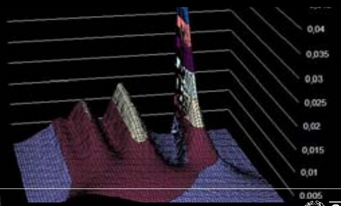
Pricing and Hedging without Fast Fourier Transform

A revisited, reliable and stable Fourier Transform Method for Affine Jump Diffusion Models



Marcello Minenna - Paolo Verzella

Structured Products Europe – Nov 3, 2005 - London

- Review of Fourier Methods in Option Pricing
- Calibration and Performance
- Greek Behavior of New FT-Q

European Call   Maturity T Terminal Spot Price S_T

In AJD models Call Price can be expressed in a form close to the canonical Black – Scholes - Merton style

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau}P_2(\Theta)$$

where

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \ge \ln[K])$$

under different martingale measures

$$P_1(\Theta), P_2(\Theta) = \Pr(\ln S_T \ge \ln[K])$$

under different martingale measures

can be

determined by using the Levy's inversion formula, i.e.:

$$\Pr(\ln S_T \ge \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi\ln[K]}\tilde{f}_j(\phi)}{i\phi}\right]d\phi$$

$$\Pr(\ln S_T \ge \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi\ln[K]}\tilde{f}_j(\phi)}{i\phi}\right]d\phi$$

requires

a close formula for the Characteristic Function of the log – terminal price, i.e.:

$$\tilde{f}_T(\phi) = E\left[e^{i\phi\ln S_T}\right]$$

$$\tilde{f}_T(\phi) = E\left[e^{i\phi\ln S_T}\right]$$

has

a closed formula for AJD models

How to compute: C_t

I

Quadrature Algorithm
FT – Q for
$P_1(\Theta), P_2(\Theta)$

a — Old FT - Q

b — New FT - Q

II

Fast Fourier Transform
FFT

Algorithms Valuation Criteria

STABILITY

The algorithm is defined stable if and only if it "closes" the quadrature scheme

spanning

the pricing formula on a vast area of the parameters set. gives a "reasonable" result different from a NaN value

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Algorithms Valuation Criteria

ACCURACY

The algorithm is defined accurate if and only if

pricing — the Call under the Black Scholes - Merton model via:

- Old FT - Q
- New FT - Q
- FFT

pricing — Call via standard Black – Scholes – Merton

up to 10^{-3} precision

Algorithms Valuation Criteria

SPEED

The algorithm is defined fast with respect to the results of the FFT algorithm

a set of 4100 prices along the strike

$P_1(\Theta), P_2(\Theta)$ through Quadrature Algorithm Old FT - Q

High Order Newton Cotes Algorithm

Up to 8th

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau}P_2(\Theta)$$

$P_1(\Theta), P_2(\Theta)$ through Quadrature Algorithm Old FT - Q

Pros (+)	Cons (-)
ACCURACY	STABILITY SPEED

$P_1(\Theta), P_2(\Theta)$ through Quadrature Algorithm New FT - Q

In order to overcome the cited problems of Old FT – Q:

$P_1(\Theta), P_2(\Theta)$ through Quadrature Algorithm New FT - Q

In order to overcome the cited problems of Old FT – Q:

- Gauss - Lobatto Quadrature Algorithm
- Re-adjustment of $\tilde{f}_T(\phi) = E\left[e^{i\phi\ln S_T}\right]$

$$C_t = S_t P_1(\Theta) - Ke^{-r\tau}P_2(\Theta)$$

$P_1(\Theta), P_2(\Theta)$ through Quadrature Algorithm New FT - Q

Pros (+)	Cons (-)
STABILITY ACCURACY	SPEED

C_t through Fast Fourier Trasform FFT

Cooley - Tukey algorithm

Applied to the equivalent formula via a recombinant FFT parameters

$$C_t = \frac{e^{-\alpha\ln K}}{\pi}\int_0^\infty e^{-i\phi\ln K}\tilde{f}_j(\phi)d\phi \quad \text{for ATM}$$

