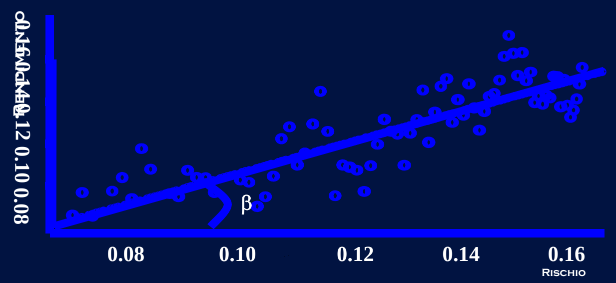
Derivatives Risk Management
and
Quant Surveillance

Columbia University
in the City of New York

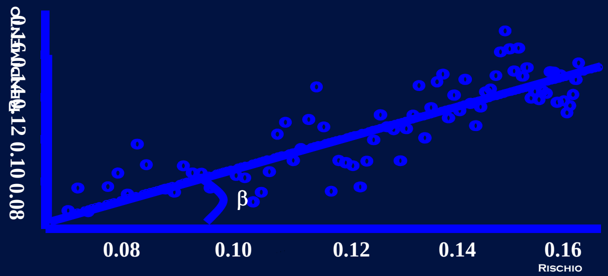
Mathematics of Finance Practitioners Seminar

RISK MANAGEMENT



... A COMPLEX SET OF GEARS RELATED TO RISK-RETURN OF FINANCIAL INSTRUMENTS

RISK MANAGEMENT



... A COMPLEX SET OF GEARS RELATED TO RISK-RETURN OF FINANCIAL INSTRUMENTS

... THAT IS SIMULATED THROUGH PROCESSES

Let us define:

S process of the stock

B process of the bond

f process of the derivative

Where:

$$f=f(S,t)$$

Let us define:

S process of the stock

B process of the bond

f process of the derivative

Where:

$$f = f(S,t)$$

Let us compute:

V replicating portfolio of the derivative

Replicating portfolio of the derivative

$$V_t = f(S,t) = N_s S_t + N_B B_t$$

Where:

N_s Number of stocks

N_B Number of bonds

Definition of the processes

Hp:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

Where:

$$dZ_t \sim \varepsilon\sqrt{dt} \qquad \varepsilon \sim N(0,1)$$

Definition of the processes

Hp:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

Where:

$$dZ_t \sim \varepsilon\sqrt{dt} \qquad \varepsilon \sim N(0,1)$$

$$\frac{dS_t}{S_t} \sim N(\mu dt, \sigma^2 dt)$$

Definition of the processes

Hp:

$$dB_t = rB_t dt$$

Whose solution is:

$$B_t = e^{rt} \qquad \forall t \in [0,T]$$

Definition of the processes

Hp:

$$dV_t = N_s dS_t + N_B dB_t$$

Where:

$$V_t = f(S,t)$$

Applying the definitions of both S and B processes

$$dV_t = N_s dS_t + N_B dB_t$$

$$dV_t = N_s \left(\mu S_t dt + \sigma S_t dZ_t\right) + N_B \left(r B_t dt\right)$$

… multiplying

$$dV_t = N_s \mu S_t dt + N_s \sigma S_t dZ_t + N_B r B_t dt$$

... MULTIPLYING

$$dV_t = N_s \mu S_t dt + N_s \sigma S_t dZ_t + N_B r B_t dt$$

COLLECTING:

$$dV_t = (N_s \mu S_t + N_B r B_t) dt + \sigma S_t N_s dZ_t$$

Let us define:

$$\mu S_t = a$$

$$\sigma S_t = b$$

Let us define:

$$\mu S_t = a$$

$$\sigma S_t = b$$

$$dS_t = a dt + b dZ_t$$

ITO's process

The SDE associated to $f=f(S,t)$ is obtained by using ITO's lemma (Brownian motion differentiating rule)

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}ds + \frac{1}{2}b^2\frac{\partial^2 f}{\partial S^2}dt$$

The SDE associated to $f=f(S,t)$ is obtained by using ITO's lemma (Brownian motion differentiating rule)

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}ds + \frac{1}{2}b^2\frac{\partial^2 f}{\partial S^2}dt$$

By Substituting the SDE associated to S we obtain:

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}\left(adt + bdZ_t\right) + \frac{1}{2}b^2\frac{\partial^2 f}{\partial S^2}dt$$

... SIMPLIFYING

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} a + \frac{1}{2} b^2 \frac{\partial^2 f}{\partial S^2} \right) dt + b \frac{\partial f}{\partial S} dZ_t$$

... SIMPLIFYING

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} a + \frac{1}{2} b^2 \frac{\partial^2 f}{\partial S^2} \right) dt + b \frac{\partial f}{\partial S} dZ_t$$

REMEMBERING:

$$\mu S_t = a$$

$$\sigma S_t = b$$

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} \mu S_t + \frac{1}{2} \left(\sigma S_t \right)^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

By recalling the hp:

$$dV_t = df$$

BY RECALLING THE HP:

$$dV_t = df$$

... LET US COMPARE THE STOCHASTIC COMPONENTS:

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} \mu S_t + \frac{1}{2} (\sigma S_t)^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

$$dV_t = (N_s \mu S_t + N_B r B_t) dt + \sigma S_t N_s dZ_t$$

... THAT IS:

$$\sigma S_t N_s dZ_t$$

$$\Updownarrow$$

$$\sigma S_t \frac{\partial f}{\partial S} dZ_t$$

… THAT IS:

$$\sigma S_t N_s dZ_t$$

$$\sigma S_t \frac{\partial f}{\partial S} dZ_t$$

$$N_s = \frac{\partial f}{\partial S}$$

By remembering:

$$V_t = f(S,t) = N_s S_t + N_B B_t$$

$$N_B = \frac{1}{B}\left(f(S,t) - N_s S\right)$$

$$N_s = \frac{\partial f}{\partial S}$$

$$N_B = \frac{1}{B}\left(f(S,t) - N_s S\right)$$

$$N_s = \frac{\partial f}{\partial S}$$

$$N_B = \frac{1}{B}\left(f(S,t) - N_s S\right)$$

$$N_B = \frac{1}{B}\left(f(S,t) - \frac{\partial f}{\partial S} S\right)$$

BY SUBSTITUTING IN THE SDE OF V

$$N_s = \frac{\partial f}{\partial S}$$

$$N_B = \frac{1}{B}\left(f(S,t) - \frac{\partial f}{\partial S} S\right)$$

$$dV_t = (N_s \mu S_t + N_B r B_t)\,dt + \sigma S_t N_s dZ_t$$

BY SUBSTITUTING IN THE SDE OF V

$$N_s = \frac{\partial f}{\partial S} \qquad N_B = \frac{1}{B}\left(f(S,t) - \frac{\partial f}{\partial S} S\right)$$

$$dV_t = (N_s \mu S_t + N_B r B_t)\,dt + \sigma S_t N_s dZ_t$$

$$dV_t = \left(\frac{\partial f}{\partial S}\mu S_t + \frac{1}{B}\left(f(S,t) - \frac{\partial f}{\partial S} S\right) r B_t\right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

Simplifying:

$$dV_t = \left(\frac{\partial f}{\partial S}\mu S_t + \frac{1}{B}\left(f(S,t) - \frac{\partial f}{\partial S}S\right) r B_t\right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

$$dV_t = \left(\frac{\partial f}{\partial S}\mu S_t + r f(S,t) - \frac{\partial f}{\partial S} r S\right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

By recalling the hp:

$$dV_t = df$$

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$$dV_t = df$$

… let us compare the deterministic components:

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S_t + \frac{1}{2}\left(\sigma S_t\right)^2 \frac{\partial^2 f}{\partial S^2}\right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

$$\Updownarrow$$

$$dV_t = \left(\frac{\partial f}{\partial S}\mu S_t + r f(S,t) - \frac{\partial f}{\partial S} r S\right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

$$\left(\frac{\partial f}{\partial t}+\frac{\partial f}{\partial S}\mu S_t+\frac{1}{2}\left(\sigma S_t\right)^2\frac{\partial^2 f}{\partial S^2}\right) = \left(\frac{\partial f}{\partial S}\mu S_t + r f(S,t) - \frac{\partial f}{\partial S} r S\right)$$

$$\left(\frac{\partial f}{\partial t}+\frac{\partial f}{\partial S}\mu S_t+\frac{1}{2}(\sigma S_t)^2\frac{\partial^2 f}{\partial S^2}\right)=\left(\frac{\partial f}{\partial S}\mu S_t+rf(S,t)-\frac{\partial f}{\partial S}rS\right)$$

$$\left(\frac{\partial f}{\partial t}+\frac{\partial f}{\partial S}rS+\frac{1}{2}(\sigma S_t)^2\frac{\partial^2 f}{\partial S^2}\right)=rf(S,t)$$

… also known as Black-Scholes PDE

… CONSIDERING THAT THE dZ COMPONENT IS THE SAME BOTH FOR dV AND df

… CONSIDERING THAT THE dZ COMPONENT IS THE SAME BOTH FOR dV AND df

BLACK-SCHOLES PDE DESCRIBES

$$f=f(S,t)$$

AS TIME ELAPSES

THE DERIVATIVE CAN BE REPLICATED BY

N_S NUMBER OF STOCKS

N_B NUMBER OF BONDS

… LET US DEFINE

$$\Theta = \frac{\partial f}{\partial t} \qquad \Delta = \frac{\partial f}{\partial S} \qquad \Gamma = \frac{\partial^2 f}{\partial S^2}$$

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$$\Theta = \frac{\partial f}{\partial t} \qquad \Delta = \frac{\partial f}{\partial S} \qquad \Gamma = \frac{\partial^2 f}{\partial S^2}$$

$$dV = df$$

$$\left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S} rS + \frac{1}{2} \left(\sigma S_t \right)^2 \frac{\partial^2 f}{\partial S^2} \right) = rf(S,t)$$

$$\left(\Theta + \Delta rS + \frac{1}{2} \left(\sigma S_t \right)^2 \Gamma \right) = rf(S,t)$$

It is important to observe that we can compute the differential expression of $f=f(S,t)$ both with Taylor's formula and with ITO's lemma obtaining the same result

Computation of df by means of Taylor's formula

COMPUTATION OF df BY MEANS OF TAYLOR'S FORMULA

LET US REMEMBER THAT dS IS OF ORDER $\sqrt{dt}$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t \qquad dZ_t \sim \varepsilon \sqrt{dt}$$

COMPUTATION OF df BY MEANS OF TAYLOR'S FORMULA

LET US REMEMBER THAT dS IS OF ORDER $\sqrt{dt}$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t \qquad dZ_t \sim \varepsilon\sqrt{dt}$$

IT IS POSSIBLE TO EXTEND TAYLOR'S FACTORIZATION TO $o(dt)$

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}dS^2 + o(dt)$$

SUBSTITUTING THE DEFINITION OF dS IN df

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}dS^2 + o(dt)$$

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}\left(\mu S_t dt + \sigma S_t dZ_t\right) + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\left(\mu S_t dt + \sigma S_t dZ_t\right)^2 + o(dt)$$

Let us focus on:

$$\left(\mu S_t dt + \sigma S_t dZ_t\right)^2$$

$$\left(\mu^2 S_t^2 dt^2 + \sigma^2 S_t^2 dt + 2\mu S_t \sigma S_t dZ_t dt\right)$$

$o(dt)$

$$\sigma^2 S_t^2 dt$$

… SIMPLIFYING

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}\left(\mu S_t dt + \sigma S_t dZ_t\right) + \frac{1}{2}\sigma^2 S_t^2 dt + o(dt)$$

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S_t + \frac{1}{2}\left(\sigma S_t\right)^2 \frac{\partial^2 f}{\partial S^2}\right)dt + \sigma S_t \frac{\partial f}{\partial S}dZ_t + o(dt)$$

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S_t + \frac{1}{2}\left(\sigma S_t\right)^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t + o(dt)$$

TAYLOR
VS.
ITO

$$df = \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S_t + \frac{1}{2}\left(\sigma S_t\right)^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \sigma S_t \frac{\partial f}{\partial S} dZ_t$$

Q.E.D.

… but if Taylor's factorization

leads to the same result of ITO's lemma

… but if ITO's lemma has shown that $df = dV$

… that is the value of a derivative can be studied by means of the value of a portfolio constituted by

N_s Number of stocks

N_B Number of bonds

... so let us use Taylor's factorization in order to study what happens when

$$f=f(S,t,\sigma)$$

without loosing generality

Derivation of df with Taylor's formula

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \frac{\partial f}{\partial \sigma}d\sigma + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}dS^2 + \frac{1}{2}\frac{\partial^2 f}{\partial \sigma^2}d\sigma^2 + + \frac{1}{2}\frac{\partial^2 f}{\partial t^2}dt^2 + \frac{\partial f}{\partial S \partial t}dtdS + ... + o(dt)$$

We can expand Taylor's factorization to $o(dt)$

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \frac{\partial f}{\partial \sigma}d\sigma + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}dS^2 + o(dt)$$

… LET US DEFINE

$$\Theta = \frac{\partial f}{\partial t} \qquad \Delta = \frac{\partial f}{\partial S} \qquad \upsilon = \frac{\partial f}{\partial \sigma} \qquad \Gamma = \frac{\partial^2 f}{\partial S^2}$$

… LET US DEFINE

$$\Theta = \frac{\partial f}{\partial t} \qquad \Delta = \frac{\partial f}{\partial S} \qquad \upsilon = \frac{\partial f}{\partial \sigma} \qquad \Gamma = \frac{\partial^2 f}{\partial S^2}$$

$$dV = df$$

$$df = \Theta dt + \Delta dS + \upsilon \; d\sigma + \frac{1}{2}\Gamma dS^2 + o(dt)$$

… because we have shown that $dV = df$

In order to avoid differences in the portfolio

... because we have shown that $dV = df$

In order to avoid differences in the portfolio

$$dV = df = 0$$

... because we have shown that $dV = df$

In order to avoid differences in the portfolio

$$dV = df = 0$$

$$0 = \Theta dt + \Delta dS + v\ d\sigma + \frac{1}{2}\Gamma dS^2$$

...HEDGING IN PRACTICE IS BASED ON THE GREEKS

$$0 = \Theta dt + \Delta dS + v \; d\sigma + \frac{1}{2}\Gamma dS^2$$

HEDGING ACTIVITY IN PRACTICE

HP: BLACK-SCHOLES' WORLD

$$C_t = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$$

$$P_t = K e^{-r(T-t)} N(-d_2) - S_t N(-d_1)$$

$$d_1 = \frac{\ln \frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln \frac{S_t}{K} + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

CALL | **PUT**

$$\Delta_{call} = N(d_1)$$

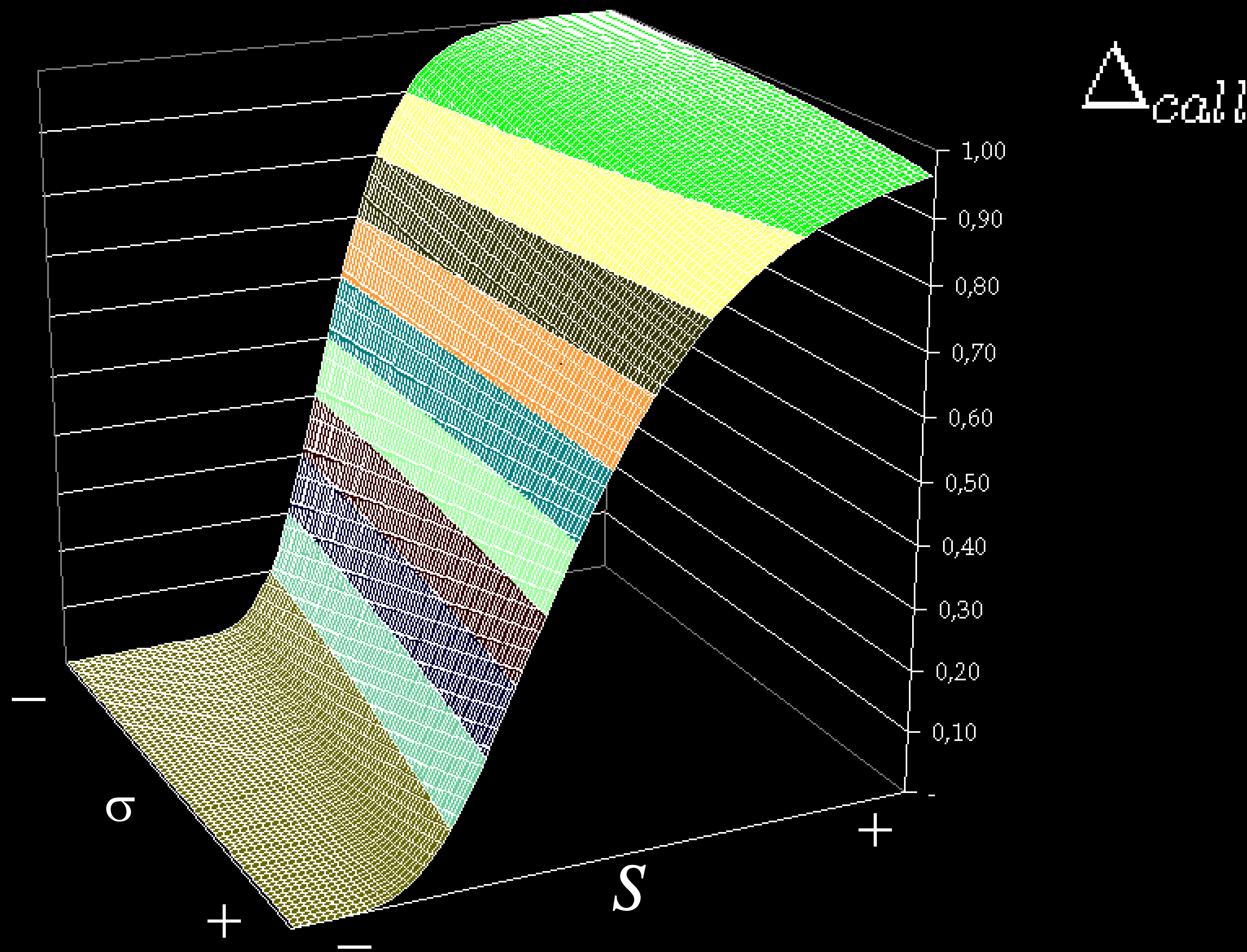
$$\Delta_{put} = N(d_1) - 1$$

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$

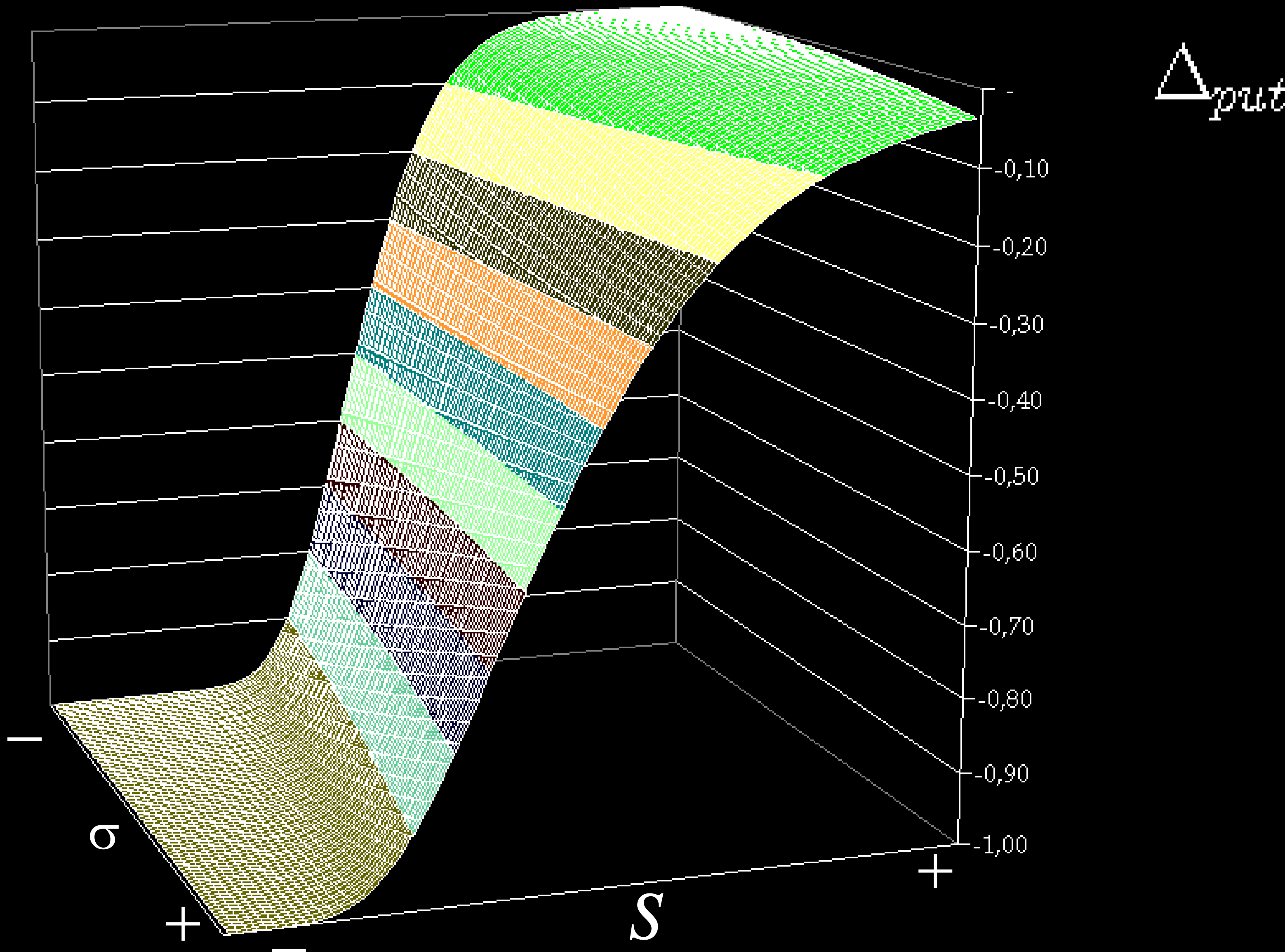
$$v = S \cdot N'(d_1)\sqrt{T-t}$$

$$\Theta_{call} = (T-t)Ke^{-r(T-t)}N(d_2)$$

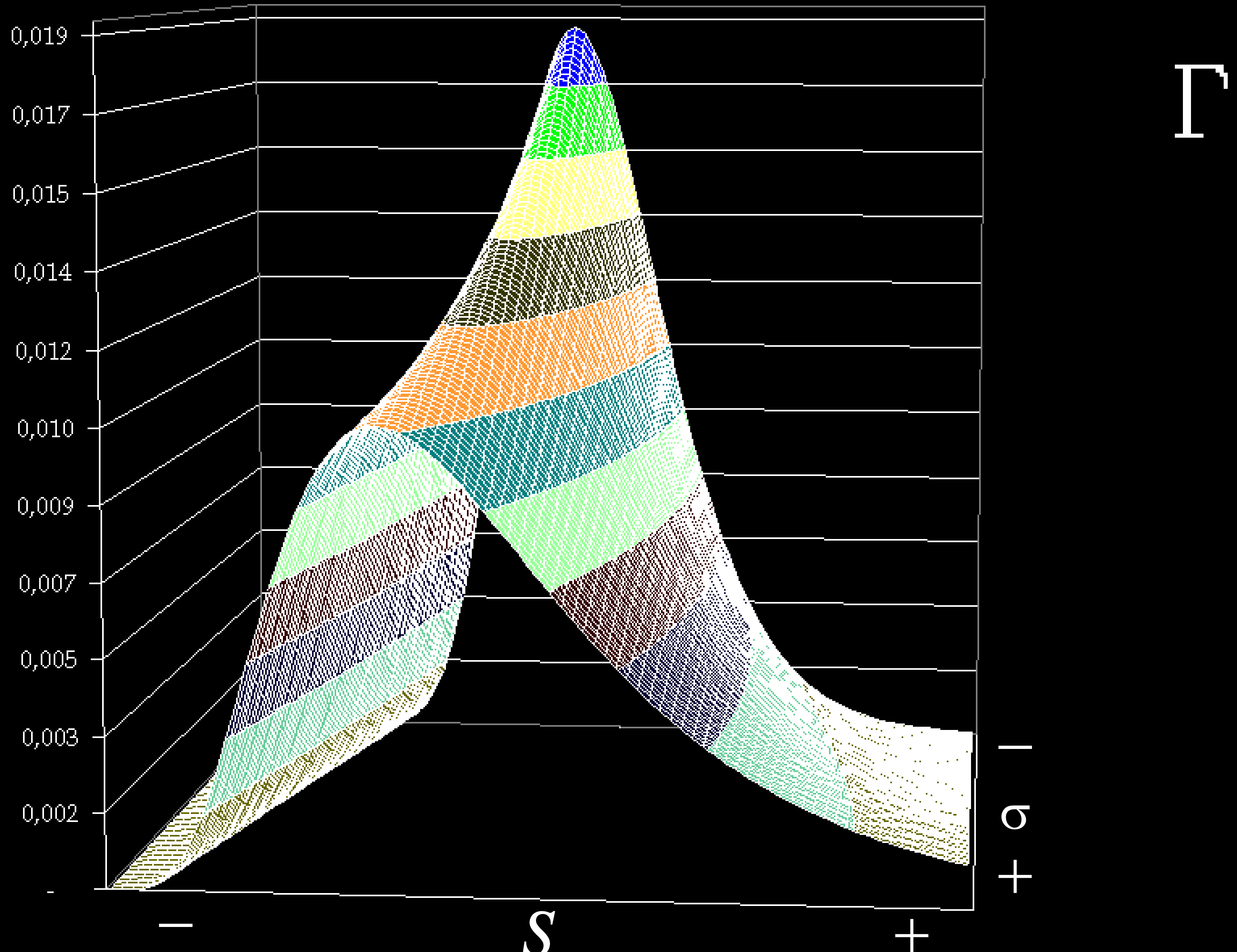
$$\Theta_{put} = -(T-t)Ke^{-r(T-t)}N(-d_2)$$

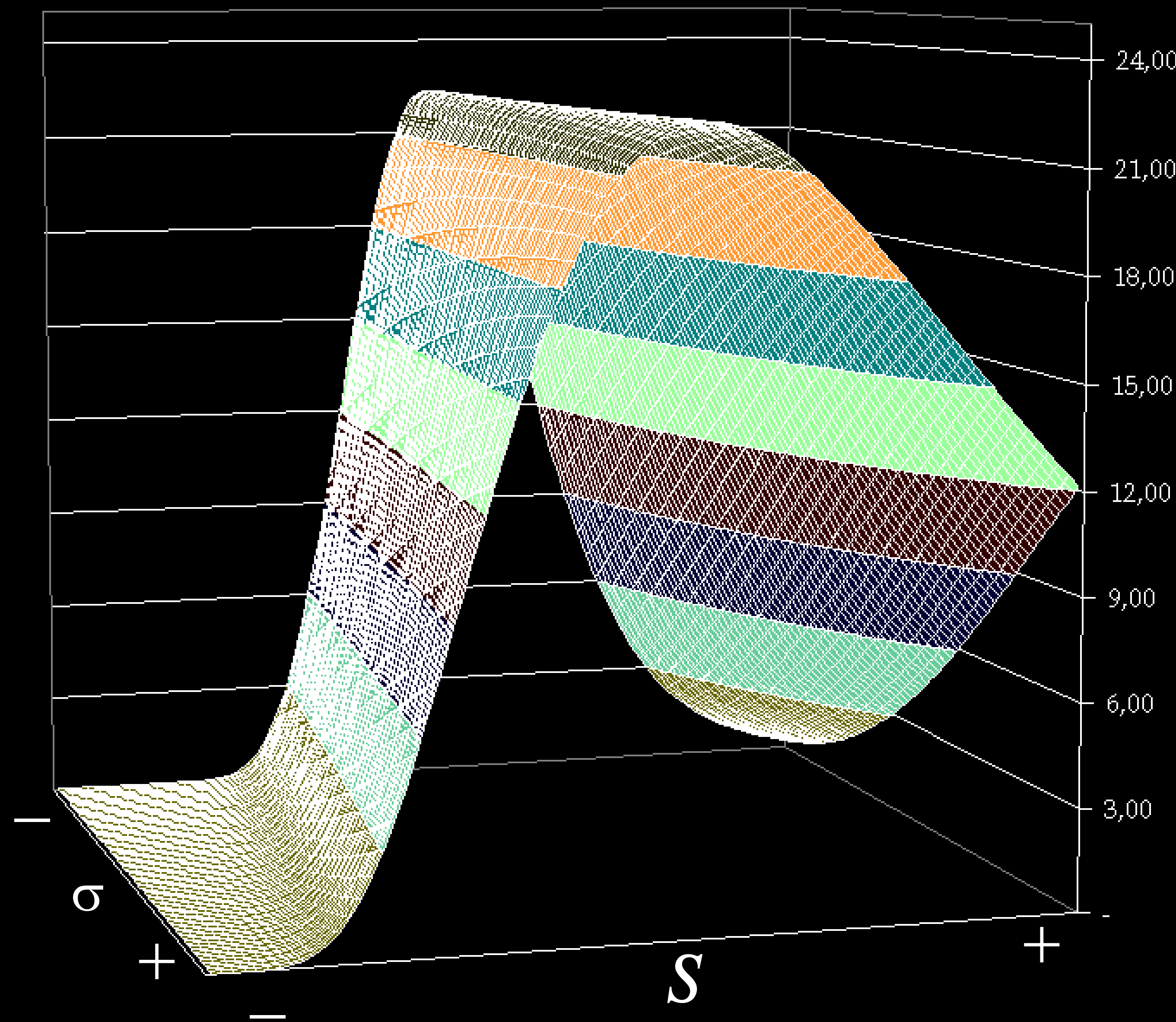

Δ_{call}
1,00
0,90
0,80
0,70
0,60
0,50
0,40
0,30
0,20
0,10
–
σ
+
–
S
+

Risk management: the greek letters

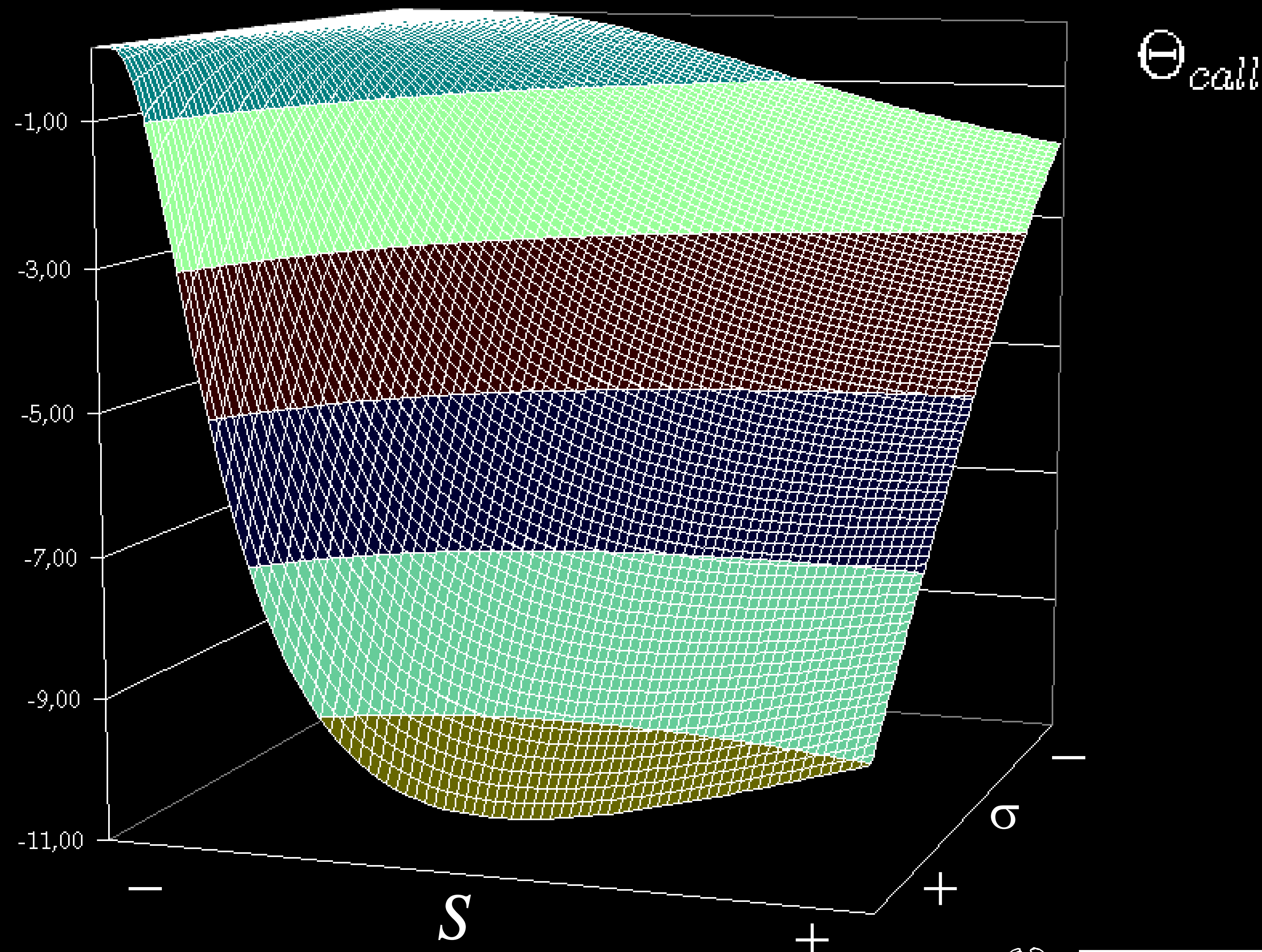


Risk management: the Greek letters

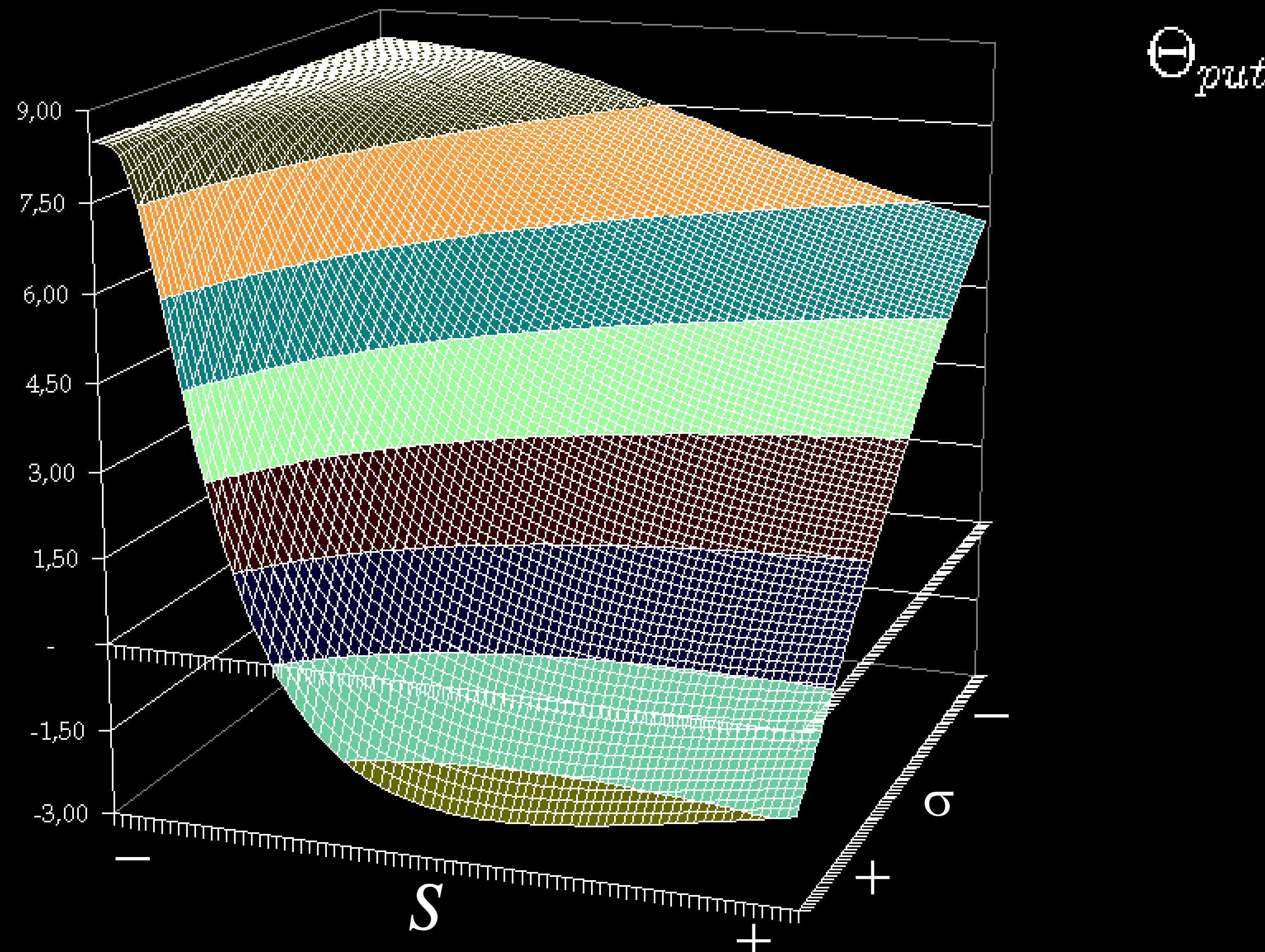



24,00
21,00
18,00
15,00
12,00
9,00
6,00
3,00
υ
–
σ
+
–
S
+

Risk management: the Greek letters



Risk management: the Greek letters



Greek letters are additive

$$\text{Porfolio Greek} = \sum_i w_i \ \text{Greek}$$

$$\sum_i w_i = 1$$

Δ hedging

Computation of df by means of a first order Taylor's formula

$$df \approx \Delta dS + o(dt)$$

At time t=0 short 1 call

At maturity the option is

IN – the money

Δ HEDGING - SHORT 1 CALL - IN – THE MONEY

Short 1000 call on 1 stock			Option and Δ			Stock and Δ				Δ Portfolio
Time Step	Time to Expiration	STOCK PRICE	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
0	0,2500	100,0	(1.000)	0,564115961	(564)	564	564	1	564	-
1	0,2375	104,0	(1.000)	0,624630657	(625)	61	625	1	625	-
2	0,2250	100,4	(1.000)	0,567671079	(568)	(57)	568	1	568	-
3	0,2125	93,8	(1.000)	0,449626897	(450)	(118)	450	1	450	-
4	0,2000	103,3	(1.000)	0,613419529	(613)	163	613	1	613	-
5	0,1875	121,6	(1.000)	0,850633639	(851)	238	851	1	851	-
6	0,1750	120,9	(1.000)	0,850534322	(851)	-	851	1	851	-
7	0,1625	120,5	(1.000)	0,853571891	(854)	3	854	1	854	-
8	0,1500	122,9	(1.000)	0,88234869	(882)	28	882	1	882	-
9	0,1375	129,0	(1.000)	0,931634606	(932)	50	932	1	932	-
10	0,1250	130,2	(1.000)	0,944999861	(945)	13	945	1	945	-
11	0,1125	126,8	(1.000)	0,935342021	(935)	(10)	935	1	935	-
12	0,1000	131,7	(1.000)	0,966714307	(967)	32	967	1	967	-
13	0,0875	139,1	(1.000)	0,989168909	(989)	22	989	1	989	-
14	0,0750	162,9	(1.000)	0,999121066	(999)	10	999	1	999	-
15	0,0625	165,4	(1.000)	0,999355248	(999)	-	999	1	999	-
16	0,0500	162,1	(1.000)	0,999494634	(999)	-	999	1	999	-
17	0,0375	162,1	(1.000)	0,999624853	(1.000)	1	1.000	1	1.000	-
18	0,0250	157,1	(1.000)	0,999750027	(1.000)	-	1.000	1	1.000	-
19	0,0125	148,4	(1.000)	0,999875008	(1.000)	-	1.000	1	1.000	-
20	0,0000	150,0	(1.000)	1	(1.000)	-	1.000	1	1.000	-

