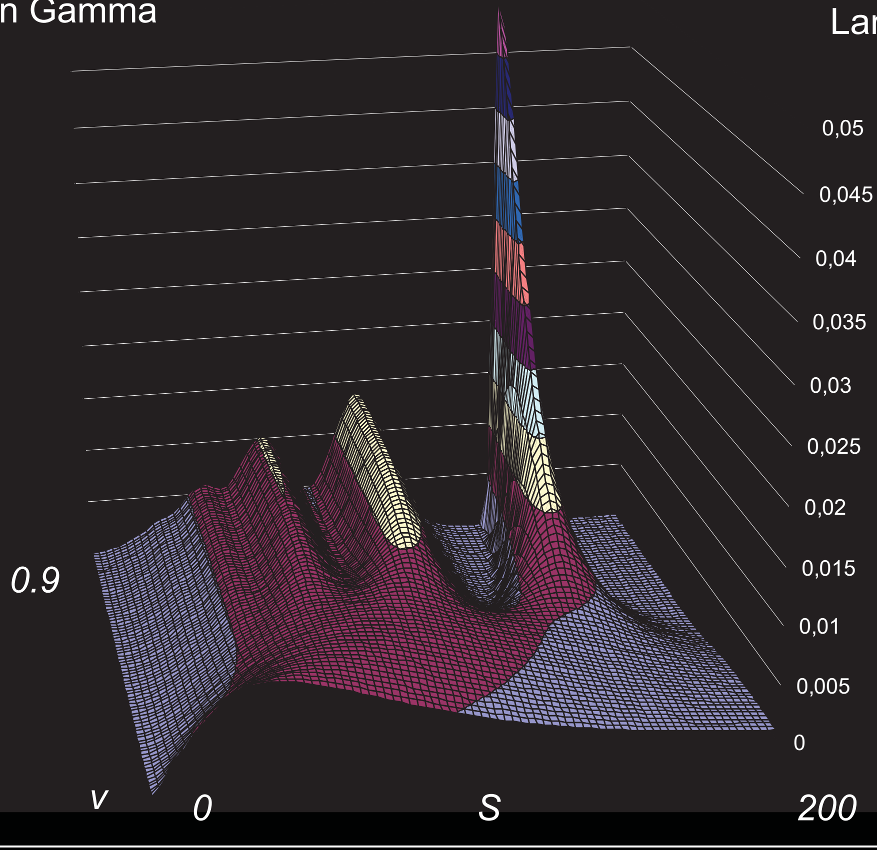


Merton Gamma

LambdaJ = 1.8

EtaJ = 0.1

MuJ= 0.5

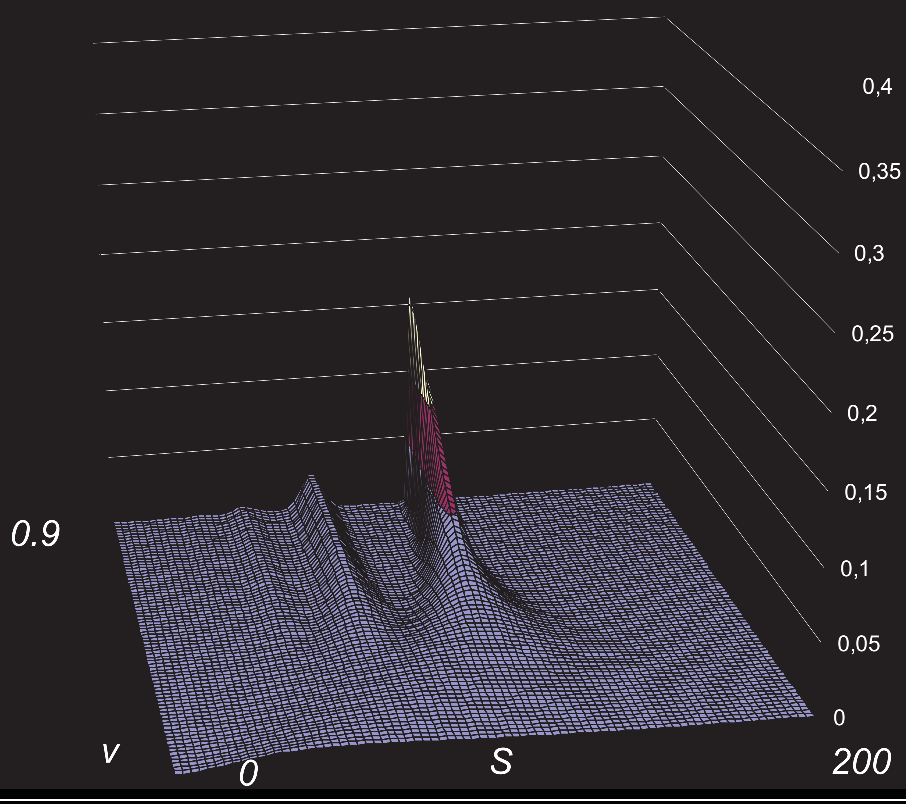


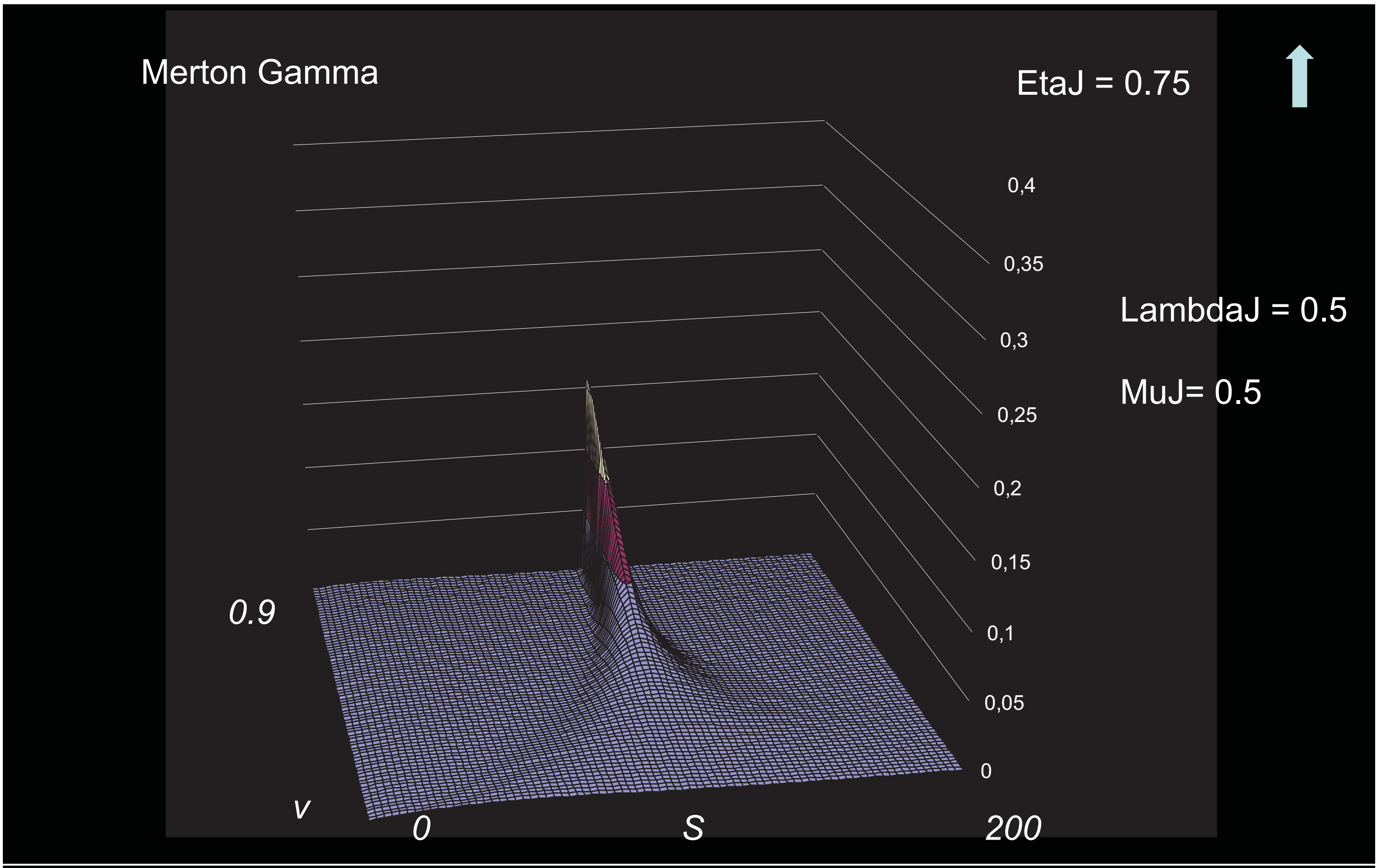
Merton Gamma