Review of Fourier Methods in Option Pricing – practice

$P_1(\Theta), P_2(\Theta)$ ⟺ through ⟺ Fast Fourier Trasform FFT

Pros (+)	Cons (-)
SPEED FASTER (up to 20 times the quadrature algorithms)	STABILITY * The formula must be changed arbitrarily according to Option moneyness ACCURACY ** the recombinant FFT parameters must be changed according to the choice of the pricing models

Syllabus of the presentation

- Review of Fourier Methods in Option Pricing
- Calibration and Performance
- Greek Behaviour of New FT-Q

The Calibration Procedure and Performance

$$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$$

I → Quadrature Algorithm FT - Q
- a → Old FT - Q
- b → New FT - Q

II → Fast Fourier Trasform FFT

The Calibration Procedure and Performance

$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$ ⟺ through ⟺ Quadrature Algorithm Old FT - Q

Pros (+)	Cons (-)
	STABILITY ACCURACY SPEED

The Calibration Procedure and Performance

$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$ ⟺ through ⟺ Fast Fourier Trasform FFT

Pros (+)	Cons (-)
SPEED	STABILITY * ACCURACY **

The Calibration Procedure and Performance

$SSE_t = \min_{v(t),\Phi} \sum_{n=1}^{N} \left[C_{Market}(S_t) - C_{AJD}(S_t) \right]^2$ ⟺ through ⟺ Quadrature Algorithm New FT - Q

Pros (+)	Cons (-)
STABILITY ACCURACY SPEED	

The Calibration Procedure and Performance

By keeping in mind that only New FT-Q is stable and accurate, some figures on speed

Original Option Pricing Formulas are used

	Heston Model	Merton Model	BCC Model
FFT	7.26 sec.	10.54 sec.	18.33 sec.
NEW FT - Q	55.12 sec.	66.48 sec.	110.39 sec.
OLD FT – Q	390.41 sec.	454.76 sec.	722.1 sec.

By now, the speed of Fourier Trasform method is closer than ever to the FFT calibration time

The Calibration Procedure and Performance

Calibration Performances using
Option Readjusted Pricing Formulas
where available

	Heston Model	Merton Model	BCC Model
FFT	7.24 sec.	10.54 sec.	18.32 sec.
NEW FT - Q	23.13 sec.	66.48 sec.	48.7 sec.
OLD FT – Q	331.6 sec.	454.76 sec.	688.5 sec.

Syllabus of the presentation

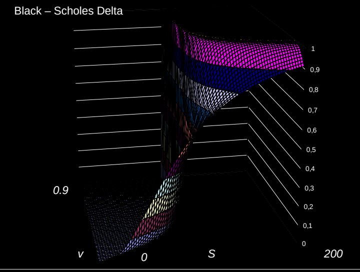
- Review of Fourier Methods in Option Pricing
- Calibration Procedure and Performance
- Greek Behaviour of New FT-Q

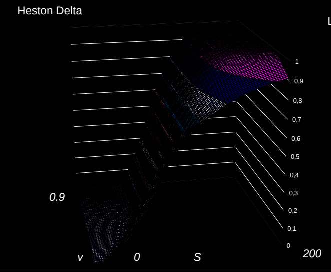
Greek behaviour of new FT-Q

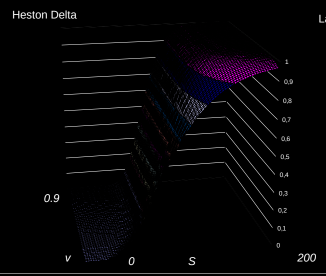
An impressive methodology to test Stability of the New FT – Quadrature algorithm is to compute Greeks

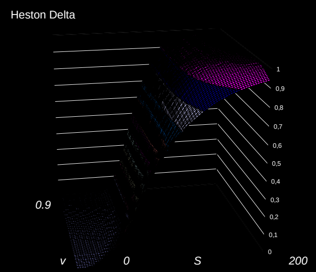
Infact, in an AJD setting the Greeks are available in closed form