Δ HEDGING - SHORT 1 CALL - IN – THE MONEY

Delta Hedging Cash Flow						Delta Hedging portfolio "A" Value				
Stock	Option	Bank			Hedging Revenue (cost)	Replicating Portfolio			Option value	Unwind value
Dollars in Stock (flow)	Cash ex Shorting/Exercising Option	Cash	Interest (flow)	Borrow (stock)		Dollars in Stock (stock)	Bank	Portfolio Value		
56.400	10.378	46.022		46.022		56.400	(46.022)	10.378	(10.378)	-
6.344		6.344	28,8	52.395		64.997	(52.395)	12.602	(12.480)	122
(5.723)		(5.723)	32,8	46.704		57.032	(46.704)	10.327	(10.060)	267
(11.072)		(11.072)	29,2	35.662		42.223	(35.662)	6.562	(6.429)	133
16.833		16.833	22,3	52.517		63.304	(52.517)	10.787	(11.167)	(380)
28.940		28.940	32,8	81.490		103.479	(81.490)	21.989	(24.517)	(2.528)
-		-	50,9	81.541		102.880	(81.541)	21.339	(23.677)	(2.338)
361		361	51,0	81.953		102.901	(81.953)	20.948	(23.089)	(2.141)
3.442		3.442	51,2	85.446		108.417	(85.446)	22.971	(24.957)	(1.986)
6.452		6.452	53,4	91.952		120.274	(91.952)	28.322	(30.315)	(1.993)
1.692		1.692	57,5	93.702		123.026	(93.702)	29.324	(31.207)	(1.883)
(1.268)		(1.268)	58,6	92.493		118.532	(92.493)	26.039	(27.818)	(1.779)
4.216		4.216	57,8	96.766		127.392	(96.766)	30.626	(32.385)	(1.759)
3.060		3.060	60,5	99.886		137.543	(99.886)	37.657	(39.460)	(1.804)
1.629		1.629	62,4	101.578		162.729	(101.578)	61.152	(63.145)	(1.994)
-		-	63,5	101.641		165.235	(101.641)	63.593	(65.609)	(2.015)
-		-	63,5	101.705		161.986	(101.705)	60.282	(62.317)	(2.036)
162		162	63,6	101.931		162.108	(101.931)	60.178	(62.235)	(2.057)
-		-	63,7	101.994		157.110	(101.994)	55.116	(57.196)	(2.080)
-		-	63,8	102.058		148.442	(102.058)	46.384	(48.486)	(2.102)
-	(100.000)	-	63,8	102.122	(2.122)	149.961	(102.122)	47.839	(49.961)	(2.122)

At time t=0 short 1 call

At maturity the option is OUT – the money

Δ HEDGING - SHORT 1 CALL - OUT — THE MONEY

Short 1000 call on 1 stock			Option and Δ			Stock and Δ				Δ Portfolio
Time Step	Time to Expiration	STOCK PRICE	Q. Opz.	Δ call	Δ call Posit.	**Stock to Buy/(Sell)**	**Warehouse**	Δ Stock	Δ Stock Posit.	Total Δ position
0	0,2500	100,0	(1.000)	0,564115961	(564)	564	564	1	564	-
1	0,2375	107,1	(1.000)	0,669595731	(670)	106	670	1	670	-
2	0,2250	98,7	(1.000)	0,539965684	(540)	(130)	540	1	540	-
3	0,2125	98,6	(1.000)	0,535439952	(535)	(5)	535	1	535	-
4	0,2000	98,1	(1.000)	0,52274553	(523)	(12)	523	1	523	-
5	0,1875	100,9	(1.000)	0,572217366	(572)	49	572	1	572	-
6	0,1750	103,8	(1.000)	0,623229667	(623)	51	623	1	623	-
7	0,1625	89,9	(1.000)	0,346231134	(346)	(277)	346	1	346	-
8	0,1500	83,0	(1.000)	0,201859233	(202)	(144)	202	1	202	-
9	0,1375	77,9	(1.000)	0,110027376	(110)	(92)	110	1	110	-
10	0,1250	74,6	(1.000)	0,061492554	(61)	(49)	61	1	61	-
11	0,1125	76,9	(1.000)	0,072830535	(73)	12	73	1	73	-
12	0,1000	70,2	(1.000)	0,016432088	(16)	(57)	16	1	16	-
13	0,0875	68,9	(1.000)	0,007800759	(8)	(8)	8	1	8	-
14	0,0750	69,5	(1.000)	0,005051823	(5)	(3)	5	1	5	-
15	0,0625	69,9	(1.000)	0,002681681	(3)	(2)	3	1	3	-
16	0,0500	64,8	(1.000)	7,23394E-05	-	(3)	-	1	-	-
17	0,0375	62,8	(1.000)	1,05616E-06	-	-	-	1	-	-
18	0,0250	63,0	(1.000)	3,36141E-09	-	-	-	1	-	-
19	0,0125	64,5	(1.000)	2,6642E-15	-	-	-	1	-	-
20	0,0000	66,7	(1.000)	0	-	-	-	1	-	-

Δ HEDGING - SHORT 1 CALL - OUT — THE MONEY

Delta Hedging Cash Flow						Delta Hedging portfolio "A" Value				
Stock	Option	Bank			Hedging Revenue (cost)	Replicating Portfolio			Option value	Unwind value
Dollars in Stock (Flow)	Cash ex Shorting/Exercising Option	Cash	Interest (Flow)	Borrow (stock)		Dollars in Stock (stock)	Bank	Portfolio Value		
56.400	10.378	46.022		46.022		56.400	(46.022)	10.378	(10.378)	-
11.355		11.355	28,8	57.406		71.772	(57.406)	14.366	(14.505)	(139)
(12.837)		(12.837)	35,9	44.605		53.324	(44.605)	8.719	(9.141)	(422)
(493)		(493)	27,9	44.140		52.762	(44.140)	8.622	(8.790)	(168)
(1.177)		(1.177)	27,6	42.991		51.282	(42.991)	8.291	(8.201)	91
4.945		4.945	26,9	47.962		57.721	(47.962)	9.759	(9.464)	295
5.294		5.294	30,0	53.286		64.674	(53.286)	11.388	(10.883)	504
(24.908)		(24.908)	33,3	28.411		31.113	(28.411)	2.702	(3.783)	(1.081)
(11.953)		(11.953)	17,8	16.476		16.768	(16.476)	292	(1.662)	(1.370)
(7.166)		(7.166)	10,3	9.321		8.568	(9.321)	(753)	(711)	(1.465)
(3.655)		(3.655)	5,8	5.672		4.550	(5.672)	(1.122)	(328)	(1.450)
923		923	3,5	6.598		5.615	(6.598)	(983)	(392)	(1.376)
(4.001)		(4.001)	4,1	2.601		1.123	(2.601)	(1.478)	(62)	(1.540)
(551)		(551)	1,6	2.051		551	(2.051)	(1.499)	(25)	(1.525)
(208)		(208)	1,3	1.844		347	(1.844)	(1.497)	(15)	(1.511)
(140)		(140)	1,2	1.705		210	(1.705)	(1.496)	(7)	(1.502)
(195)		(195)	1,1	1.512		-	(1.512)	(1.512)	(0)	(1.512)
-		-	0,9	1.513		-	(1.513)	(1.513)	(0)	(1.513)
-		-	0,9	1.514		-	(1.514)	(1.514)	0	(1.514)
-		-	0,9	1.515		-	(1.515)	(1.515)	-	(1.515)
-	-	-	0,9	1.516	(1.516)	-	(1.516)	(1.516)	-	(1.516)

Δ–Γ hedging

Computation of df by means of a second order Taylor's formula

$$df \approx \Delta dS + \frac{1}{2}\Gamma dS^2 + o(dt)$$

At time t=0 short 1 call

Let us build our portfolio in order to be Δ neutral

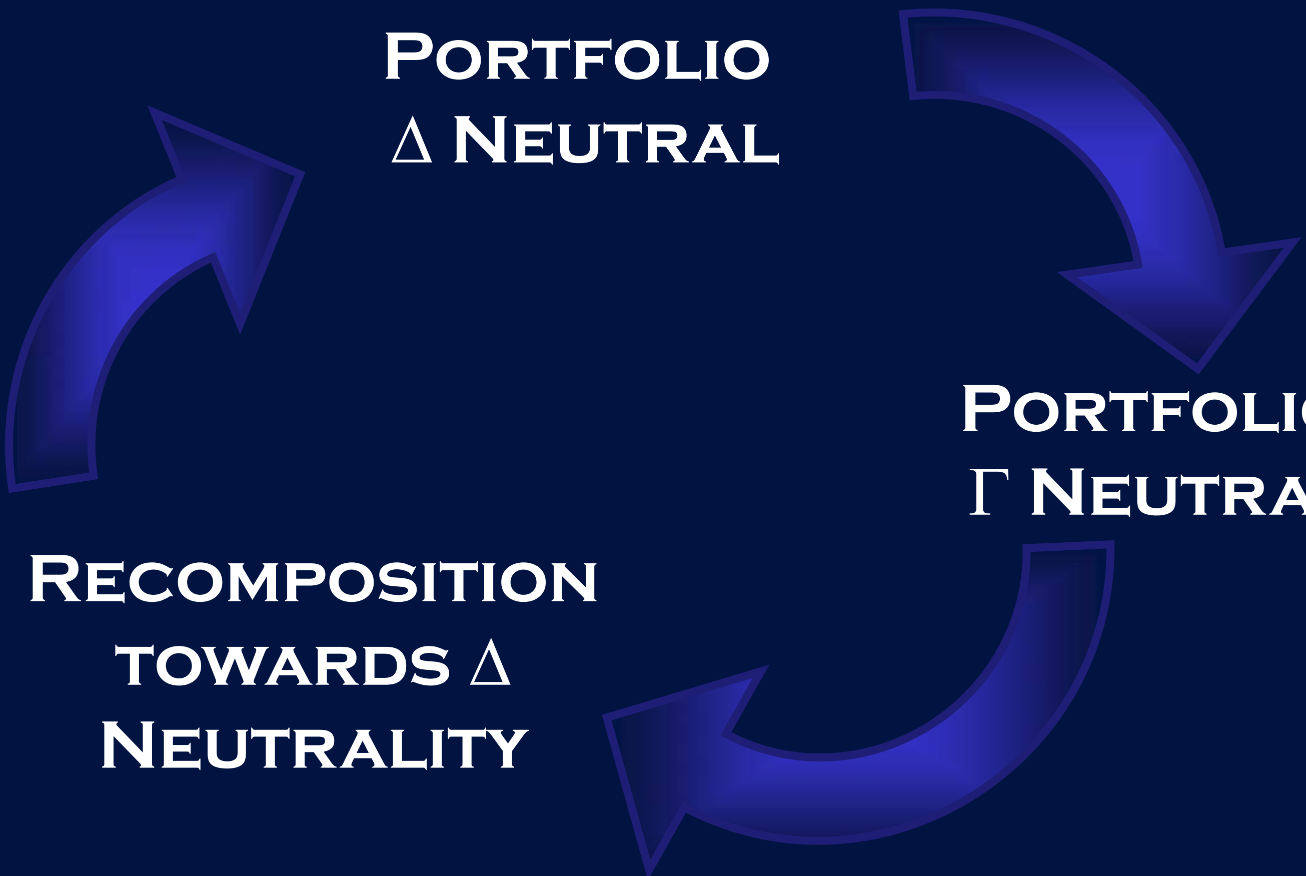
At time t=0 short 1 call

Let us build our portfolio in order to be Δ neutral

?

How can we build our portfolio in order to be also Γ neutral

… LET US FOLLOW AN ITERATIVE LOGIC



… THIS LOGIC IS CORRECT BECAUSE A STOCK'S Γ IS 0

... THIS LOGIC IS CORRECT BECAUSE A STOCK'S Γ IS 0

... IN ORDER TO LET OUR PORTFOLIO BE ALSO Γ NEUTRAL ...

... WE NEED ANOTHER OPTION

… WHAT KIND OF OPTION?

… AN OPTION THAT MATCHES THE SHORTED OPTION'S Γ

… WHAT KIND OF OPTION?

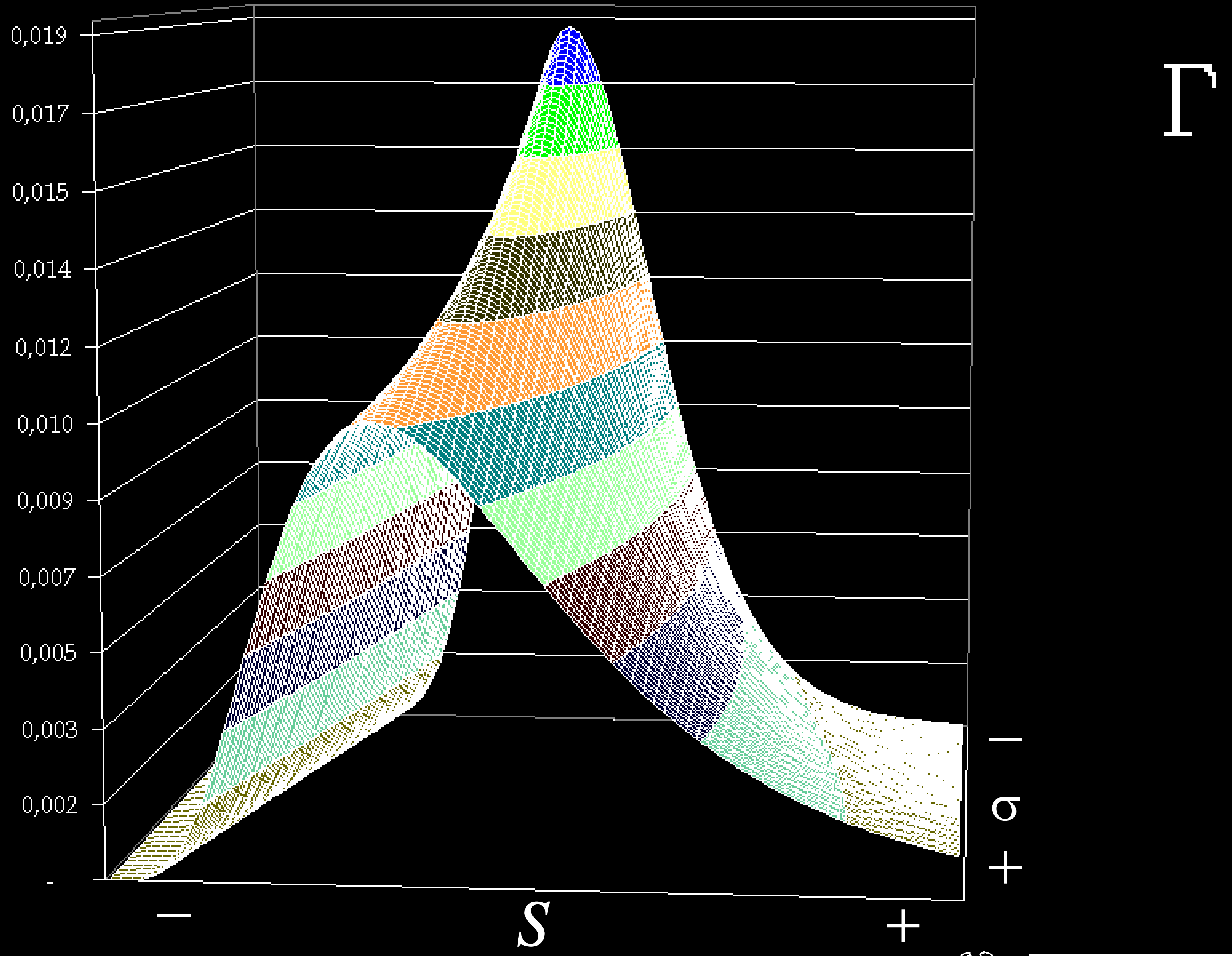
… AN OPTION THAT MATCHES THE SHORTED OPTION'S Γ

… AND THAT WOULD NOT CAUSE TOO MANY 'DEFORMATIONS' TO THE SHORTED OPTION'S DELTA

… WHAT KIND OF OPTION?

… SOME REMARKS

ATM OPTIONS HAVE THE BIGGEST Γ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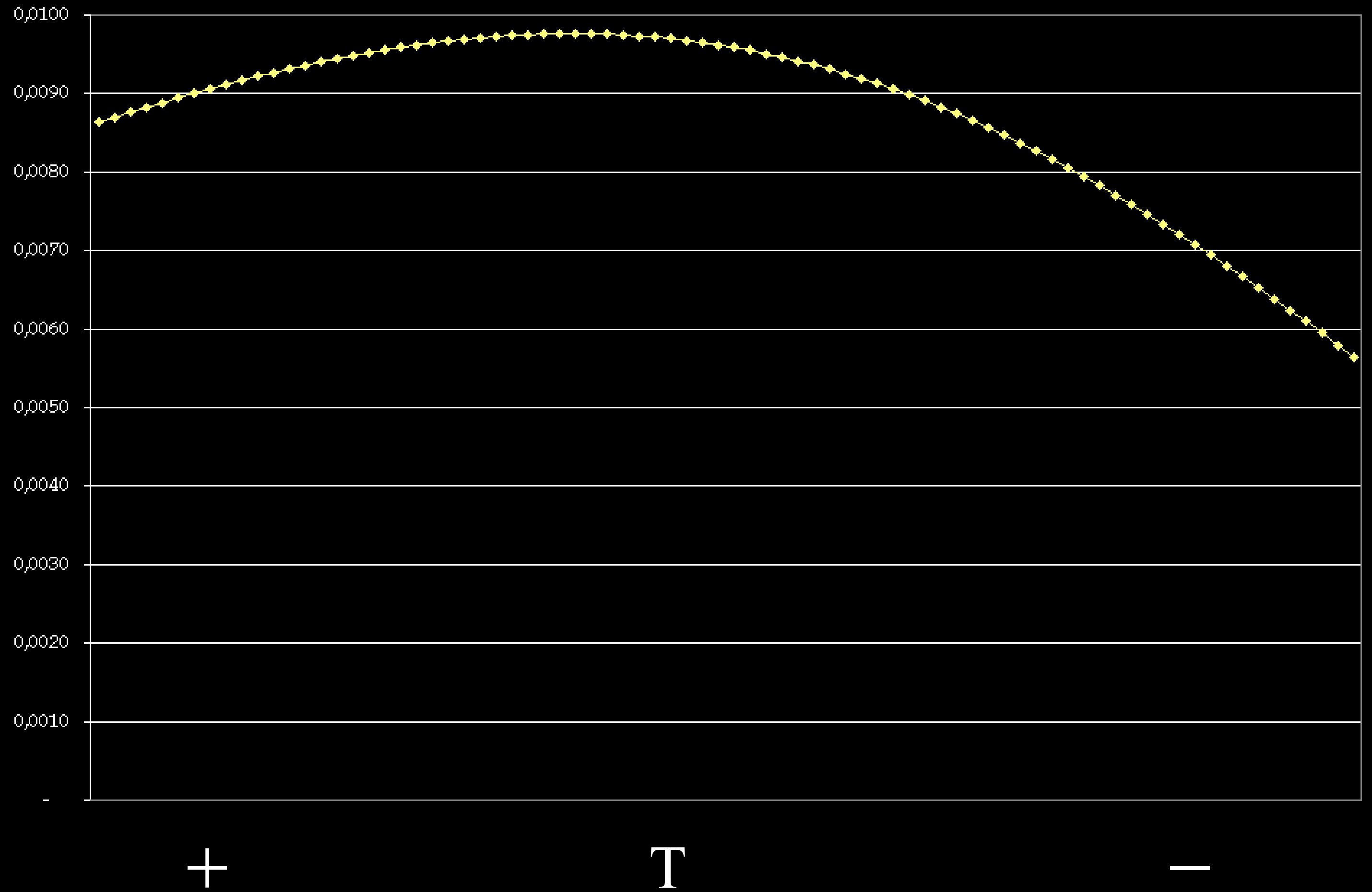
… WHAT KIND OF OPTION?

… SOME REMARKS

Γ OF AN OPTION FUNDAMENTALLY DECREASES AS TIME ELAPSES

Γ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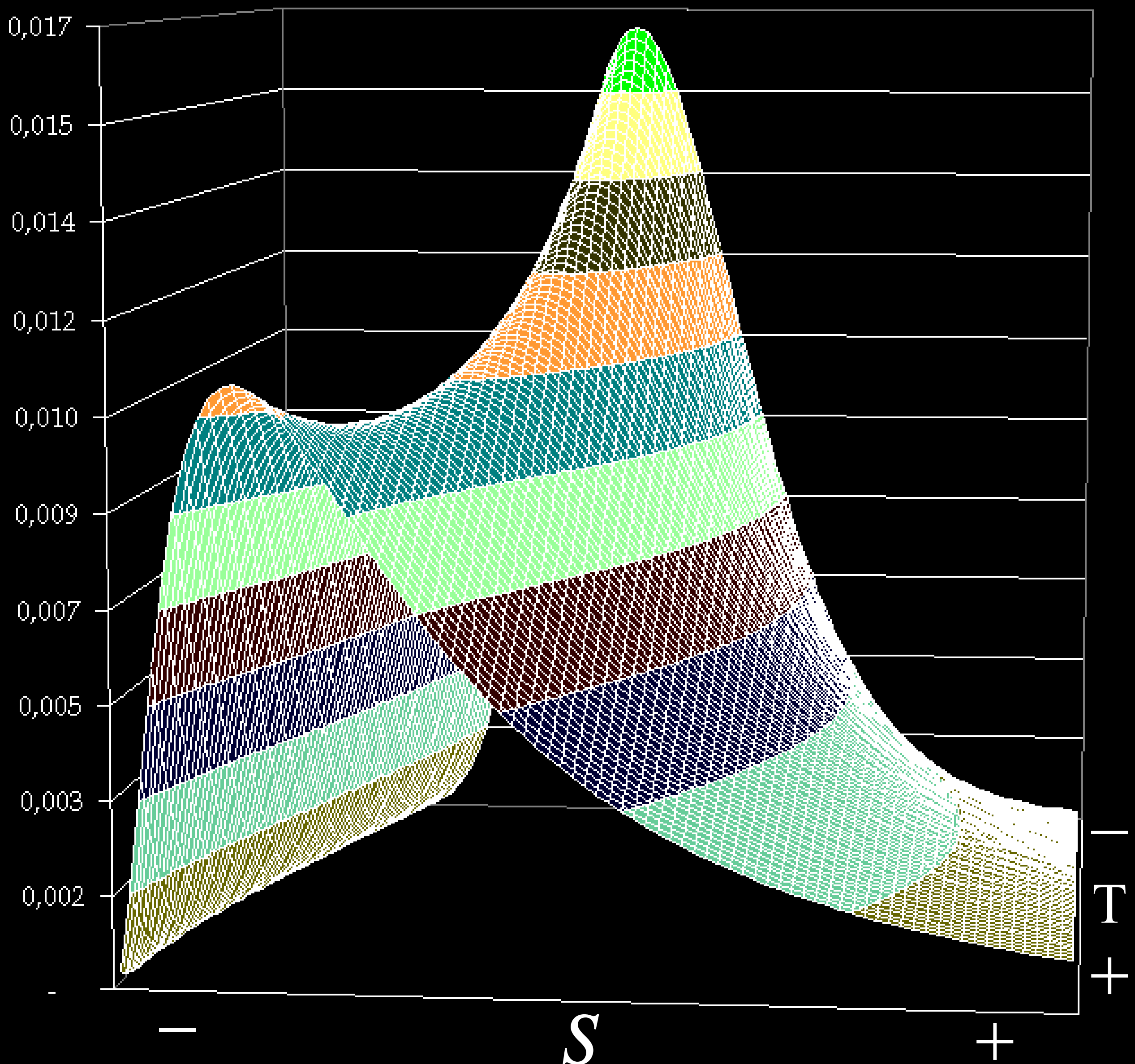
… WHAT KIND OF OPTION?

… SOME REMARKS

Γ UNDERGOES SOME DEFORMATIONS RELATED TO MONEYNESS AS TIME VARIES



Γ

$\Delta - \Gamma$ HEDGING

ATM OPTIONS HAVE THE BIGGEST Γ

Γ OF AN OPTION FUNDAMENTALLY DECREASES AS TIME ELAPSES

Γ UNDERGOES SOME DEFORMATIONS RELATED TO MONEYNESS AS TIME VARIES

LET US SELECT SHORT TERM AND ATM OPTIONS

$\Delta - \Gamma$ HEDGING

ATM OPTIONS HAVE THE BIGGEST Γ

Γ OF AN OPTION FUNDAMENTALLY DECREASES AS TIME ELAPSES

Γ UNDERGOES SOME DEFORMATIONS RELATED TO MONEYNESS AS TIME VARIES

LET US SELECT SHORT TERM AND ATM OPTIONS

=

LET US DINAMICALLY RECOMPOSE THE PORTFOLIO WITH LONG TERM AND ATM OPTIONS

$\Delta - \Gamma$ Hedging

Let us select short term and ATM options

Trade-off:

- Transation costs
- Trading strategies
- Risk limits

Let us dinamically recompose the portfolio with long term and ATM options

TIME T=0

1 SHORT CALL (W)

LET US DEFINE A Δ NEUTRAL PORTFOLIO 'A'

1 LONG OPTION (Z)

$$\Delta_A = 0$$

$$\Gamma_A = N * \Gamma_W$$

TIME T=0

PORTFOLIO B = PORTFOLIO A + N * Z

... WHAT ABOUT THE GREEK LETTERS OF B?

Time t=0

$$\Delta_B = \Delta_A + N\,\Delta_Z$$

$$\Delta_B = N\,\Delta_Z$$

TIME t=0

$$\Gamma_B = \Gamma_A + N\,\Gamma_Z$$

$$\Gamma_B = N_W \Gamma_W + N_Z \Gamma_Z$$

... IN ORDER TO OBTAIN $\Gamma_B = 0$

$$\Gamma_B = N_W \Gamma_W + N_Z \Gamma_Z$$

$$0 = N_W \Gamma_W + N_Z \Gamma_Z$$

$\Delta - \Gamma$ HEDGING TROUGH FORMULAS

... IN ORDER TO OBTAIN $\Gamma_B = 0$

$$\Gamma_B = N_W \Gamma_W + N_Z \Gamma_Z$$

$$0 = N_W \Gamma_W + N_Z \Gamma_Z$$

$$N_Z = -\frac{N_W \Gamma_W}{\Gamma_Z}$$

… THAT IS IN ORDER TO HAVE A Γ NEUTRAL PORTFOLIO

YOU SHOULD BUY $N_Z = -\frac{\Gamma_W}{\Gamma_Z}$

OPTIONS Z

… BUT IT IS NOT THE WHOLE STORY

THE NEW PORTFOLIO B WILL NOT BE Δ NEUTRAL

$$\Delta_B = N \Delta_Z$$

... BUT IT IS NOT THE WHOLE STORY

The new portfolio B will not be Δ neutral

$$\Delta_B = N \Delta_Z$$

Let us rebalance the portfolio in order to obtain this result:

$$\Delta_C = 0$$

KURPIEL & RONCALLI (1998)

$\Delta - \Gamma$ HEDGING REFERRED TO TIME HORIZONS OF 5, 1, ½ DAYS DOES NOT SUPPLY SUBSTANTIAL ADVANTAGES IN COMPARISON WITH Δ HEDGING

Time t=0

Short 1 call (w)

Let us define a portfolio Δ neutral 'A'

Long 1 call (z) with $T_z > T_w$; $K_z > K_w$

LET US HOLD THE OPTION ‘Z’ UNTIL MATURITY

AT MATURITY THE OPTION ‘W’ IS IN – THE MONEY

$\Delta - \Gamma$ HEDGING - SHORT 1 CALL - IN – THE MONEY

Short 1000 call on 1 stock			Option and Δ			Stock and Δ				Δ Portfolio
Time Step	Time to Expiration	STOCK PRICE	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
0	0,2500	100,0	(1.000)	0,564115961	(564)	564	564	1	564	-
1	0,2375	102,0	(1.000)	0,593648325	(594)	30	594	1	594	-
2	0,2250	101,9	(1.000)	0,591419714	(591)	(3)	591	1	591	-
3	0,2125	104,3	(1.000)	0,629740916	(630)	39	630	1	630	-
4	0,2000	105,9	(1.000)	0,655754583	(656)	26	656	1	656	-
5	0,1875	109,6	(1.000)	0,713190152	(713)	57	713	1	713	-
6	0,1750	109,2	(1.000)	0,710239361	(710)	(3)	710	1	710	-
7	0,1625	112,7	(1.000)	0,765213522	(765)	55	765	1	765	-
8	0,1500	112,1	(1.000)	0,762277787	(762)	(3)	762	1	762	-
9	0,1375	114,0	(1.000)	0,795097794	(795)	33	795	1	795	-
10	0,1250	116,0	(1.000)	0,828994045	(829)	34	829	1	829	-
11	0,1125	103,8	(1.000)	0,629405621	(629)	(200)	629	1	629	-
12	0,1000	97,7	(1.000)	0,482184607	(482)	(147)	482	1	482	-
13	0,0875	99,4	(1.000)	0,522140486	(522)	40	522	1	522	-
14	0,0750	92,6	(1.000)	0,31747734	(317)	(205)	317	1	317	-
15	0,0625	93,2	(1.000)	0,315146981	(315)	(2)	315	1	315	-
16	0,0500	98,6	(1.000)	0,47968815	(480)	165	480	1	480	-
17	0,0375	101,6	(1.000)	0,591235554	(591)	111	591	1	591	-
18	0,0250	104,7	(1.000)	0,737926695	(738)	147	738	1	738	-
19	0,0125	108,3	(1.000)	0,927247903	(927)	189	927	1	927	-
20	0,0000	120,1	(1.000)	1	(1.000)	73	1.000	1	1.000	-

$\Delta - \Gamma$ HEDGING - SHORT 1 CALL - IN – THE MONEY

Portfolio B = Portfolio A + II Option

Γ Portfolio "A"	Γ Portfolio "B"						Δ Port. "B"
	II Option						
Γ portafolio = Γ I Option* n.az. Underlying	II Option value	d_1	Γ II Option	n. II Option Buy	Γ II Option Tot	Γ portfolio "B"	Total Δ position
(15,70)	8,532035235	-0,021383	0,015528736	1.011	15,70263	-	496
(15,56)	9,252966775	0,047578	0,015593814	998	15,5621	-	517
(16,03)	8,927916787	0,036606	0,016022921	1.000	16,02778	-	514
(15,66)	9,931752237	0,128265	0,015959581	981	15,65784	-	539
(15,49)	10,53940858	0,189663	0,016016933	967	15,49118	-	555
(14,29)	12,45786614	0,339697	0,015333912	932	14,29063	-	589
(14,93)	11,86551334	0,322986	0,015990036	934	14,92771	-	584
(13,46)	13,85860752	0,477377	0,015071693	893	13,45713	-	609
(14,19)	13,09617795	0,457377	0,015878178	894	14,18792	-	603
(13,38)	14,05627045	0,55154	0,015501083	863	13,38009	-	611
(12,33)	15,14677459	0,657847	0,014924919	826	12,32786	-	614
(21,67)	6,98437869	0,050779	0,021689614	999	21,66644	-	519
(25,78)	3,874139145	-0,319605	0,023112366	1.115	25,7778	-	417
(27,07)	4,158819334	-0,242518	0,024624308	1.099	27,07001	-	444
(28,11)	1,658383337	-0,754865	0,021897225	1.284	28,10892	-	289
(30,49)	1,48799424	-0,780121	0,023041342	1.323	30,48717	-	288
(36,12)	2,582609126	-0,418841	0,029625236	1.219	36,11725	-	411
(39,46)	3,192341541	-0,217637	0,034269965	1.151	39,45535	-	476
(39,31)	3,991776369	0,037942	0,039296274	1.000	39,31016	-	515
(22,82)	5,305213614	0,438792	0,042324222	539	22,82201	-	361
-	15,20266822	2,445826	0,002983796	-	-	-	-

Δ – Γ HEDGING - SHORT 1 CALL - IN – THE MONEY

Δ Port. "B"	Portfolio "C" = Port. "B" + Stock f(Δ hedge of "B")				
	Stock and Δ Portfolio				Δ Portfolio "C"
Total Δ position	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
496	(496)	(496)	1	(496)	-
517	(21)	(517)	1	(517)	-
514	3	(514)	1	(514)	-
539	(25)	(539)	1	(539)	-
555	(16)	(555)	1	(555)	-
589	(34)	(589)	1	(589)	-
584	5	(584)	1	(584)	-
609	(25)	(609)	1	(609)	-
603	6	(603)	1	(603)	-
611	(8)	(611)	1	(611)	-
614	(3)	(614)	1	(614)	-
519	95	(519)	1	(519)	-
417	102	(417)	1	(417)	-
444	(27)	(444)	1	(444)	-
289	155	(289)	1	(289)	-
288	1	(288)	1	(288)	-
411	(123)	(411)	1	(411)	-
476	(65)	(476)	1	(476)	-
515	(39)	(515)	1	(515)	-
361	154	(361)	1	(361)	-
-	361	-	1	-	-

Δ – Γ HEDGING - SHORT 1 CALL - IN – THE MONEY

quantitative composition of the "C" Portfolio, value of Δ & Γ

Stock		Short Opt.	Option for Γ		Delta e Gamma	
Buy/sell	Warehouse	Short Opt.	Buy/sell	Warehouse	Δ portfolio C	Γ portfolio C
68	68	(1.000)	1.011	1.011	-	-
9	77	(1.000)	(13)	998	-	-
-	77	(1.000)	2	1.000	-	-
14	91	(1.000)	(19)	981	-	-
10	101	(1.000)	(14)	967	-	-
23	124	(1.000)	(35)	932	-	-
2	126	(1.000)	2	934	-	-
30	156	(1.000)	(41)	893	-	-
3	159	(1.000)	1	894	-	-
25	184	(1.000)	(30)	863	-	-
31	215	(1.000)	(37)	826	-	-
(105)	110	(1.000)	173	999	-	-
(45)	65	(1.000)	116	1.115	-	-
13	78	(1.000)	(16)	1.099	-	-
(50)	28	(1.000)	184	1.284	-	-
(1)	27	(1.000)	39	1.323	-	-
42	69	(1.000)	(104)	1.219	-	-
46	115	(1.000)	(68)	1.151	-	-
108	223	(1.000)	(151)	1.000	-	-
343	566	(1.000)	(461)	539	-	-
434	1.000	(1.000)	(539)	-	-	-