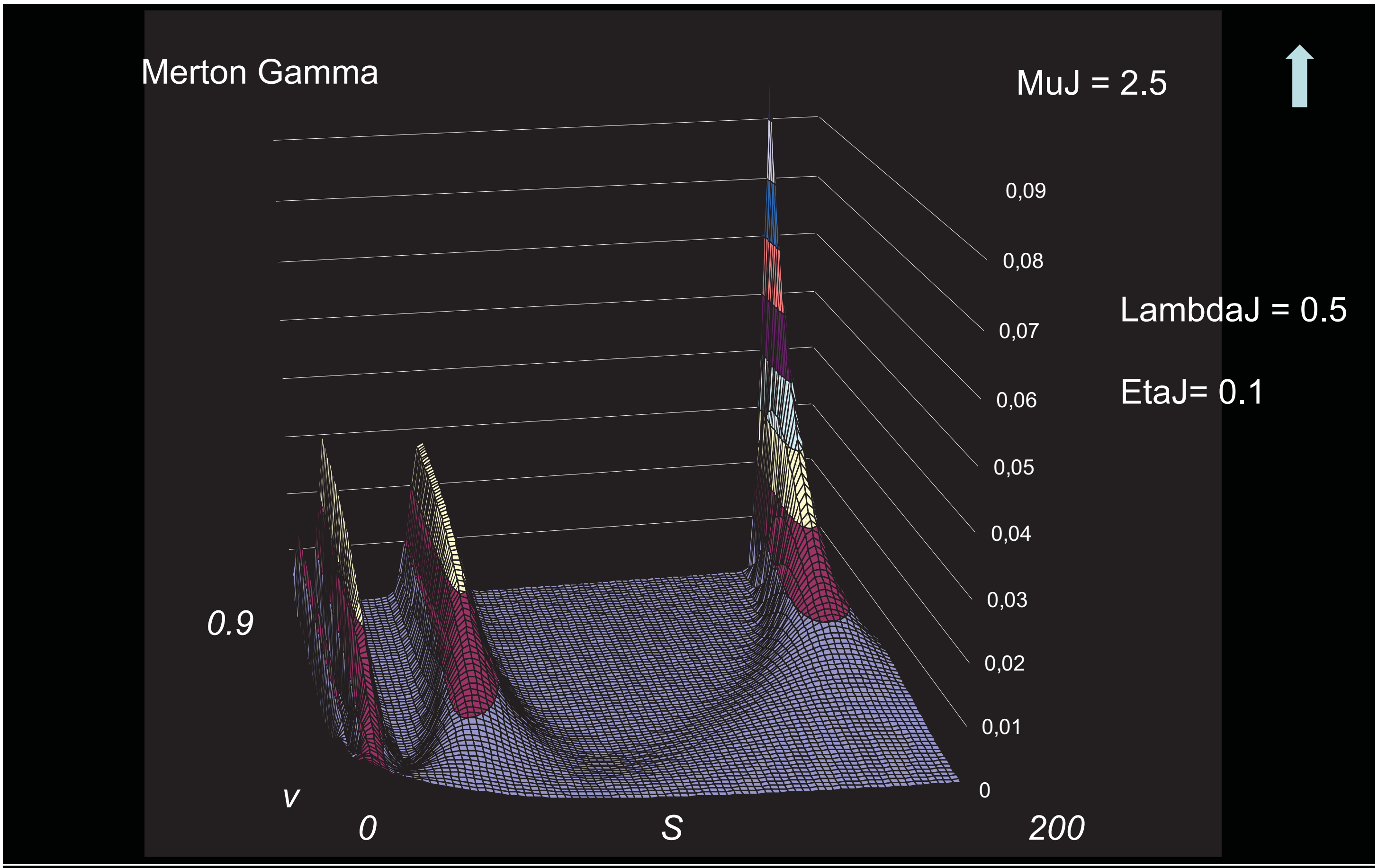
EtaJ = 0.1

LambdaJ = 0.5

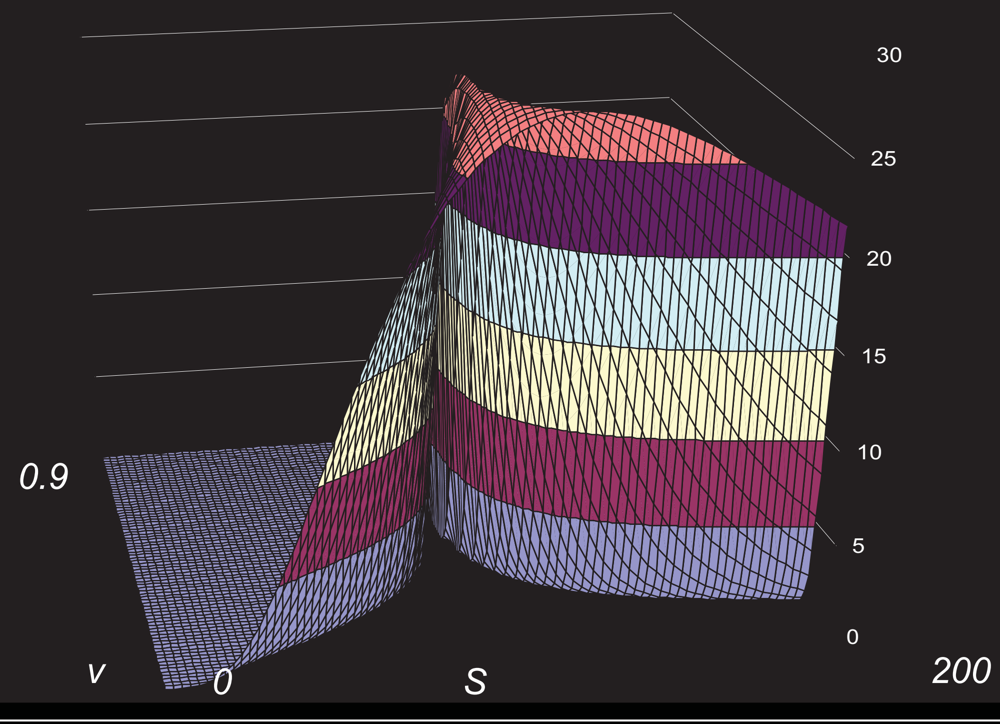
MuJ= 0.5



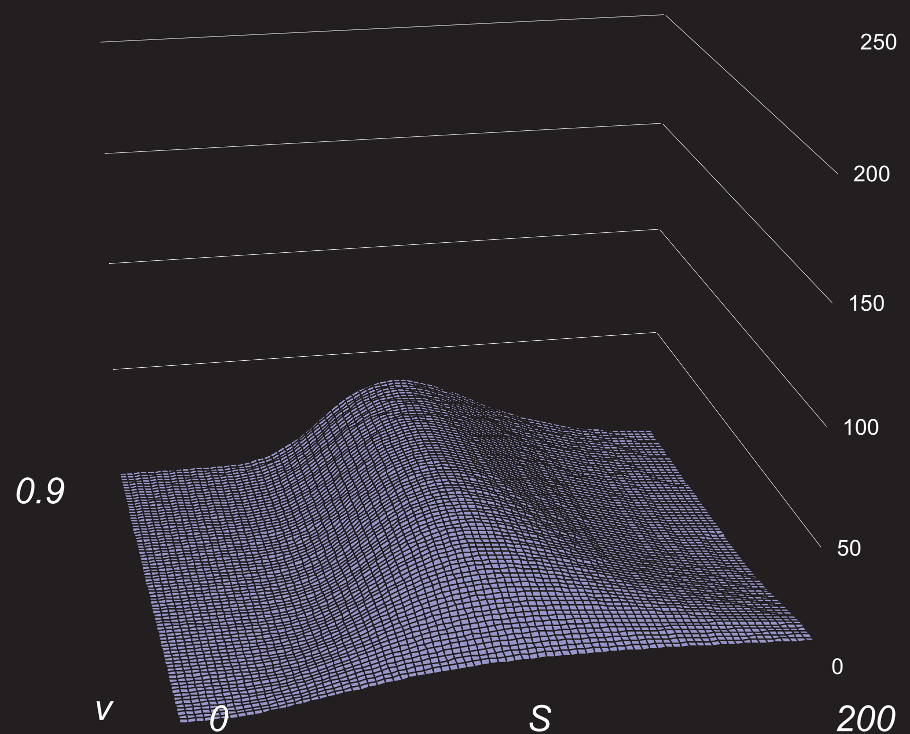

Merton Gamma
EtaJ = 0.75
LambdaJ = 0.5
MuJ= 0.5
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
0.9
v
0
S
200


Merton Gamma
MuJ = 2.5
LambdaJ = 0.5
EtaJ= 0.1
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
0.9
v
0
S
200