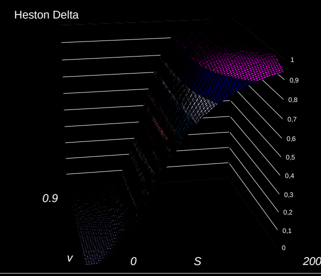
So, an extended spanning of the AJD Greeks on the parameters set is useful to assess models and test Stability

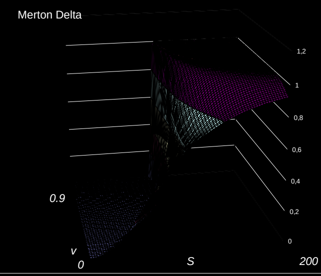


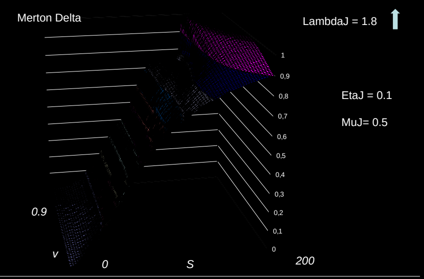












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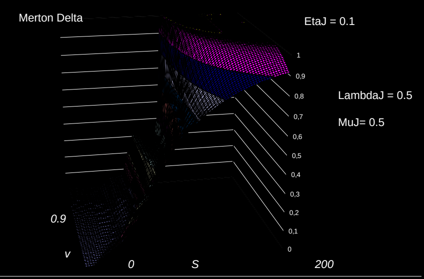

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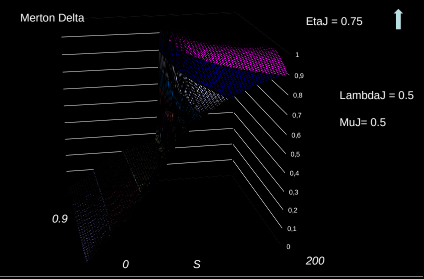

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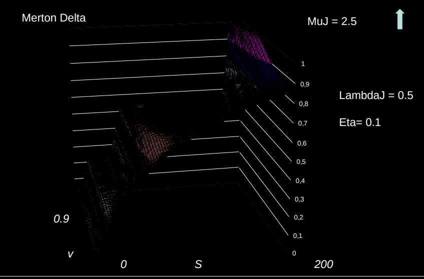

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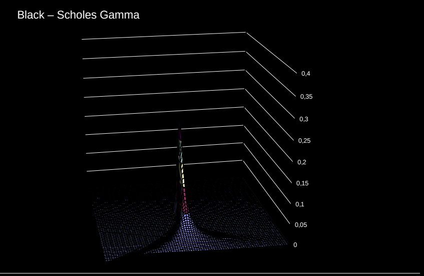

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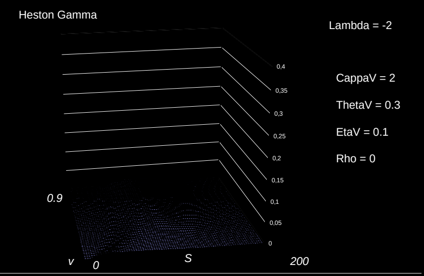

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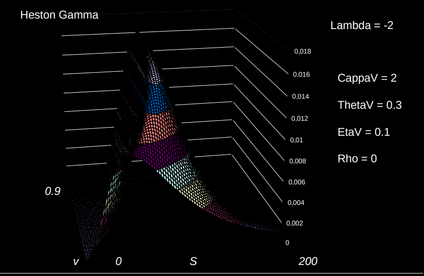

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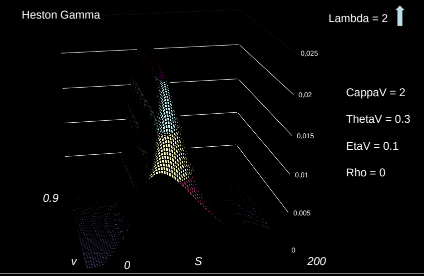