$\Delta - \Gamma$ HEDGING - SHORT 1 CALL - IN – THE MONEY

Delta Gamma Hedging Cash Flow						
Stock	Option	Opt. for Γ	Bank			
Dollars in Stock (Flow)	Cash ex Shorting/Exercising Option	Dollars in Option (Flow)	Cash	Interest (Flow)	Borrow (stock)	Hedging Revenue (cost)
6.800	10.378	8.628	5.050		5.050	
918		(122)	795	3,2	5.848	
-		21	21	3,7	5.873	
1.460		(191)	1.269	3,7	7.146	
1.059		(147)	912	4,5	8.063	
2.521		(439)	2.082	5,0	10.150	
218		19	237	6,3	10.394	
3.382		(564)	2.818	6,5	13.218	
336		9	345	8,3	13.572	
2.849		(427)	2.422	8,5	16.003	
3.595		(563)	3.032	10,0	19.044	
(10.897)		1.208	(9.689)	11,9	9.367	
(4.396)		451	(3.945)	5,9	5.427	
1.292		(67)	1.226	3,4	6.656	
(4.628)		306	(4.322)	4,2	2.338	
(93)		59	(34)	1,5	2.305	
4.142		(269)	3.873	1,4	6.180	
4.675		(217)	4.459	3,9	10.643	
11.312		(603)	10.709	6,7	21.358	
37.133		(2.446)	34.686	13,4	56.058	
52.139	(100.000)	(8.198)	43.941	35,0	100.034	34

LET US HOLD THE 'Z' OPTION UNTIL MATURITY

AT MATURITY THE OPTION 'W' IS OUT – THE MONEY

Δ HEDGING - SHORT 1 CALL - OUT — THE MONEY

Short 1000 call on 1 stock			Option and Δ			Stock and Δ				Δ Portfolio
Time Step	Time to Expiration	STOCK PRICE	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
0	0,2500	100,0	(1.000)	0,564115961	(564)	564	564	1	564	-
1	0,2375	106,1	(1.000)	0,654729124	(655)	91	655	1	655	-
2	0,2250	104,2	(1.000)	0,627365416	(627)	(28)	627	1	627	-
3	0,2125	104,9	(1.000)	0,639023423	(639)	12	639	1	639	-
4	0,2000	99,0	(1.000)	0,540155862	(540)	(99)	540	1	540	-
5	0,1875	97,9	(1.000)	0,517323489	(517)	(23)	517	1	517	-
6	0,1750	93,3	(1.000)	0,422696428	(423)	(94)	423	1	423	-
7	0,1625	87,2	(1.000)	0,292594163	(293)	(130)	293	1	293	-
8	0,1500	79,5	(1.000)	0,144979436	(145)	(148)	145	1	145	-
9	0,1375	79,7	(1.000)	0,135689732	(136)	(9)	136	1	136	-
10	0,1250	82,2	(1.000)	0,160714285	(161)	25	161	1	161	-
11	0,1125	87,2	(1.000)	0,239512401	(240)	79	240	1	240	-
12	0,1000	78,3	(1.000)	0,074592914	(75)	(165)	75	1	75	-
13	0,0875	73,1	(1.000)	0,021769947	(22)	(53)	22	1	22	-
14	0,0750	79,2	(1.000)	0,05288132	(53)	31	53	1	53	-
15	0,0625	79,0	(1.000)	0,035901243	(36)	(17)	36	1	36	-
16	0,0500	84,3	(1.000)	0,072705946	(73)	37	73	1	73	-
17	0,0375	84,3	(1.000)	0,044170375	(44)	(29)	44	1	44	-
18	0,0250	78,1	(1.000)	0,001029286	(1)	(43)	1	1	1	-
19	0,0125	74,7	(1.000)	1,09986E-07	-	(1)	-	1	-	-
20	0,0000	71,5	(1.000)	0	-	-	-	1	-	-

Δ HEDGING - SHORT 1 CALL - OUT – THE MONEY

	Portfolio B = Portfolio A + II Option						
Γ Portfolio "A"	Γ Portfolio "B"						Δ Port. "B"
	II Option						
Γ portafolio = Γ I Option n.az. Underlying	II Option value	d_1	Γ II Option	n. II Option Buy	Γ II Option Tot	Γ portfolio "B"	Total Δ position
(15,70)	8,532035235	-0,021383	0,015528736	1.011	15,70263	-	496
(14,20)	11,50385049	0,20525	0,014695255	966	14,2025	-	560
(15,26)	10,14867469	0,128089	0,015552089	982	15,26498	-	540
(15,44)	10,26284595	0,152333	0,015815246	976	15,43546	-	546
(17,89)	6,976736039	-0,102274	0,017351704	1.031	17,88561	-	472
(18,76)	6,201205895	-0,164556	0,017939034	1.046	18,7625	-	454
(20,03)	4,133050684	-0,402387	0,018178071	1.102	20,02836	-	378
(19,53)	2,172498556	-0,748035	0,016498971	1.184	19,53189	-	269
(14,80)	0,749388353	-1,247614	0,011414948	1.296	14,79729	-	137
(14,73)	0,65845741	-1,293547	0,01117497	1.318	14,73167	-	129
(16,78)	0,796600892	-1,196629	0,01277235	1.314	16,78149	-	152
(21,23)	1,339000259	-0,935054	0,016697048	1.271	21,22699	-	222
(11,38)	0,262019159	-1,638052	0,007932318	1.435	11,38223	-	73
(4,81)	0,056296405	-2,183316	0,003179027	1.513	4,808971	-	22
(9,95)	0,15339148	-1,813143	0,006580083	1.512	9,948043	-	53
(7,99)	0,091417259	-1,984494	0,005141739	1.553	7,985569	-	37
(14,67)	0,195110623	-1,67581	0,009293286	1.578	14,66809	-	74
(11,44)	0,101824554	-1,893685	0,007045	1.624	11,44363	-	47
(0,56)	0,002827193	-2,998713	0,000588448	952	0,560244	-	1
(0,00)	1,29371E-05	-4,25525	7,90033E-06	18	0,000141	-	-
-	0	-6,843081	6,77257E-12	-	-	-	-

Δ HEDGING - SHORT 1 CALL - OUT — THE MONEY

Δ Port. "B"	Portfolio "C" = Port. "B" + Stock f(Δ hedge of "B")				
	Stock and Δ Portfolio				Δ Portfolio "C"
Total Δ position	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
496	(496)	(496)	1	(496)	-
560	(64)	(560)	1	(560)	-
540	20	(540)	1	(540)	-
546	(6)	(546)	1	(546)	-
472	74	(472)	1	(472)	-
454	18	(454)	1	(454)	-
378	76	(378)	1	(378)	-
269	109	(269)	1	(269)	-
137	132	(137)	1	(137)	-
129	8	(129)	1	(129)	-
152	(23)	(152)	1	(152)	-
222	(70)	(222)	1	(222)	-
73	149	(73)	1	(73)	-
22	51	(22)	1	(22)	-
53	(31)	(53)	1	(53)	-
37	16	(37)	1	(37)	-
74	(37)	(74)	1	(74)	-
47	27	(47)	1	(47)	-
1	46	(1)	1	(1)	-
-	1	-	1	-	-
-	-	-	1	-	-

Δ HEDGING - SHORT 1 CALL - OUT – THE MONEY

quantitative composition of the "C" Portfolio, value of Δ & Γ

Stock		Short Opt.	Option for Γ		Delta e Gamma	
Buy/sell	Warehouse	Short Opt.	Buy/sell	Warehouse	Δ portfolio C	Γ portfolio C
68	68	(1.000)	1.011	1.011	-	-
27	95	(1.000)	(45)	966	-	-
(8)	87	(1.000)	15	982	-	-
6	93	(1.000)	(6)	976	-	-
(25)	68	(1.000)	55	1.031	-	-
(5)	63	(1.000)	15	1.046	-	-
(18)	45	(1.000)	56	1.102	-	-
(21)	24	(1.000)	82	1.184	-	-
(16)	8	(1.000)	112	1.296	-	-
(1)	7	(1.000)	22	1.318	-	-
2	9	(1.000)	(4)	1.314	-	-
9	18	(1.000)	(43)	1.271	-	-
(16)	2	(1.000)	164	1.435	-	-
(2)	-	(1.000)	78	1.513	-	-
-	-	(1.000)	(1)	1.512	-	-
(1)	(1)	(1.000)	41	1.553	-	-
-	(1)	(1.000)	25	1.578	-	-
(2)	(3)	(1.000)	46	1.624	-	-
3	-	(1.000)	(672)	952	-	-
-	-	(1.000)	(934)	18	-	-
-	-	(1.000)	(18)	-	-	-

Δ HEDGING - SHORT 1 CALL - OUT – THE MONEY

Delta Gamma Hedging Cash Flow						
Stock	Option	Opt. for Γ	Bank			Hedging Revenue (cost)
Dollars in Stock (Flow)	Cash ex Shorting/Exercising Option	Dollars in Option (Flow)	Cash	Interest (Flow)	Borrow (stock)	
6.800	10.378	8.628	5.050		5.050	
2.864		(515)	2.349	3,2	7.402	
(833)		153	(680)	4,6	6.727	
629		(57)	572	4,2	7.303	
(2.476)		382	(2.093)	4,6	5.214	
(490)		94	(396)	3,3	4.822	
(1.680)		231	(1.449)	3,0	3.376	
(1.832)		178	(1.654)	2,1	1.725	
(1.272)		84	(1.188)	1,1	538	
(80)		14	(65)	0,3	473	
164		(3)	161	0,3	634	
785		(57)	728	0,4	1.363	
(1.253)		43	(1.210)	0,9	153	
(146)		4	(142)	0,1	11	
-		(0)	(0)	0,0	11	
(79)		4	(75)	0,0	(64)	
-		5	5	(0,0)	(59)	
(169)		5	(164)	(0,0)	(223)	
234		(2)	232	(0,1)	9	
-		(0)	(0)	0,0	9	
-	-	-	-	0,0	9	(9)

Δ–Γ–υ HEDGING

COMPUTATION OF df BY MEANS OF A SECOND ORDER TAYLOR'S FORMULA, PAYING ATTENTION TO VOLATILITY

$$df \approx \Delta dS + \frac{1}{2}\Gamma dS^2 + \frac{\partial f}{\partial \sigma} d\sigma + o(dt)$$

At time t=0 short 1 call

Let us build our portfolio in order to be Δ neutral

AT TIME T=0 SHORT 1 CALL

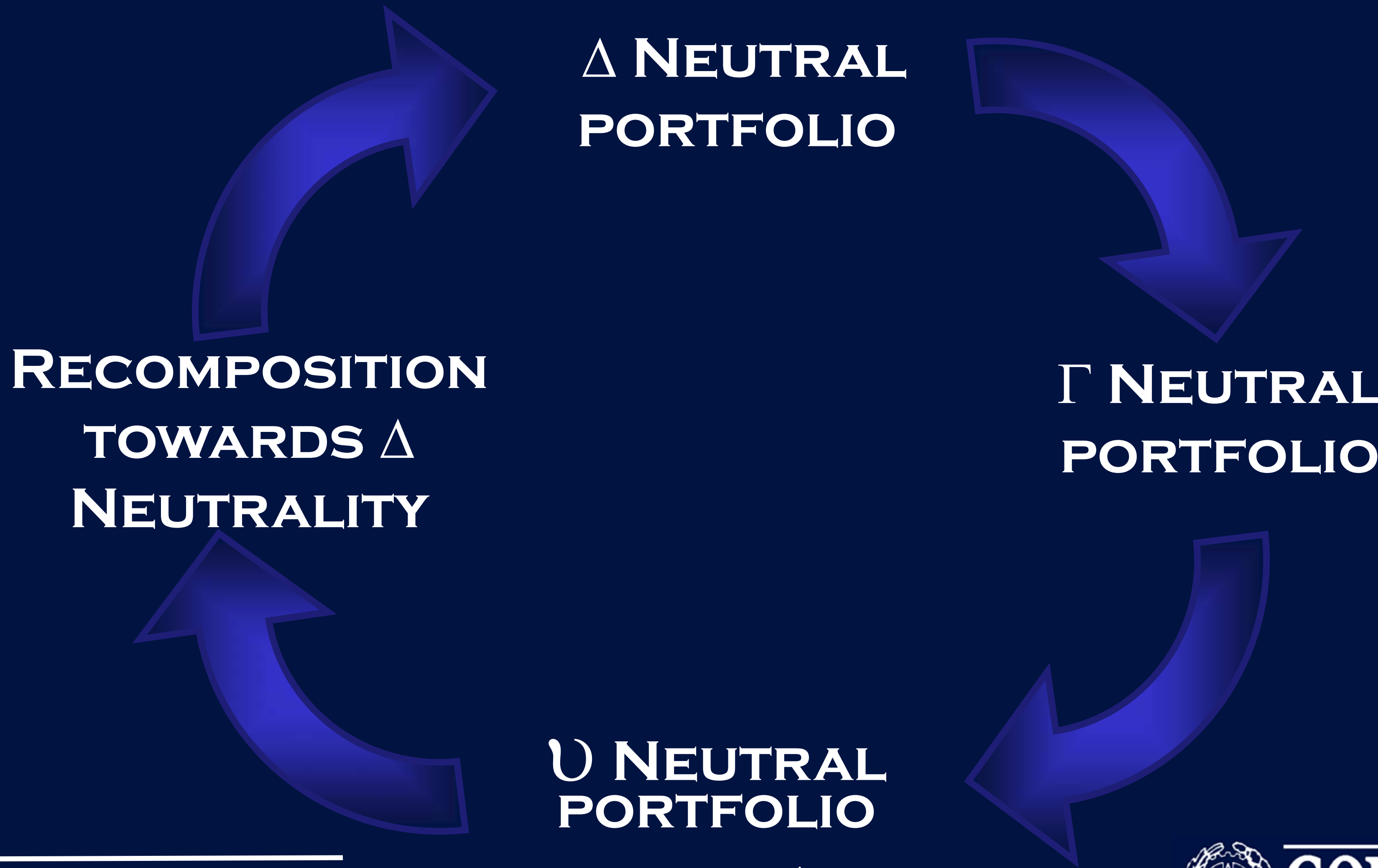
↓

LET US BUILD OUR PORTFOLIO IN ORDER TO BE Δ NEUTRAL

? ↓

HOW CAN WE BUILD OUR PORTFOLIO IN ORDER TO BE ALSO $\Gamma - \upsilon$ NEUTRAL

… LET US FOLLOW AN ITERATIVE LOGIC



... THIS LOGIC **IS NOT** CORRECT

?

... IF $\Gamma - \upsilon$ OF A STOCK IS 0

... UNFORTUNATELY THE OPTION'S Γ **IS NOT** 0

... THIS LOGIC **IS NOT** CORRECT

?

... IF $\Gamma - \upsilon$ OF A STOCK IS 0

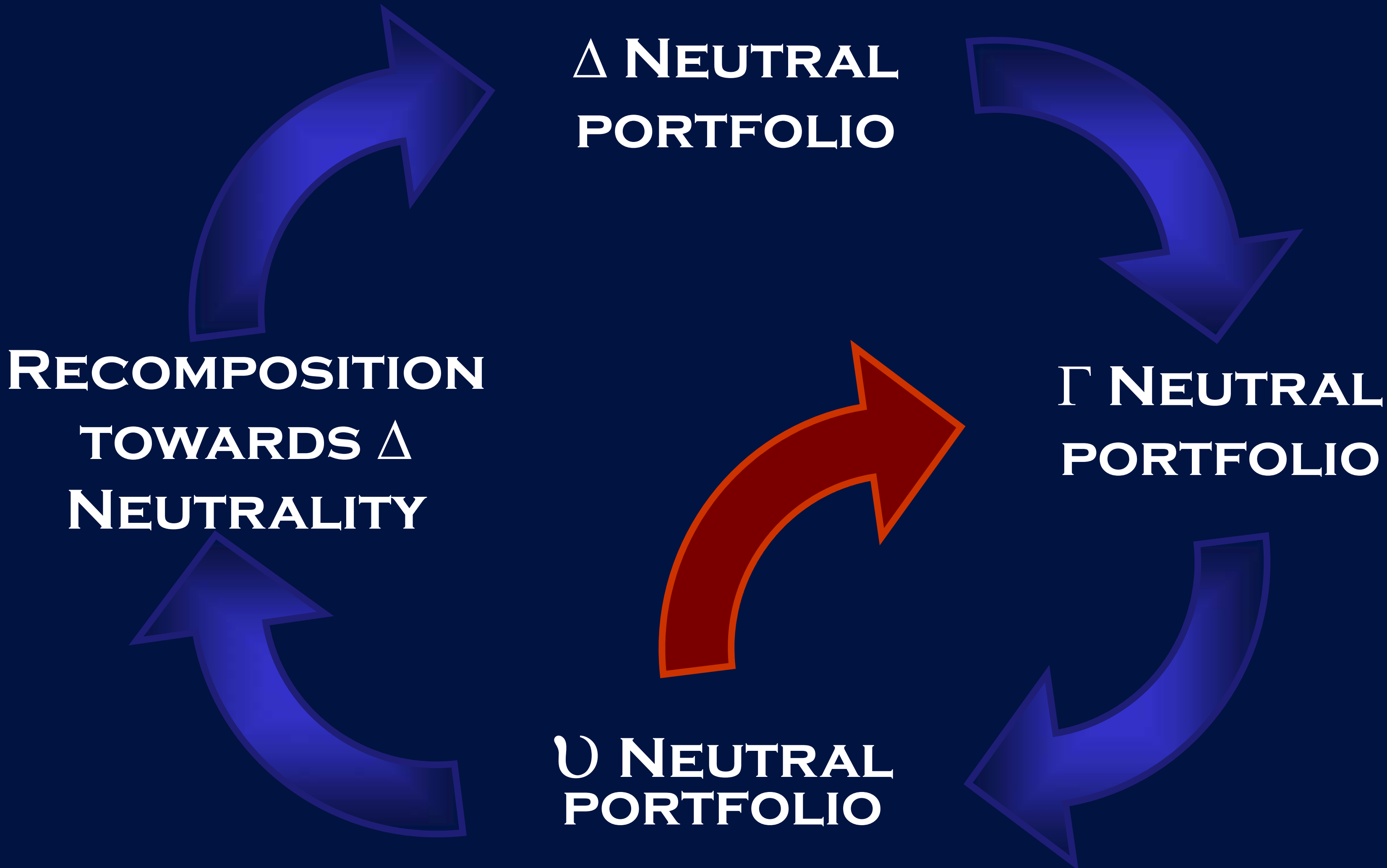
... UNFORTUNATELY THE OPTION'S Γ **IS NOT** 0

... WE NEED ANOTHER OPTION

… THEN WE WANT TO OBTAIN A JOINTLY $\Gamma - \upsilon$ NEUTRAL PORTFOLIO

… OTHERWISE WE START A LOOP WITHOUT SOLUTION

… IT FOLLOWS THE ‘VICIOUS’ LOOP



… IT FOLLOWS THE ‘VIRTUOUS’ LOOP

Δ NEUTRAL PORTFOLIO

JOINT USE OF THE 2 OPTIONS

$\Gamma - \upsilon$ NEUTRAL PORTFOLIO

RECOMPOSITION TOWARDS Δ NEUTRALITY

… WHAT KIND OF OPTIONS?

… AN OPTION THAT MATCHES THE $\Gamma - \upsilon$ 'SHORTED' OPTION

… WHAT KIND OF OPTIONS?

… AN OPTION THAT MATCHES THE $\Gamma - \upsilon$ 'SHORTED' OPTION

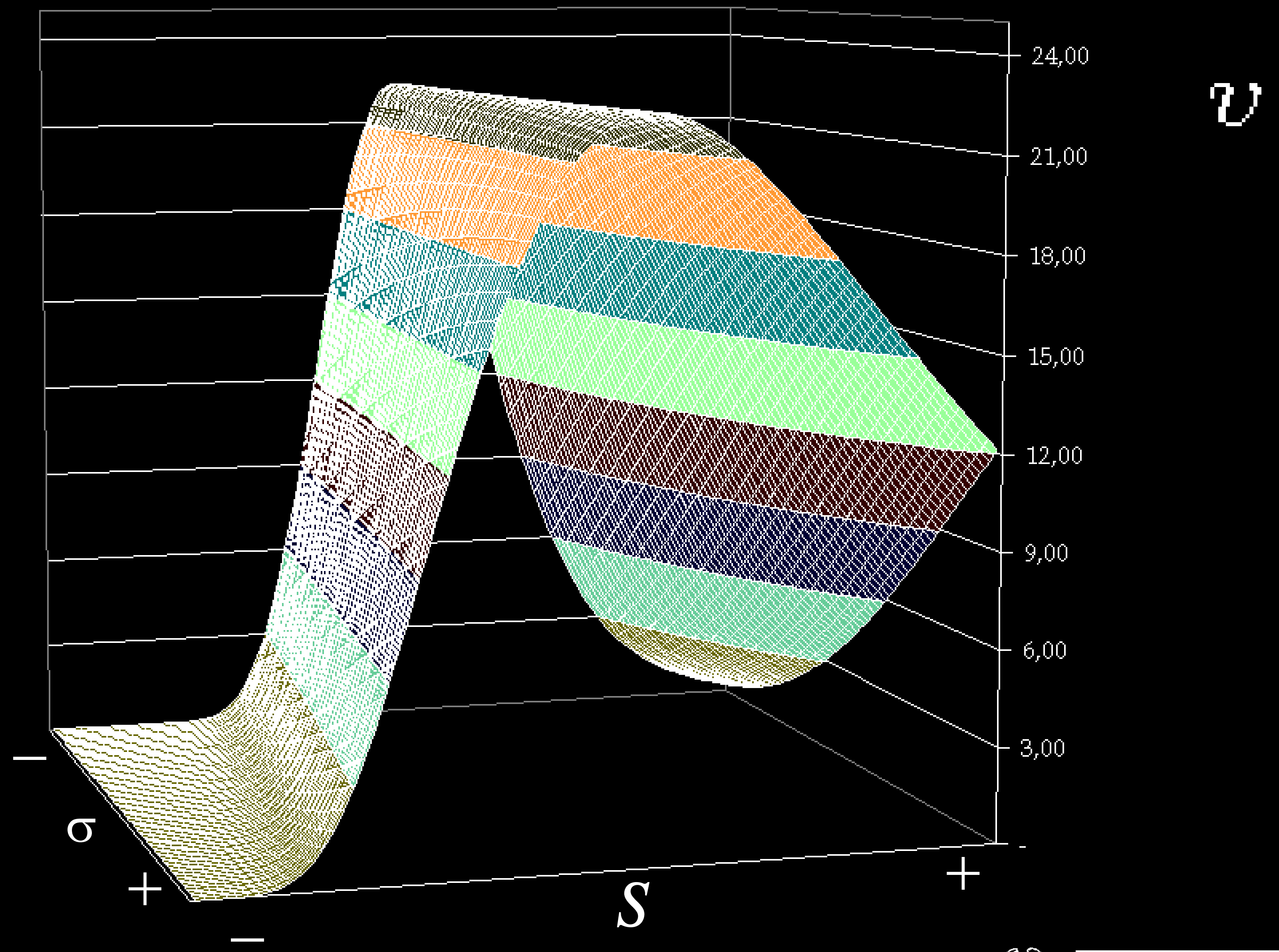
… AND THAT DOES NOT CAUSE TOO MANY 'DEFORMATIONS' TO THE DELTA OF THE 'SHORTED' OPTION

… WHAT KIND OF OPTIONS?

… SOME REMARKS

ATM OPTIONS HAVE THE BIGGEST υ

$\Delta - \Gamma - \upsilon$ HEDGING

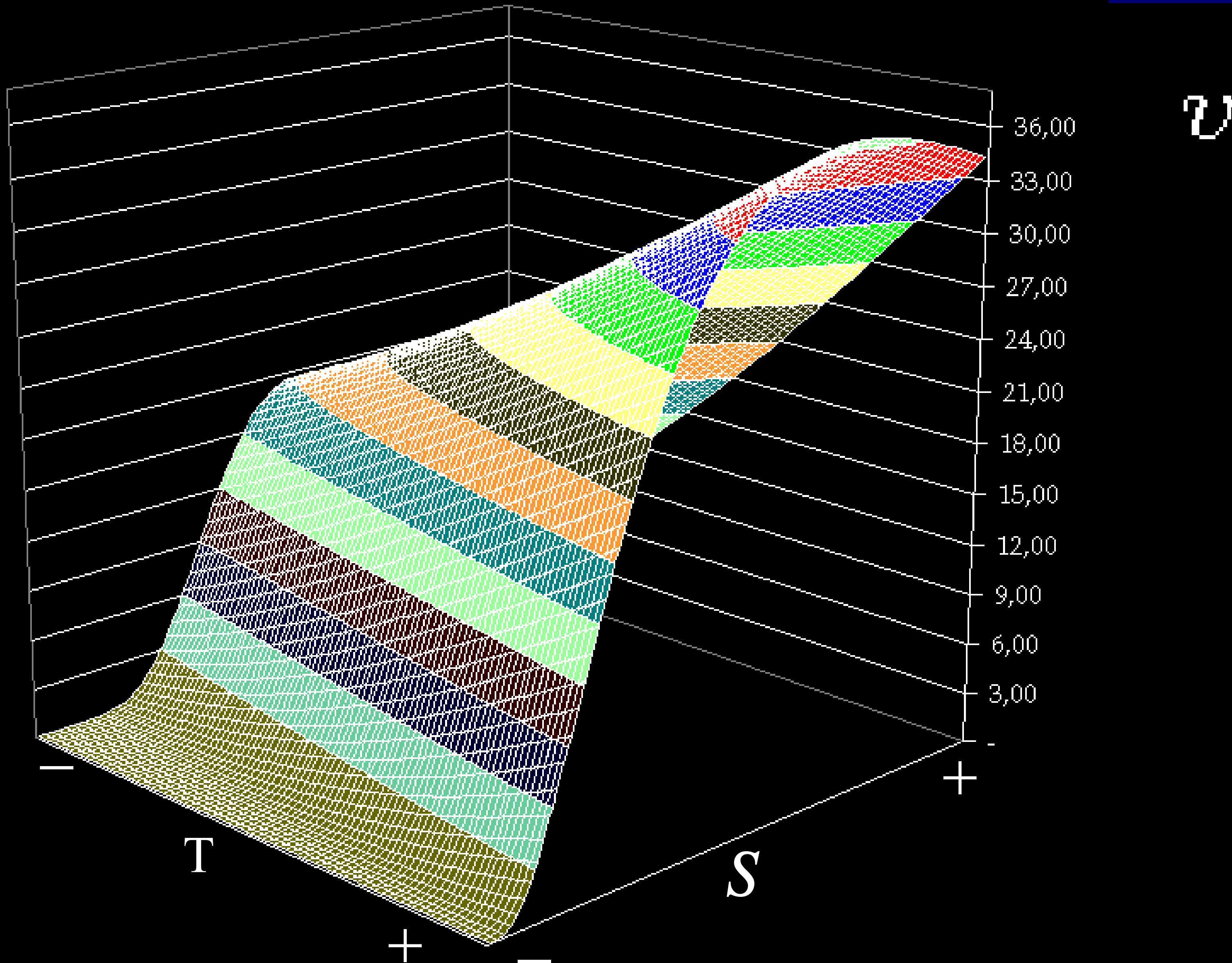


… WHAT KIND OF OPTION?

… SOME REMARKS

υ OF AN OPTION FUNDAMENTALLY DECREASES AS TIME ELAPSES

$\Delta - \Gamma - \upsilon$ HEDGING



$\Delta - \Gamma - \upsilon$ HEDGING

ATM OPTIONS HAVE THE BIGGEST υ

υ OF AN OPTION FUNDAMENTALLY DECREASES AS TIME ELAPSES

LET US DINAMICALLY RECOMPOSE THE PORTFOLIO WITH LONG TERM AND ATM OPTIONS

Let us dinamically recompose the portfolio with long term and ATM options

Let us consider

- Transation costs
- Trading strategies
- Risk limits
- .....

TIME T=0

SHORT 1 CALL (W)

LET US DEFINE A Δ NEUTRAL PORTFOLIO 'A'

LONG 1 OPTION (Z)

LONG 1 OPTION (Y)

$$\Delta_A = 0$$

$$\Gamma_A = n * \Gamma_W$$

TIME T=0

PORTFOLIO B = PORT. A + N * Z + N * Y

... GREEK LETTERS OF B?

TIME T=0

$$\Delta_B = \Delta_A + N_Z \Delta_Z + N_Y \Delta_Y$$

⇓

$$\Delta_B = N_Z \Delta_Z + N_Y \Delta_Y$$

TIME T=0

$$\Gamma_B = n_w \Gamma_w + n_z \Gamma_z + n_y \Gamma_y$$

GIVEN THAT:

$$\Gamma_A = N_W \Gamma_W$$

TIME t=0

$$\upsilon_B = n_w \upsilon_w + n_z \upsilon_z + n_y \upsilon_y$$

GIVEN THAT:

$$\upsilon_A = N_W \upsilon_W$$

... IN ORDER TO OBTAIN $\Gamma_B = \upsilon_B = 0$

...THAT IS A $\Gamma - \upsilon$ NEUTRAL PORTFOLIO

$$\begin{cases} 0 = n_w \Gamma_w + n_z \Gamma_z + n_y \Gamma_y \\ 0 = n_w \upsilon_w + n_z \upsilon_z + n_y \upsilon_y \end{cases}$$

$$\begin{cases} 0 = n_w \Gamma_w + n_z \Gamma_z + n_y \Gamma_y \\ 0 = n_w \upsilon_w + n_z \upsilon_z + n_y \upsilon_y \end{cases}$$

$$\begin{cases} n_z = \dfrac{-n_w \Gamma_w - n_y \Gamma_y}{\Gamma_z} \\ \end{cases}$$

$$\begin{cases} n_z = \dfrac{-n_w \Gamma_w - n_y \Gamma_y}{\Gamma_z} \\ \end{cases}$$

$$\begin{cases} 0 = n_w \upsilon_w + \dfrac{-n_w \Gamma_w - n_y \Gamma_y}{\Gamma_z} \upsilon_z + n_y \upsilon_y \\ \end{cases}$$

$\Delta - \Gamma - \upsilon$ HEDGING TROUGH FORMULAS

$$\begin{cases} 0 = n_w \upsilon_w + \dfrac{-n_w \Gamma_w - n_y \Gamma_y}{\Gamma_z} \upsilon_z + n_y \upsilon_y \\ \diagup \end{cases}$$

$$\begin{cases} 0 = n_w \upsilon_w \Gamma_z - n_w \Gamma_w \upsilon_z - n_y \Gamma_y \upsilon_z + n_y \upsilon_y \Gamma_z \\ \diagup \end{cases}$$

$\Delta - \Gamma - \upsilon$ HEDGING TROUGH FORMULAS

$$0 = n_w \upsilon_w \Gamma_z - n_w \Gamma_w \upsilon_z - n_y \Gamma_y \upsilon_z + n_y \upsilon_y \Gamma_z$$

$$-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z = n_y (\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)$$

$\Delta - \Gamma - \upsilon$ HEDGING TROUGH FORMULAS

$$\begin{cases} -n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z = n_y (\upsilon_y \Gamma_z - \Gamma_y \upsilon_z) \\ \text{—} \end{cases}$$

$$\begin{cases} n_y = \dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} \\ \text{—} \end{cases}$$

$$
\begin{cases}
n_y = \dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} \\[2ex]
n_z = \dfrac{-n_w \Gamma_w - n_y \Gamma_y}{\Gamma_z}
\end{cases}
$$

$$
\begin{cases}
\\
n_z = \dfrac{-n_w \Gamma_w - \left(\dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} \right) \Gamma_y}{\Gamma_z}
\end{cases}
$$

$$\begin{cases} n_z = \dfrac{-n_w \Gamma_w - \left(\dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} \right) \Gamma_y}{\Gamma_z} \\ \end{cases}$$

$$\begin{cases} \\ n_z = -\dfrac{n_w \Gamma_w}{\Gamma_z} - \left(\dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} \right) \dfrac{\Gamma_y}{\Gamma_z} \end{cases}$$

... IN ORDER TO OBTAIN $\Gamma_B = \upsilon_B = 0$

IT IS NECESSARY TO NEGOTIATE

$$\begin{cases} n_z = -\dfrac{n_w \Gamma_w}{\Gamma_z} - \left(\dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} \right) \dfrac{\Gamma_y}{\Gamma_z} & \text{Z OPTIONS} \\ n_y = \dfrac{-n_w \upsilon_w \Gamma_z + n_w \Gamma_w \upsilon_z}{(\upsilon_y \Gamma_z - \Gamma_y \upsilon_z)} & \text{Y OPTIONS} \end{cases}$$

… BUT IT IS NOT THE WHOLE STORY

THE NEW PORTFOLIO B WILL NOT BE Δ NEUTRAL

$$\Delta_B = N_Z \Delta_Z + N_Y \Delta_Y$$

… BUT IT IS NOT THE WHOLE STORY

THE NEW PORTFOLIO B WILL NOT BE Δ NEUTRAL

$$\Delta_B = N_Z \Delta_Z + N_Y \Delta_Y$$

LET US REBALANCE THE PORTFOLIO IN ORDER TO OBTAIN THE FOLLOWING RESULT:

$$\Delta_C = 0$$

KURPIEL & RONCALLI (1998)

$\Delta - \Gamma - \upsilon$ HEDGING REFERRED TO TIME HORIZONS OF 5, 1, ½ DAYS SUPPLIES SUBSTANTIAL ADVANTAGES IN PARTICULAR UNDER STOCHASTIC VOLATILITY

LET US HOLD THE OPTION 'Z' UNTIL MATURITY

AT MATURITY THE OPTION 'W' IS OUT – THE MONEY

Δ – Γ – υ HEDGING

Short 1000 call on 1 stock			Option and Δ			Stock and Δ				Δ Portfolio	Γ Portfolio "A"	υ Portfolio "A"
Time Step	Time to Expiration	STOCK PRICE	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position	Γ portfolio = Γ I Option*n.az. Underlying	υ portfolio = υ I Option*n.az. Underlying
0	0,2500	100,0	(1.000)	0,564	(564)	564	564	1	564	-	(15,70)	(19.628)
1	0,2375	99,6	(1.000)	0,557	(557)	(7)	557	1	557	-	(16,22)	(19.120)
2	0,2250	101,0	(1.000)	0,577	(577)	20	577	1	577	-	(16,30)	(18.697)
3	0,2125	107,8	(1.000)	0,683	(683)	106	683	1	683	-	(14,28)	(17.643)
4	0,2000	109,0	(1.000)	0,701	(701)	18	701	1	701	-	(14,19)	(16.847)
5	0,1875	109,1	(1.000)	0,706	(706)	5	706	1	706	-	(14,52)	(16.208)
6	0,1750	108,7	(1.000)	0,703	(703)	(3)	703	1	703	-	(15,17)	(15.681)
7	0,1625	103,6	(1.000)	0,621	(621)	(82)	621	1	621	-	(18,17)	(15.857)
8	0,1500	93,6	(1.000)	0,414	(414)	(207)	414	1	414	-	(21,48)	(14.107)
9	0,1375	91,9	(1.000)	0,370	(370)	(44)	370	1	370	-	(22,12)	(12.855)
10	0,1250	86,2	(1.000)	0,234	(234)	(136)	234	1	234	-	(20,12)	(9.342)
11	0,1125	87,6	(1.000)	0,249	(249)	15	249	1	249	-	(21,56)	(9.307)
12	0,1000	87,9	(1.000)	0,239	(239)	(10)	239	1	239	-	(22,28)	(8.610)
13	0,0875	83,6	(1.000)	0,133	(133)	(106)	133	1	133	-	(17,40)	(5.325)
14	0,0750	92,0	(1.000)	0,303	(303)	170	303	1	303	-	(27,69)	(8.797)
15	0,0625	95,0	(1.000)	0,370	(370)	67	370	1	370	-	(31,80)	(8.962)
16	0,0500	91,5	(1.000)	0,235	(235)	(135)	235	1	235	-	(30,01)	(6.277)
17	0,0375	88,6	(1.000)	0,117	(117)	(118)	117	1	117	-	(22,96)	(3.378)
18	0,0250	91,5	(1.000)	0,142	(142)	25	142	1	142	-	(31,01)	(3.245)
19	0,0125	90,8	(1.000)	0,045	(45)	(97)	45	1	45	-	(18,83)	(970)
20	0,0000	87,5	(1.000)	0,000	-	(45)	-	1	-	-	-	-