Black – Scholes Vega



Heston Vega



Lambda = -2

CappaV = 2

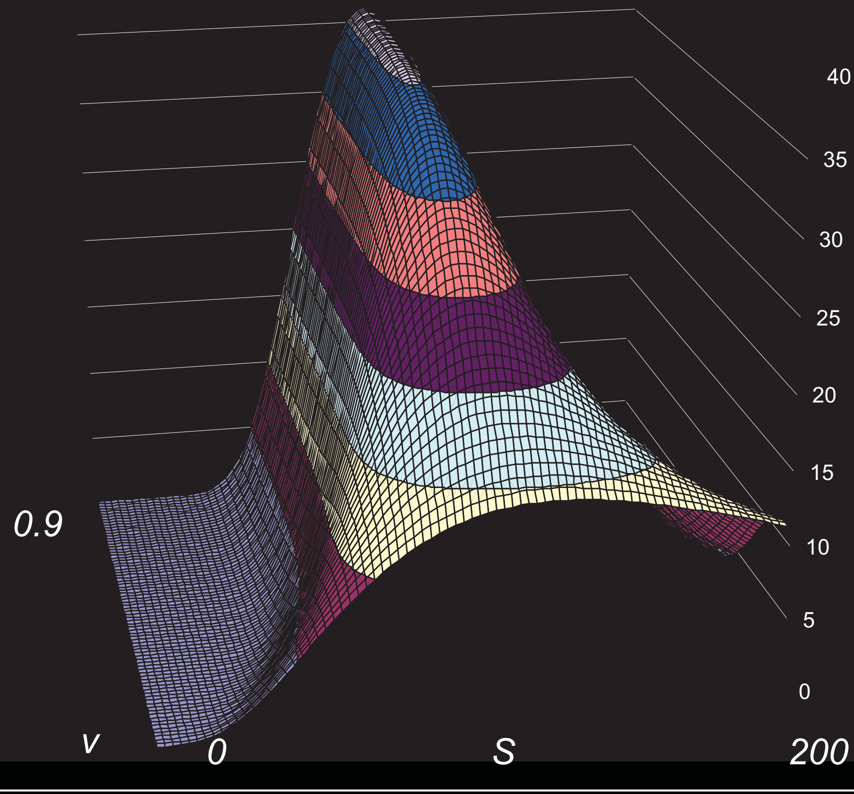
ThetaV = 0.3

EtaV = 0.1

Rho = 0

Heston Vega

Lambda = -2



CappaV = 2

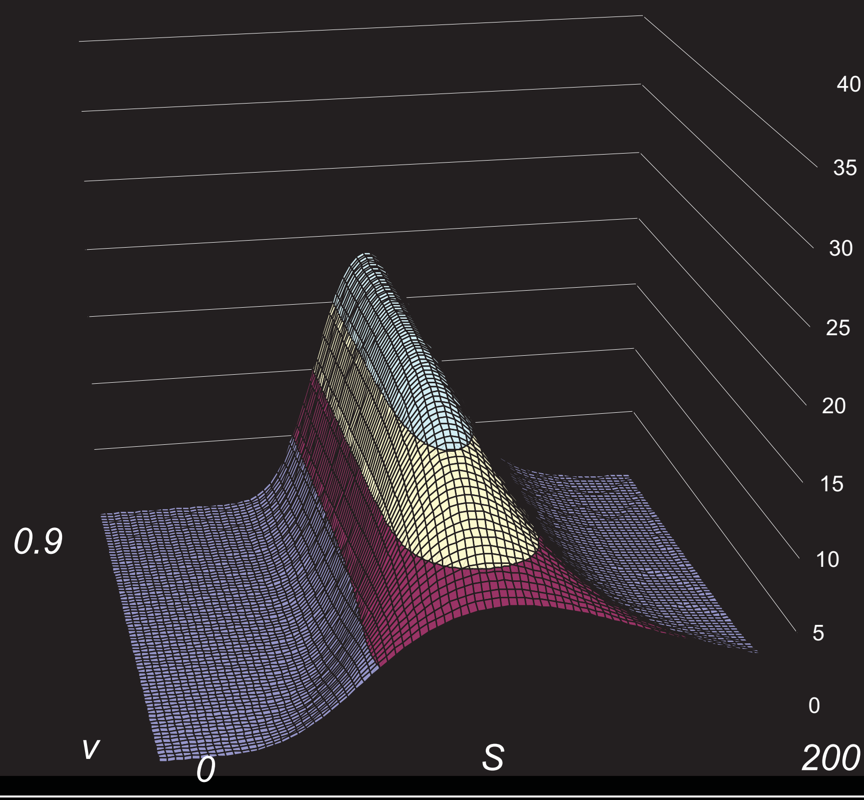