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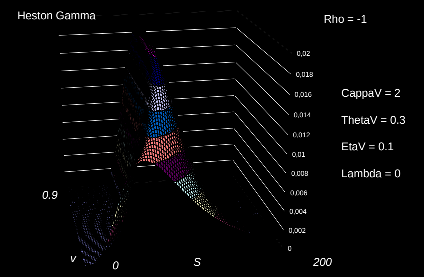

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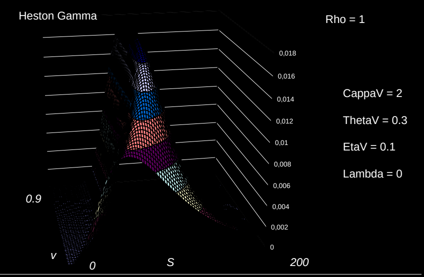

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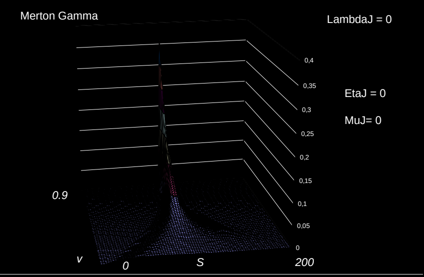

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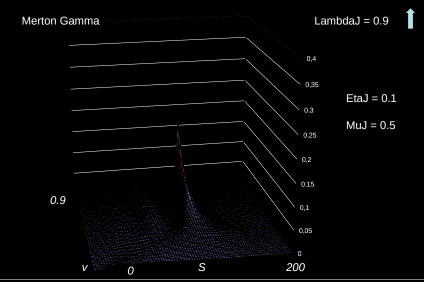

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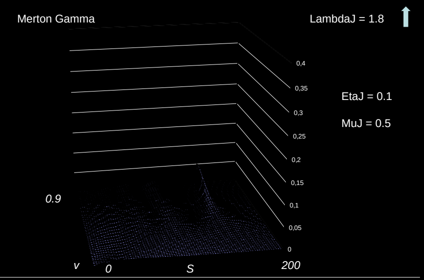

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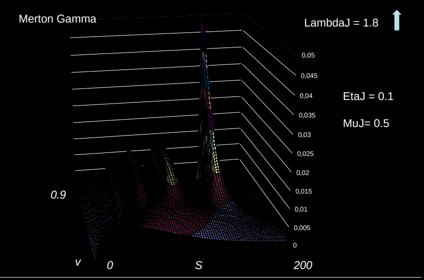

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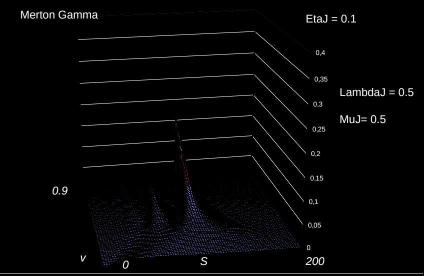

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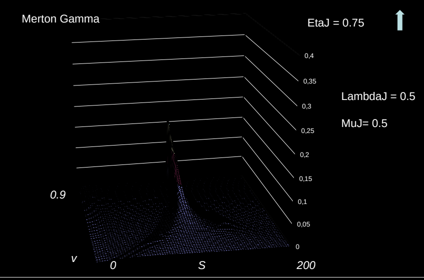

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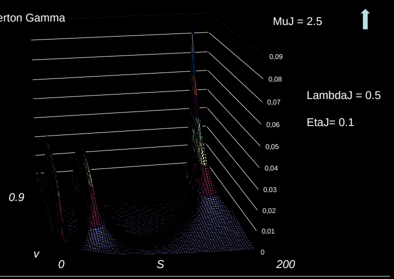
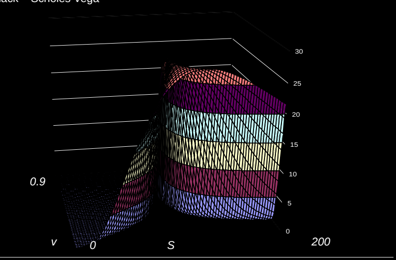
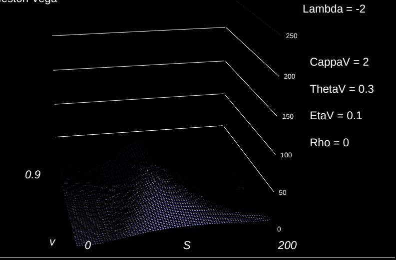
Merton Gamma