Δ – Γ – υ HEDGING

portfolio **B =** portfolio **A + II** option **+ III** option

Γ–υ Portfolio "B"														Δ Port "B"
II Option			III Option											
II Option value	Γ II option	υ II option	III Option value	Γ III option	υ III option	n. II option Buy/sell	n. III option buy/sell	Γ II option Tot	Γ III option Tot	υ II option Tot	υ III option Tot	Γ portfolio "B"	υ portfolio "B"	Total Δ position
8,5320	0,0155	20,3815	10,8995	0,0149	20,5532	2.022	(1.051)	31	(16)	41.219	(21.591)	-	-	396
8,0880	0,0160	19,8035	10,4330	0,0154	20,0694	2.033	(1.053)	32	(16)	40.253	(21.133)	-	-	388
8,4726	0,0162	19,5841	10,9321	0,0154	19,6781	2.015	(1.056)	33	(16)	39.471	(20.774)	-	-	394
11,9678	0,0150	19,6378	14,9784	0,0136	18,7267	1.903	(1.053)	29	(14)	37.361	(19.718)	-	-	435
12,3611	0,0151	19,0413	15,4827	0,0135	17,9868	1.880	(1.054)	28	(14)	35.800	(18.953)	-	-	435
12,1542	0,0155	18,4611	15,3176	0,0137	17,3926	1.873	(1.056)	29	(15)	34.577	(18.369)	-	-	429
11,5627	0,0162	17,9031	14,7232	0,0143	16,9039	1.877	(1.060)	30	(15)	33.603	(17.922)	-	-	420
8,3385	0,0183	17,2177	11,0967	0,0169	17,0414	1.984	(1.074)	36	(18)	34.153	(18.296)	-	-	385
3,6873	0,0192	13,6472	5,4801	0,0200	15,3406	2.240	(1.073)	43	(21)	30.564	(16.458)	-	-	280
2,9088	0,0191	12,1340	4,5198	0,0206	14,1752	2.311	(1.072)	44	(22)	28.047	(15.192)	-	-	250
1,3645	0,0160	8,1671	2,3963	0,0195	10,8369	2.516	(1.034)	40	(20)	20.552	(11.210)	-	-	165
1,4154	0,0170	8,1778	2,5330	0,0206	10,8703	2.529	(1.046)	43	(22)	20.683	(11.376)	-	-	170
1,2554	0,0172	7,4960	2,3531	0,0213	10,2705	2.584	(1.048)	45	(22)	19.373	(10.763)	-	-	160
0,5252	0,0124	4,3301	1,1807	0,0180	7,0894	2.811	(966)	35	(17)	12.172	(6.847)	-	-	92
1,5434	0,0214	7,9223	2,9890	0,0251	10,6184	2.591	(1.105)	55	(28)	20.527	(11.730)	-	-	182
1,9092	0,0249	8,4316	3,6923	0,0275	10,8487	2.551	(1.157)	64	(32)	21.510	(12.547)	-	-	200
0,8656	0,0207	5,4135	2,0986	0,0272	8,5335	2.899	(1.103)	60	(30)	15.693	(9.416)	-	-	128
0,3133	0,0141	2,7735	1,0744	0,0243	5,9569	3.248	(945)	46	(23)	9.008	(5.630)	-	-	63
0,3420	0,0179	2,8077	1,3338	0,0300	6,2870	3.467	(1.032)	62	(31)	9.735	(6.490)	-	-	59
0,1031	0,0113	1,1613	0,7943	0,0293	4,5360	3.342	(642)	38	(19)	3.881	(2.911)	-	-	11
0,0008	0,0005	0,0221	0,1448	0,0152	1,4579	-	-	-	-	-	-	-	-	-

$\Delta - \Gamma - \upsilon$ HEDGING

Δ Port. "B"	portfolio "C" = Port. "B" + Stock f(Δ hedge of "B")				
	Stock and Δ Portfolio				Δ Portfolio "C"
Total Δ position	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
396	(396)	(396)	1	(396)	-
388	8	(388)	1	(388)	-
394	(6)	(394)	1	(394)	-
435	(41)	(435)	1	(435)	-
435	-	(435)	1	(435)	-
429	6	(429)	1	(429)	-
420	9	(420)	1	(420)	-
385	35	(385)	1	(385)	-
280	105	(280)	1	(280)	-
250	30	(250)	1	(250)	-
165	85	(165)	1	(165)	-
170	(5)	(170)	1	(170)	-
160	10	(160)	1	(160)	-
92	68	(92)	1	(92)	-
182	(90)	(182)	1	(182)	-
200	(18)	(200)	1	(200)	-
128	72	(128)	1	(128)	-
63	65	(63)	1	(63)	-
59	4	(59)	1	(59)	-
11	48	(11)	1	(11)	-
-	11	-	1	-	-

quantitative composition of the "C" Portfolio, value of Δ & Γ & υ

Stock		Short Opt.	Option for Γ		Option for υ		Delta e Gamma		Vega
Buy/sell	Warehouse	Short Opt.	Buy/sell	Warehouse	Buy/sell	Warehouse	Δ portfolio C	Γ portfolio C	υ portfolio C
168	168	(1.000)	2.022	2.022	(1.051)	(1.051)	-	-	-
1	169	(1.000)	10	2.033	(2)	(1.053)	-	-	-
14	183	(1.000)	(17)	2.015	(3)	(1.056)	-	-	-
65	248	(1.000)	(113)	1.903	3	(1.053)	-	-	-
18	266	(1.000)	(22)	1.880	(1)	(1.054)	-	-	-
11	277	(1.000)	(7)	1.873	(2)	(1.056)	-	-	-
6	283	(1.000)	4	1.877	(4)	(1.060)	-	-	-
(47)	236	(1.000)	107	1.984	(13)	(1.074)	-	-	-
(102)	134	(1.000)	256	2.240	1	(1.073)	-	-	-
(14)	120	(1.000)	72	2.311	1	(1.072)	-	-	-
(51)	69	(1.000)	205	2.516	37	(1.034)	-	-	-
10	79	(1.000)	13	2.529	(12)	(1.046)	-	-	-
-	79	(1.000)	55	2.584	(1)	(1.048)	-	-	-
(38)	41	(1.000)	227	2.811	82	(966)	-	-	-
80	121	(1.000)	(220)	2.591	(139)	(1.105)	-	-	-
49	170	(1.000)	(40)	2.551	(52)	(1.157)	-	-	-
(63)	107	(1.000)	348	2.899	53	(1.103)	-	-	-
(53)	54	(1.000)	349	3.248	158	(945)	-	-	-
29	83	(1.000)	219	3.467	(87)	(1.032)	-	-	-
(49)	34	(1.000)	(125)	3.342	391	(642)	-	-	-
(34)	-	(1.000)	(3.342)	-	642	-	-	-	-

Delta Gamma Vega Hedging Cash Flow							
Stock	Option	Opt. for Γ	Opt. for υ	Bank			Hedging Revenue (cost)
Dollars in Stock (Flow)	Cash ex Shorting/Exercising Option	Dollars in Option (Flow)	Dollars in Option (Flow)	Cash	Interest (Flow)	Borrow (stock)	
16.800	10.378	17.255	(11.450)	12.228		12.228	
100		83	(26)	156	7,6	12.392	
1.414		(145)	(30)	1.239	7,7	13.638	
7.009		(1.352)	41	5.698	8,5	19.345	
1.961		(277)	(12)	1.673	12,1	21.029	
1.200		(87)	(37)	1.076	13,1	22.119	
652		46	(60)	638	13,8	22.771	
(4.871)		889	(149)	(4.131)	14,2	18.654	
(9.545)		944	5	(8.596)	11,7	10.069	
(1.287)		209	5	(1.073)	6,3	9.002	
(4.396)		280	89	(4.027)	5,6	4.981	
876		18	(30)	864	3,1	5.848	
-		69	(3)	66	3,7	5.918	
(3.178)		119	97	(2.962)	3,7	2.959	
7.363		(340)	(415)	6.608	1,9	9.569	
4.653		(76)	(192)	4.385	6,0	13.961	
(5.763)		301	112	(5.350)	8,7	8.619	
(4.695)		109	170	(4.416)	5,4	4.209	
2.653		75	(116)	2.612	2,6	6.824	
(4.449)		(13)	310	(4.152)	4,3	2.676	
(2.976)	-	-	93	(2.883)	1,7	(205)	205

LET US HOLD THE OPTION 'Z' UNTIL MATURITY

AT MATURITY THE OPTION 'W' IS IN – THE MONEY

$\Delta - \Gamma - \upsilon$ HEDGING

Short 1000 call on 1 stock			Option and Δ			Stock and Δ				Δ Portfolio	Γ Portfolio "A"	υ Portfolio "A"
Time Step	Time to Expiration	STOCK PRICE	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position	Γ portfolio = Γ I option *n.az. Underlying	υ portfolio = υ I option *n.az. Underlying
0	0,2500	100,0	(1.000)	0,564	(564)	564	564	1	564	-	(15,70)	(19.628)
1	0,2375	102,9	(1.000)	0,607	(607)	43	607	1	607	-	(15,28)	(19.202)
2	0,2250	96,9	(1.000)	0,508	(508)	(99)	508	1	508	-	(17,31)	(18.291)
3	0,2125	94,2	(1.000)	0,456	(456)	(52)	456	1	456	-	(18,23)	(17.176)
4	0,2000	92,4	(1.000)	0,417	(417)	(39)	417	1	417	-	(18,87)	(16.091)
5	0,1875	91,9	(1.000)	0,402	(402)	(15)	402	1	402	-	(19,41)	(15.368)
6	0,1750	97,1	(1.000)	0,499	(499)	97	499	1	499	-	(19,60)	(16.181)
7	0,1625	98,0	(1.000)	0,512	(512)	13	512	1	512	-	(20,16)	(15.723)
8	0,1500	107,3	(1.000)	0,688	(688)	176	688	1	688	-	(16,96)	(14.659)
9	0,1375	107,6	(1.000)	0,697	(697)	9	697	1	697	-	(17,45)	(13.898)
10	0,1250	113,8	(1.000)	0,801	(801)	104	801	1	801	-	(13,82)	(11.186)
11	0,1125	105,2	(1.000)	0,660	(660)	(141)	660	1	660	-	(20,72)	(12.905)
12	0,1000	105,8	(1.000)	0,677	(677)	17	677	1	677	-	(21,42)	(11.989)
13	0,0875	107,5	(1.000)	0,721	(721)	44	721	1	721	-	(21,09)	(10.668)
14	0,0750	110,8	(1.000)	0,800	(800)	79	800	1	800	-	(18,42)	(8.488)
15	0,0625	118,9	(1.000)	0,928	(928)	128	928	1	928	-	(9,17)	(4.049)
16	0,0500	128,5	(1.000)	0,989	(989)	61	989	1	989	-	(1,89)	(782)
17	0,0375	128,1	(1.000)	0,995	(995)	6	995	1	995	-	(1,03)	(317)
18	0,0250	126,1	(1.000)	0,998	(998)	3	998	1	998	-	(0,47)	(93)
19	0,0125	129,5	(1.000)	1,000	(1.000)	2	1.000	1	1.000	-	(0,00)	(0)
20	0,0000	135,4	(1.000)	1,000	(1.000)	-	1.000	1	1.000	-	-	-

Δ − Γ − υ HEDGING

Portfolio **B** = Portfolio **A** + **II** Option + **III** Option

Γ−υ Portfolio "B"														Δ Port. "B"
II Option			III Option											
II Option value	Γ II option	υ II option	III Option value	Γ III option	υ III option	n. II option Buy/sell	n. III option buy/sell	Γ II option Tot	Γ III option Tot	υ II option Tot	υ III option Tot	Γ portfolio "B"	υ portfolio "B"	Total Δ position
8,5320	0,0155	20,3815	10,8995	0,0149	20,5532	2.022	(1.051)	31	(16)	41.219	(21.591)	-	-	396
9,7225	0,0154	20,3963	12,3193	0,0145	20,1679	1.982	(1.052)	31	(15)	40.426	(21.224)	-	-	413
6,5788	0,0166	18,5301	8,7069	0,0164	19,2663	2.084	(1.055)	35	(17)	38.614	(20.323)	-	-	358
5,2071	0,0170	16,9797	7,1031	0,0173	18,1947	2.142	(1.055)	36	(18)	36.374	(19.197)	-	-	324
4,3079	0,0172	15,6141	6,0409	0,0179	17,1595	2.190	(1.055)	38	(19)	34.192	(18.102)	-	-	298
3,9135	0,0175	14,7975	5,5943	0,0184	16,4811	2.216	(1.057)	39	(19)	32.785	(17.417)	-	-	285
5,5771	0,0185	16,3570	7,7242	0,0183	17,2915	2.120	(1.069)	39	(20)	34.674	(18.493)	-	-	335
5,6346	0,0191	16,0207	7,8617	0,0188	16,8732	2.114	(1.075)	40	(20)	33.864	(18.141)	-	-	336
10,0788	0,0179	16,7351	13,1938	0,0158	15,9604	1.898	(1.072)	34	(17)	31.762	(17.102)	-	-	398
9,8899	0,0185	16,0717	13,0655	0,0162	15,2678	1.887	(1.076)	35	(17)	30.323	(16.425)	-	-	390
13,5852	0,0162	14,3971	17,3239	0,0132	12,7919	1.709	(1.049)	28	(14)	24.608	(13.423)	-	-	389
7,7618	0,0212	14,7012	10,7844	0,0189	14,3646	1.951	(1.098)	41	(21)	28.678	(15.773)	-	-	357
7,6759	0,0222	13,9702	10,7993	0,0194	13,5542	1.931	(1.106)	43	(21)	26.975	(14.986)	-	-	346
8,2416	0,0227	13,1200	11,5999	0,0191	12,4028	1.858	(1.106)	42	(21)	24.383	(13.716)	-	-	334
9,9227	0,0218	11,7366	13,6940	0,0171	10,5019	1.688	(1.078)	37	(18)	19.806	(11.318)	-	-	310
15,6275	0,0149	7,8992	20,1172	0,0102	6,2860	1.230	(902)	18	(9)	9.718	(5.669)	-	-	225
24,0642	0,0059	3,0260	28,9788	0,0036	2,2120	646	(530)	4	(2)	1.955	(1.173)	-	-	101
23,5122	0,0050	2,0470	28,4795	0,0029	1,5131	412	(349)	2	(1)	844	(528)	-	-	57
21,3530	0,0048	1,4442	26,3520	0,0028	1,1223	193	(165)	1	(0)	278	(185)	-	-	25
24,6183	0,0010	0,2100	29,6596	0,0008	0,2379	2	(1)	0	(0)	0	(0)	-	-	1
30,4471	0,0000	0,0002	35,4896	0,0000	0,0045	-	-	-	-	-	-	-	-	-

$\Delta - \Gamma - \upsilon$ HEDGING

	portfolio "C" = Port. "B" + Stock (Δ hedge of "B")				
Δ Port. "B"	Stock and Δ Portfolio				Δ Portfolio "C"
Total Δ position	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
396	(396)	(396)	1	(396)	-
413	(17)	(413)	1	(413)	-
358	55	(358)	1	(358)	-
324	34	(324)	1	(324)	-
298	26	(298)	1	(298)	-
285	13	(285)	1	(285)	-
335	(50)	(335)	1	(335)	-
336	(1)	(336)	1	(336)	-
398	(62)	(398)	1	(398)	-
390	8	(390)	1	(390)	-
389	1	(389)	1	(389)	-
357	32	(357)	1	(357)	-
346	11	(346)	1	(346)	-
334	12	(334)	1	(334)	-
310	24	(310)	1	(310)	-
225	85	(225)	1	(225)	-
101	124	(101)	1	(101)	-
57	44	(57)	1	(57)	-
25	32	(25)	1	(25)	-
1	24	(1)	1	(1)	-
-	1	-	1	-	-

$\Delta - \Gamma - \upsilon$ HEDGING

quantitative composition of the "C" Portfolio, value of Δ & Γ & υ

Stock		Short Opt.	Option for Γ		Option for υ		Delta e Gamma		Vega
Buy/sell	Warehouse	Short Opt.	Buy/sell	Warehouse	Buy/sell	Warehouse	Δ portfolio C	Γ portfolio C	υ portfolio C
168	168	(1.000)	2.022	2.022	(1.051)	(1.051)	-	-	-
26	194	(1.000)	(40)	1.982	(2)	(1.052)	-	-	-
(44)	150	(1.000)	102	2.084	(3)	(1.055)	-	-	-
(18)	132	(1.000)	58	2.142	(0)	(1.055)	-	-	-
(13)	119	(1.000)	48	2.190	0	(1.055)	-	-	-
(2)	117	(1.000)	26	2.216	(2)	(1.057)	-	-	-
47	164	(1.000)	(96)	2.120	(13)	(1.069)	-	-	-
12	176	(1.000)	(6)	2.114	(6)	(1.075)	-	-	-
114	290	(1.000)	(216)	1.898	4	(1.072)	-	-	-
17	307	(1.000)	(11)	1.887	(4)	(1.076)	-	-	-
105	412	(1.000)	(177)	1.709	26	(1.049)	-	-	-
(109)	303	(1.000)	241	1.951	(49)	(1.098)	-	-	-
28	331	(1.000)	(20)	1.931	(8)	(1.106)	-	-	-
56	387	(1.000)	(72)	1.858	(0)	(1.106)	-	-	-
103	490	(1.000)	(171)	1.688	28	(1.078)	-	-	-
213	703	(1.000)	(457)	1.230	176	(902)	-	-	-
185	888	(1.000)	(584)	646	372	(530)	-	-	-
50	938	(1.000)	(234)	412	182	(349)	-	-	-
35	973	(1.000)	(220)	193	183	(165)	-	-	-
26	999	(1.000)	(191)	2	164	(1)	-	-	-
1	1.000	(1.000)	(2)	-	1	-	-	-	-

Δ – Γ – υ HEDGING

Delta Gamma Vega Hedging Cash Flow							
Stock	**Option**	**Opt. for Γ**	**Opt. for υ**	**Bank**			**Hedging Revenue (cost)**
Dollars in Stock (Flow)	**Cash ex Shorting/Exercising Option**	**Dollars in Option** (Flow)	**Dollars in Option** (Flow)	Cash	Interest (Flow)	**Borrow (stock)**	
16.800	10.378	17.255	(11.450)	12.228		12.228	
2.674		(393)	(23)	2.259	7,6	14.494	
(4.264)		670	(22)	(3.616)	9,1	10.888	
(1.695)		304	(2)	(1.393)	6,8	9.502	
(1.201)		205	1	(994)	5,9	8.514	
(184)		101	(10)	(94)	5,3	8.425	
4.565		(534)	(98)	3.933	5,3	12.364	
1.176		(34)	(45)	1.097	7,7	13.468	
12.238		(2.176)	48	10.110	8,4	23.586	
1.830		(111)	(55)	1.664	14,7	25.265	
11.950		(2.411)	459	9.998	15,8	35.278	
(11.471)		1.874	(525)	(10.122)	22,1	25.178	
2.963		(152)	(82)	2.728	15,7	27.922	
6.021		(597)	(2)	5.422	17,5	33.361	
11.417		(1.696)	386	10.106	20,9	43.489	
25.319		(7.146)	3.538	21.711	27,2	65.227	
23.774		(14.061)	10.769	20.482	40,8	85.750	
6.406		(5.492)	5.170	6.084	53,6	91.888	
4.413		(4.693)	4.834	4.554	57,4	96.499	
3.367		(4.690)	4.860	3.537	60,3	100.096	
135	(100.000)	-	49	185	62,6	100.343	(343)

RISK MANAGEMENT

OF A FINANCIAL INSTITUTION

PROBLEMS

- RISK LIMITS
- MARKET FUNCTIONING
- OPTIONS WITHOUT CLOSED FORM SOLUTIONS

Risk Management

of a financial institution

PROBLEMS

Risk limits

Market functioning

Options without closed form solutions

Use of numeric greek letters

WHAT IS A NUMERIC GREEK LETTER?

$$\Delta = \frac{1}{2}\left(\Delta_{+1\%} + \Delta_{-1\%}\right)$$

$$\Gamma = \frac{1}{2}\left(\Gamma_{+1\%} + \Gamma_{-1\%}\right)$$

$$\upsilon = \frac{1}{2}\left(\upsilon_{+1\%} + \upsilon_{-1\%}\right)$$

... LET US LEAVE OUT THE OTHERS BECAUSE THEY ARE NOT SO IMPORTANT

Financial institutions define some bounds in terms of greek letters with reference to:

- SECURITY
- MARKET

Some definitions

- Anomalous exercise
- Payment by physical delivery
- Payment by cash settlement

Anomalous exercise

The holder of the option can partially exercise his right

Payment by phyisical settlement

The holder of the option must deliver the underlying security

Cash settlement

The holder of the option must deliver the differential by *cash*

Some Remarks

Cash settlement

Microstructure elements of the market

Knock-IN terms

SOME REMARKS

CASH SETTLEMENT

MICROSTRUCTURE ELEMENTS OF THE MARKET

KNOCK-IN TERMS

FINE TUNING

RISK MANAGEMENT

OF A FINANCIAL INSTITUTION

Some Remarks

Cash settlement

Microstructure elements of the market

Knock-IN terms

Fine Tuning

Risk Management

of a financial institution

Cases of Micromanipulation

RISK MANAGEMENT OF A FINANCIAL INSTITUTION ⟷ COVERED WARRANT

RISK MANAGEMENT OF A FINANCIAL INSTITUTION ⟷ REVERSE CONVERTIBLE/ DISCOUNT CERTIFICATE

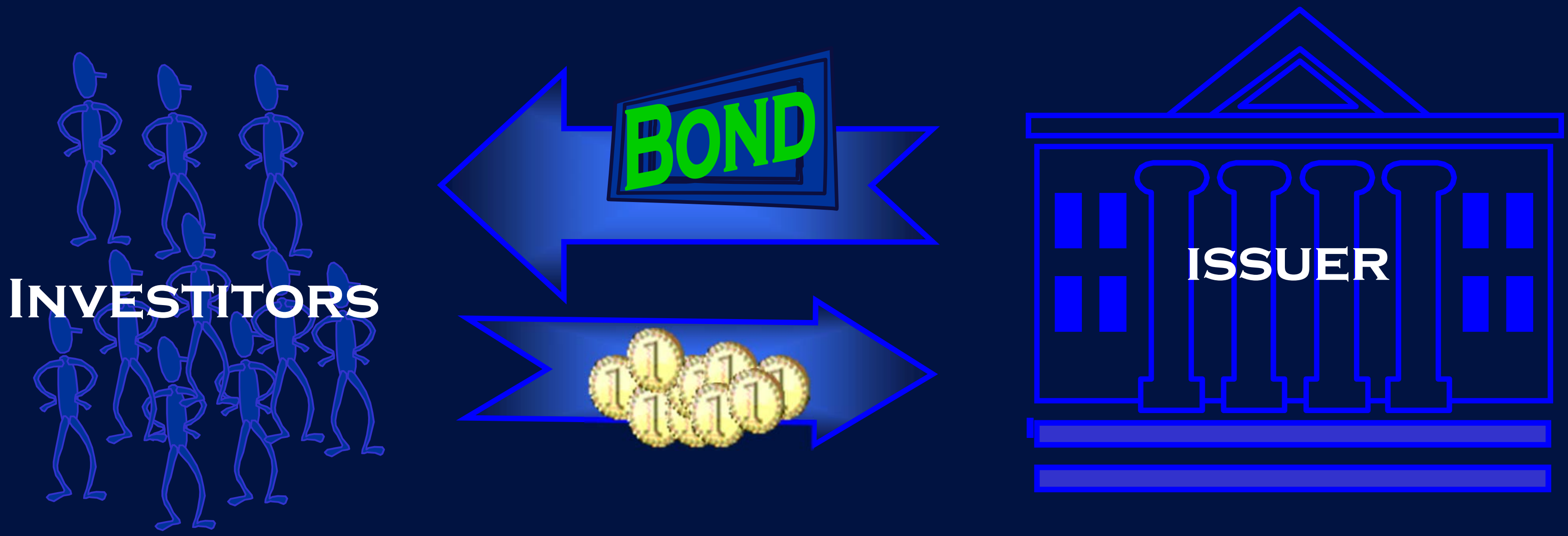
Risk Management of a financial institution ⟷ Covered Warrant