ThetaV = 0.3

EtaV = 0.1

Rho = 0

Heston Vega

Lambda = 2



CappaV = 2

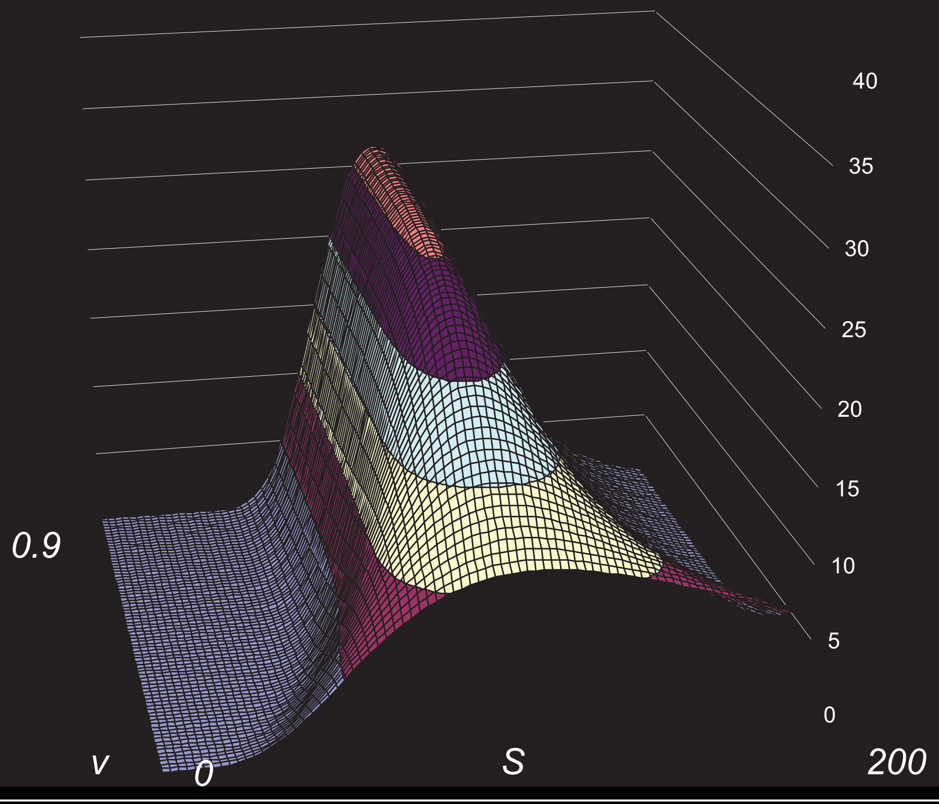
ThetaV = 0.3

EtaV = 0.1

Rho = 0

Heston Vega

Rho = -1



CappaV = 2

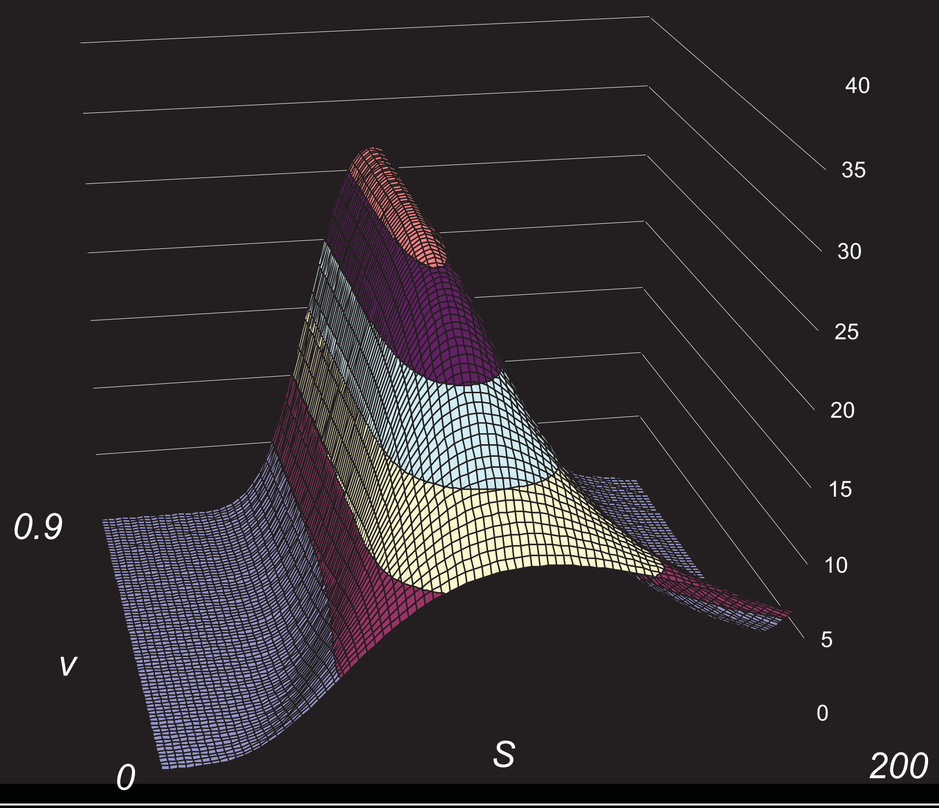