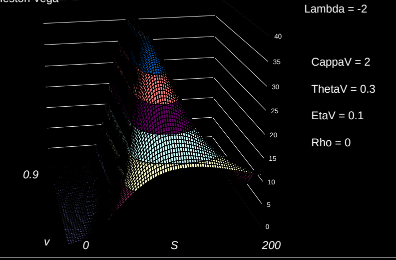
Black – Scholes Vega



Heston Vega



Heston Vega



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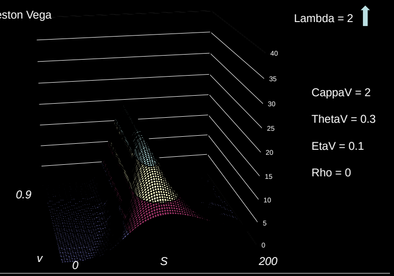

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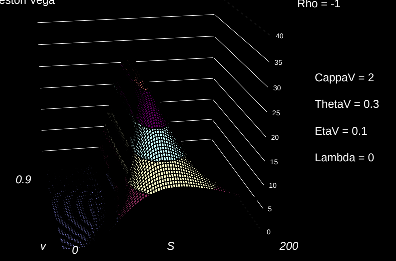

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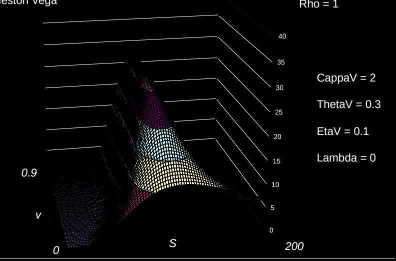

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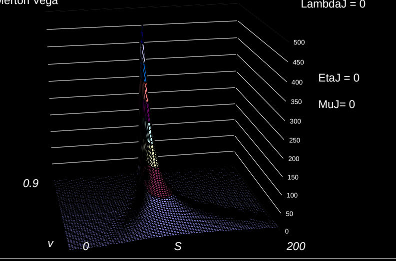

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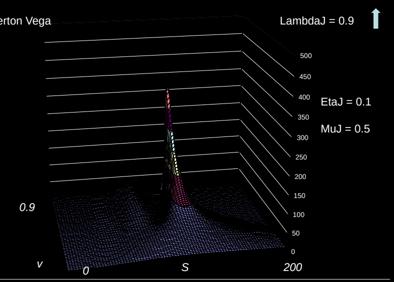

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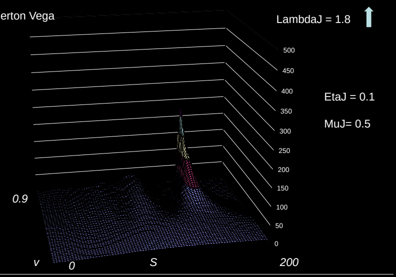

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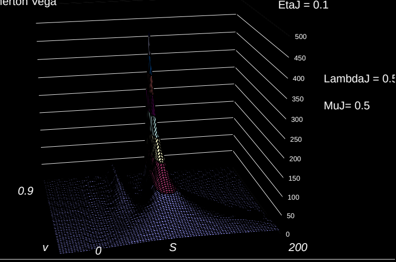

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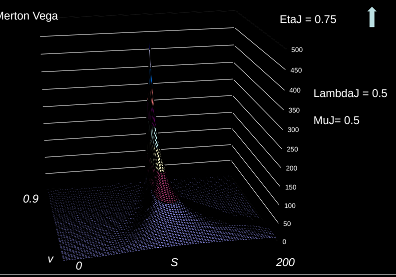

Merton Vega