Risk Management of a Financial institution ⟷ Reverse Convertible

… SOME INTRODUCTORY REMARKS …

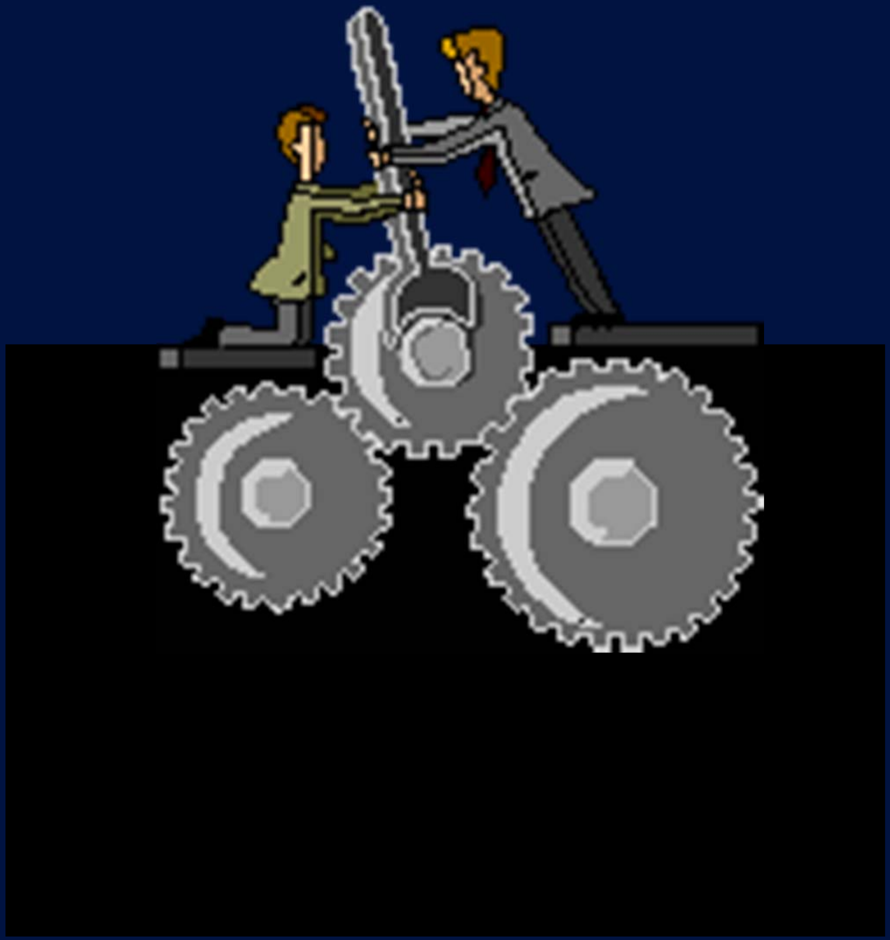
The issue

How the product is sold

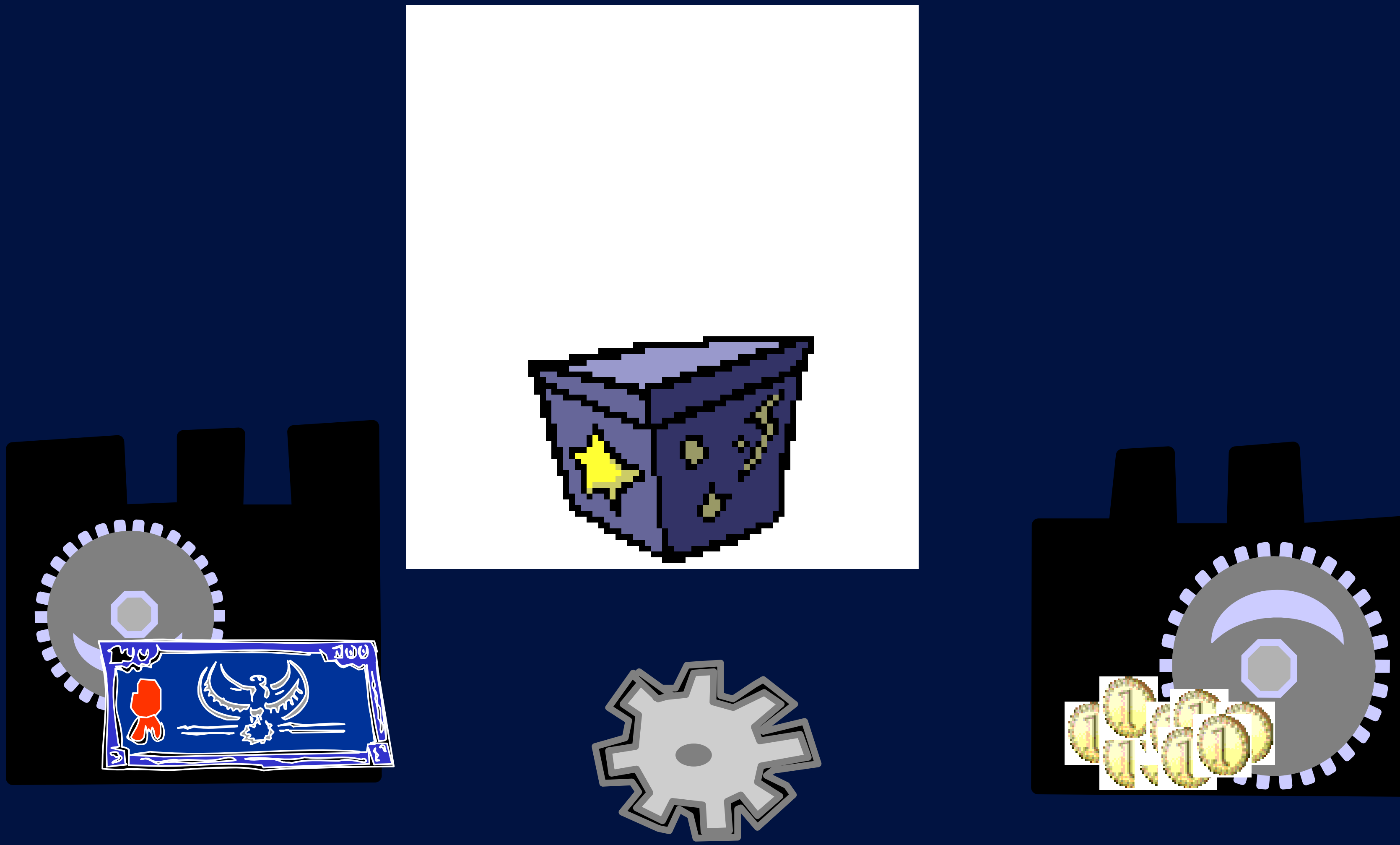


UNBUNDLING

Reverse convertible - unbundling

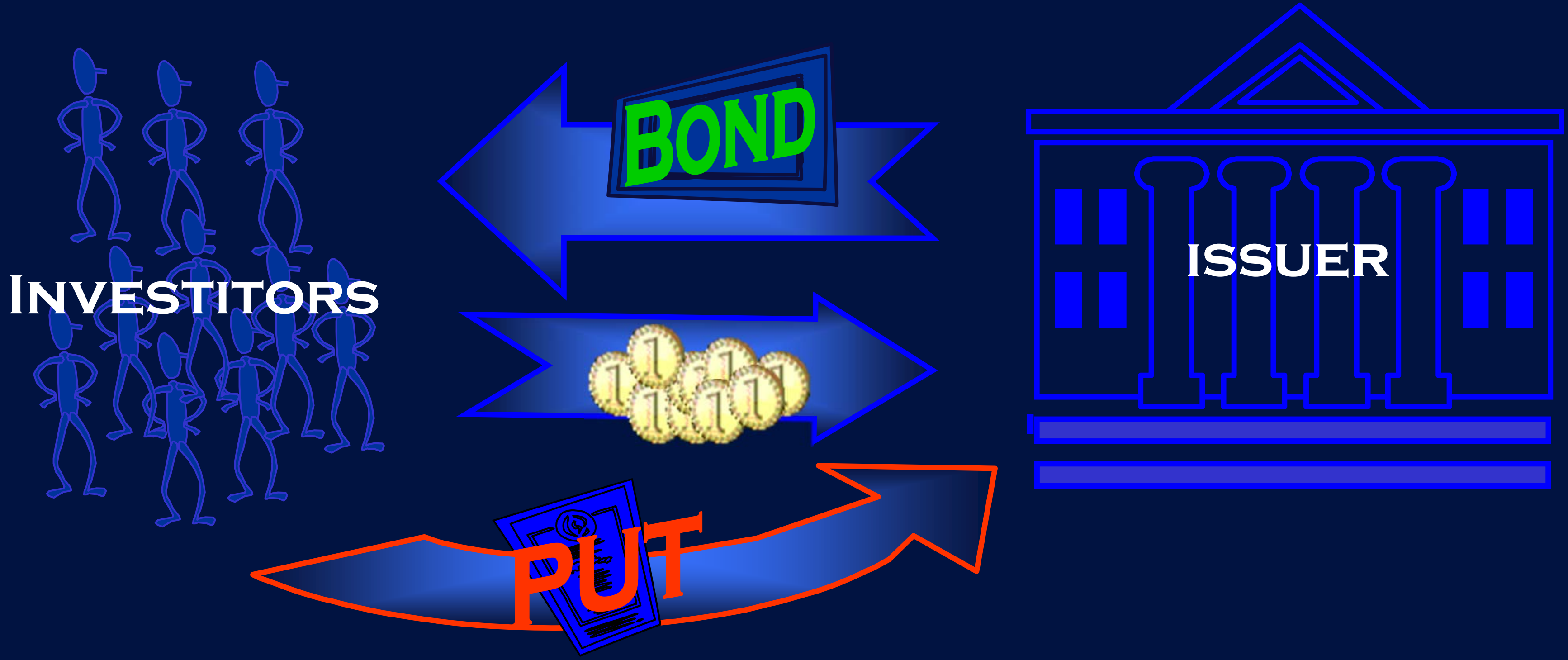


Reverse convertible - unbundling

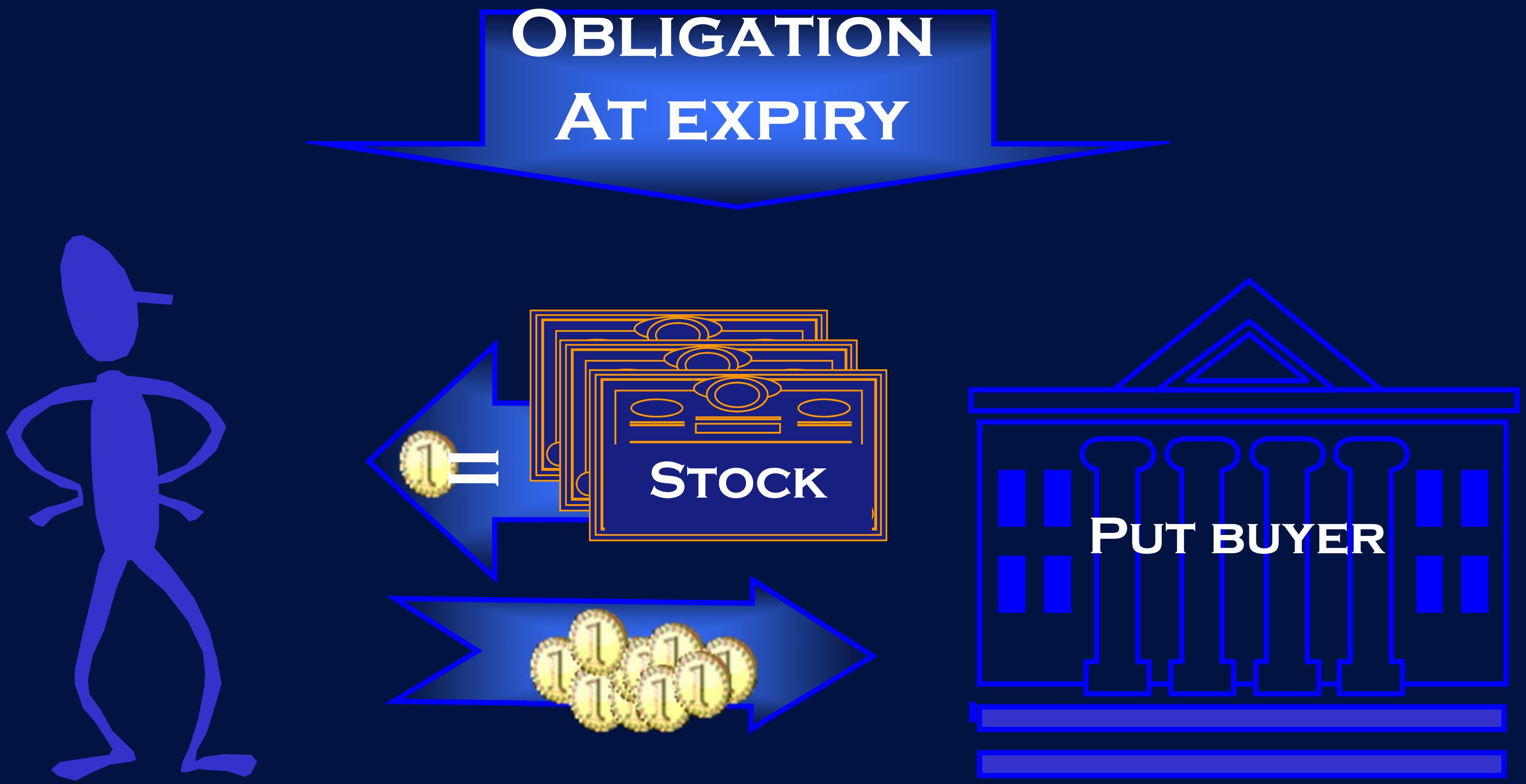


INVESTOR SHORTS A PUT

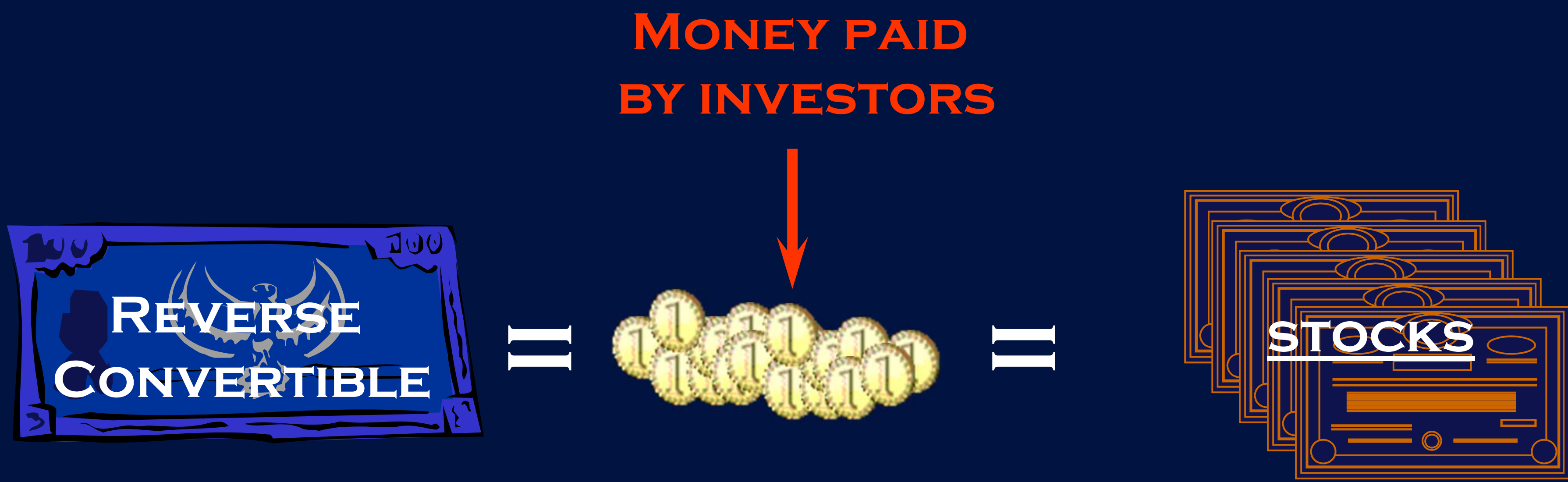
The issue



Put shorting



The structure

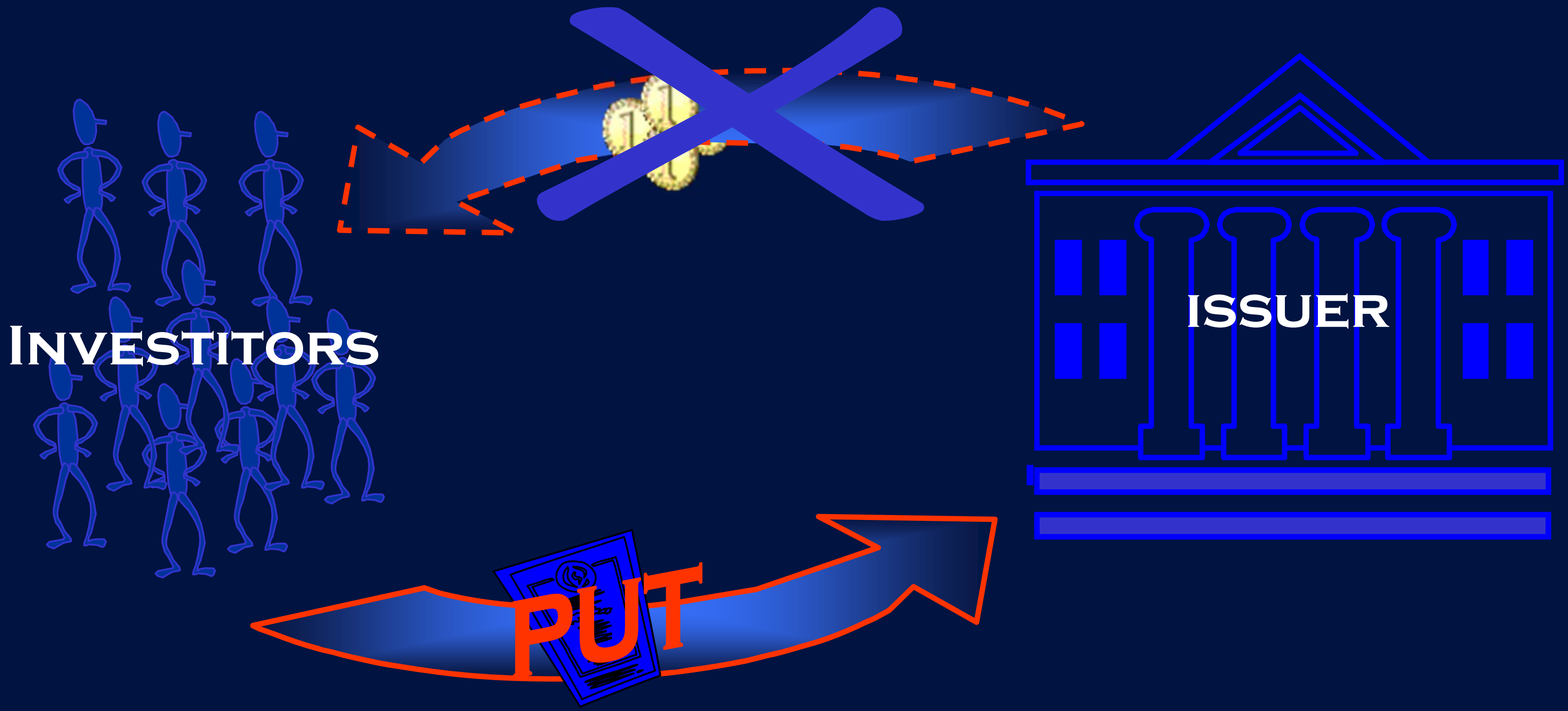


Face value

N.Stocks underlying the Put
X
Strike price (K) of the Put

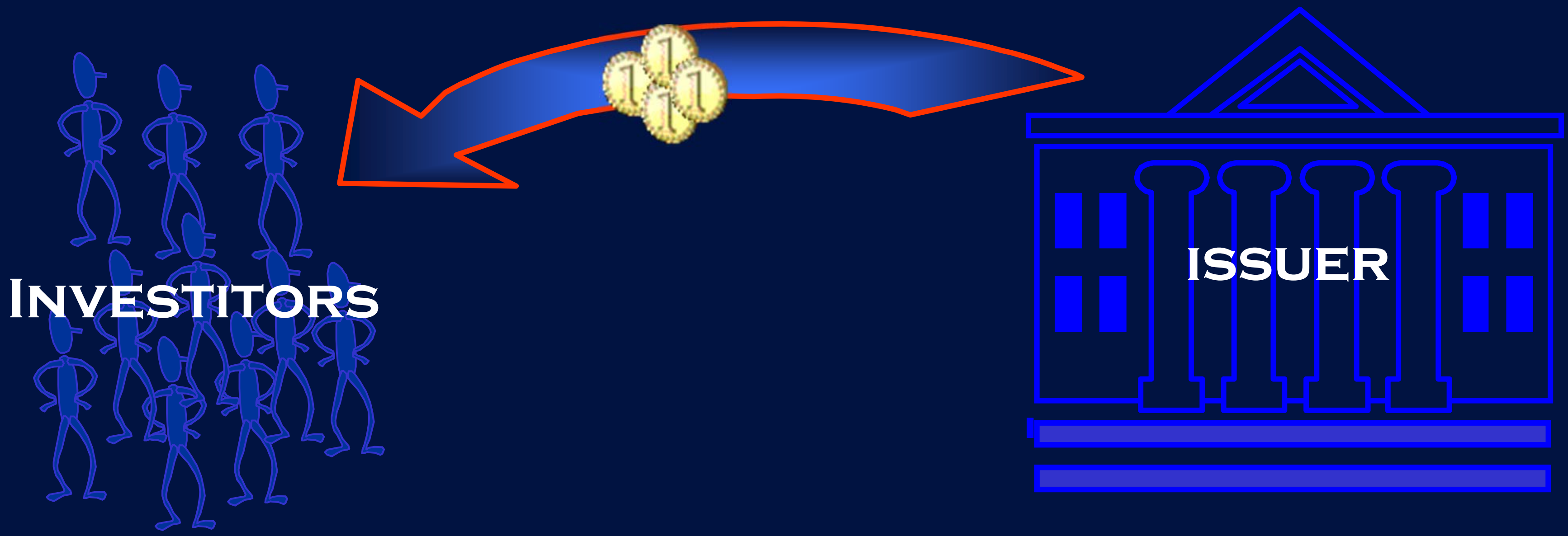
Reverse Convertible – the structure

The put premium is not received by investors when buying the product



…….BUT AT THE EXPIRY DATE

IN THE FORM OF A MAXI-COUPON

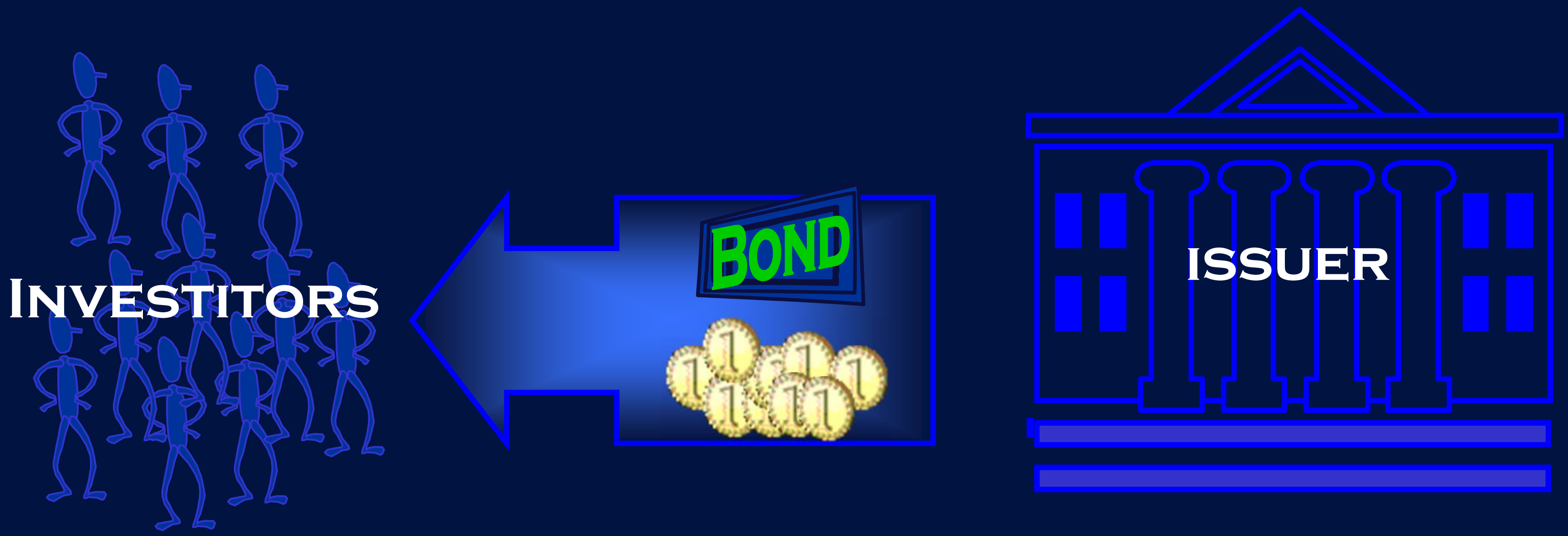


Expiry date:

option is OUT-the-money

Expiry date

Option out-of the money

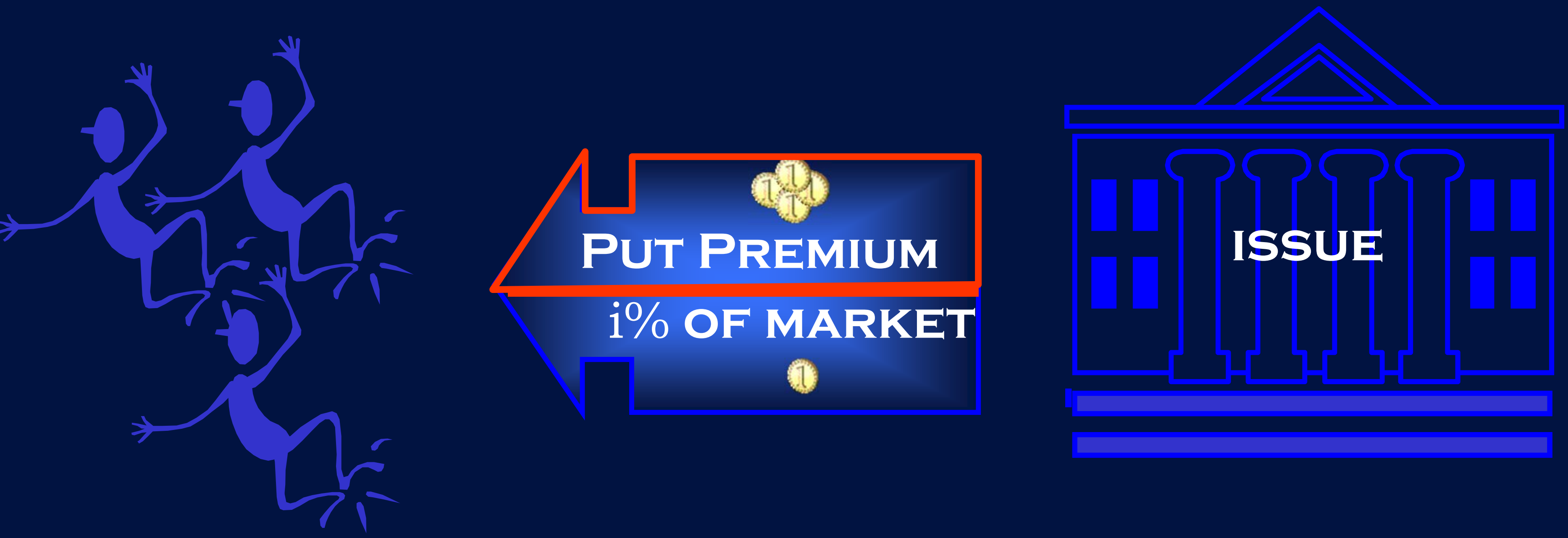


EXPIRY DATE

OPTION OUT-OF THE MONEY

Reverse Convertible – expiry date

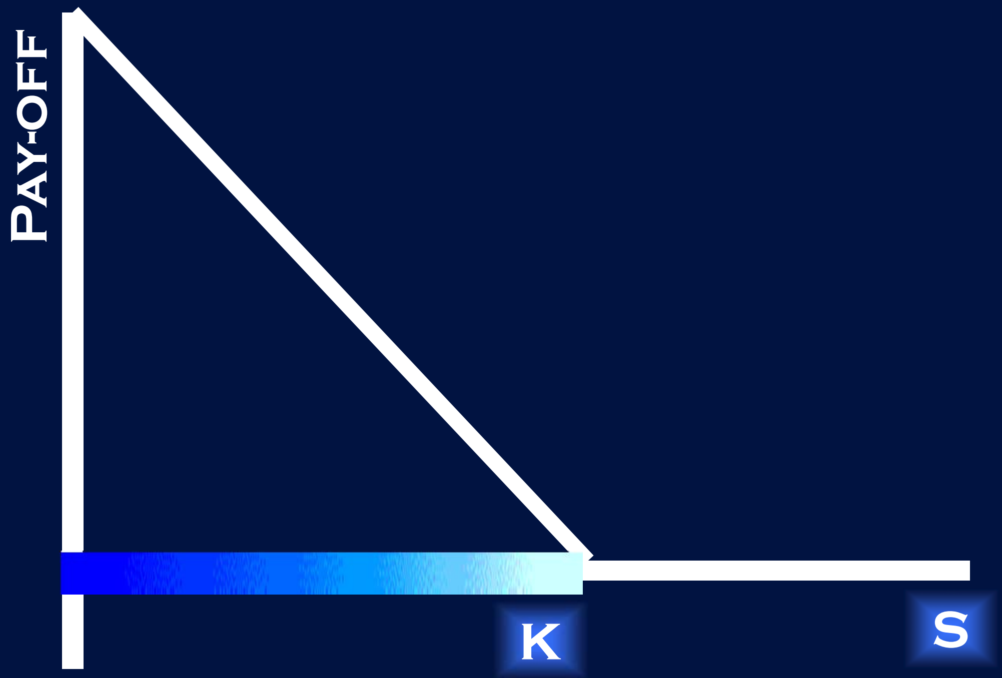
Maxi-coupon



Expiry date:

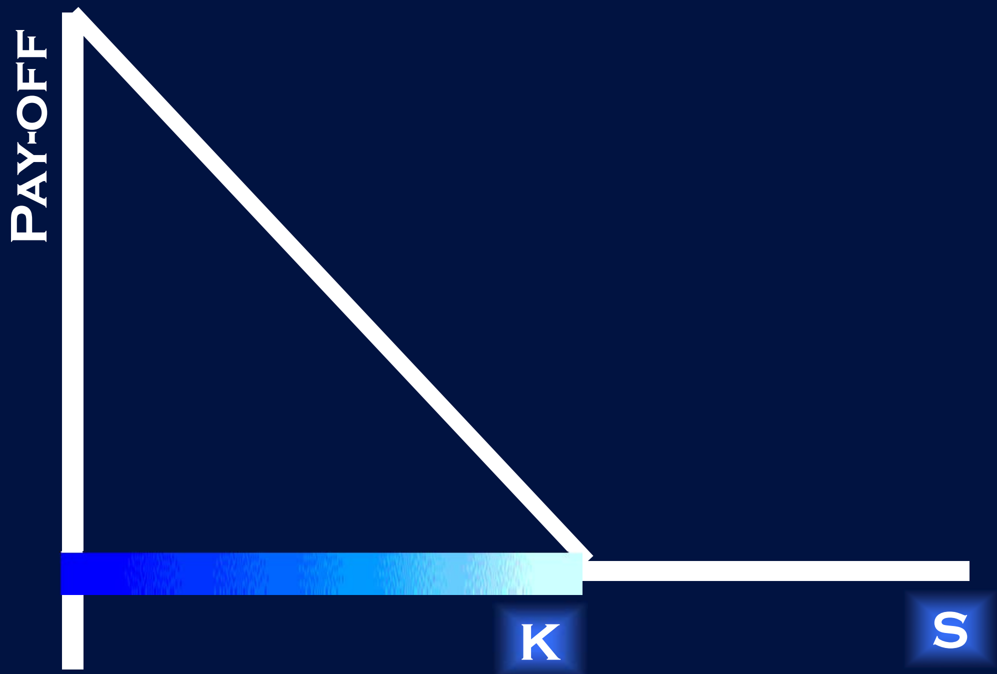
option is in-the-money

Issuer is long put.....


PAY-OFF
K
S

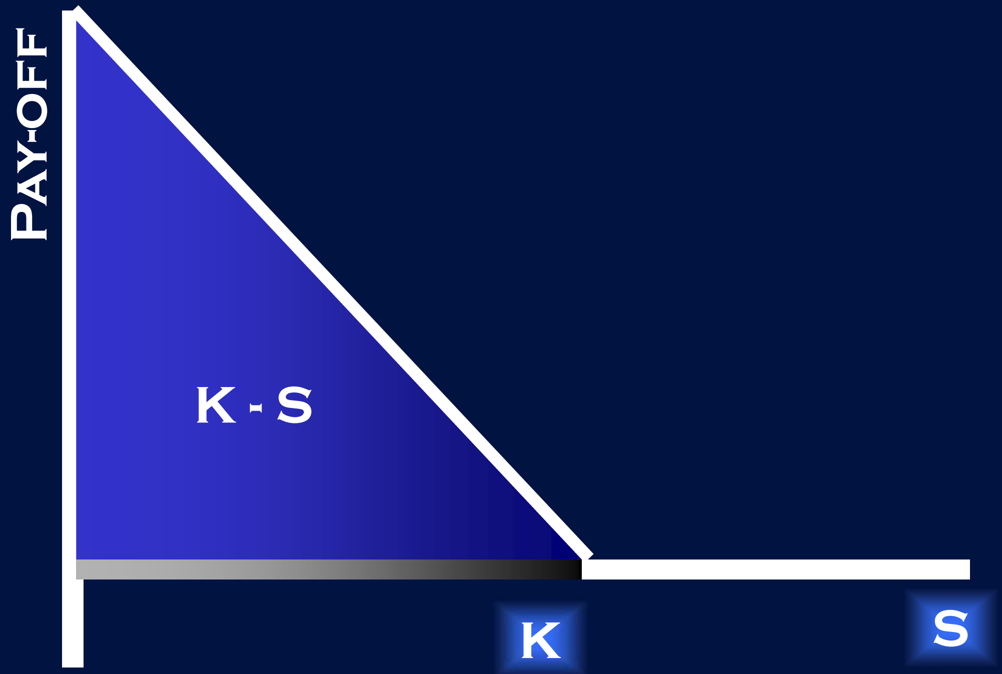

ISSUER

REVERSE CONVERTIBLE AND PUT OPTION

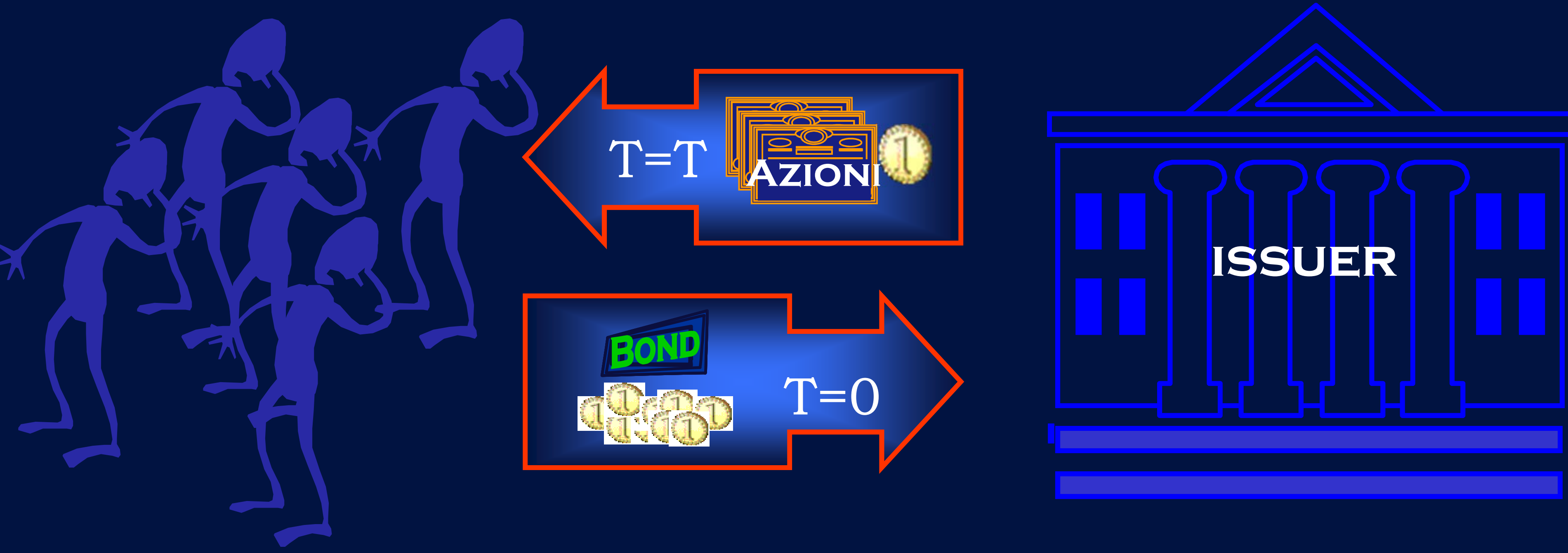


EXERCISE THE OPTION

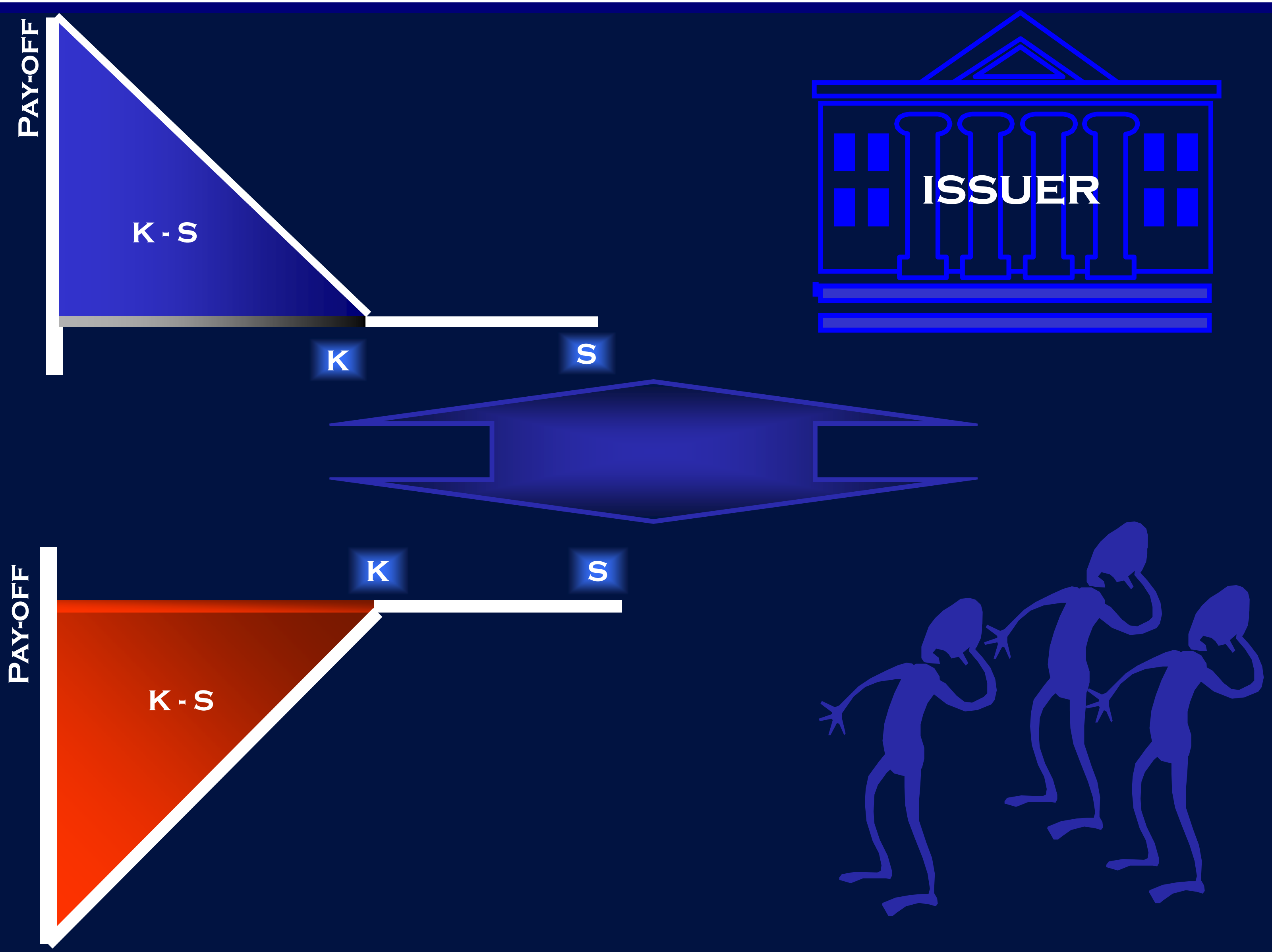
REVERSE CONVERTIBLE AND PUT OPTION



Reverse Convertible – expiry date



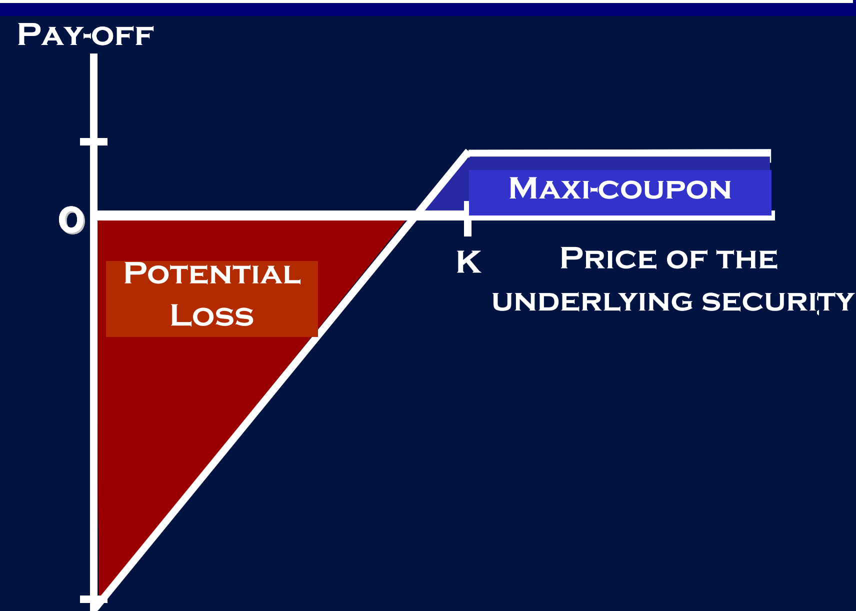
REVERSE CONVERTIBLE AND PUT OPTION

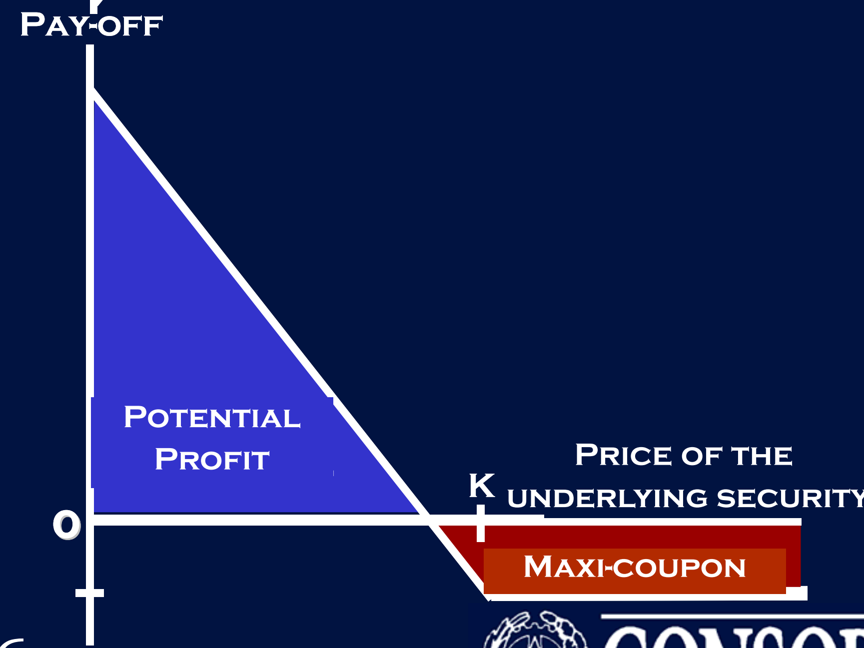


… SOME FURTHER REMARKS …

1. ISSUER'S PROFIT OR LOSS

Reverse convertible





2. THE PAY-OFF'S COMPUTATION METHOD

ARE BASED ON

NOT VERY LIQUID PRICES

AS AN EXAMPLE:

OPENING AND CLOSING PRICES

3. Tipology of structure

Plain Vanilla

Knock-IN

Settlement:

- Cash
- Physical delivery

4. hedging choises of financial institutions

They don't hedge themselves

They hedge themselves

4\. The issuer does not hedge the financial risk connected to the reverse because …

… BECAUSE HE IS AN OPTION'S BUYER …

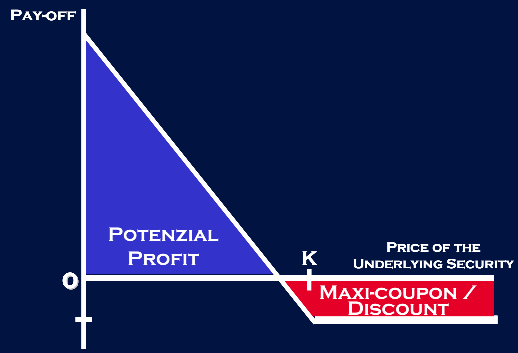
... AND BECAUSE

HE HAS FIXED

THE PRICE

REVERSE CONVERTIBLE

1. ISSUER'S PROFIT OR LOSS



2. THE PAY-OFF'S COMPUTATION METHOD

ARE BASED ON

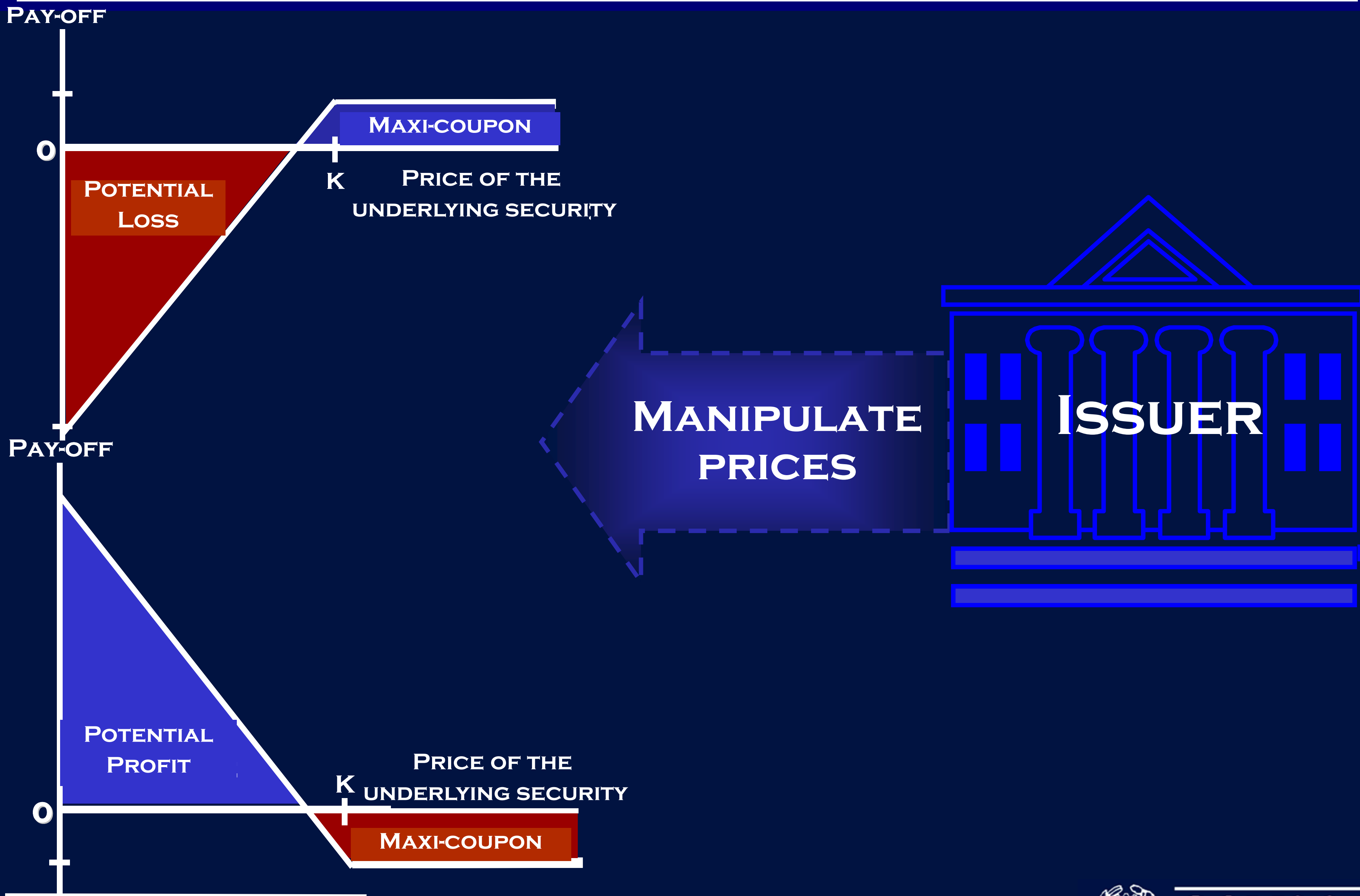
NOT VERY LIQUID PRICES

GENERATE

MANIPULATIONS

4. HEDGING LACK

Reverse convertible



4.

The issuer hedges the financial risk connected to the reverse because ...

... BECAUSE

THE PURCHASE OF AN

OPTION ...