ThetaV = 0.3

EtaV = 0.1

Lambda = 0

Heston Vega

Rho = 1



CappaV = 2

ThetaV = 0.3

EtaV = 0.1

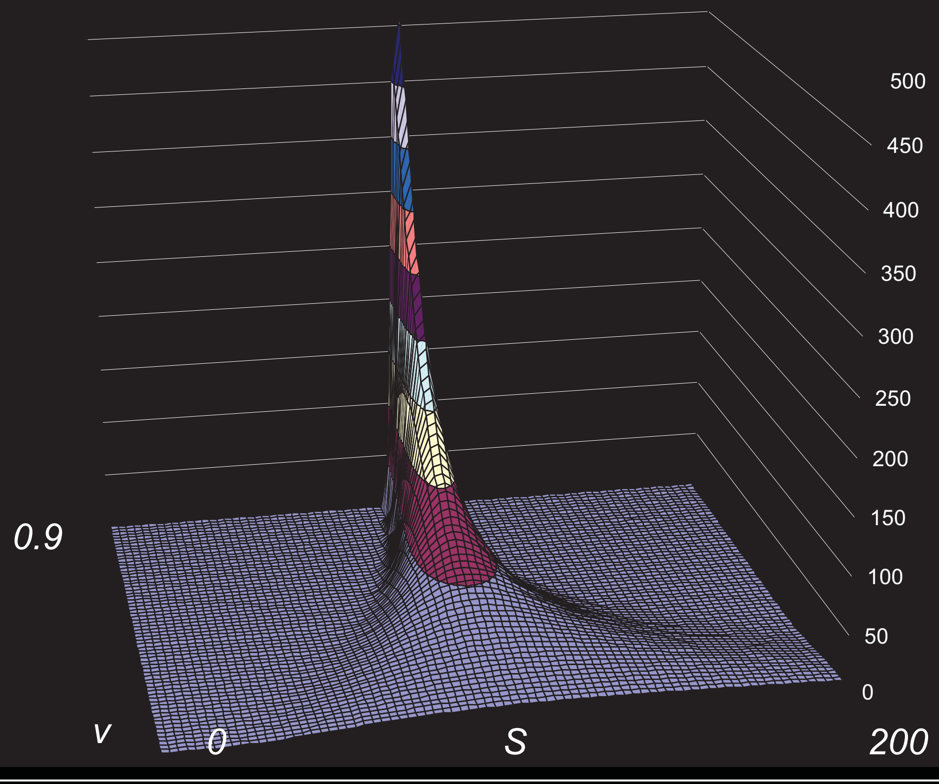
Lambda = 0

Merton Vega

LambdaJ = 0

EtaJ = 0

MuJ= 0