… IS PART OF THE MOST GENERAL SYSTEM OF RISK MANAGEMENT

Financial institutions fix limits in Terms of greek letters:

- SECURITY
- MARKET

Financial institutions fix limits in Terms of greek letters:

- SECURITY
- MARKET

The results that we are going to show are easily extensible

Case Studies

Plain Vanilla

Knock-IN

OTM

ATM

ITM

Settlement:

- Cash
- Physical settlement

Plain Vanilla

OTM

ATM

ITM

Settlement:

- Cash
- Physical settlement

Plain Vanilla

OTM

Settlement by physical delivery

Risk Management of a financial institution

micro-manipulations

Plain Vanilla

OTM

Settlement by cash

Risk Management of a financial institution

micro-manipulations

PLAIN VANILLA

ITM

SETTLEMENT BY PHYSICAL DELIVERY

RISK MANAGEMENT OF A FINANCIAL INSTITUTION

MICRO-MANIPULATIONS

Plain Vanilla

ITM

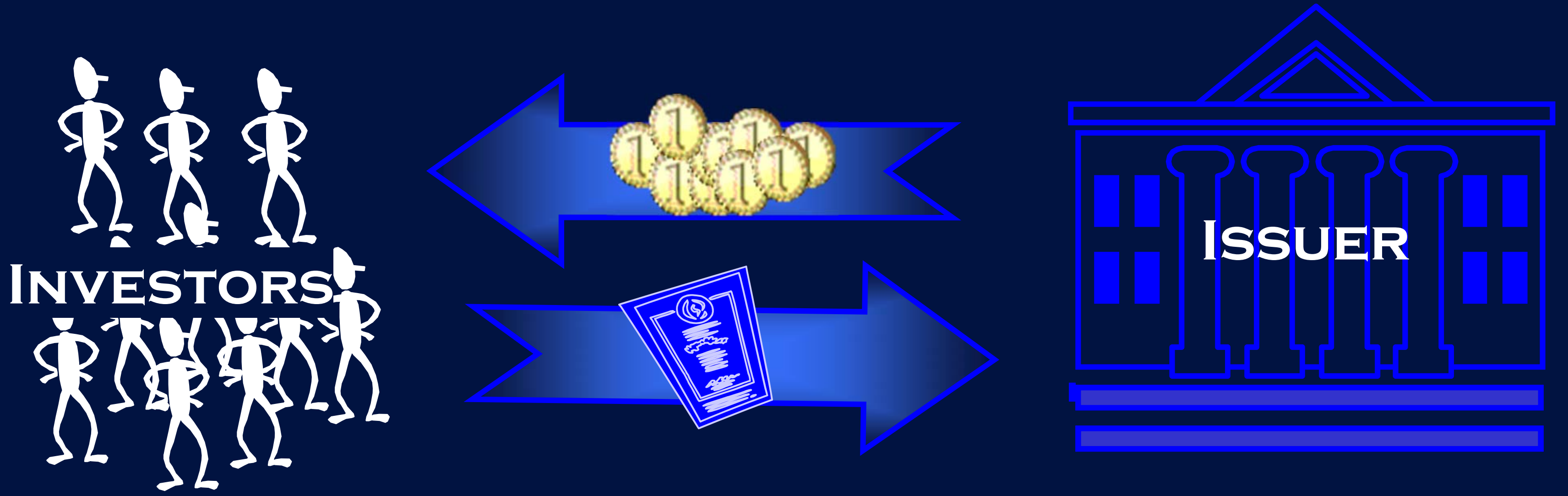
Settlement by cash

Risk Management of a financial institution

micro-manipulations

Why is there micro-manipulation?

financial institution is a Puts' net buyer



Close to maturity …

It will hold

a large amount of

the stocks underlying

the Put

At maturity the option is ITM……

It will hold <u>All</u> the stocks

underlying the Puts

In order to settle the option's pay-off ...

It will sell all the stocks

So called 'risk unwinding'

Selling all the stocks …

It will cause a fall in prices of the security

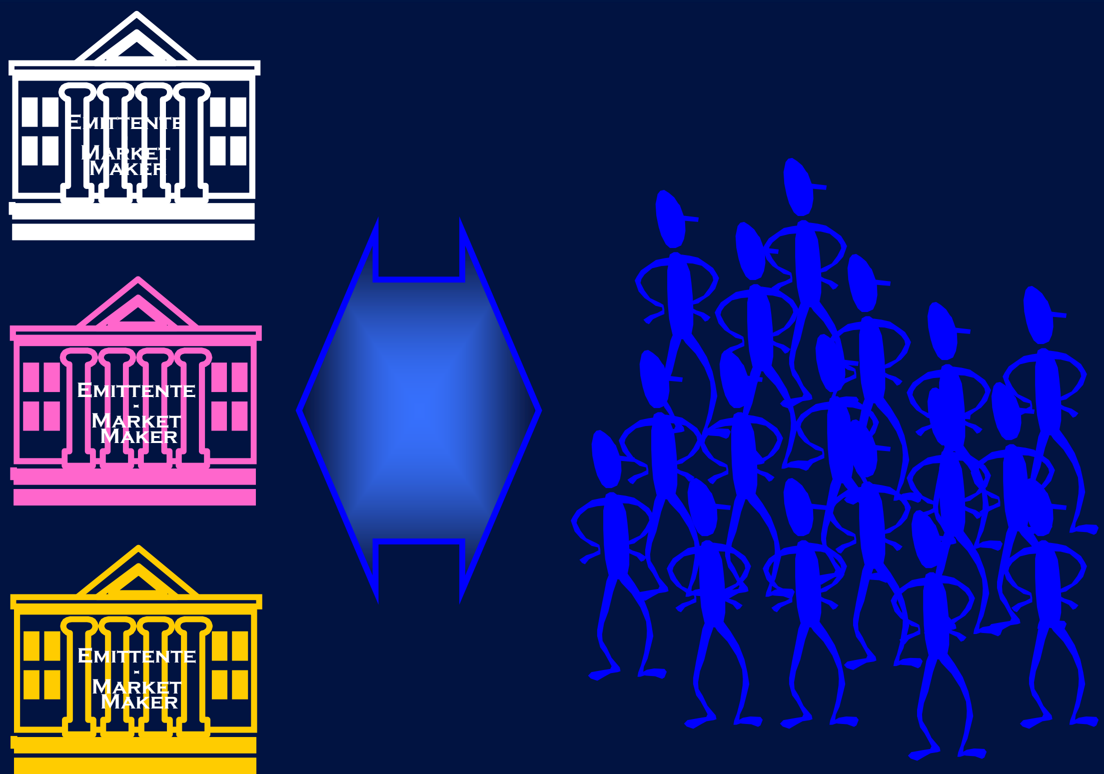
Driving a fall in prices …

It will increase

the investor's loss

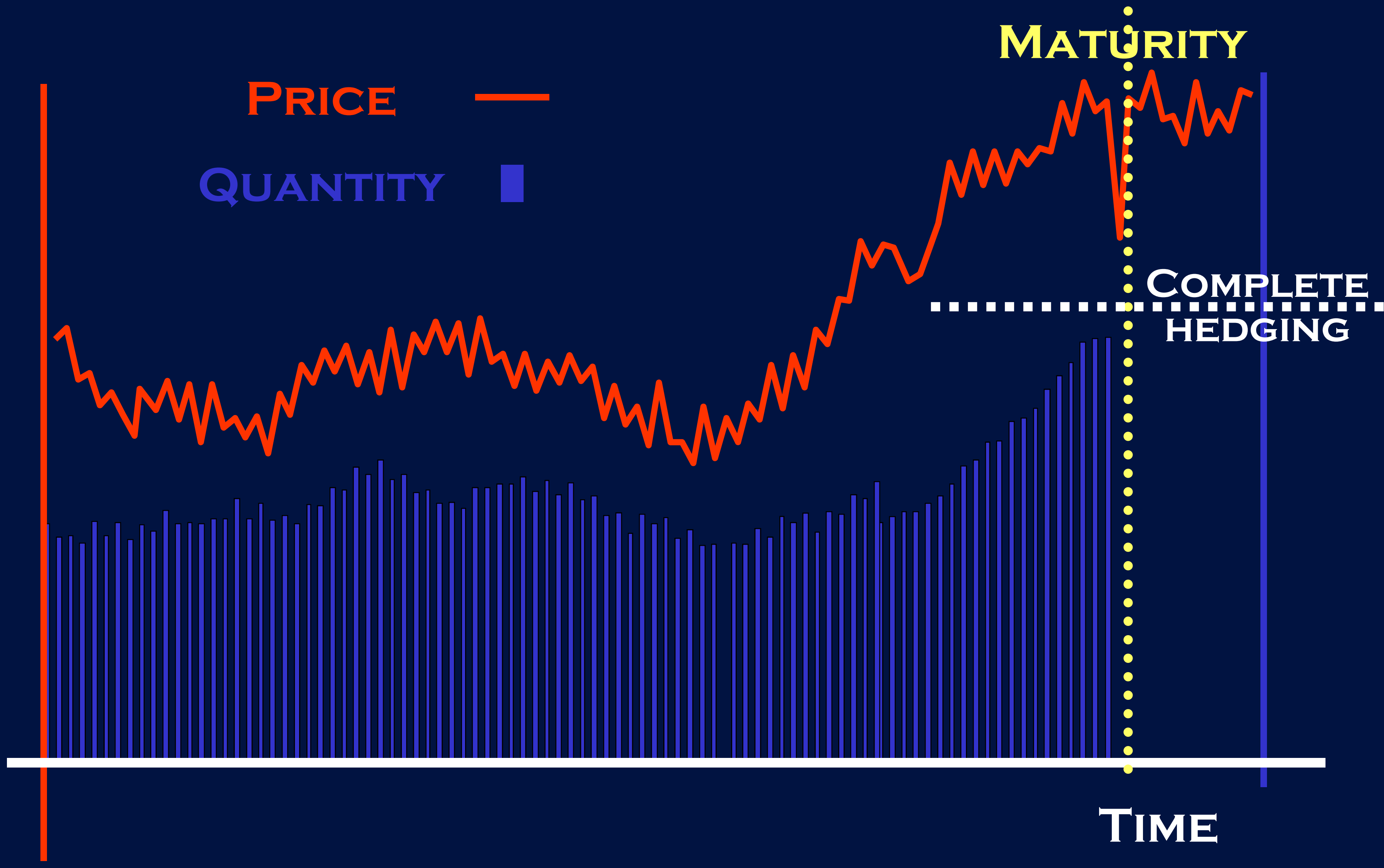
A Summary conducted through....

Delta Hedging analysis



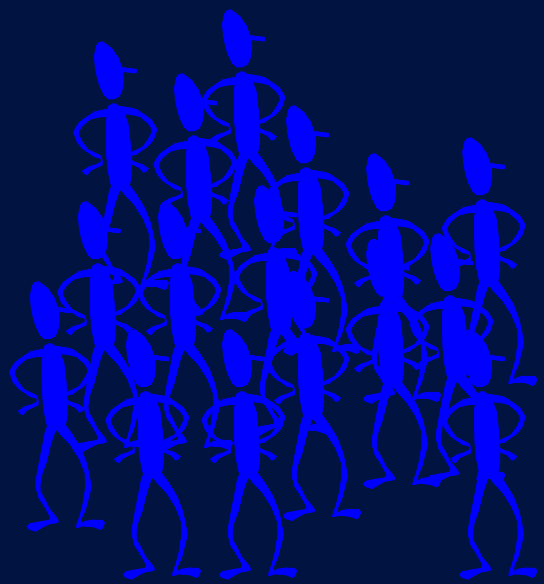
The point of View

Rev.conv. : Case Studies

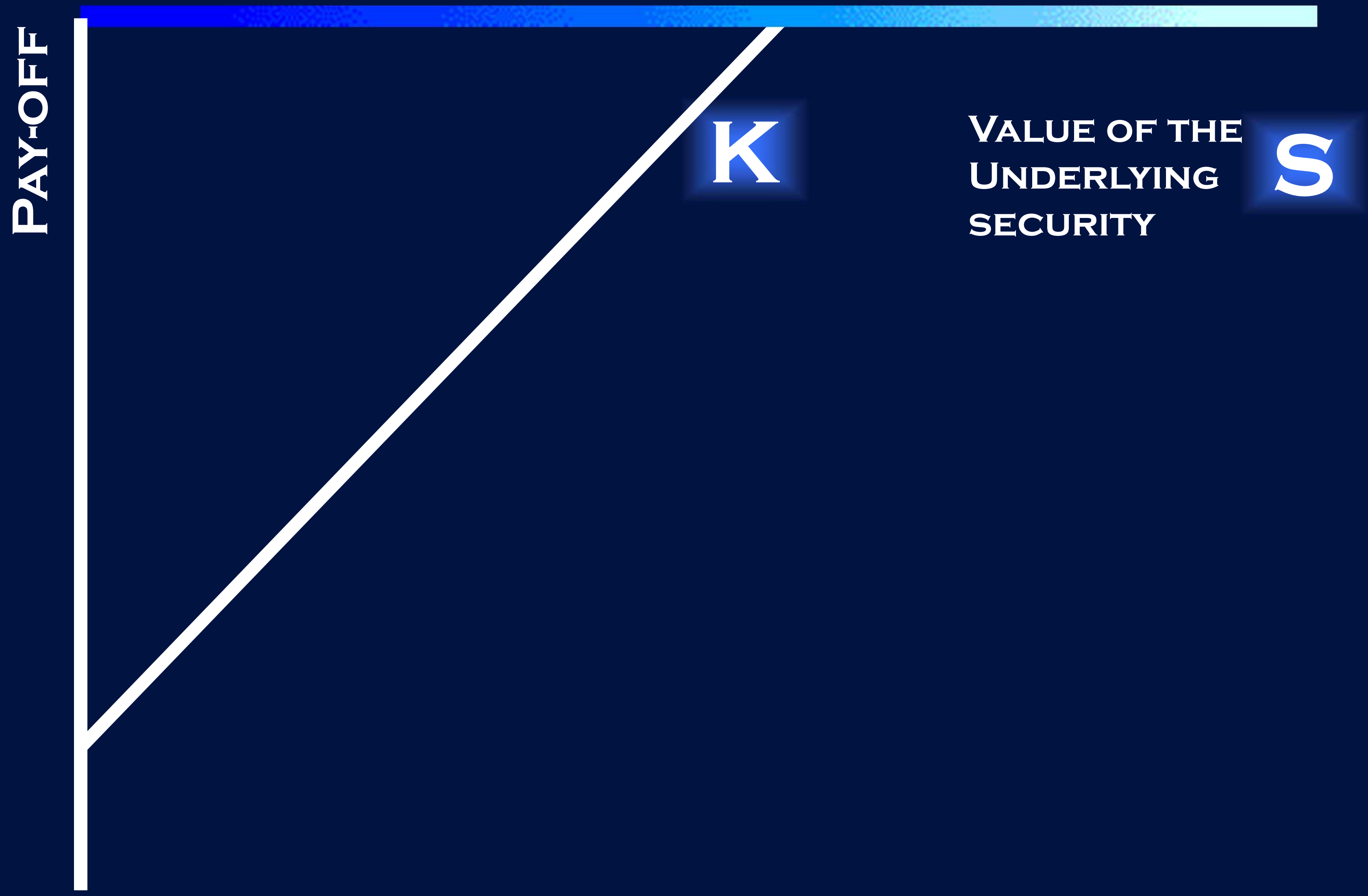


CONSOB

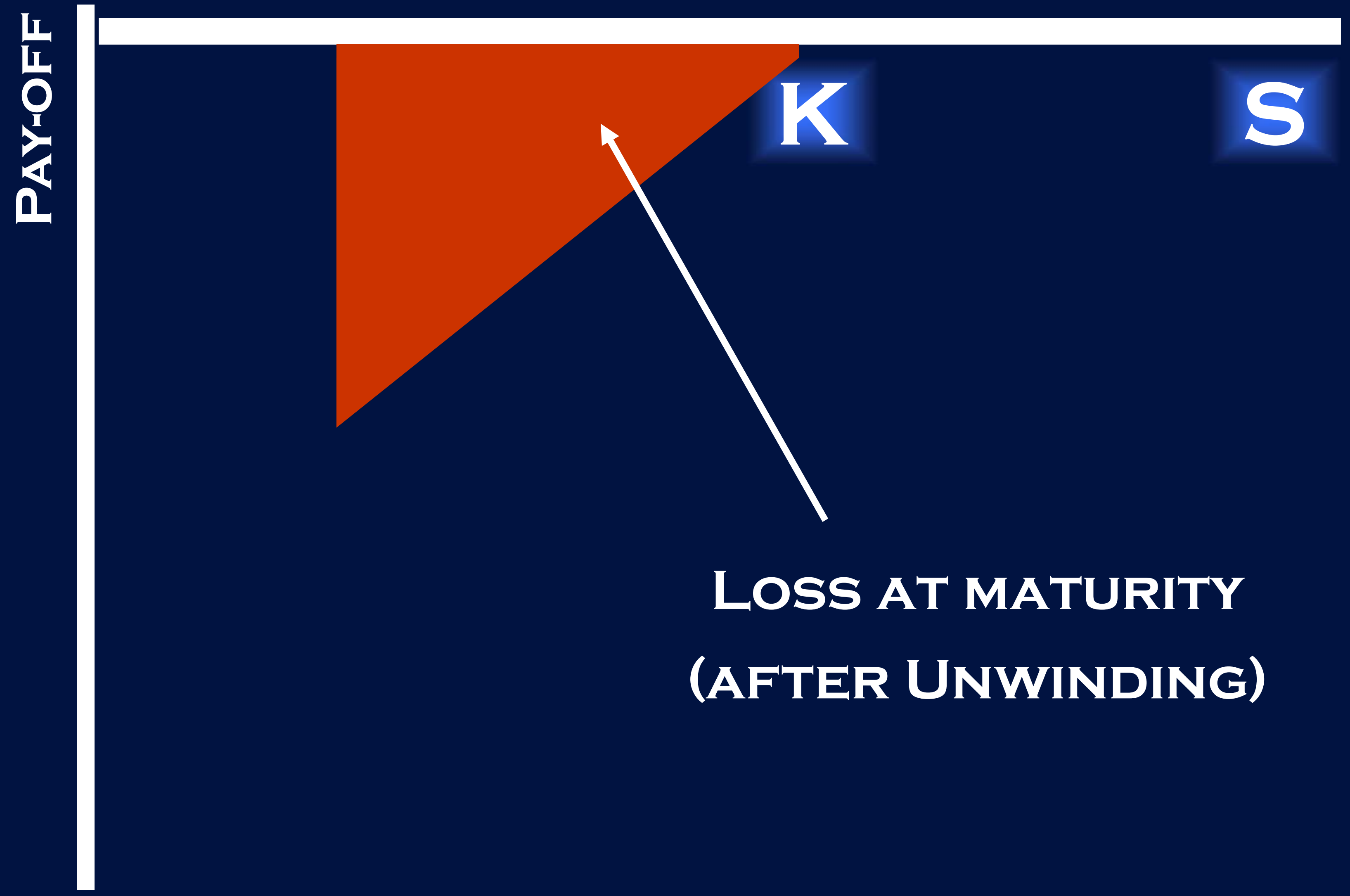
The point of View



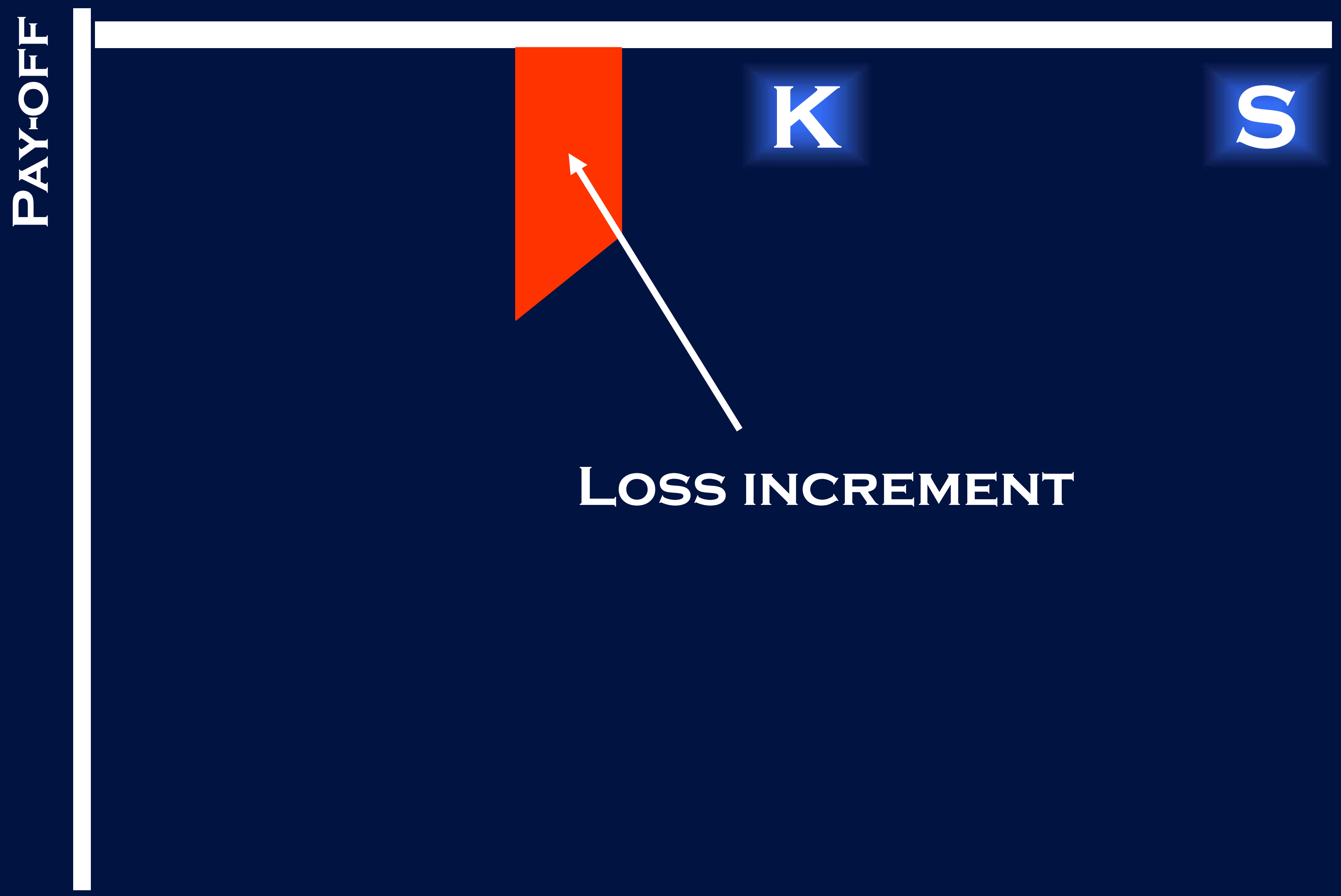
Rev.conv. : Case Studies



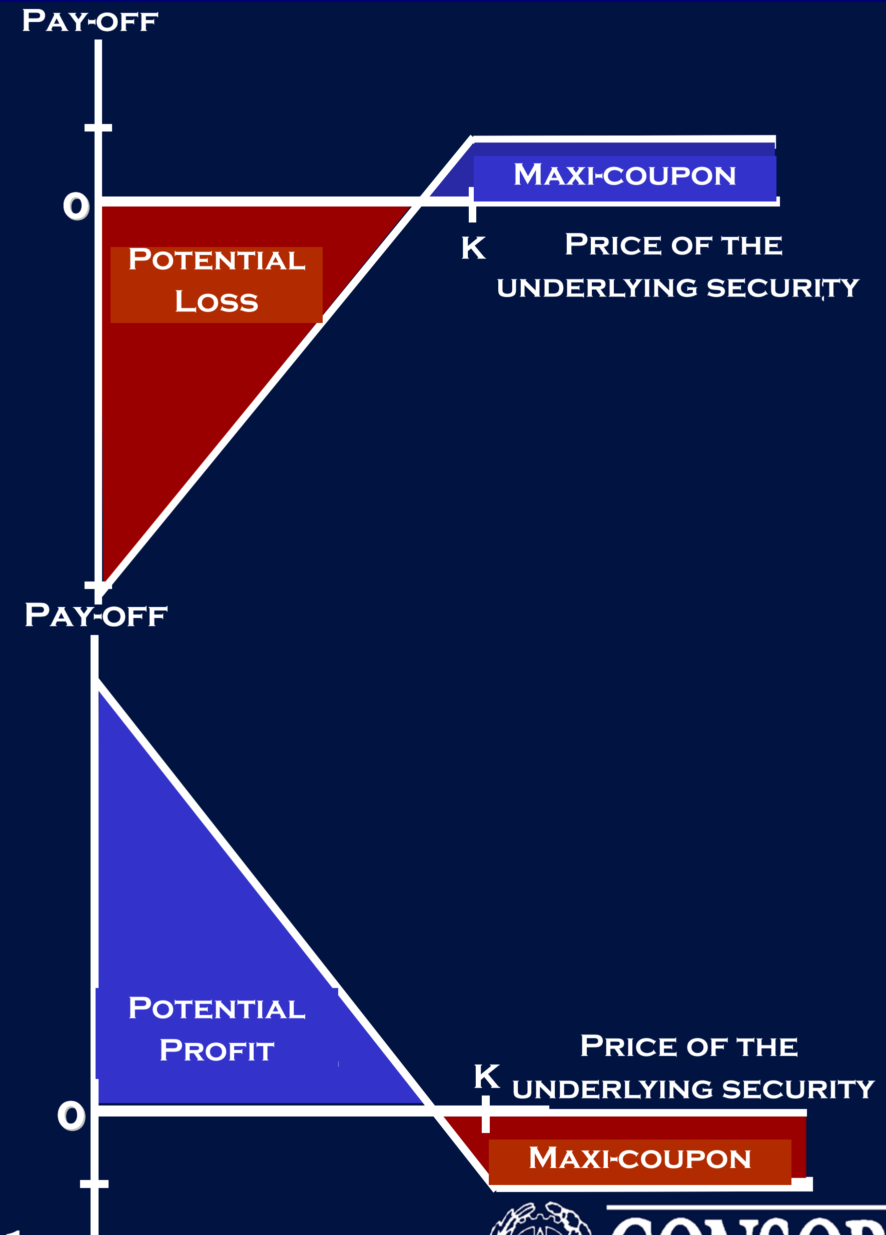
Rev.conv. : Case Studies


PAY-OFF
K
S
LOSS AT MATURITY
(AFTER UNWINDING)

Rev.conv. : Case Studies



Reverse convertible



Plain Vanilla

↓

ATM

↓

Problems connected to the so called 'view'

↓

Risk Management
of a
financial institution

↓

micro-manipulations

Financial institutions acts close both to maturity and strike

↓

They have to choose if it is (or not) the case to complete the hedging activity (so called view)

View


purchase of the security
Settlement by physical delivery
ITM

View

Settlement by physical delivery

ITM

purchase of the security

maturity

ITM

View

ITM

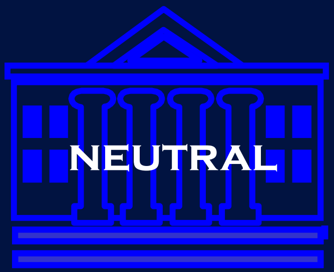
Settlement by physical delivery

purchase of the security

maturity

ITM

Delivery of the security



View

ITM

Settlement by cash

Purchase of the security

View

ITM

Settlement by cash

Purchase of the security

Maturity

ITM

Rev.conv. : Case Studies

View

ITM

Settlement by cash

Purchase of the security

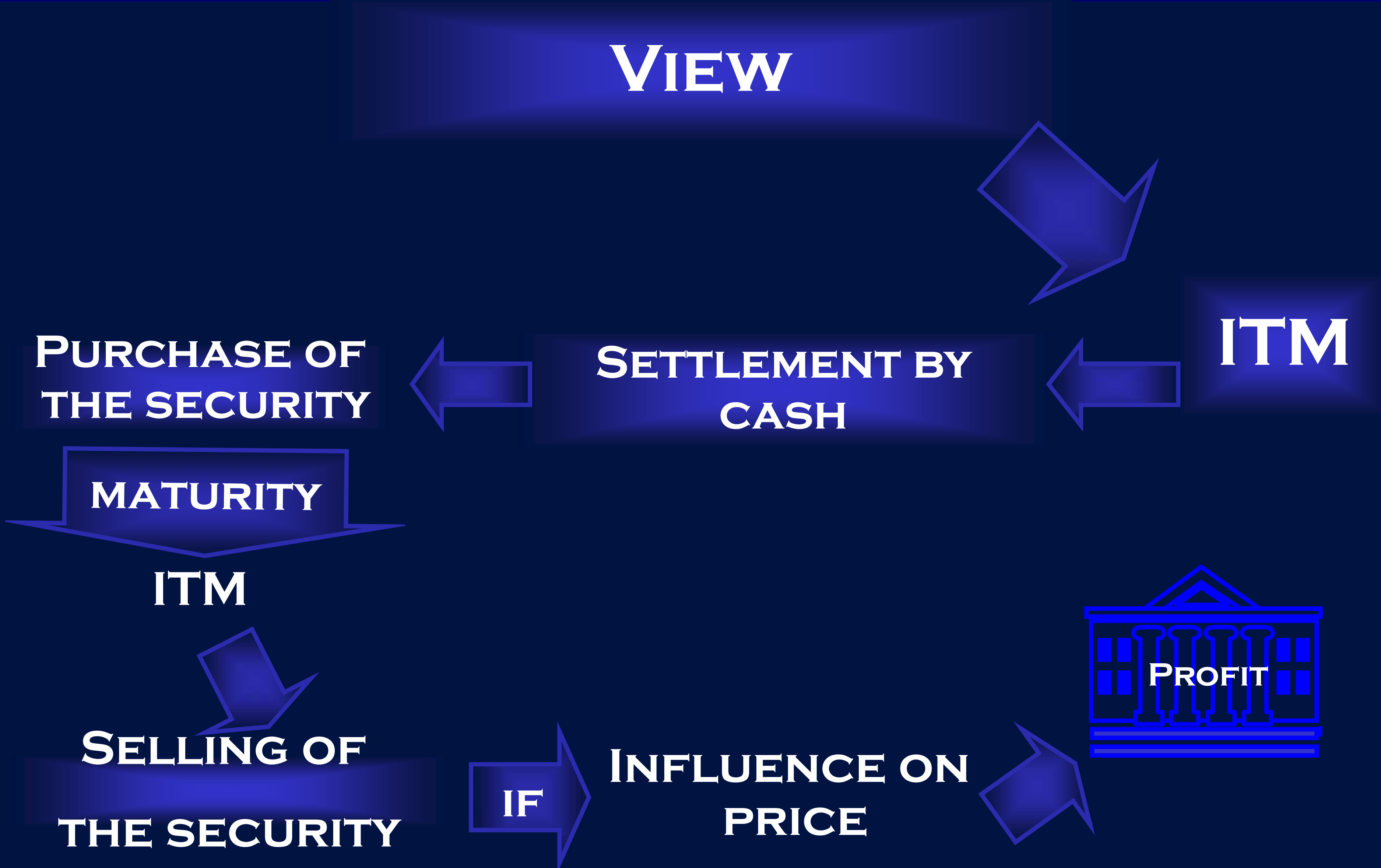
maturity

ITM

Selling of the security

if

Influence on price


View
ITM
Settlement by cash
Purchase of the security
maturity
ITM
Selling of the security
IF
Influence on price
Profit

VIEW

ITM

PURCHASE OF THE SECURITY


VIEW
ITM
PURCHASE OF THE SECURITY
MATURITY
OTM

View

ITM

Purchase of the security

maturity

OTM

Selling of the security

View

OTM

Selling of the security

VIEW

OTM

SELLING OF THE SECURITY

MATURITY

OTM

VIEW

OTM

SELLING OF THE SECURITY

MATURITY

OTM

NEUTRAL

View

OTM → Selling of the securities → Settlement by physical delivery

View

OTM

Selling of the securities

Settlement by physical delivery

Maturity

ITM

View

OTM

Selling of the securities

Settlement by physical delivery

Maturity

ITM

Purchase of the securities

View

OTM

Selling of the securities

Settlement by cash

View


OTM
Selling of the securities
Settlement by cash
maturity
ITM

View

OTM → Selling of the securities → Settlement by cash

Maturity

ITM → Settlement of the differential →

SUMMARY

View	Expiry date	Settlement	Fin.Institution
ITM	ITM	Cash Settlement	Profit
"	"	physical delivery	Neutral
ITM	OTM	Cash Settlement	Loss
"	"	physical delivery	Loss
OTM	ITM	Cash Settlement	Loss
"	"	physical delivery	Loss
OTM	OTM	Cash Settlement	Neutral
"	"	physical delivery	Neutral

ITM VIEW IS BETTER

FINANCIAL INSTITUTIONS CAN BE TEMPTED TO CAUSE THE *VIEW* TO COME TRUE

CASES OF MICROMANIPULATION

Knock-IN

<u>DEF.</u> : it is an Option such that when a barrier is crossed you have a plain vanilla one

Cases of micromanipolation

crossing of the barrier

Cases of micromanipolation

crossing of the barrier

deep ITM option

Settlement:

- Cash
- Physical delivery

Rev.conv. : Case Studies

... as a consequence of what have been said before, we will omit the

Cross-reference

See what have been said about plain vanilla options

... let us focus on

KNOCK-IN

OTM

... WE DISTINGUISH

... CLOSE TO THE BARRIER

...NOT CLOSE TO THE BARRIER

Introduction:

risk management for a Knock-in option

Numeric greek letters

Numeric greek letters

Mistakes in case of closeness to the barrier

Let us recall what a numeric greek letter is

$$\Delta = \frac{1}{2}(\Delta_{+1\%} + \Delta_{-1\%})$$

$$\Gamma = \frac{1}{2}(\Gamma_{+1\%} + \Gamma_{-1\%})$$

$$\upsilon = \frac{1}{2}(\upsilon_{+1\%} + \upsilon_{-1\%})$$

... let us omit the others because they are not very important

Why does it lead to a mistake?

Because it compares Disomogeneous quantities

Example:

Close (1%) to the barrier the formula becomes:

$$\Delta = \frac{1}{2}\left(\Delta^{+1\%} - \Delta^{-1\%}\right)$$

$$\Delta^{\pm 1\%} = \frac{P^{1}_{plain-vanilla} - P^{0}_{Knock-in}}{S^{\pm 1\%} - S^{0}}$$

Rev.conv. : Case Studies

K	H	
32,76	21,90	
price	**put**	**Δ numeric**
22,600	8,410	-326%
22,550	8,574	-328%
22,500	8,738	-330%
22,450	8,904	-332%
22,400	9,071	-334%
22,350	9,238	-335%
22,300	9,407	-337%
22,250	9,576	-338%
22,200	9,745	-340%
22,150	9,916	-341%
22,100	10,086	-329%
22,050	10,258	-303%
22,000	10,429	-276%
21,950	10,601	-248%
21,900	10,771	-221%
21,850	10,821	-193%
21,800	10,870	-165%
21,750	10,920	-137%
21,700	10,970	-109%
21,650	11,020	-100%
21,600	11,070	-100%
21,550	11,120	-100%
21,500	11,170	-100%
21,450	11,220	-100%

$$\Delta < -1$$

$\Delta < -1$ → It is a mistake

Why?

Because it compares different contingent claims

Risk Management based on greek letters referred to the Barrier options

Can be

MISLEADING

... FURTHERMORE, IT CAN BE VERY EXPANSIVE

BECAUSE IT CAN REQUIRE BOTH CONTINUOUS AND RELEVANT PORTFOLIO REBALANCES

Example:

K	H	
32,76	21,90	
price	**put**	**Δ numeric**
22,150	9,916	-341%
21,950	10,601	-248%
22,200	9,745	-340%
21,910	10,739	-226%
22,000	10,429	-276%
22,050	10,258	-303%
21,920	10,704	-232%

K	H	
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21,550	11,120	-100%
21,500	11,170	-100%
21,450	11,220	-100%

$\Delta < -1$

K	H	
32,76	21,90	

Portafoglio Re-balancing

21,910	10,739	-226%
22,000	10,429	-276%
22,050	10,258	-303%
21,920	10,704	-232%

Financial institutions do not employ traditional Δ hedging, but they use some devices, at least in case of closeness to the barrier

Devices:

- Let us fix the maximum fluctuation of Δ to -1
- Let us use Θ in order to reduce the value of Δ

… NOT close to the barrier

⇓

Δ hedging works well

... NOT close to the barrier

Δ hedging works well

Why?