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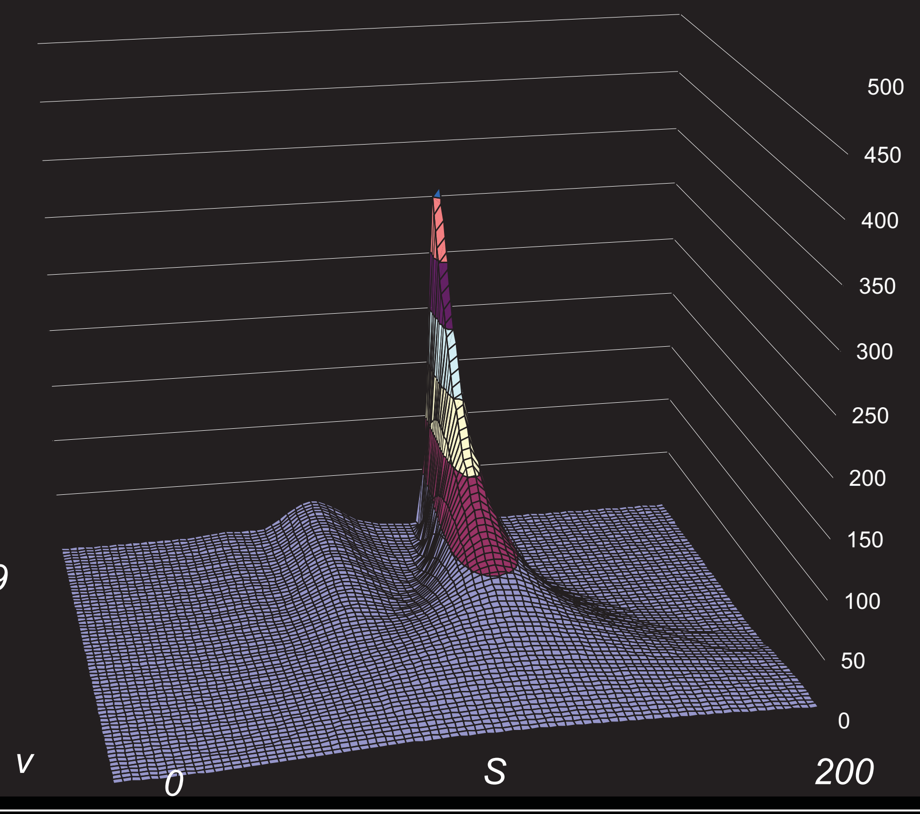
Merton Vega

LambdaJ = 0.9

EtaJ = 0.1

MuJ = 0.5



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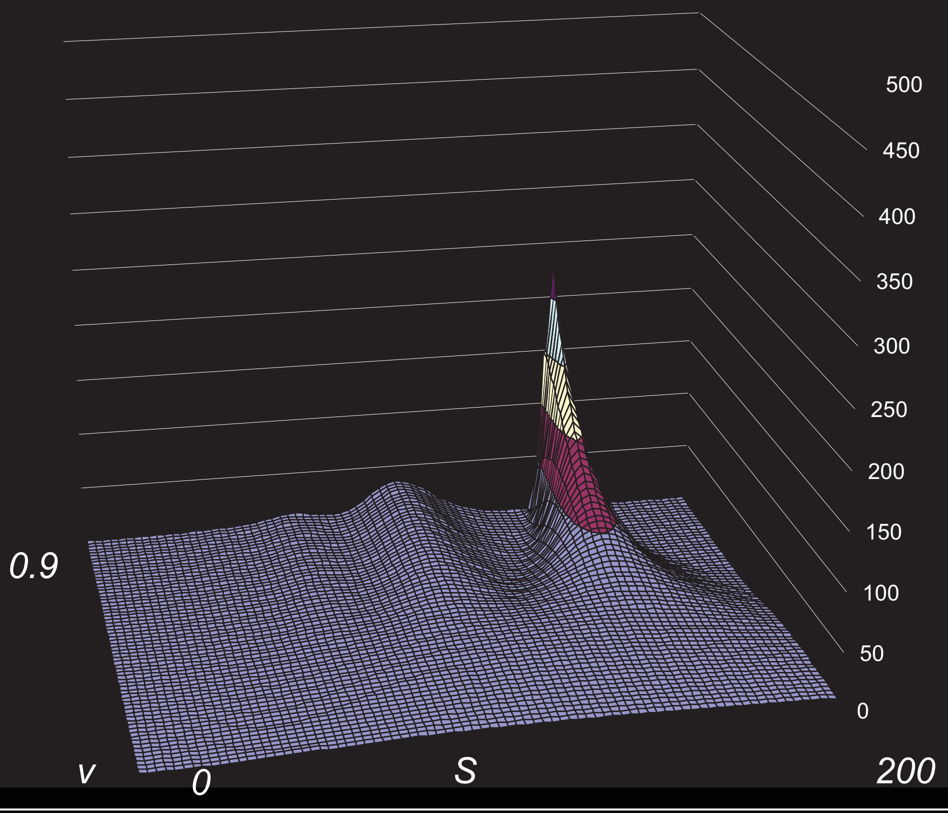
Merton Vega