Because it compares contingent claims that are equal

Example:

The formula becomes:

$$\Delta = \frac{1}{2}\left(\Delta^{+1\%} - \Delta^{-1\%}\right)$$

$$\Delta^{\pm 1\%} = \frac{P^{1}_{Knock-in} - P^{0}_{Knock-in}}{S^{\pm 1\%} - S^{0}}$$

Market manipulation targeted to cross the barrier

Very expansive

Warning in the Risk management

CLOSE TO THE BARRIER

Adjusted Δ hedging works well

Market manipulation targeted to cross the barrier

Not expansive

NO *Warning* in the Risk management

Market manipulation targeted to cross the barrier

Not expansive

NO Warning in the Risk management

The financial institution could be tempted to cross the barrier

… but it is not sure that the financial institution will adopt an adjusted Greek Hedging system

... but it is not sure that the financial institution will adopt an adjusted Greek Hedging system

... if following the traditional Hedging, close to the barrier, allows the risk management to obtain "positive" effects

… because after having realized that the activity close to the barrier leads to an <u>over-hedging,</u> the financial institution disinvests the part in excess By selling securities

... because after having realized that the activity close to the barrier leads to an over-hedging, the financial institution disinvests the part in excess by selling securities

... and this selling activity can cause a fall in price and a barrier crossing

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price	**put**	**Δ numeric**
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22,550	8,574	-328%
22,500	8,738	-330%
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$\Delta < -1$

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Portafoglio Re-balancing

21,910	10,739	-226%
22,000	10,429	-276%
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21,920	10,704	-232%

The financial institution restores the Δ hedging at its maximum value (-1) and sells the securities in excess

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21,800	10,870	-165%
21,750	10,920	-137%
21,700	10,970	-109%
21,650	11,020	-100%
21,600	11,070	[illegible]
21,550	11,120	-100%
21,500	11,170	-100%
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$\Delta < -1$

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32,76	21,90	

Portafoglio Re-balancing

21,910	10,739	-226%
22,000	10,429	-276%
22,050	10,258	-303%
21,920	10,704	-232%

The financial institution restores the Δ hedging at its maximum value (-1) and sells the securities in excess

… being close to the barrier the financial institution can reach the target to cross it

... AND WHAT HAS BEEN SAID BEFORE, HAPPENS IN AN APPARENT SITUATION OF CORRECTIVENESS

... FOR THE KNOCK-IN OPTIONS ...

… for the Knock-in options …

NOT correct

Risk Management

=

NOT correct

financial institution

Risk Management of a financial institution ⟷ Covered Warrant

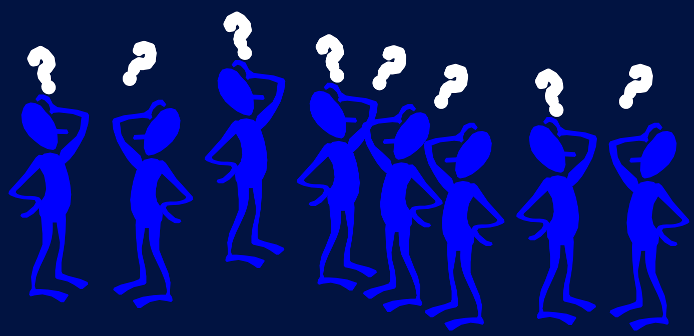
Risk Management of a financial institution ⟷ Reverse Convertible


Plain-vanilla options
Covered Warrant

After buying ...

AFTER BUYING ...

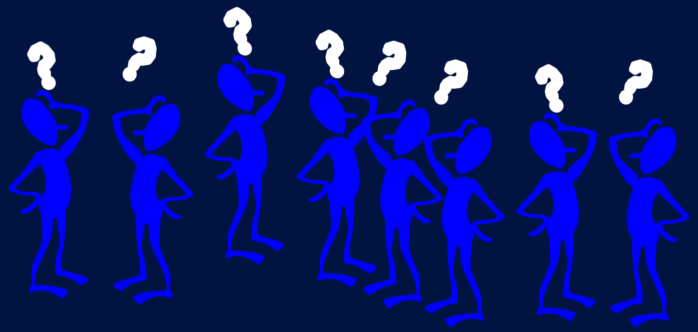
WHAT TO DO?



LET US NEGOTIATE

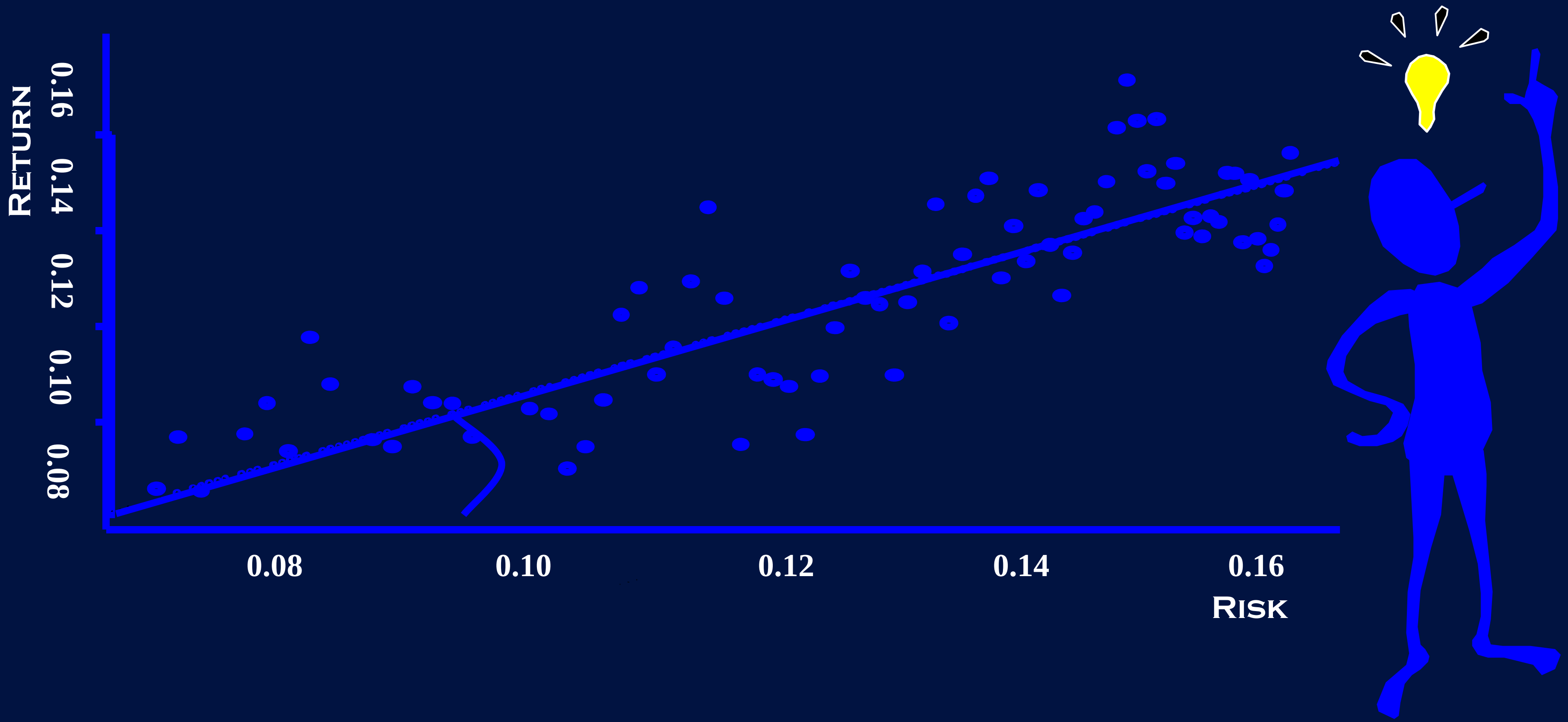
AFTER BUYING ...

WHAT TO DO?



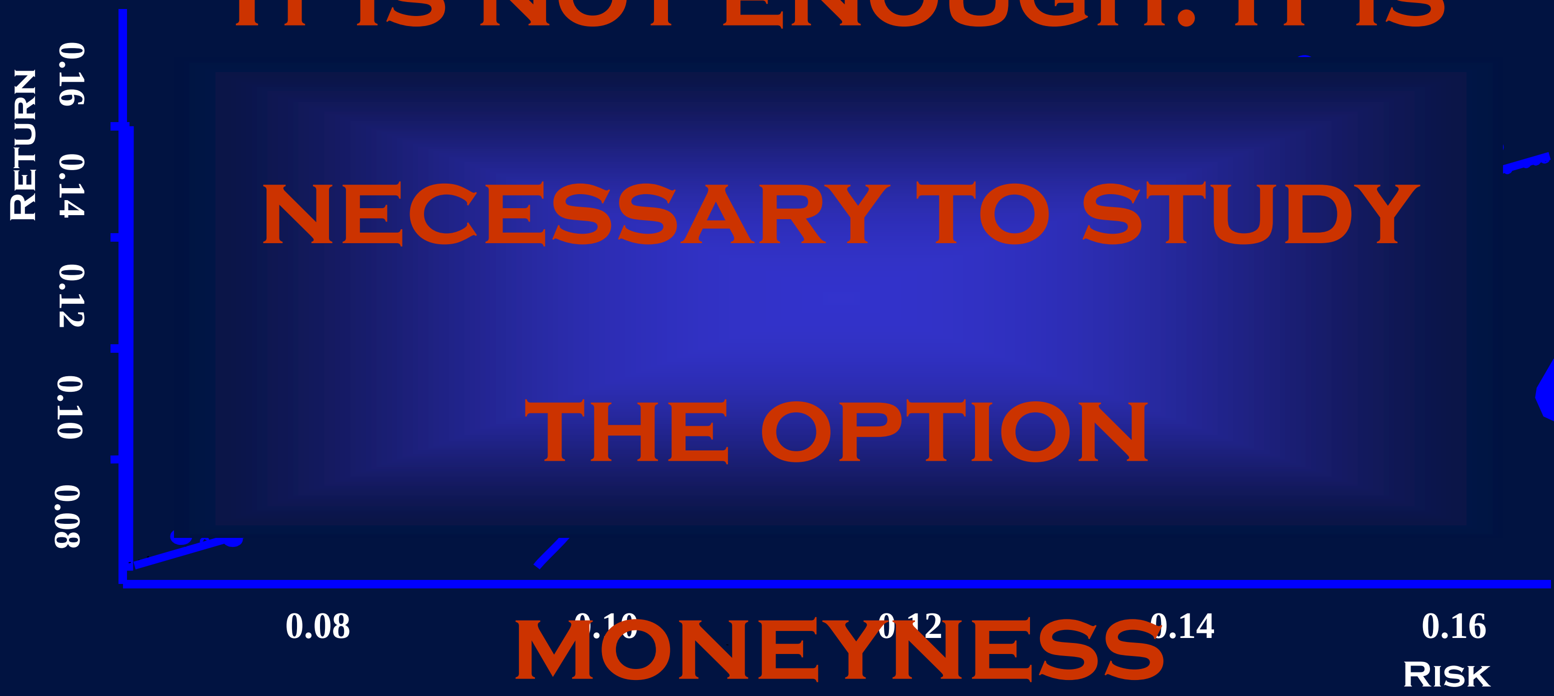
LET US WAIT FOR THE MATURITY

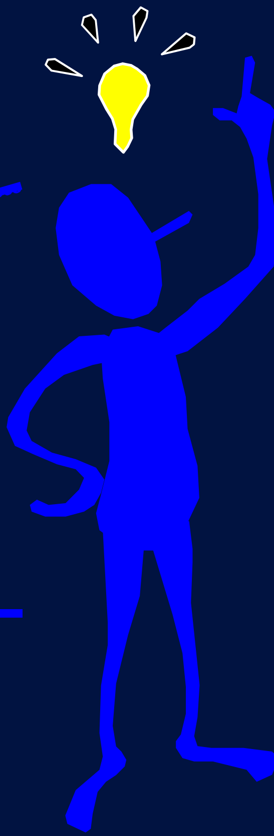
IT DEPENDS FROM INDIVIDUAL RISK – RETURN PREFERENCES



IT DEPENDS FROM INDIVIDUAL RISK – RETURN PREFERENCES

IT IS NOT ENOUGH. IT IS NECESSARY TO STUDY THE OPTION MONEYNESS





Covered Warrant – What to do after?

CW In-the-Money

CW At-the-Money

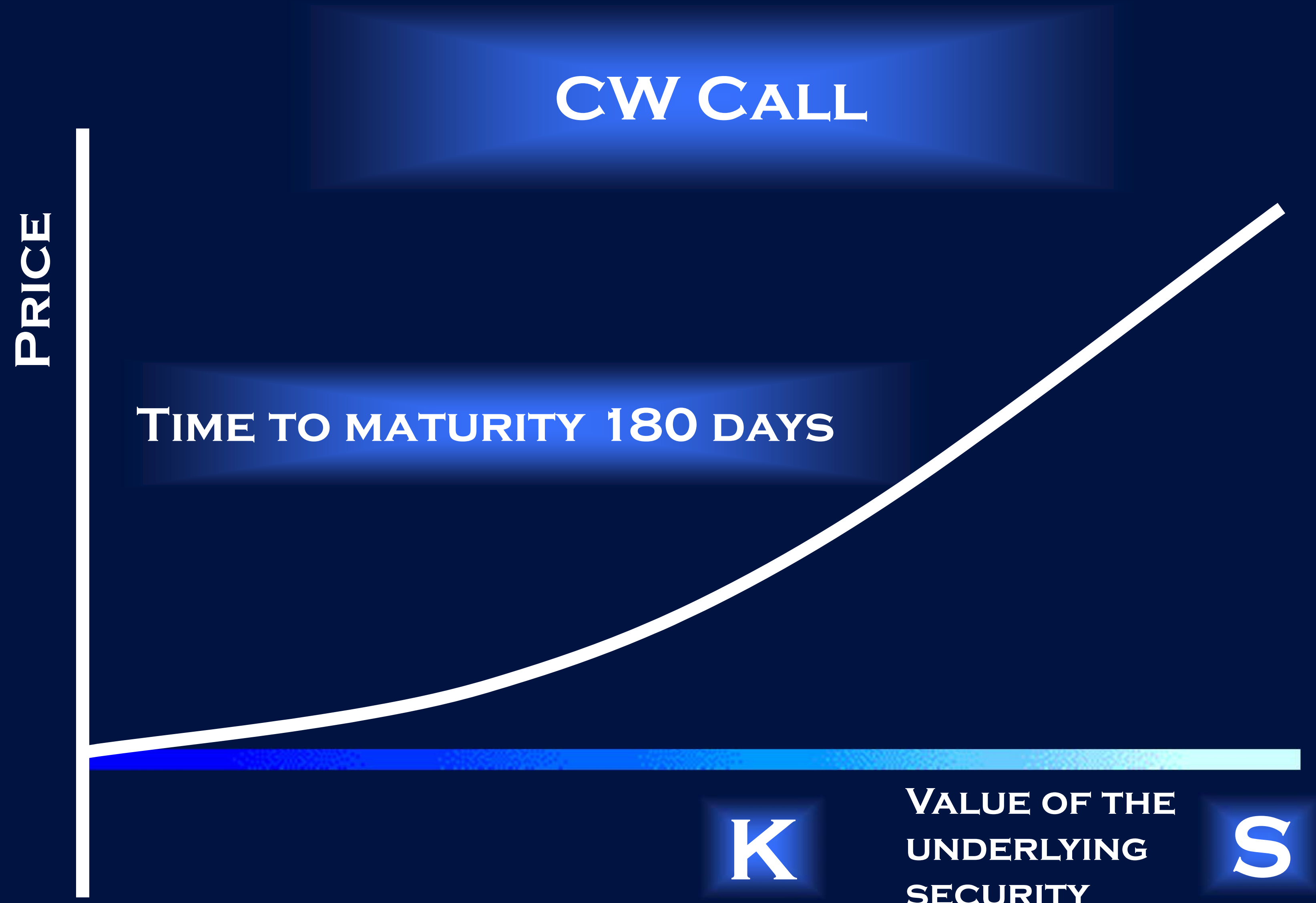
CW Out-the-Money

CW Out-the-Money – What to do after?

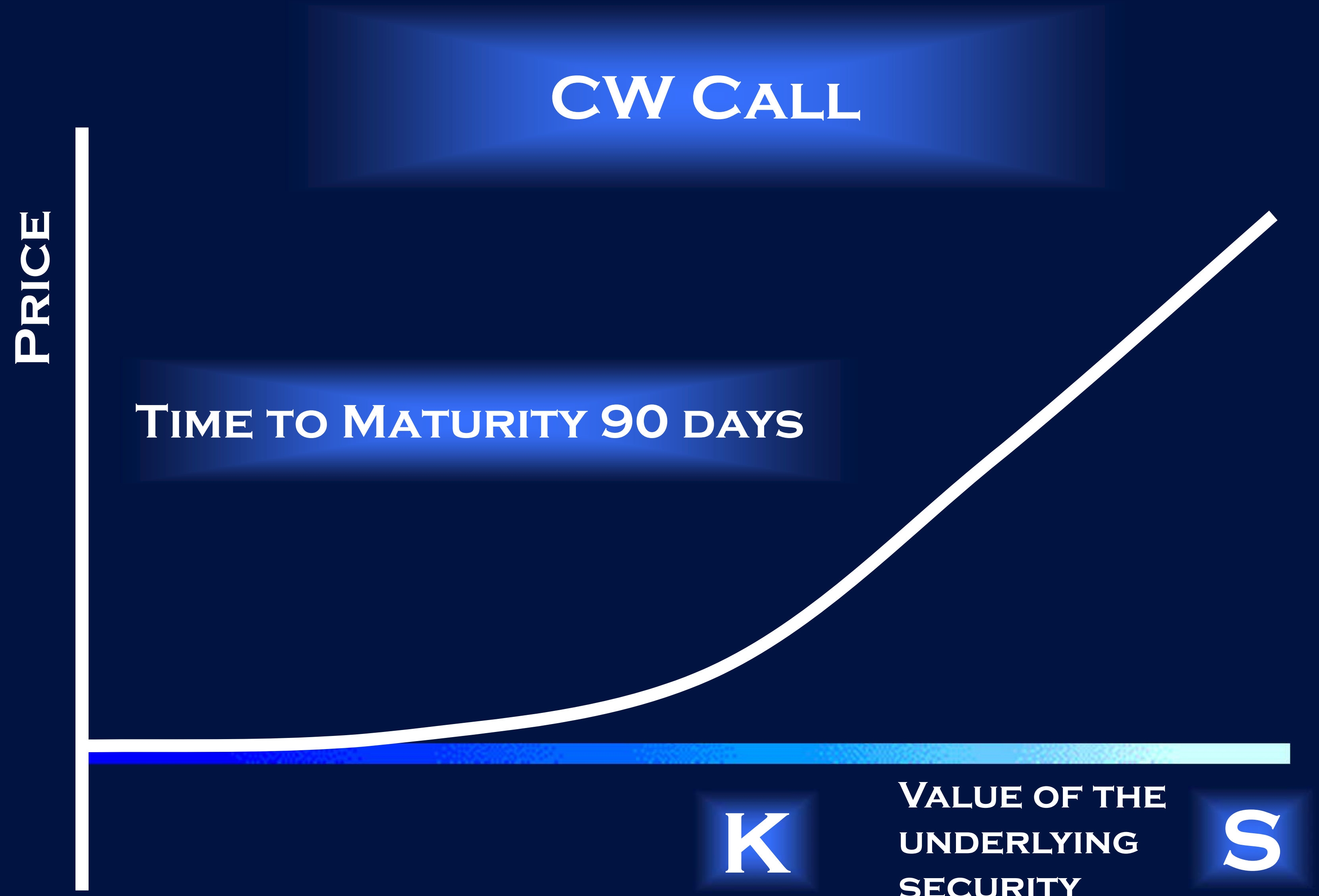
CW Out-the-Money

Problems of the Minimum Tick with regard to the trading instruments

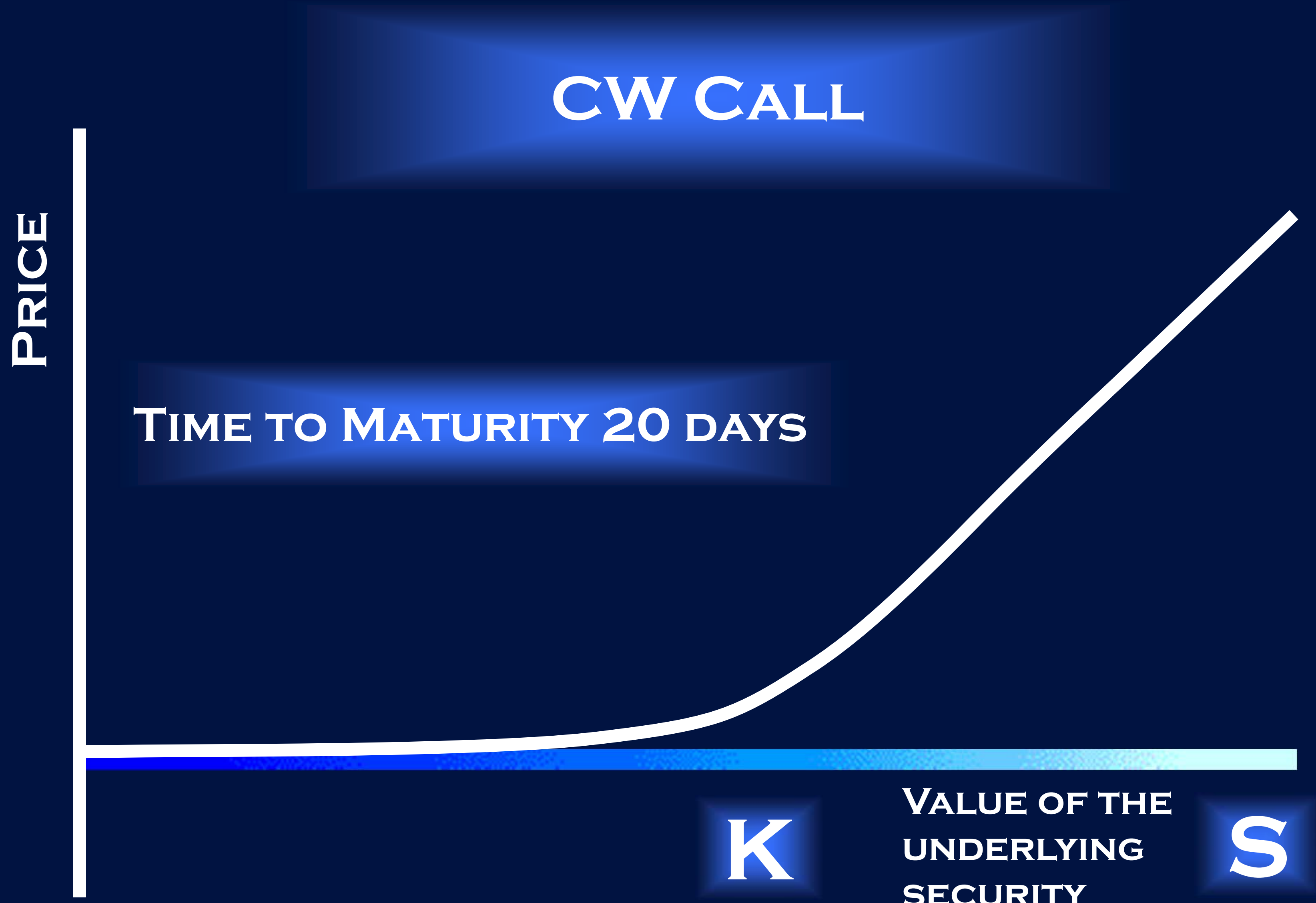
CW Out-the-Money – What to do after?



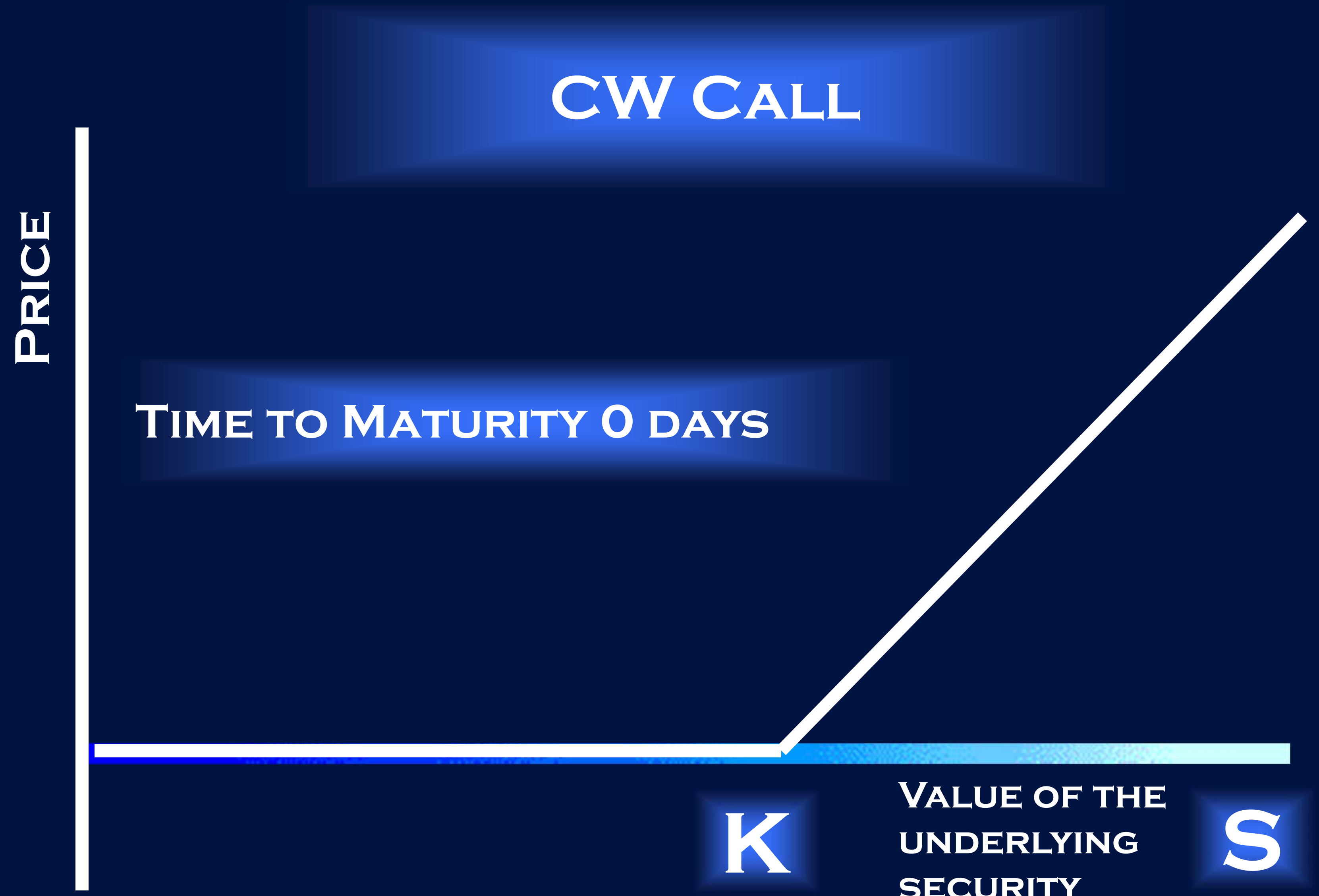
CW Out-the-Money – What to do after?



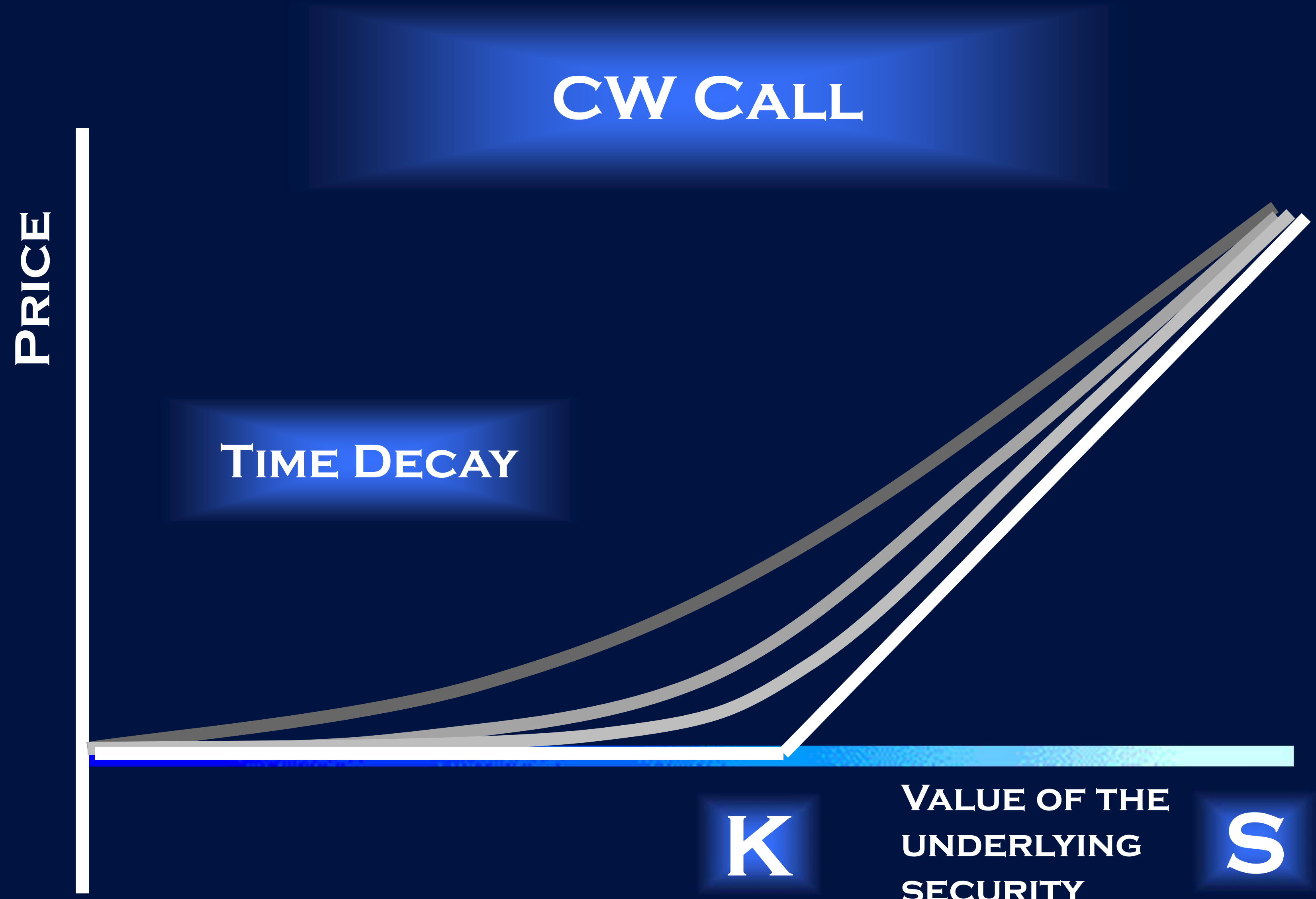
CW Out-the-Money – What to do after?



CW Out-the-Money – What to do after?



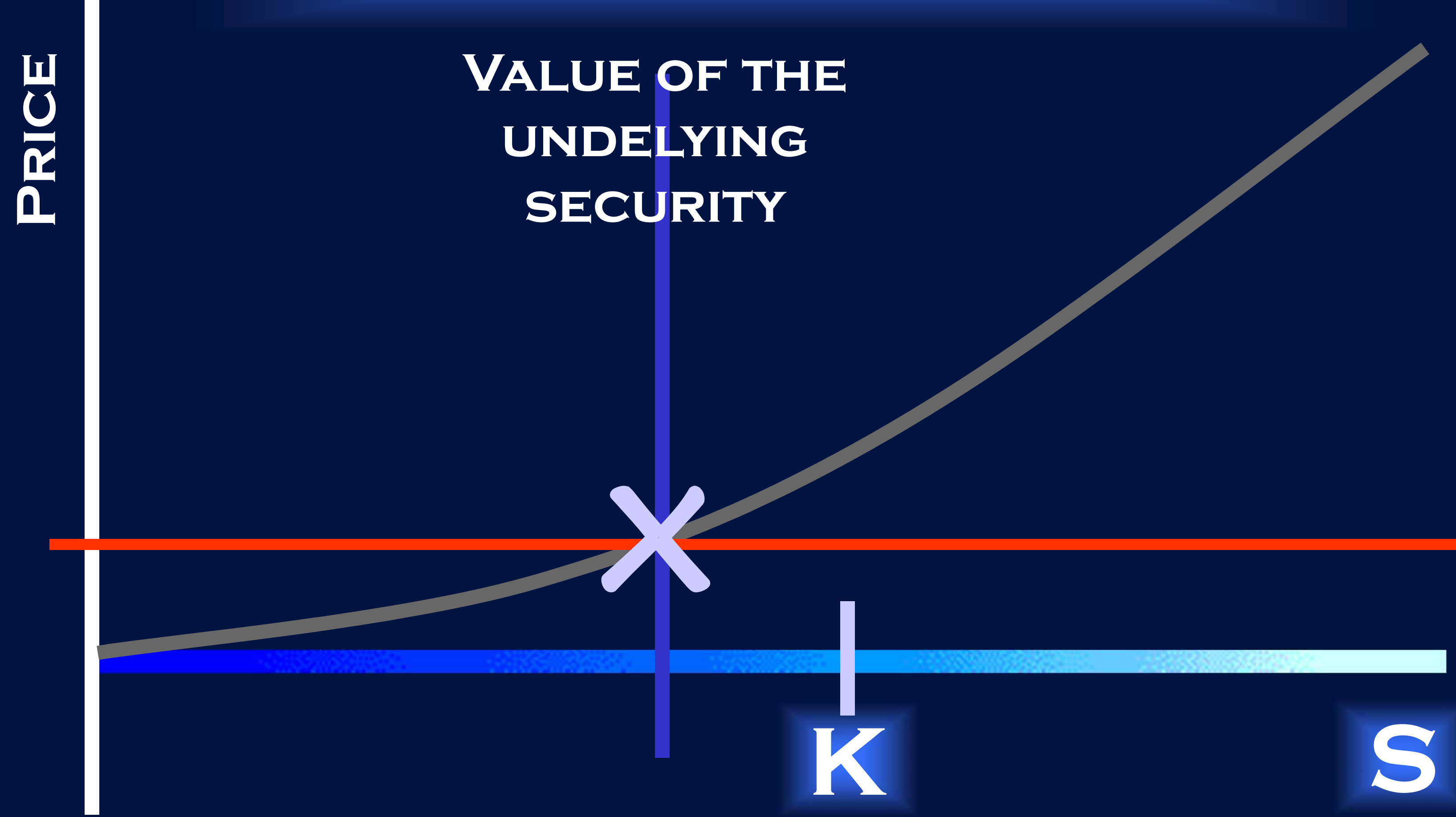
CW Out-the-Money – What to do after?



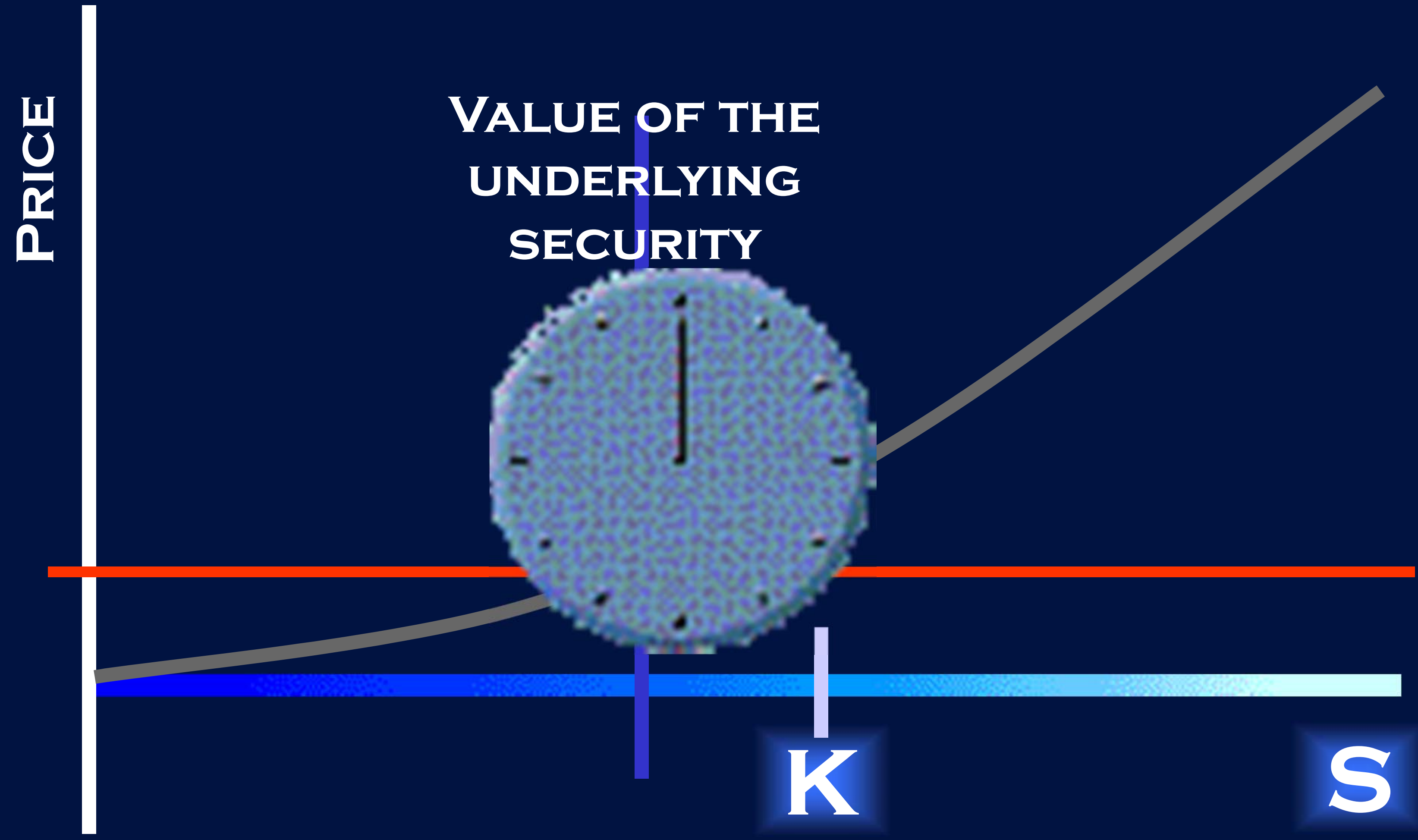
CW Out-the-Money – What to do after?