LambdaJ = 1.8

EtaJ = 0.1

MuJ= 0.5

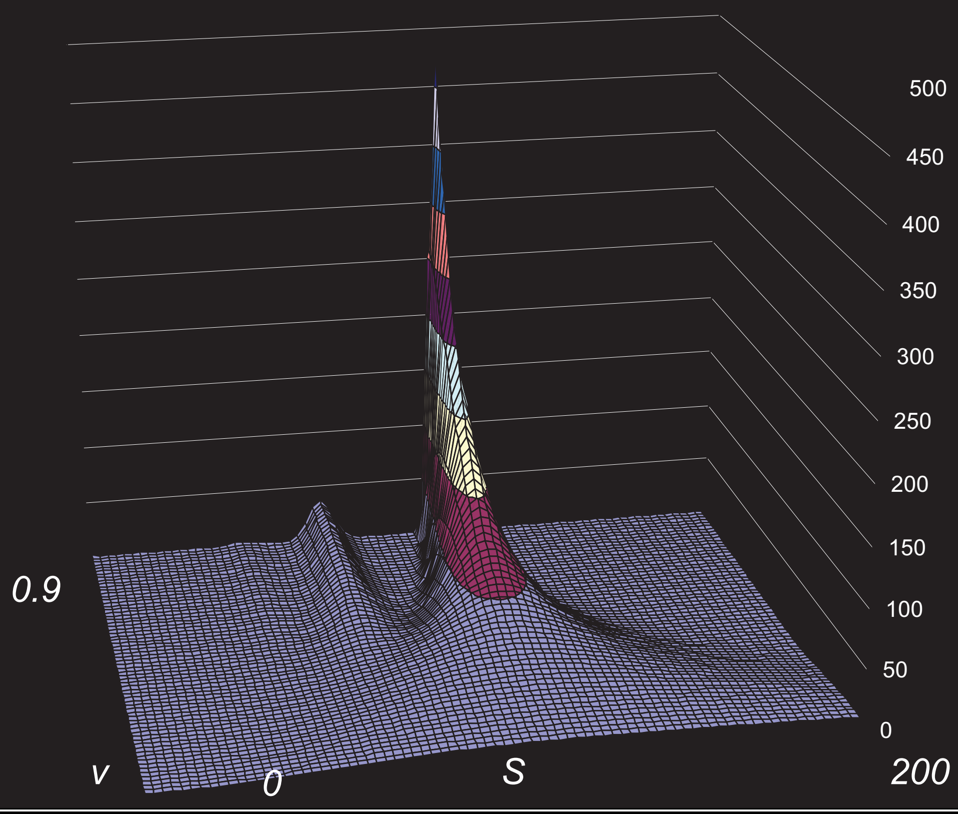


Merton Vega

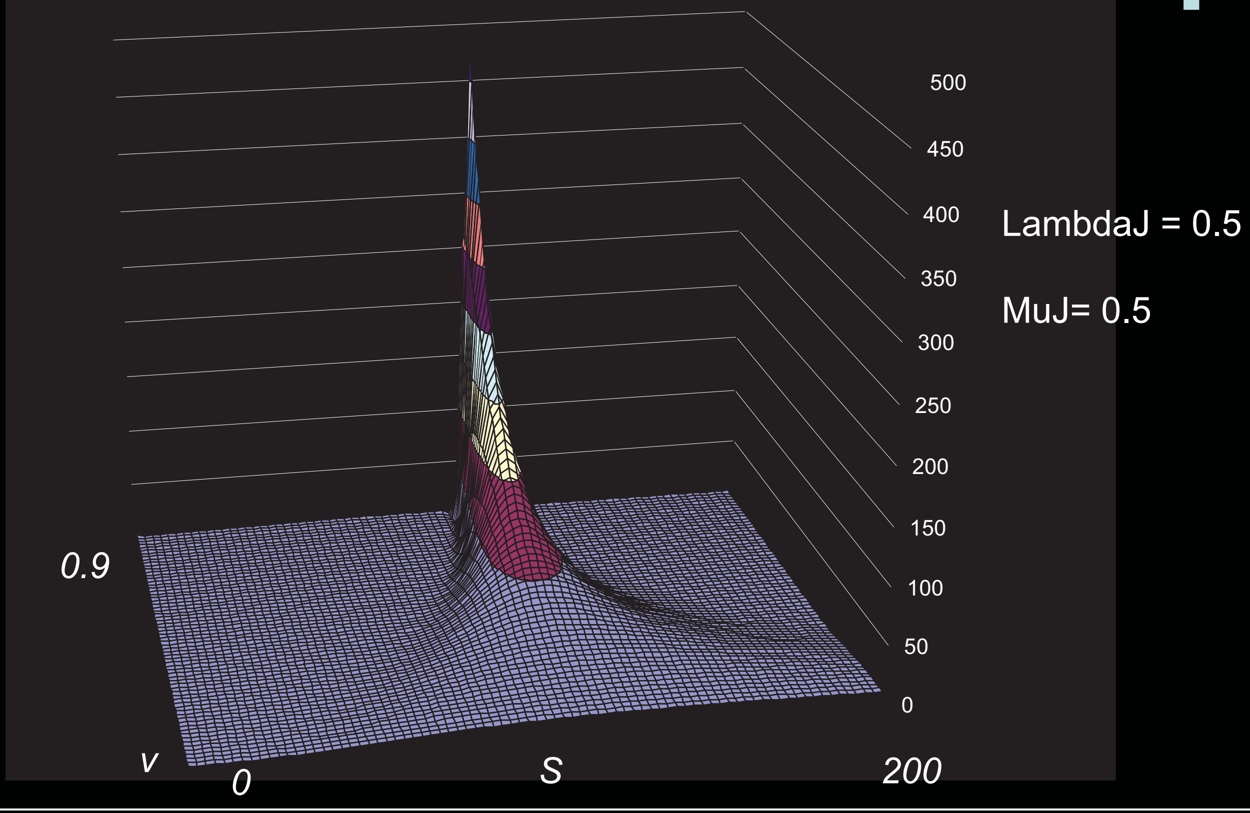
EtaJ = 0.1

LambdaJ = 0.5

MuJ= 0.5



Merton Vega



Merton Vega