CW Call

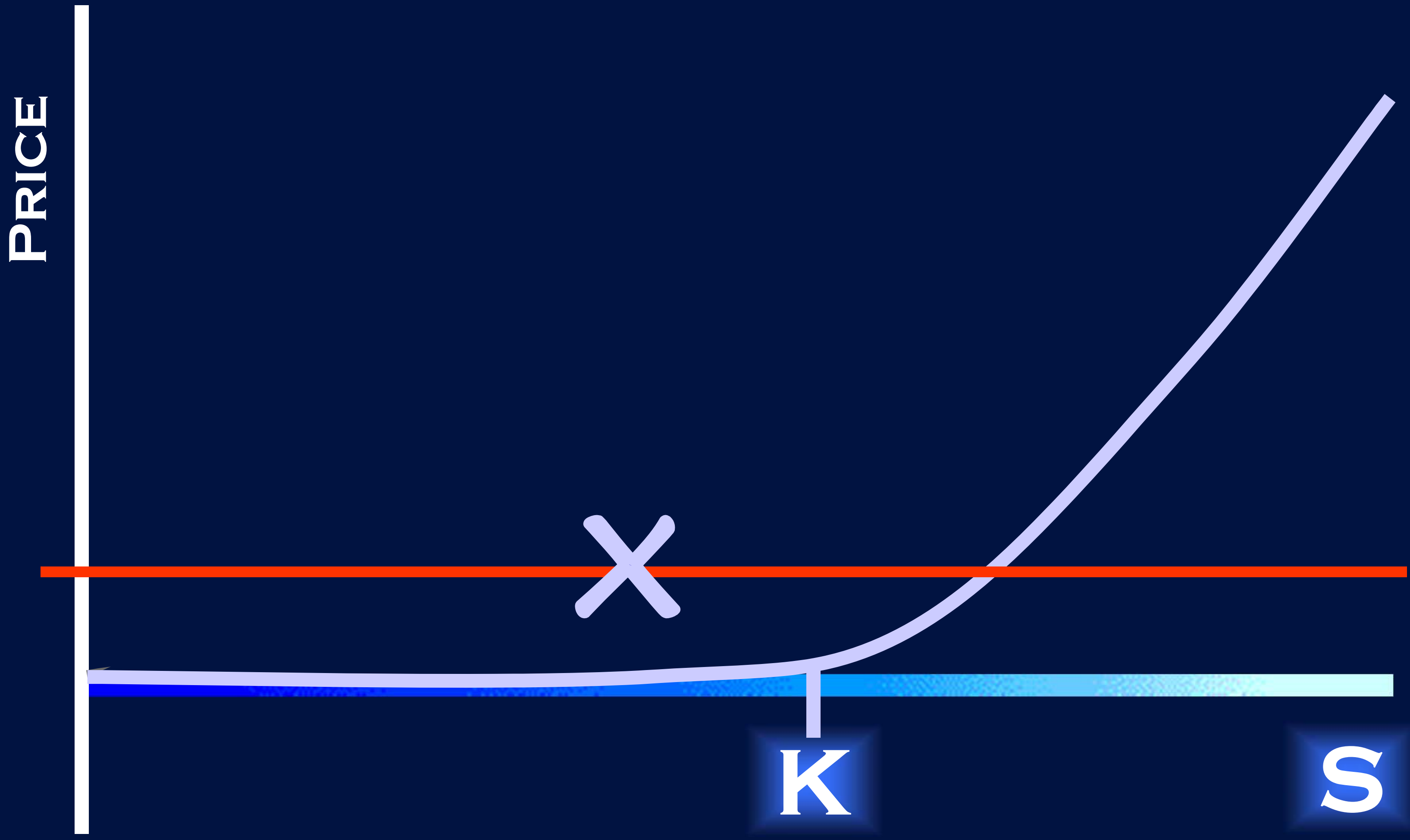
Purchase of an OTM Call CW



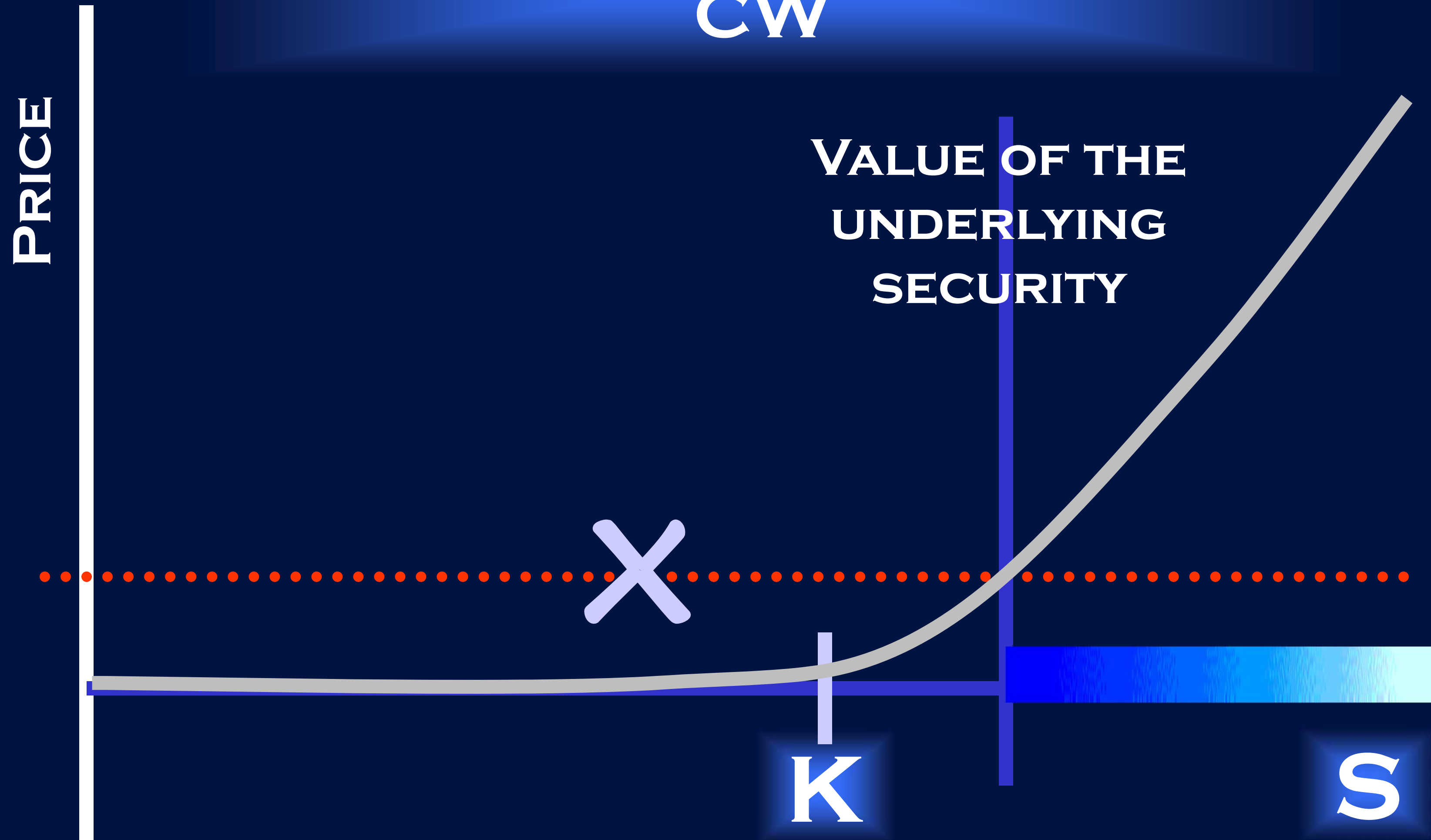
CW Out-the-Money – What to do after?



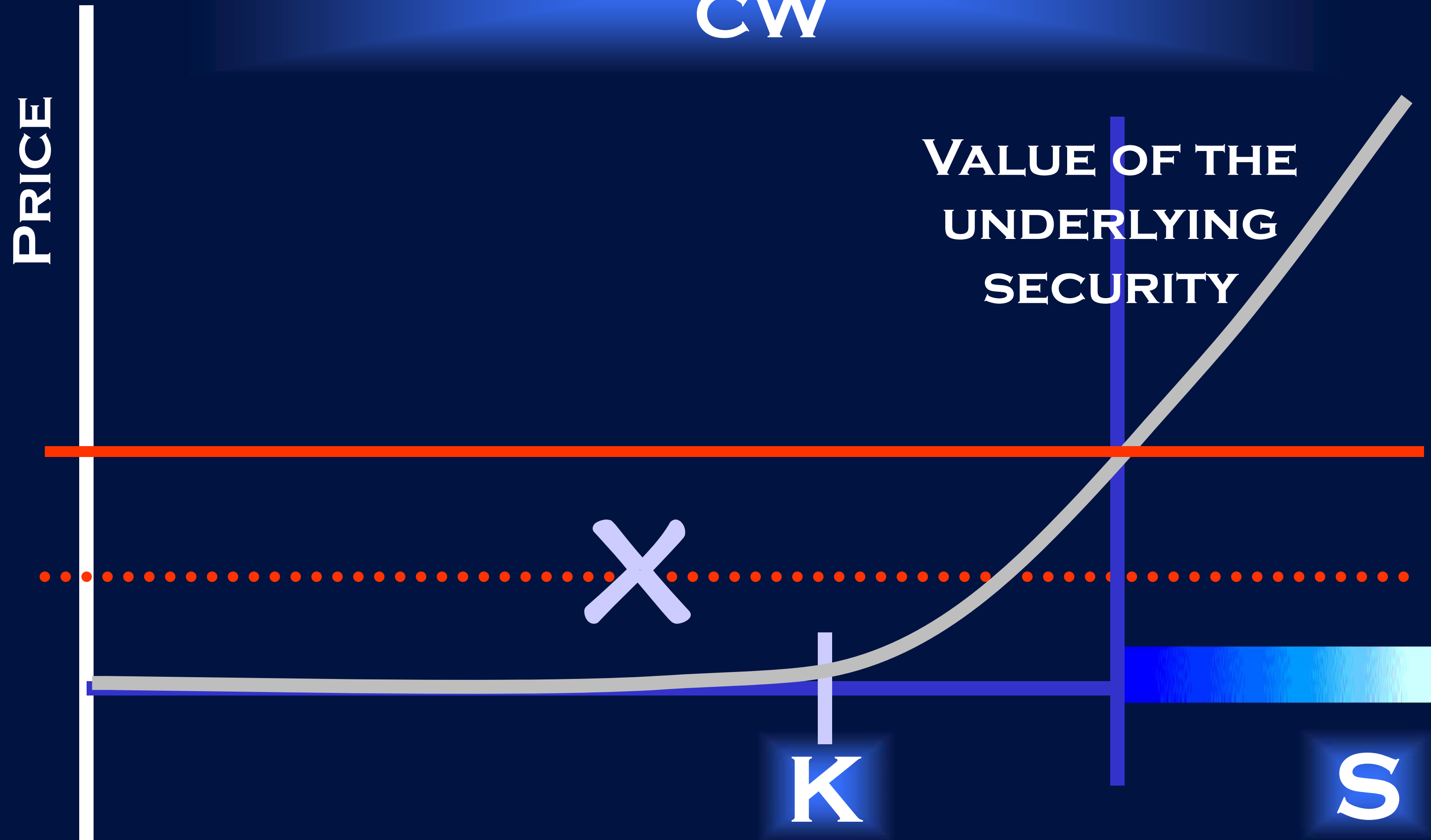
CW Out-the-Money – What to do after?



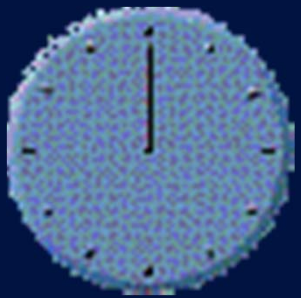
Purchase of an OTM Call CW



Purchase of an OTM Call CW



CW Out-the-Money – What to do after?



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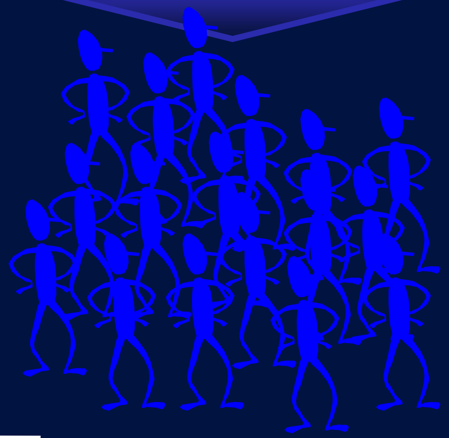
Problems regarding the trading instruments

OTM Covered Warrant hardly recover the investment value

CW In-the-Money

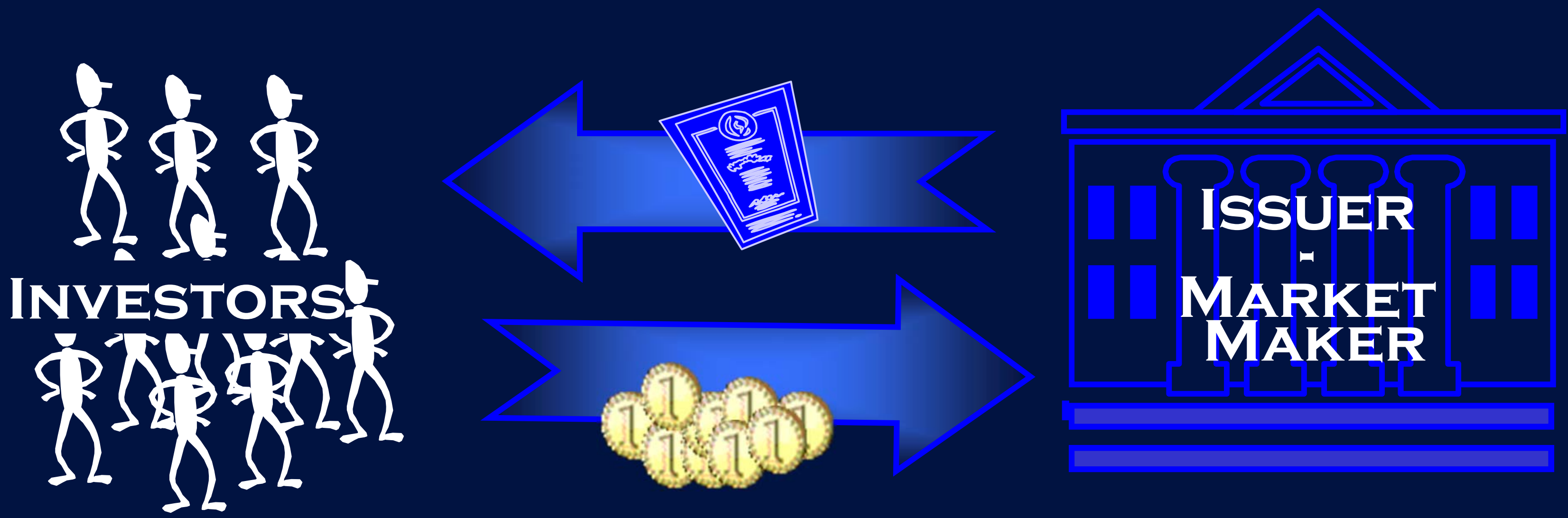
Problems regarding the Risk Management

of the financial institution



Example

financial institution is a net seller of Calls



Close to maturity …

He will hold in his portfolio

<u>a large amount</u> of the

stocks underlying the Call

CW In-the-Money – What to do after?

At maturity the option is ITM……

He will hold in his

Portfolio ALL the stocks

Underlying the Calls

CW In-the-Money – What to do after?

In order to settle the option's pay-off …

He will sell all the stocks.

so called 'risk unwinding'

CW In-the-Money – What to do after?

Selling all the stocks …

He will drive a fall in prices

DRIVING A FALL IN PRICES …

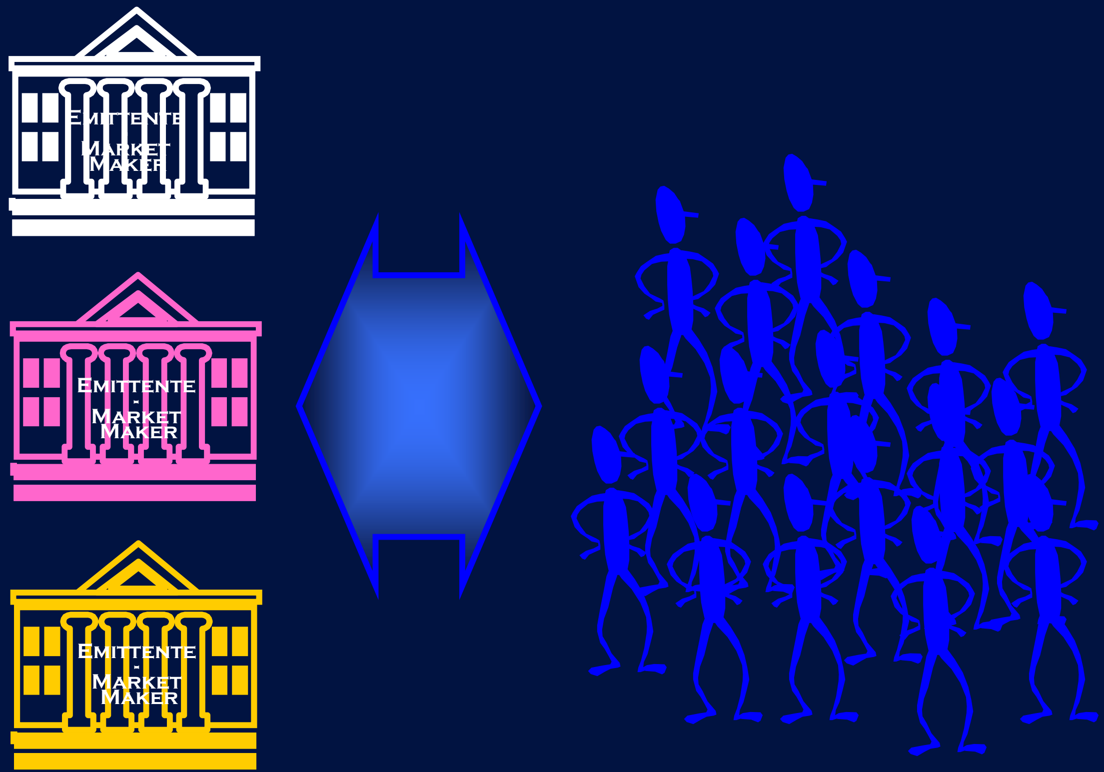
He will reduce the

investor's potential gain

All the story can be summarized through…

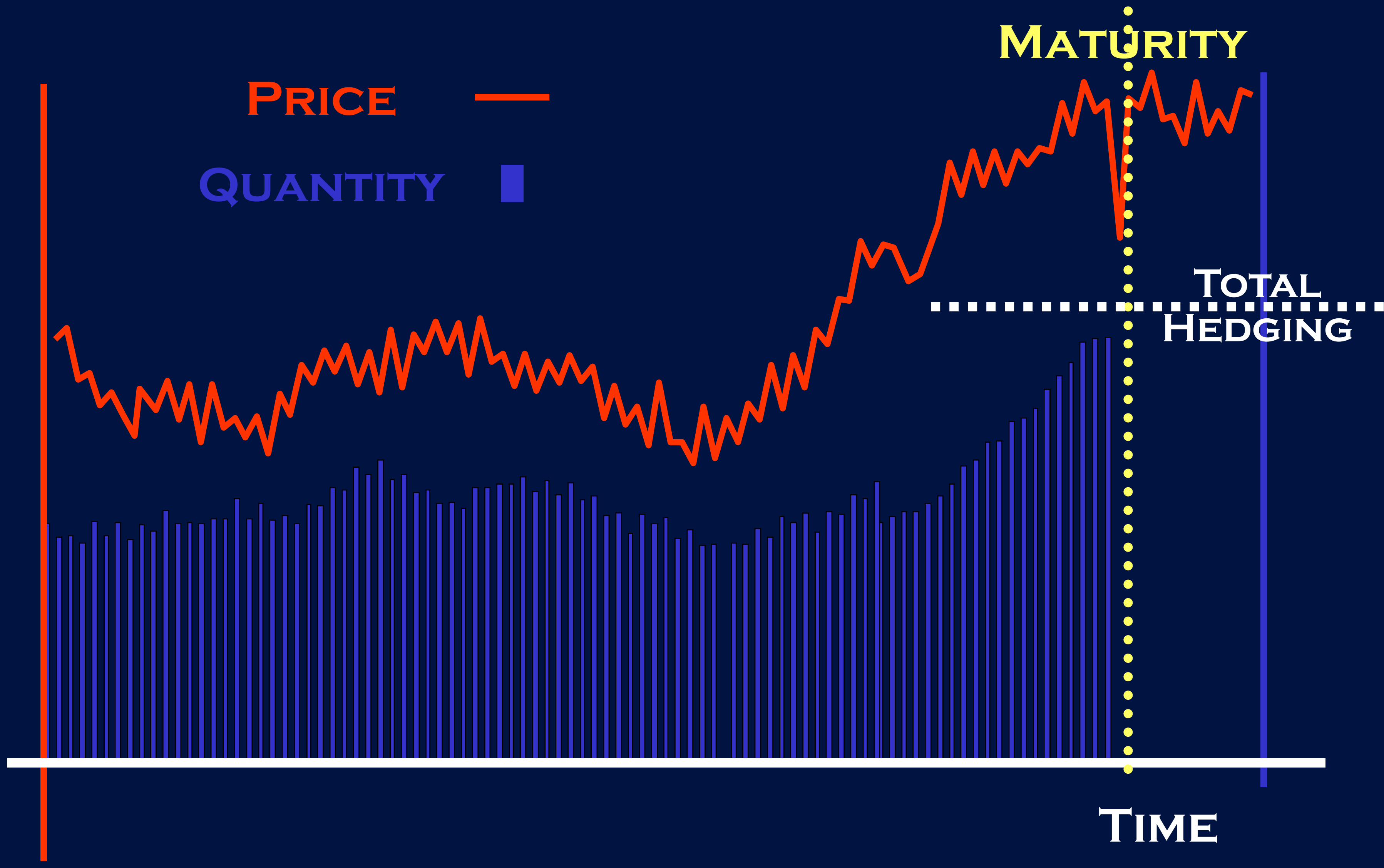
All the story can be summarized through...

Delta Hedging analysis

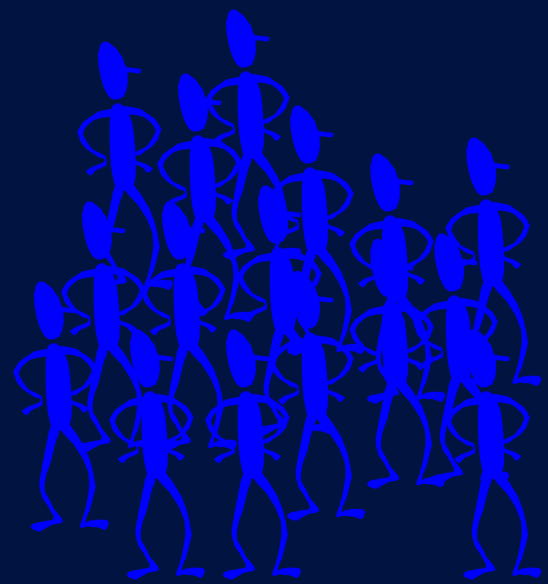


THE POINT OF VIEW

Delta Hedging on a Short Call Position

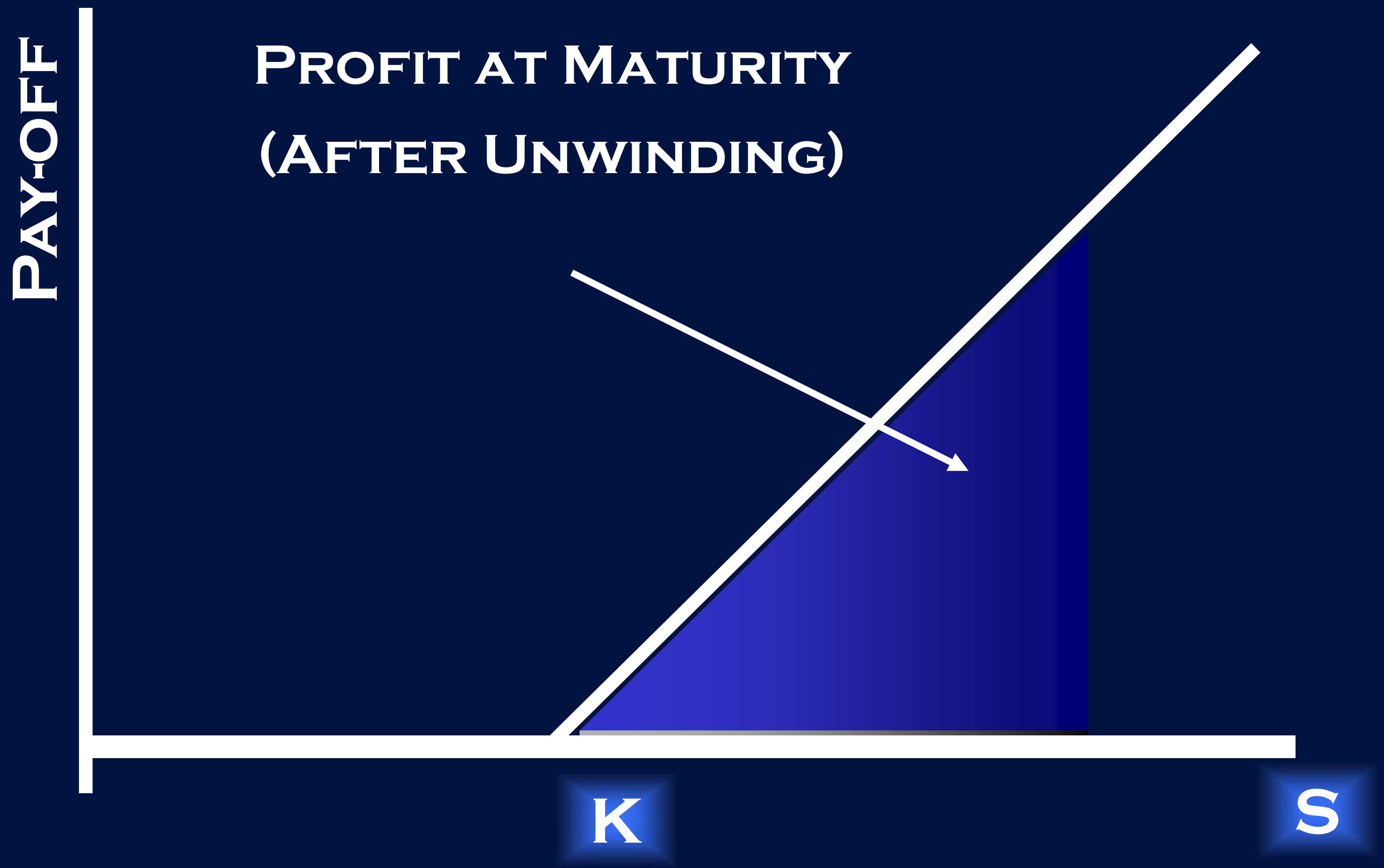


The Point of View

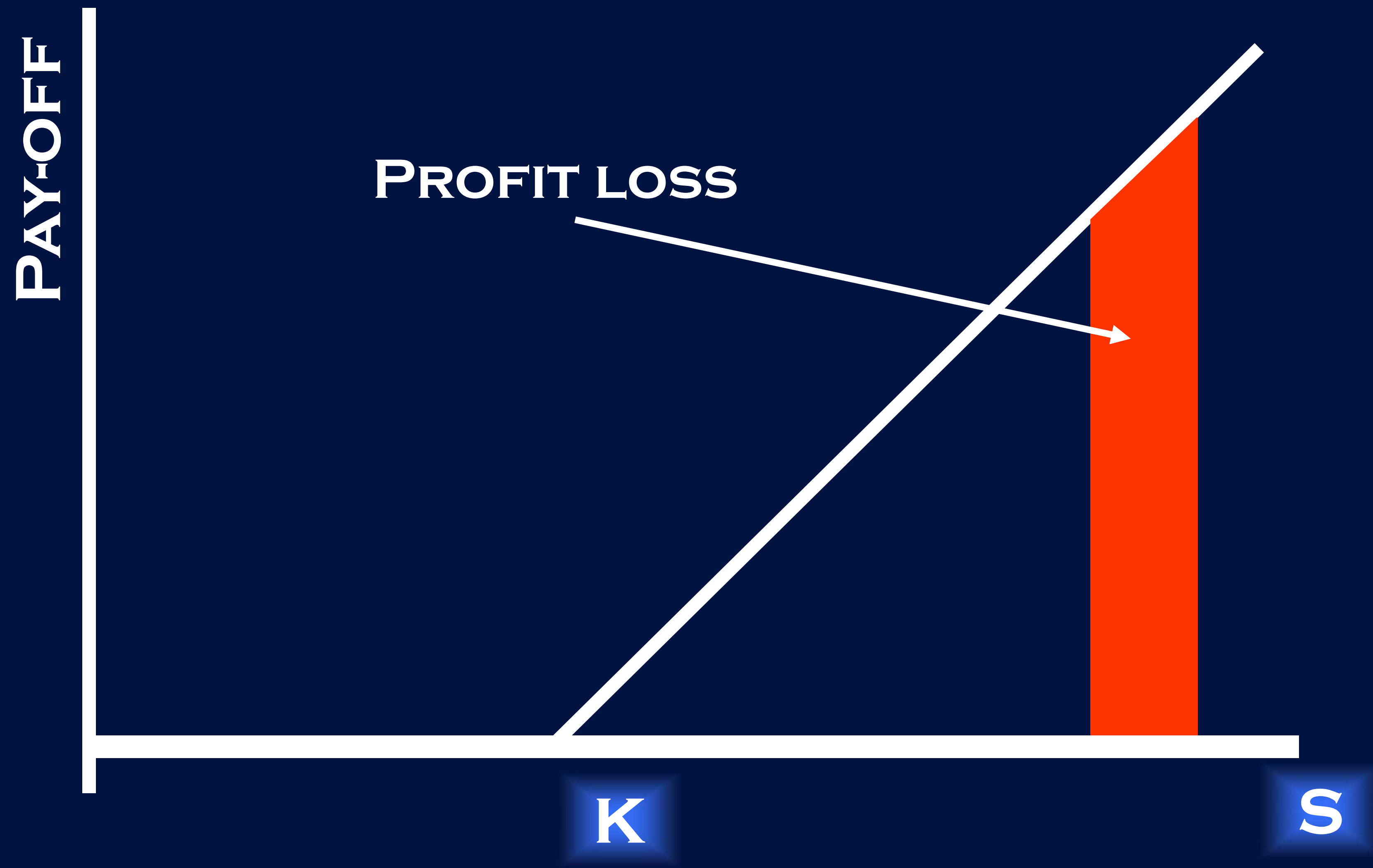



Pay-off
Profit before Maturity
(Before Unwinding)
K
S

Delta Hedging on a Short Call Position



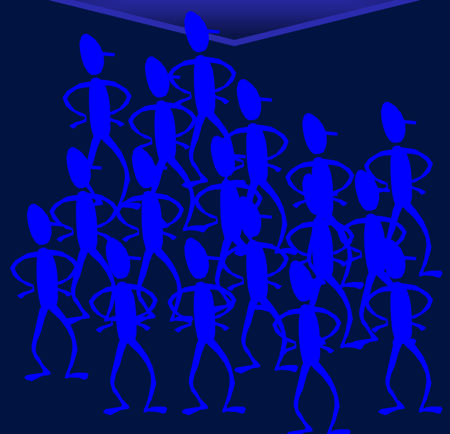
Delta Hedging on a Short Call Position



CW At-the-Money – What to do after?

CW At-the-Money

Problems connected to the so called 'view'



The stock is close

both to maturity and strike

↓

Financial institutions will have

to choose if it is the case to

complete or not the hedging

(so called view)

Problems connected to the so called 'view'

Short 1000 call on 1 stock				Option and Δ			Stock and Δ				Δ Portfolio
Time Step	Time to Expiration	STOCK PRICE	Option Value	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
0	0,2500	100,0	10,3776	(1.000)	0,56411596	(564)	564	564	1	564	-
1	0,2375	103,0	11,8661	(1.000)	0,609628	(610)	46	610	1	610	-
2	0,2250	103,7	12,0368	(1.000)	0,6207246	(621)	11	621	1	621	-
3	0,2125	101,7	10,5181	(1.000)	0,58779993	(588)	(33)	588	1	588	-
4	0,2000	96,5	7,4017	(1.000)	0,49399277	(494)	(94)	494	1	494	-
5	0,1875	95,6	6,6706	(1.000)	0,47264412	(473)	(21)	473	1	473	-
6	0,1750	98,0	7,5759	(1.000)	0,51590073	(516)	43	516	1	516	-
7	0,1625	103,7	10,4769	(1.000)	0,62159169	(622)	106	622	1	622	-
8	0,1500	104,0	10,3404	(1.000)	0,62848623	(628)	6	628	1	628	-
9	0,1375	102,0	8,7708	(1.000)	0,58982183	(590)	(38)	590	1	590	-
10	0,1250	99,0	6,7342	(1.000)	0,5231959	(523)	(67)	523	1	523	-
11	0,1125	98,0	5,8476	(1.000)	0,49554047	(496)	(27)	496	1	496	-
12	0,1000	101,0	7,0373	(1.000)	0,56586158	(566)	70	566	1	566	-
13	0,0875	103,0	7,7894	(1.000)	0,61640622	(616)	50	616	1	616	-
14	0,0750	100,5	5,8907	(1.000)	0,55119095	(551)	(65)	551	1	551	-
15	0,0625	98,0	4,0990	(1.000)	0,46817516	(468)	(83)	468	1	468	-
16	0,0500	104,0	6,9440	(1.000)	0,66410039	(664)	196	664	1	664	-
17	0,0375	99,0	3,4276	(1.000)	0,48390678	(484)	(180)	484	1	484	-
18	0,0250	104,0	5,6701	(1.000)	0,70807478	(708)	224	708	1	708	-
19	0,0125	102,0	3,4230	(1.000)	0,65206982	(652)	(56)	652	1	652	-

Problems connected to the so called 'view'

Short 1000 call on 1 stock				Option and Δ			Stock and Δ				Δ Portfolio
Time Step	Time to Expiration	STOCK PRICE	Option Value	Q. Opz.	Δ call	Δ call Posit.	Stock to Buy/(Sell)	Warehouse	Δ Stock	Δ Stock Posit.	Total Δ position
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10	0,1250	99,0	6,7342	(1.000)	0,5231959	(523)	(67)	523	1	523	-
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12	0,1000	101,0	7,0373	(1.000)	0,56586158	(566)	70	566	1	566	-
13	0,0875	103,0	7,7894	(1.000)	0,61640622	(616)	50	616	1	616	-
14	0,0750	100,5	5,8907	(1.000)	0,55119095	(551)	(65)	551	1	551	-
15	0,0625	98,0	4,0990	(1.000)	0,46817516	(468)	(83)	468	1	468	-
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18	0,0250	104,0	5,6701	(1.000)	0,70807478	(708)	224	708	1	708	-
19	0,0125	102,0	3,4230	(1.000)	0,65206982	(652)	(56)	652	1	652	-
20	0,0000	98,0	-	(1.000)	0	-	(652)	-	1	-	-

PROBLEMS CONNECTED TO THE SO CALLED 'VIEW'

Short 1000 call on 1 stock				Option and Δ			Stock and Δ				Δ Portfolio
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16	0,0500	104,0	6,9440	(1.000)	0,66410039	(664)	196	664	1	664	-
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18	0,0250	104,0	5,6701	(1.000)	0,70807478	(708)	224	708	1	708	-
19	0,0125	102,0	3,4230	(1.000)	0,65206982	(652)	(56)	652	1	652	-
20	0,0000	101,0	1,0000	(1.000)	1	(1.000)	348	1.000	1	1.000	-

A *view*'s mistake can cause a high cost to the financial institution

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Cases of micromanipulation in order to make the *view* 'come true'

Cases of micromanipulation in order to make the *view* 'come true'

Final Remarks

The quant enforcement

examines

Cases of micromanipulation

THE QUANT ENFORCEMENT

ANALYSIS

ACTIVITY OF THE FINANCIAL INSTITUTION …

The quant enforcement

VERIFY

... the financial institution's placing with reference to the bounds of Risk management

… BECAUSE FINANCIAL INSTITUTIONS SOMETIMES JUSTIFY THEIR ACTIVITY AS CONNECTED TO THE INDICATIONS OF RISK MANAGEMENT TOOLS